High-energy nucleon-nucleus scattering and cosmic-ray cross sections

Loyal Durand and Hong Pi
Department of Physics, University of Wisconsin–Madison, Madison, Wisconsin 53706
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We develop a QCD-based diffractive model for nucleon-nucleon and nucleon-nucleus cross sections in which the increase in the cross sections at high energies is driven by semihard parton-parton scattering. We use the model to calculate the absorptive cross section for protons on air at very high energies, and find excellent agreement with data from cosmic-ray experiments. The results can be used to extract $\sigma_{tot}(pp)$ from $\sigma_{abs}(p\text{-air})$ in a way which is nearly independent of the input parameters. We include an approximate treatment of single-diffractive pp scattering.

I. INTRODUCTION

The highest-energy experimental information available on nucleon-nucleon cross sections is that derived from cosmic-ray measurements of $\sigma_{abs}(p\text{-air})$, the absorption cross section for the breakup of an incoming cosmic-ray proton on a nucleus in the atmosphere. The absorption cross section has been measured for nucleon-nucleon center-of-mass energies in the range (6×10^3)–(4×10^4) GeV by the Akeno Collaboration,[1] and around 3×10^4 GeV by the Fly's Eye group[2] using different methods[3] with somewhat different but overlapping results. However, the cross sections of principal interest, $\sigma_{abs}(pp)$ and $\sigma_{tot}(pp)$, can only be extracted from $\sigma_{abs}(p\text{-air})$ using a theoretical model which relates proton-nucleus scattering to proton-proton (or more generally, nucleon-nucleon) scattering.[3] In this paper we present a QCD-based model for $\sigma_{abs}(p\text{-air})$. We show that the absorptive cross section calculated using input from a similar model for pp and $\bar{p}p$ scattering at accelerator energies[4] agrees quite well with the cosmic-ray results, and present the relations necessary to extract $\sigma_{tot}(pp)$ from $\sigma_{abs}(p\text{-air})$.

The high-energy interaction of protons with nuclei has traditionally been analyzed by means of Glauber's multiple-diffraction-scattering theory.[5] Glauber calculations have been performed and shown to give a reasonable description of the experimental data for a variety of hadron-nucleus scattering processes with the incident hadron energies in the laboratory ranging up to 300 GeV.

When applying Glauber's technique to extract $\sigma_{tot}(pp)$ from $\sigma_{abs}(p\text{-air})$, one needs to know the proton-nucleus profile function $\Gamma(b)$ which appears in the impact-parameter representation for the elastic-scattering amplitude:

$$f(t)=ip\int_0^{\infty}db\, b\Gamma(b)J_0(b\sqrt{-t}\,)\ . \tag{1}$$

This profile function has usually been parametrized as a Gaussian function,

$$\Gamma(b)=\frac{\sigma_{tot}}{4\pi B}e^{-b^2/2B}\ , \tag{2a}$$

$$B=\frac{d}{dt}\left[\ln\frac{d\sigma}{dt}\right]_{t=0}\ , \tag{2b}$$

with the total cross section $\sigma_{tot}(pp)$ and the slope parameter $B(pp)$ as its parameters.[6,7] The relation of the two parameters is model dependent, and different models of nucleon-nucleon scattering can give quite different predictions for $\sigma_{abs}(p\text{-air})$ when extrapolated to cosmic-ray energies.[3,7] Moreover, the profile function deviates from a Gaussian at the energy of the CERN collider, and the reliability of a Gaussian parametrization at much higher energies is questionable. One should, therefore, treat values of $\sigma_{tot}(pp)$ derived from cosmic-ray data with care, especially when comparing these derived "data" with models for nucleon-nucleon scattering. It is preferable as a check on dynamical models to generalize the models to the case of nucleon-nucleus scattering and to compare the calculated values of $\sigma_{abs}(p\text{-air})$ with the cosmic-ray measurements. We present such a calculation[8] in this paper for a QCD-based model of the nucleon-nucleon interaction.[4]

At very high energies, the interaction between nucleons is dominated by the strong interactions between their constituents—the quarks and gluons. The QCD-based quark-gluon parton model, in fact, gives good descriptions of the increase in the pp and $\bar{p}p$ scattering cross sections with energy,[4,9–11] and of the emergence of semisoft jets in higher-energy collider data.[12,13] A similar description should hold for proton-nucleus collisions at the same or higher nucleon-nucleon center-of-mass energies. In this paper we will, therefore, view the incident proton and the nucleons in a target nucleus as distributions of uncorrelated partons and assume that semihard interactions at the parton level dominate the proton-nucleus inelastic process. However, one cannot use the simple additive parton model to obtain the total or inelastic proton-nucleus cross section, since the model violates the constraint of partial-wave unitarity.[4,14] We, thus, use the diffraction approximation, which is consistent with the unitarity constraint, to obtain our formulation of the proton-nucleus cross sections. Our principal result is a QCD-based model for inelastic proton-nucleus scattering, which we test—with excellent results[8]—for laboratory energies of the proton from 200 to 10^9 GeV using the pp scattering parameters from Ref. 4.

