Gupta-Bleuler quantization of massive superparticle models in 6, 8, and 10 dimensions

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We consider $D=6$, 8, and 10 massive superparticle models, with the fermion mass generated by a central charge and with fermionic first-class constraints. We show that the $D=6$ and $D=8$ models with $N=2$ nonchiral supersymmetry can be covariantly quantized by using the Gupta-Bleuler method in the fermionic sector. For the $D=10$, $N=2$ model such a method implies the truncation $D=10 \rightarrow D=8$. We also show on the $D=6$ example how to perform the massless limit, and how to extend the Siegel fermionic invariance to the massive case.

I. INTRODUCTION

Superparticles are pointlike objects moving along world lines in superspace. The first proposal of a supersymmetry-covariant action for massive superparticles, made by Casalbuoni[1] in $D=4$ dimensions (see also Ref. 2), did not contain first-class fermionic constraints and, as a result, it did not lead after quantization to a massive Dirac equation for the spinors in the supermultiplet. In order to obtain first-class fermionic constraints in the $D=4$ massive superparticle model one has to add a one-dimensional supersymmetric Wess-Zumino term to the Casalbuoni action, which implies a central extension of the $D=4$ supersymmetry algebra.[3–6] The first-class fermionic constraints determine also a local fermionic invariance,[5–7] which in the massless superparticle model was found by Siegel.[8] Using the example of the $D=6$ massive model, we shall show that the local fermionic invariance for massive superparticles is the extension of the Siegel invariance to the case of an arbitrary mass parameter.

In this paper we shall consider the quantization of the massive superparticles using the covariant Gupta-Bleuler approach.[4,9,10] We recall that the application of this method requires that the constraints can be split into two complex-conjugated families $L_r, \bar{L}_r$ of first-class constraints, such that each family is closed under the canonical Poisson brackets. Thus, it is sufficient to assume the constraint conditions $L_r \mid \Phi \rangle = 0$ because they imply both $\langle \Phi \mid L_r \mid \Phi \rangle = 0$ and $\langle \Phi \mid \bar{L}_r \mid \Phi \rangle = 0$. It turns out that in this Gupta-Bleuler (GB) scheme one does not need to introduce Dirac brackets nor to abandon the canonical commutation relations even in the presence of second-class constraints.

It has already been shown[9] that the $D=4$ massive models ($N=2,4,6,8$) can be quantized by the covariant GB method and that the first-quantized theory describes massive superfields which have the correct component contents. Here we shall consider the GB quantization of massive superparticles in $D=6$, 8, and 10. We shall show that in $D=6$ and $D=8$ the method works and leads to $D=6$ and $D=8$ covariant free massive superfields. In $D=10$ the first quantization procedure breaks the manifest $D=10$ covariance. If in $D=10$ we apply the GB method to *all* fermionic constraints the $D=10$ symmetry is broken down to $D=8$ [SO(1,9)$\rightarrow$SO(1,7)], and the $D=10$ massive model is truncated after quantization to the $D=8$ superparticle model.

The mass in our superparticle models is described by a central charge characterizing a central extension. Central extensions require the existence of nontrivial cocycles defined on the unextended group. This requires nonchiral models; more specifically, in the lowest-dimensional superalgebras the supercharges are described by "Dirac" spinors having, respectively, 8 complex ($D=6$), 16 complex ($D=8$) and 32 real ($D=10$) components and, accordingly, $N=2$ supersymmetries. For the $D=4$, 6, and 8 massive superparticle models the covariant GB method can be applied because the fundamental (Weyl) spinors in these dimensions are complex, with $2^{(D/2-1)}$ components.[11] Complex $D=10$ covariant spinors can be introduced for the $N=2$ chiral model and $N=4$ nonchiral (which contains 32 left and 32 right real supercharges). For the $N=2$ nonchiral model the introduction of two complex-conjugated families of first-class fermionic constraints imposes the symmetry breaking SO(1,9)$\rightarrow$SO(1,7) and the truncation of ten-momenta to the corresponding eight-momenta.

