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Polarization effects in $e^+e^- \to Z^0\gamma$

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Polarizations of Z^0 and γ in the reaction $e^+e^- \to Z^0\gamma$ near the Z^0 peak are obtained through the density matrix of Z^0 and Stokes parameters of γ, respectively. Their dependence on the linear polarizations of incident electron-positron beams and their influence on the decay of Z^0 into two spin-$\frac{1}{2}$ fermions f and $\bar{f}$ are discussed in the standard model. It is shown that the longitudinal polarization of incident electron beam can enhance the circular polarization of the outgoing photon beam.

I. INTRODUCTION

Many Z^0's are anticipated to be produced at the e^+e^- colliding machines, the SLAC Linear Collider (SLC) and CERN's LEP, leading to ample possibilities for checking the standard model of electroweak theory and a possible deviation from it.

One interesting process to search for additional neutrino generations is the process $e^+e^- \to \nu\bar{\nu}\gamma$ (Refs. 1–3) and it is shown[3] that when the incident energy is near the Z^0 peak, the major contribution to the cross section is the Z^0 exchange, that is, near the Z^0 peak the process $e^+e^- \to Z^0\gamma$ and the subsequent decay of Z^0 is important.

The purpose of this paper is to obtain the density matrix of Z^0 and the Stokes parameters of the outgoing photon beam in the process $e^+e^- \to Z^0\gamma$ and also to obtain the cross section and the angular distribution of one of the Z^0 decay products. Once the density matrix of the Z^0 beam is obtained at their production, it can be used for its various decay processes. Here we have considered the cases that the incident e^+ and e^- beams are linearly polarized and that Z^0's decay into fermion and antifermion pairs. In particular, it is shown that the longitudinal polarization of the incident electron beam can enhance the circular polarization of the outgoing photon beam, which is free from the QED background. This can be extended to the cases that incident particles are transversely polarized and also that Z^0's decay through other processes.

II. DENSITY MATRIX

When the Z^0 and γ are produced from the e^+e^- collision, Z^0 can be identified by considering the invariant mass of its decay products and it will be relatively easy if the decay products are charged particles. The $e^+e^- \to Z^0\gamma$ process can be described in various ways. Here we consider the lowest-order process described in Fig. 1.

The Proca vector ϵ_z^μ of the Z^0 vector boson which is produced by the process of Fig. 1 can be described[4] in the standard model as

$$\epsilon_z^\mu = \frac{eg}{2}\bar{v}(p_2)\left[\frac{1}{k_1\cdot p_1}\gamma_\alpha[\epsilon_L(1-\gamma_5)+\epsilon_R(1+\gamma_5)](\not{k}_1\not{\epsilon}^* - 2p_1\cdot\epsilon^*) + \frac{1}{k_1\cdot p_2}(2p_2\cdot\epsilon^* - \not{\epsilon}^*\not{k}_1)\gamma_\alpha[\epsilon_L(1-\gamma_5)+\epsilon_R(1+\gamma_5)]\right]u(p_1)\sum_{\text{spin}} Z^{\alpha *}Z^\mu \,. \quad (1)$$

Here the wave vector is unnormalized, and $k_1 = (\omega_1, \mathbf{k}_1)$, $p_1 = (E_1, \mathbf{p}_1)$, and p_2 are momenta of outgoing photon, incoming electron, and incoming positron, respectively. Also ϵ^μ in Eq. (1) is the photon wave vector and g, ϵ_L, and ϵ_R are defined as

$$g = (\sqrt{2}G_F M^2)^{1/2} \,, \quad (2)$$

$$\epsilon_L^f = T_{3L} - Q^f \sin^2\theta_W \,, \quad (3)$$

$$\epsilon_R^f = -Q^f \sin^2\theta_W \,. \quad (4)$$

From now on, we consider the process in the e^+e^- c.m. frame neglecting the electron and positron masses. When the incident electron and positron beams are unpolarized, one obtains, after averaging over the spin states of electron and positron,