In our formulation, proton-nucleus scattering is closely related to proton-proton scattering. We, therefore, sketch the diffraction formulation of pp scattering in Sec. II. We formulate the corresponding model for proton-nucleus scattering, calculate the proton-air absorptive cross section, and compare the results with the data in Sec. III. In the Appendix we make an approximate calculation of the single-diffractive pp cross section as a separate check on some of our assumptions.

II. PROTON-PROTON SCATTERING

We will work in the impact-parameter representation, ignoring spin-dependent effects, and assuming that the pp scattering amplitude is purely imaginary at high energies. With these assumptions, we have the following expressions for the total cross section $\sigma_{\rm tot}$, the elastic cross section $\sigma_{\rm el}$, the inelastic cross section $\sigma_{\rm inel}$, and the differential elastic scattering cross section $d\sigma_{\rm el}/dt$:

$$\sigma_{\rm tot}=4\pi\int db\, b\,(1-e^{-\chi(b,s)}) , \quad (3a)$$

$$\sigma_{\rm el}=2\pi\int db\, b\,(1-e^{-\chi(b,s)})^2 , \quad (3b)$$

$$\sigma_{\rm inel}=2\pi\int db\, b\,(1-e^{-2\chi(b,s)}) , \quad (3c)$$

$$\frac{d\sigma}{dt}=\pi\left|\int_0^\infty db\, b\,(1-e^{-\chi(b,s)})J_0(b\sqrt{-t}\,)\right|^2 . \quad (3d)$$

The eikonal function $\chi(b,s)$ is real and depends on the impact parameter b and the total center-of-mass energy $\sqrt{s}$.

The factor $(1-e^{-2\chi})$ in Eq. (3c) can be interpreted semiclassically as the probability $P(b,s)$ that at least one of the two protons is broken up in a collision at impact parameter b. Thus, $e^{-2\chi}$ is the probability $\bar{P}(b,s)$ that both protons survive the collision. In Ref. 4 we calculated the QCD contribution to $\bar{P}$ using the probability-based parton model. In particular, the number of parton-parton collisions in a pp collision at impact parameter b is given in the QCD parton model by

$$n_{pp}(b,s)=\sum_{ij}\epsilon_{ij}\int d^2b'\int dx_1\int dx_2\int_{Q^2_{\rm min}} d\,|\hat{t}\,|\, f_{i,p}(x_1,|\hat{t}\,|,|\mathbf{b}-\mathbf{b}'\,|)\frac{d\hat{\sigma}_{ij}}{d\,|\hat{t}\,|}(\hat{s},\hat{t})f_{j,p}(x_2,|\hat{t}\,|,|\mathbf{b}'\,|) , \quad (4)$$

where $\epsilon_{ij}=\frac{1}{2}$ for $i=j$, and $\epsilon_{ij}=1$ otherwise. Here x_1 and x_2 are the fractions of the momenta of the parent protons carried by the partons which collide, $d\hat{\sigma}_{ij}/d\,|\hat{t}\,|$ is the cross section for scattering of partons of types i and j, $\hat{s}$ and $\hat{t}$ are the Mandelstam variables for this parton process, and $f_{i,p}(x,|\hat{t}\,|,b)dx\,d^2b$ is the number of partons of type i in the interval dx and transverse area element d^2b a distance b from the center of the proton.

A parton-parton collision will almost certainly break up at least one of the protons if the minimum momentum transfer $Q_{\rm min}$ is sufficiently large.[15] A straightforward mean-free-path argument gives the probability $\bar{P}_{\rm QCD}$ that the protons *do not* undergo such a semihard inelastic scattering at the parton level as

$$\bar{P}_{\rm QCD}(b,s)=e^{-n(b,s)} . \quad (5)$$

Allowing also for soft inelastic processes which are not describable in terms of parton-parton scattering, we find a total survival probability $\bar{P}$ for the protons given by

$$\bar{P}(b,s)=\bar{P}_{\rm QCD}(b,s)\bar{P}_{\rm soft}(b,s) \quad (6)$$

and a probability that at least one proton is broken up given by

$$P=1-\bar{P}_{\rm QCD}\bar{P}_{\rm soft} . \quad (7)$$

Comparing this expression with Eq. (3c) where P appears as $(1-e^{-2\chi})$, we find that the eikonal function χ is given by

$$\chi_{pp}(b,s)=\chi_{pp}^{\rm soft}(b,s)+\chi_{pp}^{\rm QCD}(b,s) , \quad (8)$$

where

$$\chi_{pp}^{\rm QCD}(b,s)=\tfrac{1}{2}n_{pp}(b,s) \quad (9)$$

is calculable and $\chi_{pp}^{\rm soft}$ represents the contribution from the soft hadronic processes. We have little knowledge about these soft processes. However, at high energies, semihard processes dominate and the exact form of $\chi^{\rm soft}$ is not crucial. We will parametrize it later in a simple way.