The plan of the paper is the following. Firstly, we describe and quantize the $D=6$ massive model. Because the mass does not appear in the two-dimensional generalization of superparticle models to superstrings, we consider also the formulation of our $D=6$ massive model with an auxiliary einbein field. We shall show there that in such a formulation the massless limit can be performed, and in particular that the massive extension of the Siegel transformation is generated by the canonically derived fermionic first-class constraints.[12] For $D=8$ we conclude that the massive model can be obtained either directly, by considering $N=2$, $D=8$ supersymmetry (SUSY) with central charge, or as the mentioned truncation of the $D=10$ massive model describing the degrees of freedom quantized via the GB method. Finally we shall comment on the first quantization of the "full" $D=10$ massive model.

II. THE $D=6$ MASSIVE MODEL

The $N=2$, $D=6$ nonchiral supersymmetry group is defined by the transformations

$$x'^{\mu}=x^{\mu}+a^{\mu}+i(\bar{\epsilon}\Gamma^{\mu}\Theta-\bar{\Theta}\Gamma^{\mu}\epsilon),\quad \mu=0,1,2,\ldots,5\ ,$$
$$\Theta'=\Theta+\epsilon,\quad \bar{\Theta}'=\bar{\Theta}+\bar{\epsilon}\ . \tag{2.1}$$

In (2.1) $\Theta_R=(\theta_\alpha,\theta^{\dot\alpha})$, $\epsilon_A=(\varepsilon_\alpha,\varepsilon^{\dot\alpha})$, $R=1,\ldots,8$, are complex eight-component $D=6$ Dirac spinors, consisting of two independent chiral and antichiral Weyl spinors. The position (up or down) of their Weyl indices fixes their chirality because there is no invariant metric inside each sector which could raise or lower them. The Dirac $D=6$ Γ matrices can be expressed in terms of the $D=6$ Pauli matrices in the form (see, e.g., Ref. 13)

$$\Gamma^{\mu}=\begin{bmatrix}0 & \Sigma^{\mu}_{\alpha\dot\beta}\\ \tilde{\Sigma}^{\mu\dot\alpha\beta} & 0\end{bmatrix},\quad \tilde{\Sigma}^{\mu}=(\Sigma^0,-\Sigma)\ , \tag{2.2}$$

where ($j=1,2,3$)

$$\Sigma^{\mu}=\left\{\mathbb{1}_4,\begin{bmatrix}0 & \sigma_0\\ \sigma_0 & 0\end{bmatrix},\begin{bmatrix}0 & -i\sigma^j\\ i\sigma^j & 0\end{bmatrix},\begin{bmatrix}0 & \mathbb{1}_2\\ \mathbb{1}_2 & 0\end{bmatrix},\begin{bmatrix}\mathbb{1}_2 & 0\\ 0 & -\mathbb{1}_2\end{bmatrix}\right\} \tag{2.3}$$

[we use a timelike metric $g^{\mu\nu}=(+,-,-,-,-,-)$]. We can also introduce an antisymmetric real matrix M,

$$M=\begin{bmatrix}i\sigma^2 & 0\\ 0 & i\sigma^2\end{bmatrix}\ , \tag{2.4}$$

which satisfies $M\Sigma^{\mu}M^{-1}=\Sigma^{\mu T}$, and use it to "dot" or "undot" the Weyl spinors, whose chiral or antichiral character is not changed in $D=6$ by the complex conjugation.