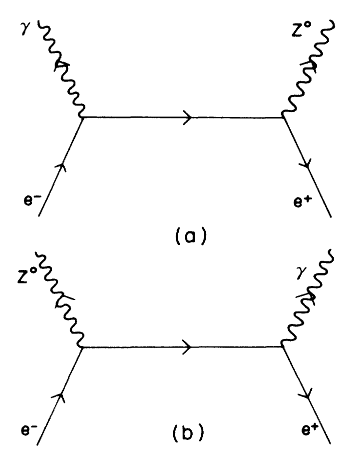

FIG. 1. Feynman diagram for the reaction $e^+e^- \to Z^0\gamma$.

$$\langle \epsilon_z^\mu \epsilon_z^{\nu *}\rangle=\frac{e^2g^2}{4m^2k_1\cdot p_1 k_1\cdot p_2}[(|\epsilon_R|^2+|\epsilon_L|^2)B_1^{\mu\nu}+(|\epsilon_R|^2-|\epsilon_L|^2)B_2^{\mu\nu}] . \tag{5}$$

Here $B_1^{\mu\nu}$ and $B_2^{\mu\nu}$ are defined as

$$B_1^{\mu\nu}=-I^{\mu\nu}(k_1\cdot k_2)^2\left[\epsilon\cdot\epsilon^*-\frac{M^2}{k_1\cdot p_1 k_1\cdot p_2}p_1\cdot\epsilon p_1\cdot\epsilon^*\right]+M^2\epsilon\cdot\epsilon^* K^\mu K^\nu-2k_1\cdot p_1 k_1\cdot p_2[(\epsilon^\mu+\Sigma^\mu)(\epsilon^\nu+\Sigma^\nu)^*+\text{c.c.}]$$
$$-\tfrac{1}{2}[(k_1\cdot k_2)^2+(k_1\cdot\Delta)^2][(\epsilon^\mu+\Sigma^\mu)(\epsilon^\nu+\Sigma^\nu)^*-\text{c.c.}]-\frac{M^2k_1\cdot\Delta k_1\cdot k_2}{2k_1\cdot p_1 k_1\cdot p_2}[p_1\cdot\epsilon(K^\mu\epsilon^{\nu *}-K^\nu\epsilon^{\mu *})-\text{c.c.}] , \tag{6a}$$

$$B_2^{\mu\nu}=i\left[-I^{\mu\nu}k_1\cdot\Delta\,\epsilon(k_1k_2\epsilon\epsilon^*)+\frac{1}{2}\left[\sigma^\mu\epsilon^\nu(k_1\epsilon\epsilon^*)+\sigma^\nu\epsilon^\mu(k_1\epsilon\epsilon^*)+\frac{1}{M^2}(\sigma^\mu k_2^\nu+\sigma^\nu k_2^\mu)\epsilon(k_1k_2\epsilon\,\epsilon^*)\right]\right.$$
$$+k_1\cdot k_2[\epsilon^\mu\epsilon^\nu(p_1p_2\epsilon^*)+\epsilon^\nu\epsilon^\mu(p_1p_2\epsilon^*)-\text{c.c.}]-\frac{k_1\cdot k_2}{M^2}[(\epsilon^\mu k_2^\nu+\epsilon^\nu k_2^\mu)\epsilon(p_1k_1k_2\epsilon^*)-\text{c.c.}]$$
$$\left.+k_1\cdot\Delta\,\epsilon\cdot\epsilon^*\epsilon^{\mu\nu}(k_1k_2)-k_1\cdot k_2[p_1\cdot\epsilon\,\epsilon^{\mu\nu}(k_2\epsilon^*)+\text{c.c.}]-\frac{k_1\cdot k_2}{k_1\cdot p_1 k_1\cdot p_2}p_1\cdot\epsilon p_1\cdot\epsilon^*\epsilon^{\mu\nu}(k_2\sigma)\right] , \tag{6b}$$

where the following notation is introduced:

$$I^{\mu\nu}=-g^{\mu\nu}+\frac{k_2^\mu k_2^\nu}{M^2} , \tag{7}$$

$$K^\mu=k_1^\mu-\frac{k_1\cdot k_2}{M^2}k_2^\mu , \tag{8}$$

$$\Delta^\mu=p_1^\mu-p_2^\mu , \tag{9}$$

$$\sigma^\mu=k_1\cdot\Delta\,k_1^\mu+k_1\cdot k_2\Delta^\mu , \tag{10}$$

$$\Sigma^\mu=\frac{p_1\cdot\epsilon}{2k_1\cdot p_1 k_1\cdot p_2}\sigma^\mu , \tag{11}$$

$$\epsilon^{\mu\nu}(ab)=\epsilon^{\mu\nu\lambda\tau}(a_\lambda b_\tau) , \tag{12a}$$

$$\epsilon^\mu(abc)=\epsilon^{\mu\nu\lambda\tau}(a_\nu b_\lambda c_\tau) , \tag{12b}$$

$$\epsilon(abcd)=\epsilon^{\mu\nu\lambda\tau}(a_\mu b_\nu c_\lambda d_\tau) , \tag{12c}$$

and $k_2^\mu=(\omega_2,\mathbf{k}_2)$, the momentum of Z^0, satisfies $k_2^2=M^2$.

Then the density matrix $\rho^{\mu\nu}$ of Z^0 can be obtained:[4]

$$\rho^{\mu\nu}=-\langle\epsilon^\mu\epsilon^{\nu *}\rangle/g_{\mu\nu}\langle\epsilon^\mu\epsilon^{\nu *}\rangle=\tfrac{1}{3}I^{\mu\nu}-\frac{i}{2M}\epsilon^{\mu\nu\lambda\tau}k_{2\lambda}\mathcal{P}_\tau-\tfrac{1}{2}Q^{\mu\nu} . \tag{13}$$

Explicitly one obtains the denominator of Eq. (13) from Eqs. (5) and (6) as

$$-g_{\mu\nu}\langle\epsilon_z^\mu\epsilon_z^{\nu *}\rangle=\frac{-e^2g^2A_0}{2m^2k_1\cdot p_1 k_1\cdot p_2} , \tag{14}$$

where a subscript 0 implies that incident particles are unpolarized and A_0 is defined as

$$A_0=(|\epsilon_R|^2+|\epsilon_L|^2)\left[\epsilon\cdot\epsilon^*[(k_1\cdot k_2)^2-2k_1\cdot p_1 k_1\cdot p_2]-\frac{M^2(k_1\cdot k_2)^2}{k_1\cdot p_1 k_1\cdot p_2}p_1\cdot\epsilon p_1\cdot\epsilon^*\right]+i(|\epsilon_R|^2-|\epsilon_L|^2)k_1\cdot k_2\epsilon(\Delta k_2\epsilon\epsilon^*) . \tag{15}$$

The polarization vector $\mathcal{P}_0^\mu$ and polarization tensor $Q_0^{\mu\nu}$ of Z^0 are given by

$$\mathcal{P}_0^\mu=-\frac{i}{M}\epsilon^{\mu\alpha\beta\tau}\rho_{\alpha\beta}k_{2\tau}=\frac{1}{2MA_0}[(|\epsilon_R|^2+|\epsilon_L|^2)R_1^\mu+(|\epsilon_R|^2-|\epsilon_L|^2)R_2^\mu] , \tag{16}$$

where R_1^μ and R_2^μ are defined as

$$R_1^{\mu}=i\left[[(k_1\cdot k_2)^2+(k_1\cdot\Delta)^2]\left\{\epsilon^{\mu}(k_2\epsilon\epsilon^*)+\frac{k_1\cdot k_2}{2k_1\cdot p_1 k_1\cdot p_2}[p_1\cdot\epsilon\,\epsilon^{\mu}(k_2\Delta\epsilon^*)-p_1\cdot\epsilon^*\epsilon^{\mu}(k_2\Delta\epsilon)]\right\}\right.$$
$$\left.+\frac{k_1\cdot\Delta}{2k_1\cdot p_1 k_1\cdot p_2}[(k_1\cdot k_2)^2+(k_1\cdot\Delta)^2+2M^2 k_1\cdot k_2][p_1\cdot\epsilon^*\epsilon^{\mu}(k_1k_2\epsilon)-p_1\cdot\epsilon\,\epsilon^{\mu}(k_1k_2\epsilon^*)]\right], \tag{17a}$$