To calculate $n(b,s)$ or $\chi^{\rm QCD}$, we assume that the parton distribution function $f_{i,p}(x,|\hat{t}\,|,b)$ in Eq. (4) factors,

$$f_{i,p}(x,|\hat{t}\,|,b)\approx f_{i,p}(x,|\hat{t}\,|)\rho_p(b) , \quad (10)$$

where $f_{i,p}(x,|\hat{t}\,|)$ is the usual parton distribution function of a parton and $\rho_p(b)$ describes the spatial distribution of the well-localized partons at the impact parameter b. This factorization is expected to hold for $x\ll 1$, the dominant region in the integrations in Eq. (4). We further assume that $\rho_p(b)$ can be approximated by the distribution function determined from the proton electric form factor $G_E(k_\perp^2)$,

$$\rho_p(b)\simeq\frac{1}{(2\pi)^2}\int d^2k_\perp G_E(k_\perp^2)e^{i\mathbf{k}_\perp\cdot\mathbf{b}} , \quad (11)$$

where[16]

$$G_E(k_\perp^2)\simeq(1+k_\perp^2/\nu^2)^{-2},\quad \nu^2\simeq 0.71({\rm GeV}/c)^2 . \quad (12)$$

Substituting Eqs. (10) and (11) into Eq. (4) we find that

$$\chi_{pp}^{\rm QCD}(b,s)=\tfrac{1}{2}A(b)\sigma_{\rm QCD}(s) , \quad (13)$$

where $A(b)$ represents the effective density of the overlapping parton distributions in the colliding protons:

$$A(b)=\int d^2b'\rho_p(|\mathbf{b}-\mathbf{b}'\,|)\rho_p(|\mathbf{b}'\,|)$$
$$=\frac{\nu^2}{12\pi}\tfrac{1}{8}(\nu b)^3K_3(\nu b) , \quad (14)$$

where $K_n(x)$ is the hyperbolic Bessel function which vanishes exponentially for $x\to\infty$ and $\sigma_{\rm QCD}(s)$ is the parton-parton elastic-scattering cross section. It is sufficiently accurate for our purposes to keep only the leading $|\hat{t}\,|^{-2}$ contributions to the parton cross sections. With this approximation,

$$\sigma_{\rm QCD}(s)=2\int_{2\nu/\sqrt{s}}dx_1\int_{x_1}^{1}dx_2\int_{Q_{\min}^2}^{x_1x_2s/2}d\,|\hat{t}\,|\,\frac{9\pi\alpha_s^2(\hat{t}\,)}{2\hat{t}^{\,2}}F(x_1,\hat{t}\,)F(x_2,\hat{t}\,)\ , \tag{15}$$

where $F(x,\hat{t}\,)$ is given in terms of the gluon and quark distributions in the proton by

$$F(x,\hat{t}\,)=G(x,\hat{t}\,)+\tfrac{4}{9}\sum_i[Q_i(x,\hat{t}\,)+\bar{Q}_i(x,\hat{t}\,)]\ . \tag{16}$$

We note that the largest contributions to $\sigma_{\rm QCD}$ for large $\sqrt{s}$ come from very small values of x, and that the gluon contributions are dominant in this region.

The lower limit on the x_1 integration in Eq. (15) represents in approximate form the observation that the parton distributions cease to increase for $\frac{1}{2}x\sqrt{s}<\nu$ where ν^{-1} is a characteristic radius of the proton. [A better, but more complicated way of implementing the restriction is to replace $F(x,|\hat{t}\,|)$ by $xF(x,|\hat{t}\,|)/(x^2+4\nu^2/s)^{1/2}$, and to use the kinematic lower limit $x_{\min}=2Q_{\min}^2/s$ appropriate for gluons. The difference is not significant.] The choice of $Q_{\min}$ is restricted by the condition $Q_{\min}>\nu$ which is necessary for a parton to be confined laterally in the parent proton and by the requirement that it be large enough in magnitude that perturbation theory can be expected to hold approximately.

To parametrize $\chi_{pp}^{\rm soft}$, we use the diffraction scattering model introduced by Durand and Lipes[17] and Chou and Yang[18] in which $\chi_{\rm soft}$ is of the form $\chi_{\rm soft}=A(b)C(s)$, where $C(s)$ is a function which varies slowly with the energy $\sqrt{s}$ at accelerator energies. This weak energy dependence can be compensated for by an adjustment in the cutoff parameter $Q_{\min}$ in Eq. (15). We, therefore, ignore it and take

$$\chi_{pp}^{\rm soft}=C_0A(b)=\tfrac{1}{2}\sigma_0A(b)\ , \tag{17}$$

where σ_0 will be fixed at a value which reproduces the pp total cross section at $\sqrt{s}=30.6$ GeV where the QCD contribution is relatively small, $\sigma_0\approx 123\ {\rm GeV}^{-2}=47.9$ mb.

At this point we have completely specified the eikonal function for pp scattering:

$$\chi_{pp}(b,s)=\tfrac{1}{2}A(b)[\sigma_0+\sigma_{\rm QCD}(s)]\ . \tag{18}$$

The pp scattering cross sections were calculated in Ref. 4 using the parton distributions of Eichten, Hinchliffe, Lane, and Quigg[19] (set 1 with $\Lambda_{\overline{\rm MS}}=200$ MeV, where $\overline{\rm MS}$ denotes the modified minimal subtraction scheme), $Q_{\min}^2=2\ ({\rm GeV}/c)^2$, and a cutoff $x_{\min}=Q_{\min}/\sqrt{s}$ on the x integration. The model gives a very good overall fit to pp scattering to the highest CERN collider energies, including the observed rise in the total and elastic scattering cross sections from the CERN ISR energy range, the increase in the ratio $\sigma_{\rm el}/\sigma_{\rm tot}$, and the diffraction structure observed in $d\sigma_{\rm el}/dt$ (Refs. 4, 9–11). The total cross sections calculated at cosmic-ray energies[4] agree well with the cross sections extracted from the data by conventional methods. In this paper we will use the theoretically preferable cutoff $x_{\min}=2\nu/\sqrt{s}$ shown in Eq. (15) and $Q_{\min}^2=2.2\ ({\rm GeV}/c)^2$. The resulting changes in the pp cross sections are quite small.

