Numerical analysis of thermal fluctuations in new inflation

Hume A. Feldman
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794
(Received 22 January 1988)

The effects of thermal fluctuations on the evolution of the scalar field in a preinflationary period are explored numerically to complement a previous analytical investigation. The analysis is applicable to a weakly coupled scalar field in a double-well potential in new inflation. A critical value of the strength of the coupling to the thermal bath is found, above which thermal fluctuations do not allow the inflationary period to begin. The evolution of the scalar field is followed until the onset of the inflationary period.

I. INTRODUCTION

The inflationary description of the evolution of the Universe at its very early epoch[1–3] has proven to be a most successful approach. The inflationary scenario succeeded in explaining at least ten different problems both in particle physics and in cosmology. In particular the inflationary-universe scenario explains the large-scale homogeneity, isotropy, and flatness of the observed part of the Universe; problems that plagued cosmologists for many years. Thus, inflationary-universe models have become very popular and been studied extensively by many authors; for review articles see Ref. 4.

The model we investigate in this paper is the so-called new inflationary scenario.[2] In this model an initial horizon size domain nucleates and grows quasiexponentially (i.e., inflates) for a time sufficiently long such that it becomes many orders of magnitude larger than the observable part of the Universe.

A standard example of new inflation is to consider the behavior of some scalar field $\phi(t,\mathbf{x})$ (sometimes called the inflaton) that is very weakly self-coupled[5] and is characterized by a double-well potential $V(\phi)$ (Fig. 1). The potential $V(\phi)$ has a local maximum at $\phi=0$ and global minima at $\phi=\pm\sigma$, where σ is the symmetry-breaking scale; $\sigma < M_{\rm Pl}$ where $M_{\rm Pl}$ is the Planck mass. The scalar field ϕ obeys the classical Klein-Gordon equation of motion:

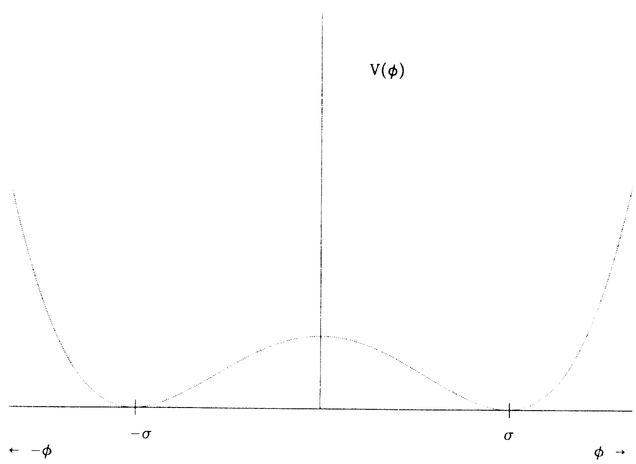

FIG. 1. The double-well potential $V(\phi)=\lambda_\phi(\phi^2-\sigma^2)^2$ used in this paper.

$$\ddot{\phi}+3H\dot{\phi}-a^{-2}(t)\nabla^2\phi=-V'(\phi)\ , \tag{1}$$

where $a(t)$ is the scale factor of the Universe that varies as $a\sim t^{1/2}$ for a radiation-dominated universe, $H(t)=\dot{a}(t)/a(t)$ is the Hubble parameter and $V'(\phi)=\partial V(\phi)/\partial\phi$. Further, we assume that the space-time metric is a flat Friedmann-Robertson-Walker (FRW) one:

$$ds^2=dt^2-a^2(t)(dx^2+dy^2+dz^2)\ . \tag{2}$$

It is appreciated that since the scalar field ϕ is very weakly self-coupled, the equation of state for ϕ,

$$\begin{aligned}\rho(\phi)&=\tfrac{1}{2}\dot{\phi}^2+\tfrac{1}{2}(\nabla\phi)^2+V(\phi)\ ,\\ P(\phi)&=\tfrac{1}{2}\dot{\phi}^2-\tfrac{1}{6}(\nabla\phi)^2-V(\phi)\ ,\end{aligned} \tag{3}$$

where ρ is the energy density and P is the pressure, will be dominated at early times by the spatial gradient and kinetic terms[6] and so inflation will not occur.