$$R_2^{\mu}=-2M^2\left\{k_1\cdot\Delta\,\epsilon\cdot\epsilon^* K^{\mu}+k_1\cdot k_2(p_1\cdot\epsilon\,\epsilon^{\mu *}+p_1\cdot\epsilon^*\epsilon^{\mu})+\frac{k_1\cdot k_2}{k_1\cdot p_1 k_1\cdot p_2}p_1\cdot\epsilon p_1\cdot\epsilon^*(k_1\cdot\Delta\,k_1^{\mu}+k_1\cdot k_2\Delta^{\mu})\right\}, \tag{17b}$$

and

$$Q_0^{\mu\nu}=-\tfrac{1}{3}I^{\mu\nu}+\frac{1}{A_0}[(|\epsilon_R|^2+|\epsilon_L|^2)G_1^{\mu\nu}+(|\epsilon_R|^2-|\epsilon_L|^2)G_2^{\mu\nu}], \tag{18}$$

where $G_1^{\mu\nu}$ and $G_2^{\mu\nu}$ are defined as

$$G_1^{\mu\nu}=M^2\epsilon\cdot\epsilon^* K^{\mu}K^{\nu}-2k_1\cdot p_1 k_1\cdot p_2\{\epsilon\cdot\epsilon^* I^{\mu\nu}+[(\epsilon^{\mu}+\Sigma^{\mu})(\epsilon^{\nu}+\Sigma^{\nu})^*+\text{c.c.}]\}, \tag{19a}$$

$$G_2^{\mu\nu}=i\left[2k_1\cdot k_2\epsilon(p_1p_2\epsilon\epsilon^*)I^{\mu\nu}+\frac{1}{2}\left\{\sigma^{\mu}\epsilon^{\nu}(k_1\epsilon\epsilon^*)+\sigma^{\nu}\epsilon^{\mu}(k_1\epsilon\epsilon^*)+\frac{1}{M^2}(\sigma^{\mu}k_2^{\nu}+\sigma^{\nu}k_2^{\mu})\epsilon(k_1k_2\epsilon\epsilon^*)\right.\right.$$
$$\left.\left.+k_1\cdot k_2[\epsilon^{\mu}\epsilon^{\nu}(p_1p_2\epsilon^*)+\epsilon^{\nu}\epsilon^{\mu}(p_1p_2\epsilon^*)-\text{c.c.}]-\frac{1}{M^2}[(\epsilon^{\mu}k_2^{\nu}+\epsilon^{\nu}k_2^{\mu})\epsilon(p_1k_1k_2\epsilon^*)-\text{c.c.}]\right\}\right]. \tag{19b}$$

III. PHOTON POLARIZATION

When the Z^0 polarization is not measured in the process $e^+e^-\to Z^0\gamma$, only Eq. (14) needs to be considered. Our method[5] of density-matrix formalism for the photon is applied here to obtain the outgoing photon polarization. The photon polarization can be specified by Stokes parameters ξ_i $(i=1,2,3)$ which are contained in the photon density matrix in the helicity-state basis as

$$\rho_{\lambda\lambda'}=\tfrac{1}{2}\left[\delta_{\lambda\lambda'}+\frac{i}{2}\xi_1(\lambda-\lambda')+\tfrac{1}{2}\xi_2(\lambda+\lambda')+\tfrac{1}{2}\xi_3(\lambda\lambda'-1)\right]$$

or

$$\rho=\frac{1}{2}\begin{bmatrix}1+\xi_2 & -\xi_3+i\xi_1\\ -\xi_3-i\xi_1 & 1-\xi_2\end{bmatrix}. \tag{20}$$