III. PROTON-NUCLEUS SCATTERING

A. Theory

It is straightforward to extend our model of pp scattering to proton-nucleus (pA) scattering. We will treat the nucleus as a collection of independent nucleons which can be regarded in the calculation of semihard interactions as being composed of partons. We will work in the center-of-mass frame of the incoming proton and an "average" target nucleon with a fraction $1/A$ of the momentum of the nucleus, and will neglect the effects of Fermi motion in the nucleus. These effects average out to good approximation, and are unimportant for our purposes. The parton distribution of the nucleus will, thus, be taken as a convolution of the distribution ρ_A of nucleons in the nucleus with the distribution ρ_a of partons in a free nucleon:

$$f_{i,A}(x,|\hat{t}\,|,b)=\frac{1}{A}\sum_{a=1}^{A}\int d^2r_\perp dz\,f_{i,a}(x,|\hat{t}\,|)\times\rho_a(|\mathbf{b}-\mathbf{r}_\perp|)\rho_A(\mathbf{r}_\perp,z)\ , \tag{19}$$

where

$$\int d^2r_\perp dz\,\rho_A(\mathbf{r}_\perp,z)=A\ . \tag{20}$$

The foregoing picture neglects the small effect of nuclear binding on the parton distribution (the European Muon Collaboration effect[20]). However, the observed effect is quite small for the small-x part of the distribution function[20] which gives the main contribution to our final results. Its neglect is, therefore, unimportant. We will also neglect the small difference between the parton distributions of the neutron and proton, and use the proton distribution for both, again a good approximation at small x where gluons are dominant. Then

$$f_{i,A}(x,|\hat{t}\,|,b)=f_{i,N}(x,|\hat{t}\,|)\int d^2r_\perp dz\,\rho_p(|\mathbf{b}-\mathbf{r}_\perp|)\times\rho_A(\mathbf{r}_\perp,z)\ . \tag{21}$$

The calculation of the contribution of semihard parton scattering processes to the eikonal function for pA scattering is now straightforward and gives

$$\chi_{pA}^{\rm QCD}(b,s)=\int d^2r_\perp dz\,\rho_A(\mathbf{r}_\perp,z)\chi_{pp}^{\rm QCD}(|\mathbf{b}-\mathbf{r}_\perp|)=\tfrac{1}{2}\tilde{A}(b)\sigma_{\rm QCD}(s)\ , \tag{22}$$

where

$$\tilde{A}(b)=\int d^2r_\perp dz\,\rho_A(\mathbf{r}_\perp,z)A(|\mathbf{b}-\mathbf{r}_\perp|)\ . \tag{23}$$

$A(b)$ is defined in Eq. (14), and $\sigma_{\rm QCD}$ is the semihard parton-parton scattering cross section defined in Eq. (15), evaluated for s the square of the total center-of-mass energy of the proton-nucleon system, $s\approx 2m_pE_{{\rm lab},p}$

$\approx 2(M_A/A)E_{\text{lab},p}$.

We will treat the soft scattering contribution to χ_{pA} as approximately energy independent, as in our treatment of *pp* scattering, and parametrize it as

$$\chi_{pA}^{\text{soft}}(b,s) \approx \tfrac{1}{2}\tilde{A}(b)\tilde{\sigma}_0 \ . \tag{24}$$

However, the constant $\tilde{\sigma}_0$ which appears in this expression is not the same as σ_0 in *pp* scattering, but is expected to be smaller. The cosmic-ray and accelerator experiments on $\sigma_{\text{abs}}(pA)$ are not sensitive to processes in which a target nucleon is broken up softly, but the incident nucleon is not (single-diffractive dissociation of a target particle, with the incident particle retaining most of its energy). Soft diffractive processes can also lead to excitation and deexcitation of a parton in successive collisions in an extended target: time dilation effects preclude the decay of the excited state inside an "air" nucleus for beam energies in the 100-GeV range, and the soft processes must be treated as coherent. This effect increases the probability that a proton can traverse a nucleus without breakup ("inelastic shielding"[21]). The net effect of these two corrections is to decrease the effective soft contribution to χ_{pA}. However, the semihard parton interactions remain incoherent and can be treated as before. At sufficiently high energies, the parton interactions are dominant and the uncertainty in the treatment of the soft effects is unimportant. We consider the single-diffractive effects further in the Appendix, but the results of that approximate discussion are not needed in the remainder of the paper.