A mechanism to solve this problem was proposed by Albrecht and Brandenberger.[7] It was shown both analytically[7] and by numerical studies[8] that the dynamics of the scalar field is governed by the expansion of the Universe and by the nonlinear forces due to the potential terms in the equation of motion (1). The Hubble expansion rate acts as a damping force on the amplitude of the scalar field. Thus if the effects of the potential force are small, the scalar field ϕ will relax to its spatial average $\langle\phi\rangle=0$ due to the symmetry of the potential. Then an inflationary period, which is sufficiently long to solve the cosmological problems, will commence.[7] That happens in such a way that at some critical temperature T_c the scalar field $\phi(\mathbf{x})$ is confined to $\phi\sim 0$, the potential energy $V(0)$ starts to dominate the energy-momentum tensor and hence both the kinetic and the spatial gradient terms will become negligible relative to $V(0)$. When that occurs we see from Eq. (3) that $P(\phi)=-\rho(\phi)$ and the usual initial conditions for new inflation are reproduced.

In a recent paper[9] we studied the evolution of $\phi(t,\mathbf{x})$

when the scalar field ϕ is coupled to a radiation bath. Theoretical limits on the strength of the interaction between ϕ and the radiation bath were derived. A critical value was found for the coupling constant in the Lagrangian that describes the interaction, below which the effects of the thermal fluctuations are small and thus do not change significantly the evolution of the scalar field ϕ. ϕ relaxes to its spatial average, that is, to a region where $\phi=0$ and so inflation is possible. The work in Ref. 9 was predominantly analytical, this paper aims to complement it, confirm its validity, and present more exact limits on the problem.

When we couple the scalar field to the radiation bath, in effect we add to the equation of motion (1) a random-force term F_R. The Klein-Gordon equation thus becomes

$$\ddot{\phi}+3H\dot{\phi}-a^{-2}(t)\nabla^2\phi=-V'(\phi)+F_R \ . \tag{4}$$

Quantum fluctuations of the scalar field ϕ were considered and studied in many papers[10–17] as the source for the random-force term F_R in the equation of motion (4). In the above publications a local patch was investigated, it was assumed that the domain in question is homogeneous such that both the kinetic and the spatial gradient terms could be neglected, thus the equation of motion was reduced to a first-order differential equation. This approach is called the slow-rolling approximation. In a previous paper[9] and in this one we do not take this approach and investigate the full equation of motion (4) which is second order in time and second order in space.

We study the evolution of the scalar field obeying the equation of motion (4) in the preinflationary period. Our initial time is $t_i=t_{\rm Pl}$ where $t_{\rm Pl}=M_{\rm Pl}^{-1}$ is the Planck time. We follow the evolution of ϕ up to $t=t_f$ which is the time at which the preinflationary stage ends and the inflationary period begins.

Both in Ref. 9 and this paper the random force F_R is taken to be due to classical fluctuations of the scalar field ϕ that are caused by the interaction with the radiation bath. Other authors have done a quantum-mechanical analysis of the evolution of the scalar field ϕ in the slow-rolling phase transition in new inflation.[18–20]

In Sec. II we will discuss some of the ideas that this paper is based upon and summarize the results of the analytical approach.[9] The numerical approach will be presented in Sec. III and conditions for the solution of the equation of motion will be discussed. In Sec. IV the initial conditions are discussed. The numerical results are presented in Sec. V and are shown to agree quite well with the analytical predictions. The conclusions are given in Sec. VI.

The space-time metric used in this paper is that of a flat FRW (2) universe. The units used throughout are the natural units in which $\hbar=k_B=c=G=M_{\rm Pl}=1$.

II. PRELIMINARY DISCUSSION

In recent publications[7,8] detailed studies of the dynamical relaxation of the scalar field in the preinflationary period were made. As can be seen from Eq. (1), there are two conflicting forces that influence the dynamical evolution of the inflaton. One is the effect due to the Hubble damping force that tends to red-shift the wavelength of the scalar field ϕ and dampen its magnitude. The other is due to the nonlinear effects of the potential that tend to cause inhomogeneities in the field. It was shown that if the parameters in the potential were chosen appropriately, the Hubble damping force dominates and the scalar field becomes homogeneous enough and localized near the local maximum of the potential for the inflationary state to begin. To enter the inflationary period and to assure that inflation is long enough we need to have the conditions that the potential energy $V(\phi)$ dominates the energy-momentum tensor and both the kinetic terms and the spatial gradient terms associated with the scalar field ϕ are negligible. This is the so-called slow-rolling phase described in the Introduction. When these conditions are satisfied we get from Eq. (3) that $P(\phi)=-\rho(\phi)$ and inflation begins.