Let $Q^{\alpha},Q_{\dot\alpha}$, $Q_{\alpha},Q^{\dot\alpha}$ be the generators associated with ε_α, $\varepsilon^{\dot\alpha}$, $\varepsilon^{\alpha}=(\varepsilon^{\dot\alpha})^*$, and $\varepsilon_{\dot\alpha}=(\varepsilon_\alpha)^*$. We may define

$$Q^{\alpha}_1=M^{\alpha}{}_{\dot\beta}Q^{\dot\beta},\quad Q^1_{\alpha}=Q_{\alpha}\ ,$$
$$Q^{\alpha}_2=Q^{\alpha},\quad Q^2_{\alpha}=(M^{-1})_{\alpha}{}^{\dot\beta}Q_{\dot\beta}\ , \tag{2.5}$$

and $(\Sigma^{\mu})_{\alpha\dot\alpha}M^{\dot\alpha}{}_{\beta}=\Sigma^{\mu}_{\alpha\beta}=-\Sigma^{\mu}_{\beta\alpha}$, $\Sigma^{\alpha\beta}=M^{\alpha}{}_{\dot\beta}\tilde{\Sigma}^{\dot\beta\alpha}$. The Q^{a}_{α} and Q^{b}_{β} satisfy the superalgebra

$$\{Q^{a}_{\alpha},Q^{b}_{\beta}\}=2i\varepsilon^{ab}(\Sigma^{\mu})_{\alpha\beta}\partial_{\mu}\quad (N=1\ \text{chiral})\ , \tag{2.6a}$$

$$\{Q^{\alpha}_{a},Q^{\beta}_{b}\}=2i\varepsilon_{ab}(\Sigma^{\mu})^{\alpha\beta}\partial_{\mu}\quad (N=1\ \text{antichiral})\ , \tag{2.6b}$$

$$\{Q^{a}_{\alpha},Q^{\beta}_{b}\}=0\ . \tag{2.6c}$$

The four real central charges of the $N=2$, $D=6$ superalgebra can be introduced by replacing (2.6c) by

$$\{Q^{a}_{\alpha},Q^{\beta}_{b}\}=\delta^{\beta}_{\alpha}\varepsilon^{ac}(z_0+i\sigma\cdot\mathbf{z})_{cb}\ . \tag{2.7}$$

In order to have the $D=6$ mass parameter we shall assume $z_0=m$, $\mathbf{z}=0$. This implies defining the new (extended) group law by (2.1) *together with*

$$\phi'=\phi+\alpha-im(\bar{\epsilon}\Theta-\bar{\Theta}\epsilon)\ , \tag{2.8}$$

where ϕ is the central variable, α is the central parameter, and the term in m is the cocycle of the extension whose existence requires a nonchiral supersymmetry. Returning to the eight-dimensional spinors, the algebra for (2.1) and (2.8) can be written as

$$\{Q_R,\bar{Q}_S\}=2i(\Gamma^{\mu})_{RS}\partial_{\mu}-2im\,\delta_{RS}\Xi\ ,$$
$$\{Q_R,Q_S\}=0=\{\bar{Q}_R,\bar{Q}_S\}\ , \tag{2.9}$$

where Ξ is the central generator: $\Xi=\partial/\partial\phi$.

The massive superparticle model is built from the invariant forms on the group (2.1) and (2.8). The invariant line and central elements are given by the one-forms

$$\omega^{\mu}=dx^{\mu}+i(d\bar{\Theta}\Gamma^{\mu}\Theta-\bar{\Theta}\Gamma^{\mu}d\Theta)\ ,$$
$$\omega_{\phi}=d\phi-im(d\bar{\Theta}\Theta-\bar{\Theta}d\Theta)\ . \tag{2.10}$$

These group-invariant forms are sufficient to build the Lagrangian of our model, which is given by

$$L=m(\dot\omega_{\mu}\dot\omega^{\mu})^{1/2}-im(\dot{\bar{\Theta}}\Theta-\bar{\Theta}\dot{\Theta})\ , \tag{2.11}$$

where $\dot\omega^{\mu}\equiv\omega^{\mu}/d\tau$, etc.[14] The momenta are given by

$$p_{\mu}=\frac{\partial L}{\partial\dot x^{\mu}}=m(\dot\omega^{\mu}\dot\omega_{\mu})^{-1/2}\omega_{\mu}\ , \tag{2.11a}$$