We will include the foregoing effects in our calculation by defining an eikonal function $\tilde{\chi}_{pA}$ for absorptive processes which involve the breakup of the incoming proton. We write the effective eikonal function $\tilde{\chi}_{pp}$ for a proton in a nucleus as

$$\tilde{\chi}_{pp}(b,s) = \tfrac{1}{2}(\tilde{\sigma}_0 + \sigma_{\text{QCD}})A(b) \tag{25}$$

and calculate $\tilde{\chi}_{pA}$ by averaging over the spatial distribution of nucleons in the nucleus as above, with the result

$$\tilde{\chi}_{pA}(b,s) = \tfrac{1}{2}(\sigma_0 + \sigma_{\text{QCD}})\tilde{A}(b) \ . \tag{26}$$

The proton-air absorptive cross section is then

$$\sigma_{\text{abs}}(pA) = 2\pi\int_0^\infty db\, b\,(1 - e^{-2\tilde{\chi}_{pA}(b,s)}) \ . \tag{27}$$

The total inelastic cross section $\sigma_{\text{inel}}(pA)$ includes additional contributions from single diffraction with breakup of a target nucleon, and quasielastic scattering of the incident particle from a target nucleon with breakup of the nucleus, but no particle production.

B. Calculations and results

We have calculated $\sigma_{\text{abs}}(pA)$ for light nuclei ($A \le 16$) using both shell-model and Gaussian representations for the nuclear density $\rho_A(\mathbf{r})$. For the shell model,

$$\rho_A(\mathbf{r}) = \frac{4}{\pi^{3/2}a_0^3}\left[1 + \tfrac{1}{6}(A-4)\frac{r^2}{a_0^2}\right]e^{-r^2/a_0^2} \ , \tag{28}$$

where a_0^2 is related to the mean-square charge radii of the nucleus and the proton by

$$\langle r_{\text{ch}}^2\rangle_A = \left[\frac{5}{2} - \frac{4}{A}\right]a_0^2 + \langle r_{\text{ch}}^2\rangle_p \ . \tag{29}$$

In this case,

$$\tilde{A}(b) = \frac{\nu^2}{12\pi a_0^2}\int_0^\infty db'\,b'\,e^{-(b^2+b'^2)/a_0^2}(\nu b')^3K_3(\nu b') \times \left[\left[1 + \frac{1}{6a_0^2}(A-4)(b^2+b'^2+\tfrac{1}{2}a_0^2)\right]I_0\left[\frac{2bb'}{a_0^2}\right] - \frac{1}{3a_0^2}(A-4)bb'I_1\left[\frac{2bb'}{a_0^2}\right]\right] , \tag{30}$$

where K_n and I_n are hyperbolic Bessel functions. For the Gaussian model

$$\rho_A(\mathbf{r}) = \frac{A}{\pi^{3/2}a^3}e^{-r^2/a^2} \ , \tag{31}$$

where

$$\langle r_{\text{ch}}^2\rangle_A = \tfrac{3}{2}a^2 + \langle r_{\text{ch}}^2\rangle_p \tag{32}$$

and

$$\tilde{A}(b) = \frac{\nu^2 A}{48\pi a^2}\int_0^\infty db'\,b'\,e^{-(b^2+b'^2)/a^2}(\nu b')^3K_3(\nu b') \times I_0\left[\frac{2bb'}{a^2}\right] \ . \tag{33}$$

The nuclear parameters for our calculation were determined using measured values[22] of the charge radii of ^{14}N, ^{16}O, and the proton ($\langle r_{\text{ch}}^2\rangle_{\text{N}}^{1/2} = 2.55 \pm 0.02$ fm, $\langle r_{\text{ch}}^2\rangle_{O}^{1/2} = 2.72 \pm 0.02$ fm, $\langle r_{\text{ch}}^2\rangle_p^{1/2} = 0.81 \pm 0.02$ fm). The parameter $\tilde{\sigma}_0 = 78$ GeV^{-2} was determined by fitting the absorptive cross sections for protons in ^{7}Li and ^{12}C measured by Carroll *et al.*[23] at 280 GeV/*c*. The fit is excellent, with theoretical values of $\sigma_{\text{abs}}(p\text{-C})$ and $\sigma_{\text{abs}}(p\text{-Li})$ of 224 and 157 mb, respectively, compared to experimental values of 225 ± 7 mb and 156 ± 5 mb. As would be expected from the discussion above, the value of $\tilde{\sigma}_0$(78 GeV^{-2}) is smaller than the value 123 GeV^{-2} of the soft scattering parameter σ_0 used in our fit to the *pp* total cross section at $\sqrt{s} = 30.6$ GeV. The QCD contribution σ_{QCD} in Eq. (25) was calculated as described in Sec. II with $Q_{\min}^2 = 2.2$ (GeV/*c*)^2.

The results of our calculations for protons on air (taken as 21% O and 79% N) are shown in Figs. 1–5. In Fig. 1 we compare the absorptive cross section $\sigma_{\text{abs}}(p\text{-air})$ calculated with the parameters above to data for incident laboratory energies of 280 to 10^9 GeV for the proton. The

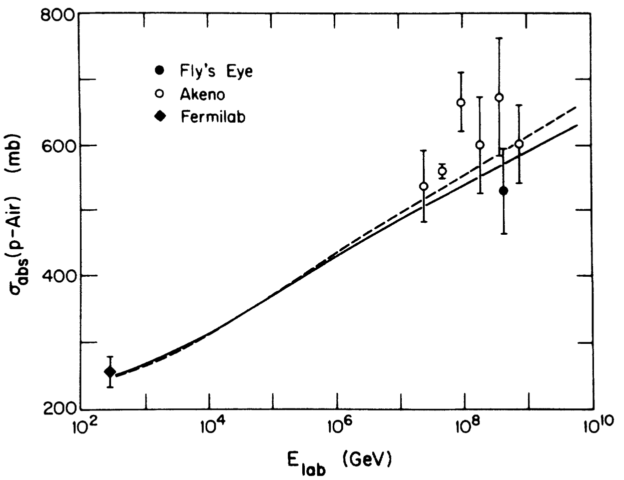

FIG. 1. The absorptive cross section for protons on air calculated using Eq. (27) with shell-model (solid line) and Gaussian (dashed line) nuclear densities. The data are from Refs. 1, 2, and 23.