In a recent publication[9] and in this one we add another force to the equation of motion [Eq. (4)]. We study the effect of this force in a similar way to the analysis presented in Ref. 7, that is, we compare this additional force to the Hubble force and see if the effect of this force prevents dynamical relaxation of the scalar field ϕ and thus prevents inflation. The random force is being generated by coupling the scalar field ϕ to the thermal bath. Since the temperature of the thermal radiation is very high during the preinflationary period we assume that the scalar field ϕ is influenced by it thereby producing thermal fluctuations of the field.

To describe the effects of thermal fluctuations on the evolution of the scalar field we couple ϕ to the thermal bath that is manifested by N scalar fields ψ. The Lagrangian $\mathcal{L}_I$ that describes the interaction between the inflaton ϕ and the thermal field ψ is

$$\mathcal{L}_I=\tfrac{1}{2}\lambda N\phi^2\psi^2 \ , \tag{5}$$

where λ is the coupling constant that describes the strength of the interaction between ϕ and the thermal bath. We find limits on λ such that the effects of the thermal fluctuation are small enough and the dynamical evolution of the scalar field ϕ produces conditions compatible with inflation.

To find limits on the coupling constant that describes the interaction strength between the inflaton field ϕ and the thermal bath we need to get an estimate of the magnitude of the fluctuations of ϕ due to the radiation bath. In Ref. 9 we derived an upper limit on the value of interaction strength, that is, we found the largest λ for which the coupling of the scalar field ϕ to the thermal bath would not prevent inflation. To do that we transformed to a free conformally coupled scalar field in conformal time. The equation of motion in momentum space was derived and allowed us to find a critical coupling strength λ_c using the perturbative Green's-function analysis:[7,9]

$$\lambda_c \sim \sigma \ . \tag{6}$$

The inflationary period is entered only for values of λ smaller than λ_c. For $\lambda>\lambda_c$ there is no inflation.

III. NUMERICAL APPROACH

The main investigative tool of this paper is the numerical integration of the equation of motion (4) for the scalar field $\phi(t,\mathbf{x})$ in a given flat FRW metric (2) background. The potential chosen here is the double-well potential

$$V(\phi(t,\mathbf{x}))=\lambda_\phi(\phi^2-\sigma^2)^2 , \quad (7)$$

where λ_ϕ is the self-coupling constant that, because of cosmological considerations, must be very small[5] and σ is the symmetry-breaking scale: $\sigma < M_{\rm Pl}$.

We begin the analysis in a hot radiation-dominated phase at the Planck time $t_{\rm Pl}=M_{\rm Pl}^{-1}=1$ in our units. At these high temperatures the energy-momentum tensor is dominated by the energy density of the radiation, thus it is sensible to use the flat FRW metric. For a radiation-dominated expanding universe the scale factor $a(t)$ is proportional to $t^{1/2}$. We let the scalar field ϕ evolve up to the time t_f for which

$$\begin{aligned} V(\phi(t,\mathbf{x})) &> \tfrac{1}{2}[\partial_0\phi(t,\mathbf{x})]^2 ,\\ V(\phi(t,\mathbf{x})) &> \tfrac{1}{2}[\partial_i\phi(t,\mathbf{x})]^2 , \end{aligned} \quad (8)$$

or, equivalently, as can be seen from Eq. (3),

$$P(\phi(t,\mathbf{x}))\simeq-\rho(\phi(t,\mathbf{x})) . \quad (9)$$

The above equation is called the de Sitter equation.

In our analysis the scalar field ϕ starts as a plane wave in the x direction:

$$\phi(t,\mathbf{x})=A \sin kx, \quad \dot\phi(t,\mathbf{x})=0 , \quad (10$$

where A is the amplitude of the wave and k is the wave number. The choice of a plane wave as the initial condition for $\phi(t,\mathbf{x})$ allows us to treat the problem in one space and one time dimension. Thus the equation of motion (4) reduces to a two-coordinate second-order differential equation.