$$\Pi_{\Theta}=\frac{\partial L}{\partial\Theta}=i\bar{\Theta}(\not p-m)\ ,$$
$$\Pi_{\bar\Theta}=\frac{\partial L}{\partial\bar\Theta}=i(\not p-m)\Theta\ , \tag{2.11b}$$

where $\not p=\Gamma^{\mu}p_{\mu}$ and $\bar{\Pi}_{\Theta}=-\Pi_{\bar\Theta}$ (anti-Hermiticity). The model implies the following set of primary constraints:

$$\Phi=p^2-m^2=0\ , \tag{2.12a}$$

$$G_{\Theta}=\Pi_{\Theta}-i\bar{\Theta}(\not p-m)=0\ , \tag{2.12b}$$

$$G_{\bar\Theta}=\Pi_{\bar\Theta}-i(\not p-m)\Theta=0\ . \tag{2.12c}$$

To perform the quantization by means of the GB method[4,9,10] we choose as a maximal set of first-class constraints (2.12a) and (2.12b) together with $C_{\bar\Theta}$ below:

$$C_{\Theta}=G_{\Theta}(\not p+m)$$
$$=\Pi_{\Theta}(\not p+m)-i\bar{\Theta}(p^2-m^2)=0\ , \tag{2.13a}$$

$$C_{\bar\Theta}=(\not p+m)G_{\bar\Theta}$$
$$=(\not p+m)\Pi_{\bar\Theta}-i(p^2-m^2)\Theta=0\ , \tag{2.13b}$$

where the last terms vanish on account of (2.12a). We now introduce the Schrödinger representation for the quantized space variables acting on a generic "superwave function" $\Phi(x,\Theta,\bar\Theta)$. We perform the quantization in two steps.

(1) Off-shell quantization:

$$G_\Theta \mid \Psi\rangle = 0 \Longrightarrow G_\Theta \Phi^{\rm off} = 0 \ , \tag{2.14}$$

which leads to off-shell superfields

$$\Phi^{\rm off}(x,\Theta,\bar\Theta) = \exp[\bar\Theta(i\Gamma^\mu\partial_\mu - m)\Theta]\Phi(x,\bar\Theta) \ . \tag{2.15}$$

(2) On-shell quantization. Imposing now (2.12a) and (2.13b) we obtain

$$(\Box + m^2)\Phi(x,\bar\Theta) = 0 \ ,$$
$$(i\not p + m)\frac{\partial}{\partial\bar\Theta}\Phi(x,\bar\Theta) = 0 \ . \tag{2.16}$$

One might note here that (2.13a) does not produce any new condition. Reverting to the Weyl spinors notation we find that the *on-shell* degrees of freedom are characterized by a $N=1$, $D=6$ superfield $\Phi(x,\theta^\alpha)$. Its component expansion

$$\Phi(x,\theta^\alpha) = \phi + \psi_\alpha\theta^\alpha + A_{\alpha\beta}\theta^\alpha\theta^\beta + \chi^\alpha\varepsilon_{\alpha\beta\gamma\delta}\theta^\alpha\theta^\beta\theta^\gamma\theta^\delta + D\varepsilon_{\alpha\beta\gamma\delta}\theta^\alpha\theta^\beta\theta^\gamma\theta^\delta \tag{2.17}$$

shows that the on-shell contents of the model are one scalar, one pseudoscalar, one six-vector, and one chiral and one antichiral $D=6$ Weyl spinor; in all, 8 + 8 complex degrees of freedom. One can say that Eqs. (2.16) describe the $N=2$, $D=6$ massive hypermultiplet, which leads after dimensional reduction to the $N=4$, $D=4$ massive vector multiplet, discussed firstly in Ref. 15 and obtained by first quantization of the massive superparticle model for $N=4$, $D=4$, in Ref. 9.