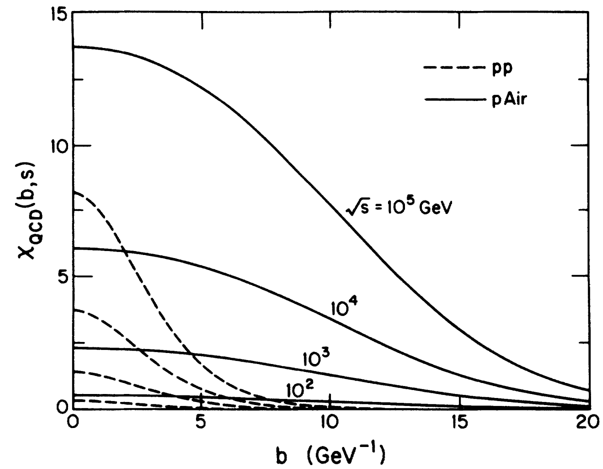

FIG. 3. The contribution to $\chi(b,s)$ from semihard QCD processes for proton-proton scattering $[\chi_{\rm QCD}=\frac{1}{2}\sigma_{\rm QCD}A(b)]$ and proton-nucleus scattering $[\chi_{\rm QCD}=\frac{1}{2}\sigma_{\rm QCD}\tilde{A}(b)]$ with the effective atomic weight $A=14.4$ for "air." The QCD and soft contributions to $\chi_{p\text{-air}}$ are roughly equal for $\sqrt{s}=350$ GeV.

agreement of the calculated cross section with the Akeno and Fly's Eye data is excellent for both the Gaussian and the (preferred) shell-model nuclear densities. The two cross sections differ by 22 mb for $\sqrt{s}=5\times10^8$ GeV, less than the present uncertainty in the data.

We should emphasize that we have *not* tuned the model to the high-energy cosmic-ray data, but fixed the QCD parameter $Q_{\rm min}$ (and σ_0 in χ_{pp}) by fitting $\sigma_{\rm tot}(pp)$ at $\sqrt{s}=30.6$ and 546 GeV. The modified parameter $\tilde{\sigma}_0$ was determined from the p-Li and p-C absorptive cross sections at $E_{{\rm lab},p}=280$ GeV (Ref. 23). The rapid rise in $\sigma_{\rm abs}(p\text{-air})$ with $E_{\rm lab}$ is a prediction of the model, the result of the rapid increase in $\sigma_{\rm QCD}$ with increasing $\sqrt{s}$.

The insensitivity of the high-energy theoretical results to changes in the treatment of the soft scattering is illustrated in Fig. 2 where we show $\sigma_{\rm abs}(p\text{-air})$ calculated as above, and with $\tilde{\sigma}_0$ replaced by σ_0, the soft scattering parameter of Sec. II. The differences are large at low energies, but are essentially negligible at the highest energies shown. This is the result of the rapid increase in $\chi_{\rm QCD}=\sigma_{\rm QCD}(s)\tilde{A}(b)$, the QCD contribution to $\tilde{\chi}_{pA}$, with increasing s. This increase is shown in Fig. 3 for both proton-proton and proton-air scattering. The soft contributions to χ_{pp} or $\tilde{\chi}_{pA}$ remain constant in our model, or at most, increase in magnitude much less rapidly than $\chi_{\rm QCD}$. The soft and QCD contributions to χ_{pA} are equal for $\sqrt{s}\approx350$ GeV, and the QCD contribution is dominant at higher energies.

The total absorptive profile function $(1-e^{-2\tilde{\chi}})$ is shown in Fig. 4. The nucleus is extremely "black" at

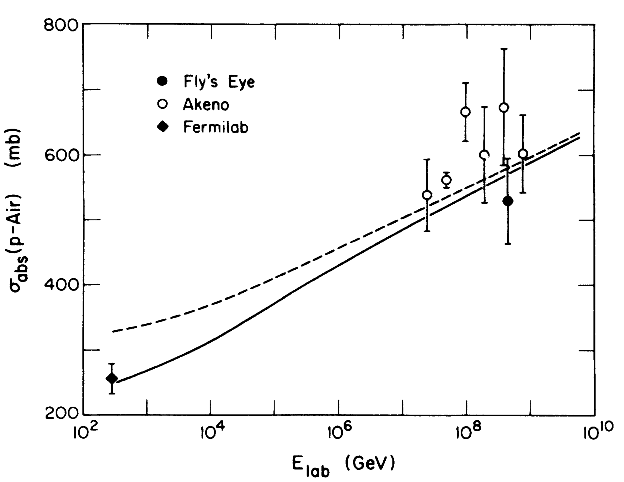

FIG. 2. The absorptive cross section for protons on air calculated using $\tilde{\sigma}_0=78$ GeV^{-2} (solid line) and with $\tilde{\sigma}_0$ replaced by the value $\sigma_0=123$ GeV^{-2} used for proton-proton scattering. The difference represents the correction for inelastic shielding and unobserved diffraction dissociation of a target nucleon without breakup of the incident proton.