The photon density matrix can be obtained explicitly from Eq. (14) after replacing $\epsilon^{\mu}\epsilon^{\nu *}$ in the equation by

$$\epsilon^i(\lambda)\epsilon^j(\lambda')^*=\frac{1}{2}\left[(\delta^{ij}-\hat{\mathbf{k}}_1^i\hat{\mathbf{k}}_1^j)\delta_{\lambda\lambda'}-\frac{i}{2}(\lambda+\lambda')\epsilon^{ijk}\hat{\mathbf{k}}_1^k-\frac{i}{2}(\lambda-\lambda')[\hat{a}^i(\hat{\mathbf{k}}_1\times\hat{\mathbf{a}})^j+\hat{\mathbf{a}}^j(\hat{\mathbf{k}}_1\times\hat{\mathbf{a}})^i]\right.$$
$$\left.+\tfrac{1}{2}(\lambda\lambda'-1)[\hat{a}^i\hat{a}^j-(\hat{\mathbf{k}}_1\times\hat{\mathbf{a}})^i(\hat{\mathbf{k}}_1\times\hat{\mathbf{a}})^j]\right]; \tag{21}$$

here $\hat{\mathbf{k}}_1$ is the unit vector along $\mathbf{k}_1$ and $\hat{\mathbf{a}}$ is a unit vector perpendicular to $\mathbf{k}_1$. In particular, if $\hat{\mathbf{a}}$ is chosen to be normal to the reaction plane of $e^+e^-\to Z^0\gamma$, one can obtain the following relations from Eq. (21):

$$\epsilon(\lambda')\cdot\epsilon(\lambda)^*=-\delta_{\lambda\lambda'}, \tag{22a}$$

$$p_1\cdot\epsilon(\lambda')p_1\cdot\epsilon(\lambda)^*=\tfrac{1}{2}(\mathbf{p}_1\times\hat{\mathbf{k}}_1)^2[\delta_{\lambda\lambda'}-\tfrac{1}{2}(\lambda\lambda'-1)], \tag{22b}$$

$$\epsilon(\Delta k_2\epsilon\epsilon^*)=i(\lambda+\lambda')\omega_2(\mathbf{p}_1\cdot\hat{\mathbf{k}}_1), \tag{22c}$$

and, therefore, Eq. (14) becomes

$$
\begin{aligned}
|M|^2(\lambda\lambda')_{Z^0 unp} &= -g_{\mu\nu}\langle \epsilon_z^\mu \epsilon_z^{\nu*}\rangle \\
&= \frac{e^2g^2(|\epsilon_R|^2+|\epsilon_L|^2)}{2m^2 k_1\cdot p_1 k_1\cdot p_2}\left[\delta_{\lambda\lambda'}\left[(k_1\cdot p_1)^2+(k_1\cdot p_2)^2+\frac{M^2(k_1\cdot k_2)^2}{2k_1\cdot p_1 k_1\cdot p_2}(\mathbf{p}_1\times\hat{\mathbf{k}}_1)^2\right]\right. \\
&\qquad -\tfrac{1}{2}(\lambda\lambda'-1)\frac{M^2(k_1\cdot k_2)^2}{2k_1\cdot p_1 k_1\cdot p_2}(\mathbf{p}_1\times\hat{\mathbf{k}}_1)^2 \\
&\qquad \left. +\tfrac{1}{2}(\lambda+\lambda')\frac{(|\epsilon_R|^2-|\epsilon_L|^2)}{(|\epsilon_R|^2+|\epsilon_L|^2)}\cdot 2\omega_2(k_1\cdot k_2)\mathbf{p}_1\cdot\hat{\mathbf{k}}_1\right] . \qquad (23)
\end{aligned}
$$

The first term of Eq. (23) gives the contribution to the reaction when the photon polarization is not measured. The differential cross section is then[6]