To solve the second-order differential equation of motion (4) we define

$$\chi(t,x)\equiv\dot\phi(t,x) ; \quad (11)$$

thus we get two coupled first-order differential equations:

$$\begin{aligned} \dot\phi(t,x)&=\chi(t,x) ,\\ \dot\chi(t,x)&=\chi(t,x)-3H(t)\chi(t,x)+a^{-2}(t)\frac{\partial^2\phi(t,x)}{\partial x^2}\\ &\quad -V'(\phi(t,x))+F_R(\phi(t,x)) . \end{aligned} \quad (12)$$

For the potential term we use Eq. (7) and for the random-force term we use

$$\begin{aligned} F_R&=\epsilon(t,x)\lambda N\psi^2(t)\phi(t,x)\\ &=\epsilon(t,x)\lambda N\left[\frac{\pi^2}{30}\right]^{1/2}T_i^2\left[\frac{T}{T_i}\right]^2\phi(t,x) , \end{aligned} \quad (13)$$

where T is the temperature, T_i is the initial temperature, and $\epsilon(t,x)$ is a random number

$$\epsilon(t,x)\in[-1,1] \quad (14)$$

which is different for every space-time point.

IV. INITIAL CONDITIONS

The choice of initial values for the problem is made using the "quasithermal" conditions discussed in the first paper of Ref. 8. We demand that both the potential energy $V(\phi(t_i,x))$ and the spatial gradient $\partial^2\phi(t_i,x)/\partial x^2$ are initially of the order of the Planck density $M_{\rm Pl}^4$ (=1 in our units). Using the potential in Eq. (7) and the initial conditions on the scalar field $\phi(t_i,x)$ given in Eq. (10) the amplitude A is found to be

$$A\sim\lambda_\phi^{-1/4} \quad (15)$$

and the wave number k is

$$k\sim\lambda_\phi^{1/4} . \quad (16)$$

To find the final time for the preinflationary period we demand that the potential energy for $\phi=0$ in comoving coordinates will be larger than both the spatial gradient and the kinetic terms [Eq. (8)]. That is,

$$A^2k^2a^{-4}(t_f)\le V(0) , \quad (17)$$

which for $a(t)\sim t^{1/2}$ yields

$$t_f\sim\lambda_\phi^{-1/2}\sigma^{-2} . \quad (18)$$

We choose the self-coupling λ_ϕ and σ to be

$$\lambda_\phi=5\times10^{-3}, \quad \sigma=2\times10^{-1} , \quad (19)$$

in order to minimize the computer time since, as can be seen from Eq. (18), t_f is strongly dependent on σ.

The time steps are constant in conformal time and are chosen to be one-hundredth of the initial time. We restricted our analysis to one-wavelength excitations and thus we take our spatial interval to be $x\in[0,2\pi k^{-1}]$. We use 100 spatial mesh points for our analysis.

The random number $\epsilon(t,x)$ used for the force term F_R [Eq. (14)] was generated using the Ridge computer library Random Number Generators RAND and SRAND.

V. NUMERICAL ANALYSIS

The equations for the numerical analysis are the two first-order-in-time differential equations given in Eq. (12) which we integrate numerically. We are using the double-well potential $V(\phi)$ [Eq. (7)] and the random-force term F_R given in Eq. (13). The force F_R is generated using a random-number generator. We employ the initial conditions [Eq. (10)] and periodic boundary conditions. One hundred mesh points were chosen for the spatial grid, various other spatial resolutions were used to check the validity of this choice. The spatial grid points are constant in comoving coordinates and the time steps are constant in conformal time.

The parameters used in this problem are the self-coupling constant λ_ϕ, the symmetry-breaking scale σ that determines the shape of the potential $V(\phi)$ [Eq. (7)], the

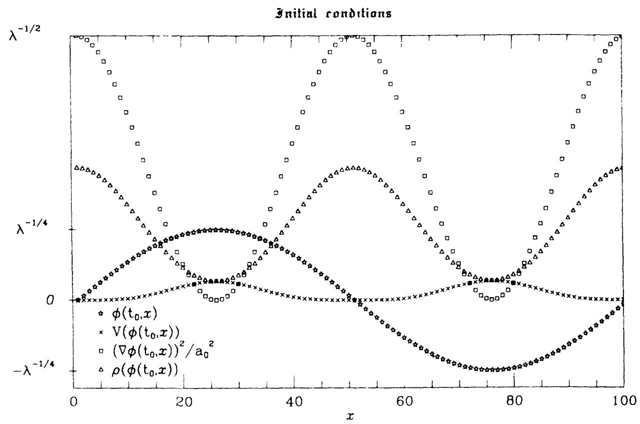

FIG. 2. The initial configuration where the scalar field $\phi(t_i,x)$ initially has an amplitude $A=\lambda^{-1/4}$.

amplitude A, and the wave number k of the scalar field ϕ [Eq. (10)], the number of particle species N that interact with ϕ and the coupling constant λ, which is the characteristic strength of the interaction between the scalar field ϕ with the thermal bath.