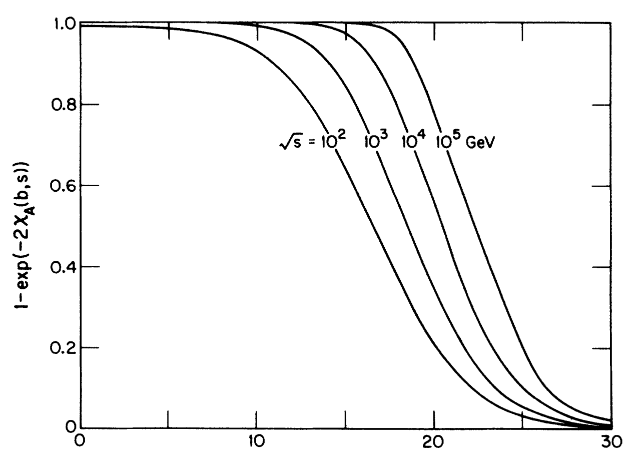

FIG. 4. The inelastic profile function for proton-air scattering as a function of energy. The increase in the profile function at large b with increasing $\sqrt{s}$ results from the growth of the cross section for semihard parton-parton interactions, $\sigma_{\rm QCD}$.

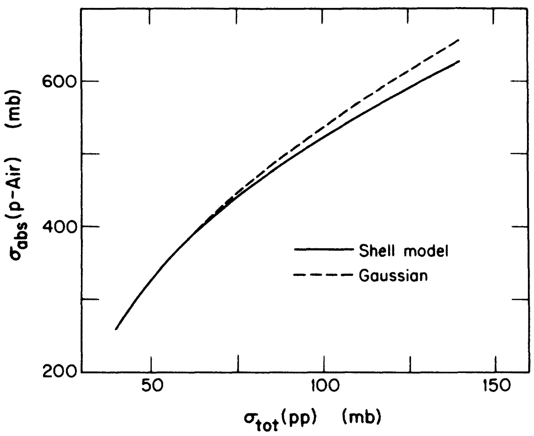

FIG. 5. The relation between $\sigma_{tot}(pp)$ and $\sigma_{abs}(p\text{-air})$ for shell-model (solid line) and Gaussian (dotted line) nuclear densities calculated assuming that $\sigma_0-\tilde{\sigma}_0=45\ \text{GeV}^{-2}$, the value determined at $E_{\text{lab},p}=280$ GeV. The curves are otherwise independent of σ_{QCD} or the division of χ_{pp} and $\tilde{\chi}_{p\text{-air}}$ into soft and hard contributions.

small impact parameters as would be expected, and the sensitivity of $\sigma_{abs}(pA)$ to the details of the proton-proton scattering is confined to the peripheral region. This has the effect that $\sigma_{abs}(pA)$ is less sensitive to changes in $\sigma_{tot}(pp)$ for large values of the latter than it is for small values. This is shown in Fig. 5. The same effect is evident in the curves given by Yodh in Ref. 7 for a standard Glauber multiple-diffraction-scattering calculation using the Gaussian profile function for the nucleon given in Eq. (2a), and is also emphasized in the work of Kopeliovich, Nikolaev, and Potashnikova.[24]

The curve in Fig. 5 encompasses the essential geometrical features of the model through the shape factors $A(b)$ and $\tilde{A}(b)$ for the proton and the "air" nucleus. The curve was calculated assuming that $\sigma_0-\tilde{\sigma}_0\approx 45\ \text{GeV}^{-2}$ as determined in our fits to $\sigma_{tot}(pp)$ at $\sqrt{s}=30.6$ GeV and to $\sigma_{abs}(pA)$ at $E_{\text{lab},p}=280$ GeV, but is actually independent of the details of the QCD calculation or the division of the χ's into soft and hard contributions provided the factors in brackets in Eqs. (18) and (26) differ by 45 GeV^{-2}. Using this curve with the shell-model nuclear densities, we find that the Fly's Eye value[2] of 540±50 mb for $\sigma_{abs}(p\text{-air})$ (which is probably to be interpreted as a lower bound[25]) corresponds to $\sigma_{tot}(pp)=106\pm20$ mb at $\sqrt{s}\approx 30$ TeV, a somewhat smaller value than that usually quoted,[2] 120±15 mb. Our results differ markedly in this respect from the results of Kopeliovich, Nikolaev, and Potashnikova,[24] who need a quite large value of $\sigma_{tot}(pp)$ (approximately 164±30 mb) to fit the proton-air absorption cross section. The reason for the difference is not clear.