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{2s^2}(s-M^2)\left[\frac{2(s^2+M^4)}{\sin^2\theta(s-M^2)^2}-1\right]F(\theta_W) , \qquad (24)$$

where s is $(p_1+p_2)^2$, θ is the angle of the outgoing photon relative to the incident electron (Fig. 2), and $F(\theta_W)$ is defined as

$$F(\theta_W)=\frac{4g^2(|\epsilon_R|^2+|\epsilon_L|^2)}{e^2}=\frac{\frac{1}{4}-\sin^2\theta_W+2\sin^4\theta_W}{\sin^2\theta_W\cos^2\theta_W} . \qquad (25)$$

Then the total Born cross section becomes[7]

$$\sigma=\frac{2\pi\alpha^2}{s^2}F(\theta_W)\left[\frac{s^2+M^4}{s-M^2}\ln\left[\frac{s}{m^2}\right]-(s-M^2)\right] . \qquad (26)$$

The Stokes parameters which specify the polarization of the outgoing photon can be obtained from Eqs. (20) and (23):

$$\xi_1=0 , \qquad (27a)$$

$$\xi_2=\frac{|\epsilon_R|^2-|\epsilon_L|^2}{|\epsilon_R|^2+|\epsilon_L|^2}\left[\frac{2\cos\theta\left[1-\frac{M^4}{s^2}\right]}{\frac{4M^2}{s}+\left[1-\frac{M^2}{s}\right]^2(1+\cos^2\theta)}\right] , \qquad (27b)$$

$$\xi_3=-\frac{1}{1+\left[1-\frac{M^2}{s}\right]^2(1+\cos^2\theta)\frac{s}{4M^2}} . \qquad (27c)$$

The result of ξ_2 which characterizes the circular polarization of the outgoing photon beam has been obtained by Rekalo[8] in the reaction $e^+e^-\to\nu\bar{\nu}\gamma$. The linear polarization of the outgoing photon is independent of the electroweak

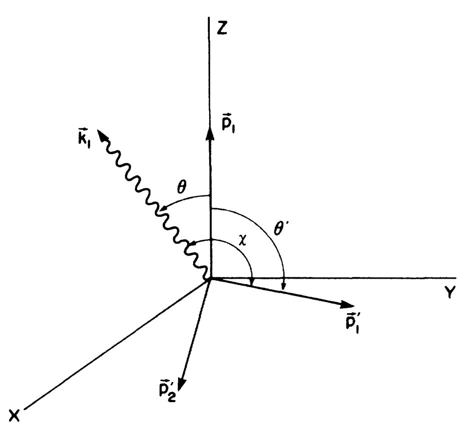

FIG. 2. Choice of coordinate axes in the c.m. frame of $e^-(p_1)$ and $e^+(p_2)$ to describe the reactions $e^+e^-\to Z^0(k_2)\gamma(k_1)$ and $Z^0\to f(p_1')\bar{f}(p_2')$.

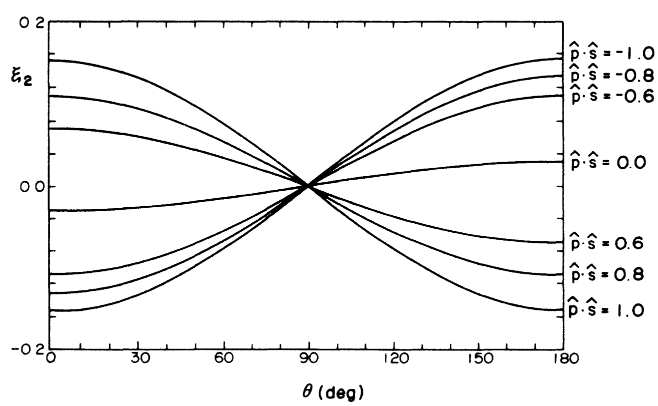

FIG. 3. The dependence of the degree of circular polarization (ξ_2) on θ for the cases of $\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1=-1.0$, -0.8, -0.6, 0, 0.6, 0.8, and 1.0.

couplings when the incident beams are unpolarized.

It is obvious that the polarization of the outgoing photon will be changed if the polarization of Z^0's are considered and the decay distribution of Z^0's depends on the polarization of the outgoing photon beam.