III. SIEGEL INVARIANCE AND MASS

The Lagrangian (2.11) vanishes in the limit $m \to 0$. To introduce a Lagrangian formulation of the massive superparticle which allows for a zero-mass limit we introduce an auxiliary einbein variable, and rewrite (2.11) as

$$I = \tfrac{1}{2}\int d\tau \left[\frac{1}{e}\dot\omega^\mu\dot\omega_\mu + m^2 e - 2im(\dot{\bar\Theta}\Theta - \bar\Theta\dot\Theta)\right] \ . \tag{3.1}$$

Of course, by putting the einbein "on shell" $[e = (\dot\omega_\mu\dot\omega^\mu)^{1/2}/m]$ one gets again (2.11). The canonical momenta are given by

$$p_\mu = \dot\omega_\mu / e, \quad \Pi_e = 0 \tag{3.2}$$

plus (2.11b); the mass-shell condition (2.12a) is obtained as the secondary constraint which ensures that $\Pi_e = 0$ is preserved in time.

Let us consider the transformation of the superspace variables generated by the constraints (2.13a) and (2.13b) in the following way:

$$\begin{aligned} \delta\Theta &= \{i\bar C\kappa,\Theta\} = i(\not p + m)\kappa \ , \\ \delta\bar\Theta &= -\{i\bar\kappa C,\bar\Theta\} = -i\bar\kappa(\not p + m) \ , \\ \delta x^\mu &= \{i(\bar C\kappa + \bar\kappa C), x^\mu\} \\ &= i(\bar\Theta\Gamma^\mu\delta\Theta - \delta\bar\Theta\Gamma^\mu\Theta) \ , \end{aligned} \tag{3.3}$$

where $\kappa(\tau)$ is a local fermionic parameter; notice the relative minus sign for δx^μ as given above for $\delta\Theta,\delta\bar\Theta$ and in (2.1) for $\epsilon,\bar\epsilon$. It follows from (3.3) that

$$\delta\omega^2 = 4i(\dot{\bar\Theta}\not p\delta\Theta - \delta\bar\Theta\not p\dot\Theta) \tag{3.4}$$

and that the action (3.1) is invariant for

$$\delta e = -4(\dot{\bar\Theta}\kappa + \bar\kappa\dot\Theta) \ . \tag{3.5}$$

Thus, we see that in the limit $m \to 0$ (3.1) leads to the $N=2$ Brink-Schwarz massless superpaticle whose action is (see Refs. 2 and 16)

$$I = \tfrac{1}{2}\int d\tau \frac{1}{e}\dot\omega^\mu\dot\omega_\mu \tag{3.6}$$

and that in the limit $m \to 0$ the transformations (3.3) and (3.5) give the Siegel transformations (Ref. 8; see also Ref. 17).

Our model may be written also for $N=1$, $D=6$ supersymmetry if m is set equal to zero. Indeed, no extension cocycle [(2.8)] or Wess-Zumino term [(2.11)] can be written for $N=1$; there are no central extensions of the chiral superalgebra. The invariant line element for chiral superspace is given by

$$\omega^\mu = dx^\mu + i(d\theta_{\dot\alpha}\tilde\Sigma^{\mu\dot\alpha\beta}\theta_\beta - \theta_{\dot\beta}\tilde\Sigma^{\dot\beta\alpha}d\theta_\alpha) \tag{3.7}$$

and substituted in (3.6) defines the massless action. We conclude from the $D=6$ example that (1) nonchiral massless superparticle models can be made massive without losing the local fermionic invariance if the mass is properly introduced as a central charge and (2) chiral massless superparticle models can be obtained from the above massive models by taking the limit $m \to 0$ and retaining one chirality sector.