IV. REMARKS

The calculation of $\sigma_{abs}(p\text{-air})$ given here provides a direct check on the QCD-based model for proton-proton scattering developed in Ref. 4. The cross sections calculated using parameters determined at CERN collider and Fermilab energies increase rapidly with $\sqrt{s}$, and are in excellent agreement with the cosmic-ray cross sections measured by the Akeno and Fly's Eye groups[1,2] at nucleon-nucleon center-of-mass energies of $\sqrt{s}=(6\times10^3)-(4\times10^4)$ GeV. Conversely, the model provides a definite way of converting measured cosmic-ray absorption cross sections into proton-proton total cross sections, the quantities of basic interest. This conversion depends on the shapes of the proton and the nucleus, but is almost independent of the division of the eikonal functions into soft- and hard-scattering contributions and their energy dependence.

We remark finally that the present model for proton-nucleus scattering can clearly be extended to a model for nucleus-nucleus scattering. The model would be applicable at very high energies where semihard parton-parton interactions are dominant, but would require further development at accelerator energies.

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APPENDIX

In this appendix we consider an approximate, probabilistic treatment of the correction to the eikonal function for the effects of single-diffractive scattering in which a target nucleon is broken up, but the incident proton is not. We will neglect the effects of inelastic screening which is expected to proceed mainly through diffractive excitation of a low mass excited state of the proton in one collision, followed by deexcitation in a later collision in the nucleus.

The diffractive dissociation of a target nucleon is a soft process. We may extract its contribution to the soft part of the nucleon-nucleon eikonal function by writing the probability $\bar{P}_{\text{soft}}(pp)$ that neither nucleon is broken up by a soft process in a single collision as

$$\bar{P}_{\text{soft}}(pp)=\bar{P}^{\,2}_{\text{SD}}\bar{P}^{\,\prime}_{\text{soft}}\ , \tag{A1}$$

that is, as the product of the probabilities that there was no single-diffractive process involving either nucleon separately and the probability that there was no soft process which breaks up both nucleons. This is clearly a semiclassical approximation. With it, we find that

$$\chi^{\text{soft}}_{pp}=2\chi^{\text{SD}}_{pp}+\chi'_{pp}\equiv\chi^{\text{SD}}_{pp}+\tilde{\chi}^{\,\text{soft}}_{pp}\ . \tag{A2}$$

The probability that the *incident* nucleon survives the collision irrespective of the fate of the target nucleon is then given by

$$\bar{P}_{\text{inc}}=\bar{P}_{\text{SD}}\bar{P}^{\,\prime}_{\text{soft}}\bar{P}_{\text{QCD}}=e^{-2\chi_{pp}+\chi^{\text{SD}}_{pp}}\equiv e^{-2\tilde{\chi}_{pp}}\ . \tag{A3}$$

As a check on this approximate description of single-diffractive processes, we calculated the single-diffractive cross section for the antiproton in $\bar{p}p$ scattering using the expression

$$\sigma_{\text{SD}}=2\pi\int_0^\infty db\, b(1-e^{-2\chi^{\text{SD}}_{pp}})e^{-2\tilde{\chi}_{pp}}\ , \tag{A4}$$

where we identified $\tilde{\chi}_{pp}$ with the corrected eikonal function in Eq. (25), and χ_{pp}^{SD} with the difference between χ_{pp} and $\tilde{\chi}_{pp}$ according to Eq. (A2),

$$\chi_{pp}^{SD} = \chi_{pp}^{soft} - \tilde{\chi}_{pp}^{soft} = \tfrac{1}{2}(\sigma_0 - \tilde{\sigma}_0) A(b) \ . \tag{A5}$$

(This identification neglects the possible effects of inelastic screening.) The factors $(1-e^{-2\chi_{SD}})$ and $e^{-2\tilde{\chi}}$ in Eq. (A4) are, respectively, the semiclassical probability that the target particle is broken up by a single-diffractive process, and the probability that there are no additional soft or QCD processes which break up the incident particle.

The results of the calculation, $\sigma_{SD}=8.3$, 6.9, 5.7, 5.0, and 2.7 mb for $\sqrt{s}=50$, 200, 546, 900, and 10^4 GeV, are in reasonable agreement with measurements at the CERN ISR, Ref. 26, $\sigma_{SD} \approx 7.2 \pm 0.3$ mb for $\sqrt{s}=35-60$ GeV, and the CERN collider, $\sigma_{SD}=4.8\pm0.5\pm0.8$ mb (UA5, Ref. 27, 200 GeV), 9.4 ± 0.7 mb (UA4, Ref. 28, 546 GeV), and $7.8\pm0.5\pm1.1$ mb (UA5, Ref. 27, 900 GeV), especially considering the strong dependence of the experimental results on the rapidity gap used to define single-diffractive events. (For example, if the rapidity gap in the UA4 measurement[28] is increased from 3.2 to 6 units to nearly separate the forward and backward hemispheres, σ_{SD} decreases from 9.4 mb to $\lesssim 6$ mb at $\sqrt{s}=546$ GeV.) However, the fit to the data would be improved somewhat by a slow increase of χ_{pp}^{SD} with $\sqrt{s}$, a possible energy dependence which we have neglected The resulting change in $\tilde{\chi}_{pp}$ can be compensated to a considerable extent by a change in Q_{min} in Eq. (15), and the changes in the high-energy cross sections are not large if σ_{tot} is required to fit the ISR and CERN collider data.

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