IV. EFFECTS OF INCIDENT PARTICLE POLARIZATION

Since experiments on the positron and polarized electron beams are planned at the SLC, it would be useful to consider the $e^+e^-\to Z^0\gamma$ reaction via polarized electron beams. At high energy, one can write the electron and positron projection operators as

$$u(p_1s_1)\bar{u}(p_1s_1)\simeq\frac{\not{p}_1}{4m}(1-\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1\gamma_5-\not{s}_{1T}\gamma_5)\ , \tag{28}$$

$$v(p_2s_2)\bar{v}(p_2s_2)\simeq\frac{\not{p}_2}{4m}(1+\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2\gamma_5-\not{s}_{2T}\gamma_5)\ , \tag{29}$$

where $\hat{\mathbf{s}}_1$ and $\hat{\mathbf{s}}_2$ are spin vectors of electron and positron in their rest frame, respectively.

If the linearly polarized incident beams are considered, the results given in Secs. II and III can be extended accordingly. The $\langle\epsilon_z^\mu\epsilon_z^{\nu*}\rangle$ for Z^0 now becomes

$$\langle\epsilon_z^\mu\epsilon_z^{\nu*}\rangle=\frac{e^2g^2}{4m^2k_1\cdot p_1k_1\cdot p_2}\{(1-\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2)[(|\epsilon_R|^2+|\epsilon_L|^2)B_1^{\mu\nu}+(|\epsilon_R|^2-|\epsilon_L|^2)B_2^{\mu\nu}]$$
$$+(\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1-\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2)[(|\epsilon_R|^2-|\epsilon_L|^2)B_1^{\mu\nu}+(|\epsilon_R|^2+|\epsilon_L|^2)B_2^{\mu\nu}]\}\ . \tag{30}$$

In particular, for the unpolarized Z^0 one obtains, instead of Eq. (14),

$$-g_{\mu\nu}\langle\epsilon_z^\mu\epsilon_z^{\nu*}\rangle=\frac{-e^2g^2\tilde{A}}{2m^2k_1\cdot p_1k_1\cdot p_2}\ , \tag{31}$$

where $\tilde{A}$ is defined as

$$\tilde{A}=(1-\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2)A_0+(\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1-\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2)A_1 \tag{32}$$

and A_1 is the same as A_0 except that $(|\epsilon_R|^2+|\epsilon_L|^2)$ and $(|\epsilon_R|^2-|\epsilon_L|^2)$ are interchanged in Eq. (15), i.e.,

$$A_1=(|\epsilon_R|^2-|\epsilon_L|^2)\left[\epsilon\cdot\epsilon^*[(k_1\cdot k_2)^2-2k_1\cdot p_1k_1\cdot p_2]-\frac{M^2(k_1\cdot k_2)^2}{k_1\cdot p_1k_1\cdot p_2}p_1\cdot\epsilon p_1\cdot\epsilon^*\right]+i(|\epsilon_R|^2+|\epsilon_L|^2)k_1\cdot k_2\epsilon(\Delta k_2\epsilon\epsilon^*). \tag{15'}$$

Also using Eqs. (21) and (22), one obtains

$$-g_{\mu\nu}\langle\epsilon_z^\mu\epsilon_z^{\nu*}\rangle=\frac{e^2g^2}{2m^2k_1\cdot p_1k_1\cdot p_2}\Bigg\{[(1-\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2)(|\epsilon_R|^2+|\epsilon_L|^2)+(\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1-\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2)(|\epsilon_R|^2-|\epsilon_L|^2)]$$
$$\times\left[\delta_{\lambda\lambda'}\left[(k_1\cdot p_1)^2+(k_1\cdot p_2)^2+\frac{M^2(k_1\cdot k_2)^2}{2k_1\cdot p_1k_1\cdot p_2}(\mathbf{p}_1\times\hat{\mathbf{k}}_1)^2\right]\right.$$
$$\left.-\tfrac{1}{2}(\lambda\lambda'-1)\frac{M^2(k_1\cdot k_2)^2}{2k_1\cdot p_1k_1\cdot p_2}(\mathbf{p}_1\times\hat{\mathbf{k}}_1)^2\right]$$
$$+[(1-\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2)(|\epsilon_R|^2-|\epsilon_L|^2)+(\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1-\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2)(|\epsilon_R|^2+|\epsilon_L|^2)]$$
$$\times\tfrac{1}{2}(\lambda+\lambda')\cdot2\omega_2(k_1\cdot k_2)\mathbf{p}_1\cdot\hat{\mathbf{k}}_1\Bigg\}\ . \tag{33}$$