We follow the evolution of the scalar field ϕ and plot its amplitude as a function of the spatial coordinate x and the time t. We follow the time evolution of the energy density $\rho(\phi)$ and the pressure $P(\phi)$ [Eq. (3)] for a given spatial point and observe the validity of the de Sitter equation (9) around the final time t_f (the onset of inflation). We vary the strength of the coupling constant λ of the interaction between the scalar field ϕ and the thermal bath through its critical value λ_c and see the effect it has on the evolution of the field ϕ and on the equation of state (3).

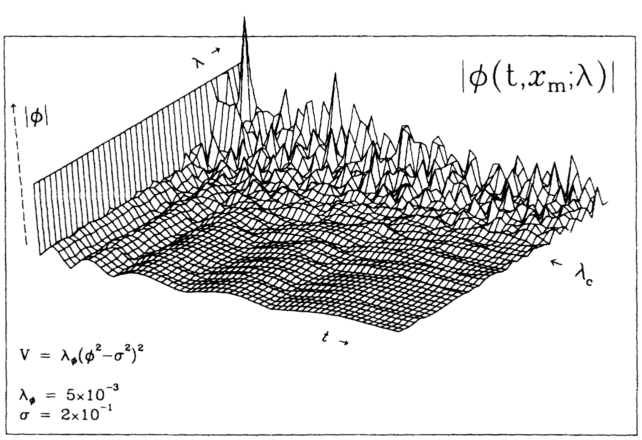

FIG. 3. A three-dimensional plot of the absolute value of the scalar field $|\phi(t,x_m;\lambda)|$ as a function of the time t and the coupling constant λ, at a spatial point x_m that is the maximum of the initial wave. The parameters used are $\lambda_\phi=5\times10^{-3}$ and $\sigma=2\times10^{-1}$. The coupling constant was varied in the interval $\lambda\in[0,5\sigma]$.