IV. $D=8$ MODEL FROM $D=10$

The $N=2$ nonchiral $D=10$ supersymmetry group is given by

$$x'^\mu = x^\mu + a^\mu + i\bar\epsilon\Gamma^\mu\Theta, \quad \Theta' = \Theta + \epsilon \ , \tag{4.1a}$$

where the $\Theta = (\theta_A, \theta^B)$, $\epsilon = (\varepsilon_A, \varepsilon^B)$, $A,B = 1,\ldots,16$, are real 32-dimensional spinors made out of two 16-dimensional Majorana-Weyl (chiral and antichiral) *independent* spinors characterized by the position of the index. The central extension is achieved by adding

$$\phi' = \phi + \alpha - im\bar\epsilon\Theta \tag{4.1b}$$

to (4.1a). The associated graded algebra is given by

$$\begin{aligned} &\{Q_A, Q_B\} = \Sigma^\mu_{AB}\partial_\mu, \quad \{Q^A, Q^B\} = \tilde\Sigma^{\mu AB}\partial_\mu \ , \\ &\{Q_A, Q^B\} = -2im\delta^B_A\Xi, \quad [\Xi, \text{any } Q] = 0 \ , \end{aligned} \tag{4.2}$$

where $\Sigma^\mu, \tilde\Sigma = (\Sigma^0, -\Sigma)$ are the $D=10$ Pauli matrices, which are symmetric because they are Hermitian and real; if $m=0$, (4.2) splits into a $N=1$ chiral and a $N=1$ antichiral superalgebra. We choose the following form for the 16×16 Σ matrices:

$$\Sigma^\mu=\left\{\begin{bmatrix}\mathbf{1}_8 & 0\\ 0 & \mathbf{1}_8\end{bmatrix},\begin{bmatrix}0 & \sigma^r\\ -\sigma^r & 0\end{bmatrix},\begin{bmatrix}0 & \mathbf{1}_8\\ \mathbf{1}_8 & 0\end{bmatrix},\begin{bmatrix}\mathbf{1}_8 & 0\\ 0 & -\mathbf{1}_8\end{bmatrix}\right\}$$

$$=\{\Sigma^0,\Sigma^r\ (r=1,\ldots,7),\Sigma^8,\Sigma^9\}\ , \qquad (4.3)$$

where (Σ^r,Σ^8) can be considered the $D=8$ Euclidean Γ matrices and $(\sigma^r)^T=-\sigma^r$.

The invariant one-forms required to build the model are given by

$$\omega^\mu=dx^\mu+i(d\theta_A\tilde{\Sigma}^{AB}\theta_B+d\theta^A\Sigma_{AB}\theta^B)\ ,$$
$$\omega_\phi=d\phi-im(d\theta^A\theta_A+d\theta_A\theta^A)\ , \qquad (4.4)$$

from which the Lagrangian for our $D=10$ model is constructed:

$$L=m(\dot\omega^\mu\dot\omega_\mu)^{1/2}-im\dot{\bar\Theta}\Theta\ . \qquad (4.5)$$

The $D=10$ fermionic constraints G have now the form

$$G_{\theta_A}=\Pi_{\theta_A}-i(\tilde{\Sigma}^{\mu AB}p_\mu\theta_B-m\theta^A)=0\ ,$$
$$G_{\theta^A}=\Pi_{\theta^A}-i(\theta^B\Sigma^\mu_{BA}p_\mu-m\theta_A)=0\ . \qquad (4.6)$$

Let us now split the set (4.6) into four eight-component constraints:

$$G_{\eta_1}=\Pi_{\eta_1}-i(p^0+p^9)\eta_1-i(\sigma\cdot\mathbf{p})\eta_2-ip^8\eta_2+im\theta^1=0\ ,$$
$$G_{\eta_2}=\Pi_{\eta_2}-i(p^0-p^9)\eta_2+i(\sigma\cdot\mathbf{p})\eta_1-ip^8\eta_1+im\theta^2=0\ ,$$
$$G_{\theta^1}=\Pi_{\theta^1}-i(p^0-p^9)\theta^1+i(\sigma\cdot\mathbf{p})\theta^2+ip^8\theta^2+im\eta_1=0\ , \qquad (4.7)$$
$$G_{\theta^2}=\Pi_{\theta^2}-i(p^0+p^9)\theta^2-i(\sigma\cdot\mathbf{p})\theta^1+ip^8\theta^1+im\eta_2=0\ ,$$