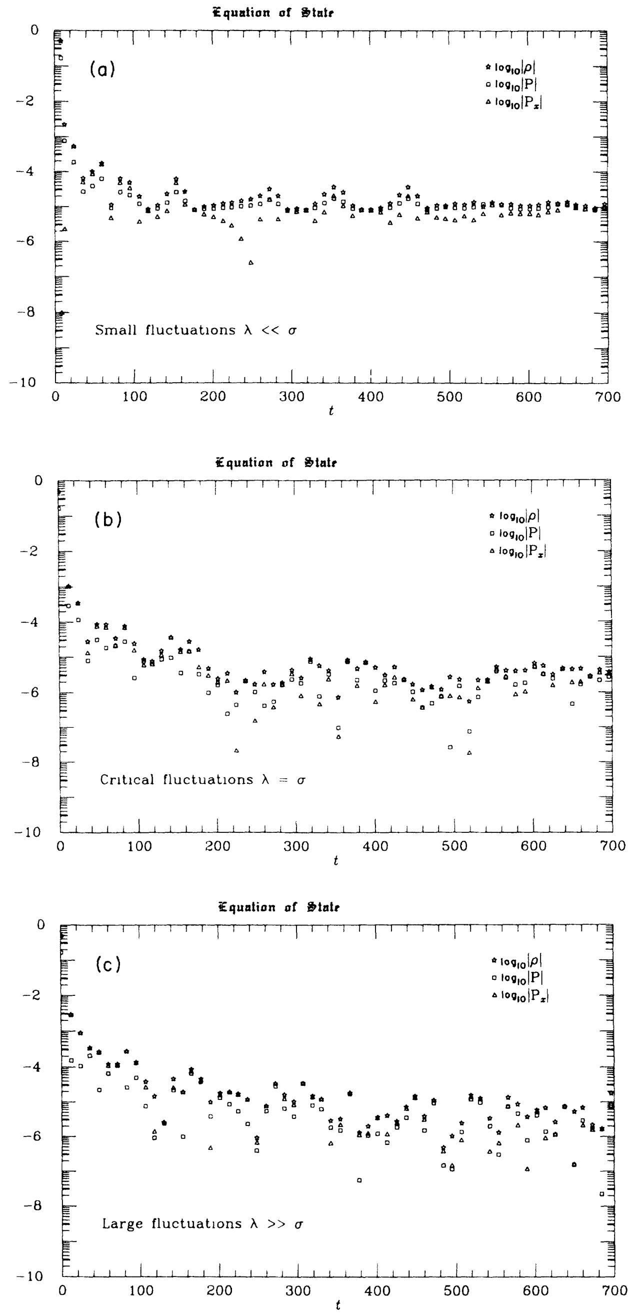

FIG. 4. The equation of state for the energy density of the scalar field $\rho(\phi(t,x_m))$, the pressure $P(\phi(t,x_m))$, and the pressure $P_x(\phi(t,x_m))$ in the direction of the standing wave as a function of time at a spatial point that is the maximum of the initial wave. The parameters used are $\lambda_\phi=5\times10^{-3}$ and $\sigma=2\times10^{-1}$. (a) shows the equation of state for a small coupling to the thermal bath, in (b) we see the equation of state for a critical coupling, and (c) shows the equation of state for a large coupling.

The initial conditions are presented in Fig. 2. The scalar field ϕ, the potential $V(\phi)$, the spatial gradient $\nabla^2\phi/a^2$, and the energy density of the scalar field $\rho(\phi)$ are shown as functions of the space coordinate x at the initial time $t_i=t_{\rm Pl}$ the Planck time.

Figure 3 shows a three-dimensional plot of the evolution of the absolute value of the scalar field $|\phi(t,x_m;\lambda)|$ as a function of t at a particular spatial point x_m as we vary the coupling constant λ through its critical value. We chose $N=1$, $\lambda_\phi=5\times10^{-3}$, and $\sigma=2\times10^{-1}$. The initial amplitude is $A=\lambda_\phi^{-1/4}$ and the wave number is $k=\lambda_\phi^{1/4}$. We started at $\lambda=0$, i.e., no interaction with the thermal bath, and followed the evolution up to $\lambda=5\sigma$. We can see that for $\lambda<\lambda_c=\sigma$ the amplitude at the end of the run is more than an order of magnitude smaller than the initial amplitude, that means that the scalar field is localized to a large degree about $\phi=0$. For $\lambda>\lambda_c$ it is clear that such localization does not occur.

The equation of state for the energy density $\rho(\phi)$, the pressure $P(\phi)$, and the pressure $P_x(\phi)$ in the direction of the standing wave, for three characteristic values of λ, is shown as a function of time at a particular spatial point x_m in Figs. 4(a)–4(c). Figure 4(a) is a plot of the time evolution of the absolute value of the pressure and the energy density for $\lambda \ll \lambda_c$, Fig. 4(b) shows $|\rho(\phi)|$ and $|P(\phi)|$ for $\lambda=\lambda_c$, and Fig. 4(c) shows the evolution for $\lambda \gg \lambda_c$. Again we see that for small λ we get the de Sitter condition (9) that states that $P(\phi)\simeq-\rho(\phi)$, whereas for large λ this condition is not satisfied.

Three-dimensional plots of the evolution of the scalar field $\phi(t,x)$ as a function of space x and time t are presented in Figs. 5(a)–5(d) for $\lambda=0$, $\lambda \ll \lambda_c$, $\lambda=\lambda_c$, and $\lambda \gg \lambda_c$, respectively. We clearly see that for large λ there is no relaxation of the field and thus we get no inflation.

VI. CONCLUSIONS

The evolution of the scalar field that drives inflation was studied in the preinflationary period in the presence of classical thermal fluctuations appearing as a random-force term in the equation of motion. The analysis was done in one-space one-time coordinate using the Klein-Gordon equation of motion. We integrated the equations numerically in the new inflationary scenario and found upper limits on the strength of the coupling constant that determines the magnitude of the thermal fluctuations.

We used the double-well potential in our analysis and found that the symmetry-breaking scale σ in the potential gives an upper limit on the coupling of the scalar field to the thermal bath:

$$\lambda_c \sim \sigma \; . \tag{20}$$

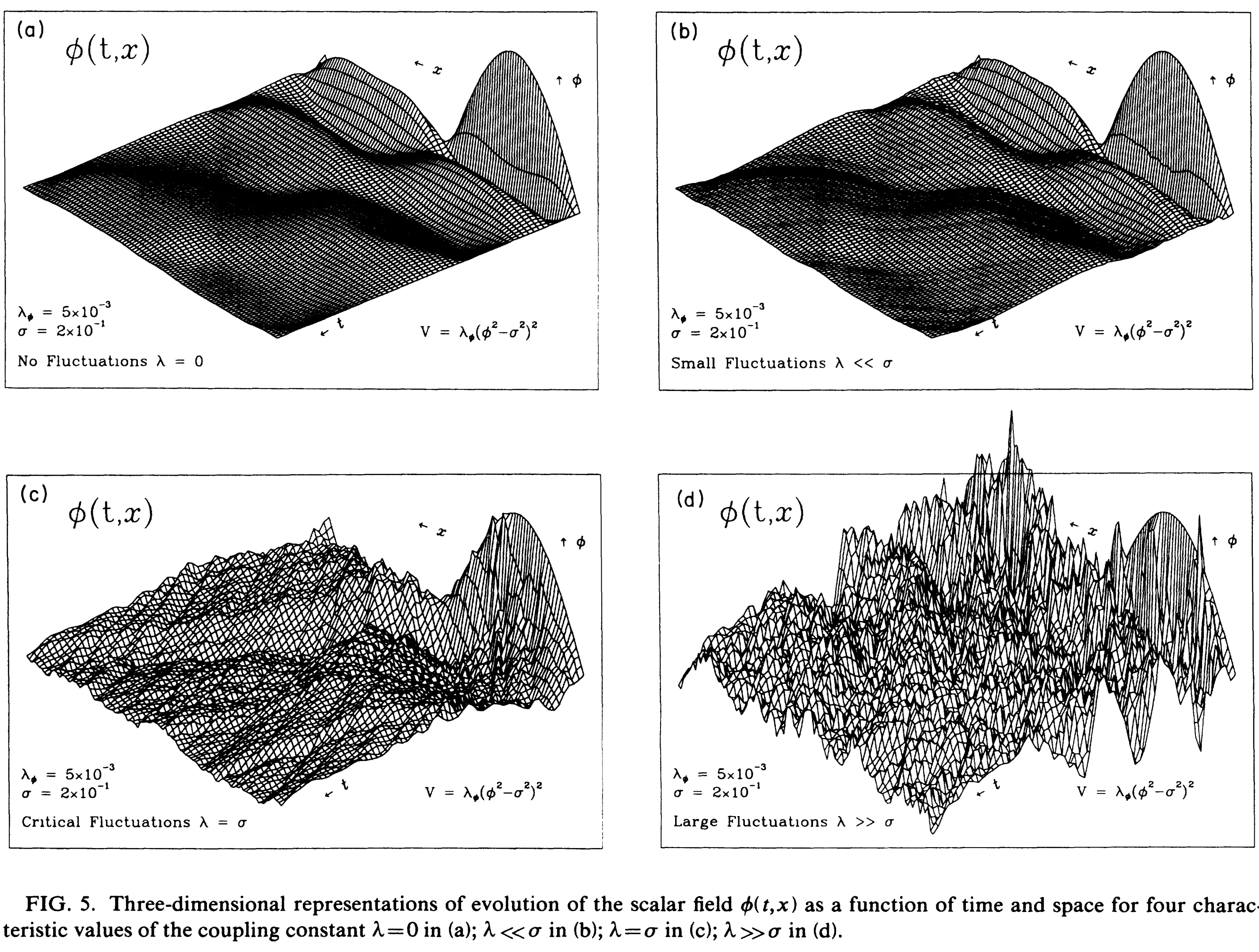

FIG. 5. Three-dimensional representations of evolution of the scalar field $\phi(t,x)$ as a function of time and space for four characteristic values of the coupling constant $\lambda=0$ in (a); $\lambda \ll \sigma$ in (b); $\lambda=\sigma$ in (c); $\lambda \gg \sigma$ in (d).

For larger values of λ dynamical relaxation does not occur, the equation of state does not obey the de Sitter equation $P(\phi)\simeq -\rho(\phi)$, and inflation does not begin. For smaller values of λ we do get dynamical relaxation and the de Sitter equation describes accurately the equation of state.