where $\theta^A\equiv(\theta^1,\theta^2)$, $\theta_A\equiv(\theta_1,\theta_2)$, and θ^i,θ_i $(i=1,2)$ are real SO(8) spinors. In order to use the fermionic constraints as GB quantization conditions we now introduce the eight-component complex SO(1,7) spinors

$$(\xi,\xi^*)=\frac{1}{\sqrt{2}}(\theta_1\pm i\theta_2)\ ,$$
$$(\rho,\rho^*)=\frac{1}{\sqrt{2}}(\theta^1\pm i\theta^2) \qquad (4.8)$$

and their associated SO(1,7) covariant fermionic constraints

$$G_{\xi,\xi^*}=\frac{1}{\sqrt{2}}(G_{\theta_1}\mp iG_{\theta_2})\ ,$$
$$G_{\rho,\rho^*}=\frac{1}{\sqrt{2}}(G_{\theta^1}\mp iG_{\theta^2})\ . \qquad (4.9)$$

Computing their Poisson brackets

$$\{G_\xi,G_\xi\}=2(p^8+ip^9)\ ,$$
$$\{G_\rho,G_\rho\}=2(p^8+ip^9)\ , \qquad (4.10)$$
$$\{G_\xi,G_\rho\}=0\ ,$$

we see that the model can be quantized by the GB procedure if $p^8=p^9=0$. Introducing a 16-component $D=8$ spinor $\Psi=\binom{\xi}{\rho}$ and SO(1,7) Dirac matrices, the constraints (4.7) can be written as

$$G_{\bar\Psi}=\Pi_{\bar\Psi}-i(\not p-m)\Psi=0\ ,$$
$$G_\Psi=\Pi_\Psi-i\bar\Psi(\not p-m)=0\ , \qquad (4.11)$$

where now Γ^μ belongs to the SO(1,7) Dirac algebra. (4.11) also leads to the constraints

$$C_{\bar\Psi}=(\not p+m)G_{\bar\Psi}=(\not p+m)\Pi_{\bar\Psi}=0\ ,$$
$$C_\Psi=G_\Psi(\not p+m)=\Pi_\Psi(\not p+m)=0\ , \qquad (4.12)$$

if we use the mass-shell condition $p^2=m^2$.

The GB quantization proceeds now as before, Φ^{off} is determined by $G_\Psi\Phi^{\text{off}}(x,\xi,\rho)=0$, and Φ^{on} satisfies $C_{\bar\Psi}\Phi^{\text{on}}=0$, where, due to the condition $p^8=p^9=0$, p contains only SO(1,7) components. The off-shell superfield is a holomorphic superfield depending on 16 complex Grassmann coordinates, and the on-shell superfield is described by an unconstrained holomorphic superfield depending on an 8-component SO(1,7) spinor ($N=2$, $D=8$ hypermultiplet) with $2^8=256$ complex degrees of freedom. Interestingly enough, this number of 128 bosonic and 128 fermionic complex degrees of freedom is in accordance with the number which corresponds to the $D=10$, $N=2$ massive multiplet (see, e.g. Ref. 18), an equality which is explained by the same real dimension $(=16)$ of the $D=8$ (8 complex) and $D=10$ (16 real) Weyl spinors for the Minkowski metric.

To conclude, we would like to add that in order to quantize all the degrees of freedom of the $D=10$ massive superparticle without changing the canonical Poisson brackets one can apply a "mixed" method: firstly, one splits the 32 real fermionic constraints of the $D=10$, $N=2$ massive model into 16 real first-class constraints and 16 real second-class constraints, which breaks O(9,1) to O(8)×O(1,1), and then introduces the Gupta-Bleuler method in the sector of second-class constraints. Such a method of quantization was presented recently in Refs. 19 and 20 for the massless case, and the O(8) symmetry in the sector of second-class constraints was broken to O(6)×O(2). It appears that such a split into first- and second-class constraints and the structure of second-class constraints permitting the application of the GB method is not spoiled by the presence of the $D=10$ mass parameter.

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