The constraint on λ_c given in (20) is weaker than

$$\lambda_\phi \simeq \sigma^2 , \tag{21}$$

which is the constraint on the coupling constant in the potential that is required by dynamical relaxation.[8] It is much weaker than the one that comes from the magnitude of the energy-density fluctuations. Thus these results do not impose any unreasonable new limits on the parameters in the new inflationary scenario.

This paper complements and verifies the analytical investigation given in Ref. 9. We found that the critical value of the coupling constant λ_c is very close to the theoretical predictions. Thus we conclude that thermal fluctuations do not affect the evolution of the scalar field in a major way and new inflation can be realized in our model with a mild constraint on the strength of the fluctuations.

ACKNOWLEDGMENTS

I wish to thank R. Brandenberger for many conversations and discussions that led to the research done in this paper. Further I would like to thank J. Smith for his support and J. Schecker for his help in generating the computer plots.

[1]A. Guth, Phys. Rev. D **23**, 347 (1981).
[2]A. Linde, Phys. Lett. **108B**, 389 (1982); A. Albrecht and P. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
[3]A. Linde, Phys. Lett. **129B**, 177 (1983).
[4]For review articles, see A. Linde, Rep. Prog. Phys. **47**, 925 (1984); R. Brandenberger, Rev. Mod. Phys. **57**, 1 (1985); M. Turner, in *The Inflationary Paradigm,* proceedings of the Cargèse School on Fundamental Physics and Cosmology, edited by J. Audouze and J. Tran Thanh Van (Editions Frontières, Gif-sur-Yvette, France, 1985); R. Brandenberger, Int. J. Mod. Phys. **A2**, 77 (1987).
[5]V. Mukhanov and G. Chibisov, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 549 (1981) [JETP Lett. **33**, 532 (1981)]; A. Guth and S-Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982); S. Hawking, Phys. Lett. **115B**, 295 (1982); A. Starobinsky, *ibid.* **117B**, 174 (1982); J. Bardeen, P. Steinhardt, and M. Turner, Phys. Rev. D **28**, 679 (1983); R. Brandenberger and R. Kahn, *ibid.* **29**, 2172 (1984).
[6]G. Mazenko, W. Unruh, and R. Wald, Phys. Rev. D **31**, 373 (1985).
[7]A. Albrecht and R. Brandenberger, Phys. Rev. D **31**, 1225 (1985).
[8]A. Albrecht, R. Brandenberger, and R. Matzner, Phys. Rev. D **32**, 1280 (1985); **35**, 429 (1987); H. Kurki-Suonio, J. Centrella, R. Matzner, and J. Wilson, *ibid.* **35**, 435 (1987).
[9]R. Brandenberger, H. Feldman, and J. MacGibbon, Phys. Rev. D **37**, 2071 (1988).
[10]F. Graziani and K. Olynyk, Fermilab Report No. Pub-85/175-T, 1985 (unpublished).
[11]A. Starobinsky, in *Field Theory, Quantum Gravity and Strings,* edited by H. J. de Vega and N. Sanchez (Lecture Notes in Physics, Vol. 246) (Springer, Heidelberg, 1986).
[12]A. Goncharov and A. Linde, Zh. Eksp. Teor. Fiz. **92**, 1137 (1987) [Sov. Phys. JETP **65**, 635 (1987)]; Fiz. Elem. Chastits At. Yadra **17**, 837 (1987) [Sov. J. Part. Nucl. **17**, 369 (1986)].
[13]J. Bardeen and G. Bublik, Class. Quantum Gravit. **4**, 573 (1987).
[14]S-J. Rey, Nucl. Phys. **B284**, 706 (1987).
[15]M. Pollock, Trieste Report No. IC/86/65, 1986 (unpublished).
[16]D. Coule and K. Olynyk, Mod. Phys. Lett. **A2**, 811 (1987).
[17]M. Sasaki, Y. Nambu, and K-I. Nakao, RIFP Hiroshima Report No. RRK-87-22, 1987 (unpublished).
[18]M. Evans and J. McCarthy, Phys. Rev. D **31**, 1799 (1985).
[19]A. Guth and S-Y. Pi, Phys. Rev. D **32**, 1899 (1985).
[20]A. Ringwald, Phys. Rev. D **36**, 2598 (1987).