The Stokes parameters of the outgoing photon beam in this case are

$$\xi_1=0\ , \tag{34a}$$

$$\xi_2=\frac{(1-\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2)(|\epsilon_R|^2-|\epsilon_L|^2)+(\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1-\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2)(|\epsilon_R|^2+|\epsilon_L|^2)}{(1-\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2)(|\epsilon_R|^2+|\epsilon_L|^2)+(\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1-\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2)(|\epsilon_R|^2-|\epsilon_L|^2)}\left[\frac{2\cos\theta\left[1-\frac{M^4}{s^2}\right]}{\frac{4M^2}{s}+\left[1-\frac{M^2}{s}\right]^2(1+\cos^2\theta)}\right], \tag{34b}$$

$$\xi_3=-\frac{1}{1+\left[1-\frac{M^2}{s}\right]^2(1+\cos^2\theta)\frac{s}{4M^2}}\ . \tag{34c}$$

Therefore, only the circular polarization is affected by the incident linear polarizations. When the positron beam is unpolarized and the electron beam is polarized, ξ_2 can be increased compared to the case that both are unpolarized as shown in Fig. 3. From Eqs. (30), (33), and (34), Eqs. (5), (23), and (27) can be recovered by setting $\hat{\mathbf{s}}_1=\hat{\mathbf{s}}_2=0$.

The polarization vector $\mathcal{P}^\mu$ and polarization tensor $Q^{\mu\nu}$ in this case become

$$\mathcal{P}^\mu=\frac{1}{2M\tilde{A}}\Big\{(1-\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2)[(|\epsilon_R|^2+|\epsilon_L|^2)R_1^\mu+(|\epsilon_R|^2-|\epsilon_L|^2)R_2^\mu]$$
$$+(\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1-\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2)[(|\epsilon_R|^2-|\epsilon_L|^2)R_1^\mu+(|\epsilon_R|^2+|\epsilon_L|^2)R_2^\mu]\Big\}, \tag{35}$$

$$Q^{\mu\nu}=-\tfrac{1}{3}I^{\mu\nu}+\frac{1}{\tilde{A}}\Big\{(1-\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2)[(|\epsilon_R|^2+|\epsilon_L|^2)G_1^{\mu\nu}+(|\epsilon_R|^2-|\epsilon_L|^2)G_2^{\mu\nu}]$$
$$+(\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1-\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2)[(|\epsilon_R|^2-|\epsilon_L|^2)G_1^{\mu\nu}+(|\epsilon_R|^2+|\epsilon_L|^2)G_2^{\mu\nu}]\Big\}\ . \tag{36}$$

V. DECAY DISTRIBUTION OF Z^0

Once Z^0's are produced, they will decay into various decay modes and one can calculate the decay distribution according to the models which specify the processes.

In particular, when Z^0 is produced by e^+e^- near its peak and it decays into two fermions $f\bar{f}$ where f can be ν, e, μ, τ, or quarks such as u, d, s, b (and also t if the toponium mass is smaller than the Z^0 mass), the transition amplitude for the process in the standard model of electroweak theory becomes

$$M=\frac{eg^2}{2}\bar{v}(p_2)\Big\{\frac{1}{k_1\cdot p_1}\gamma_\mu[\epsilon_L(1-\gamma_5)+\epsilon_R(1+\gamma_5)](\not{k}_1\not{\epsilon}^*-2p_1\cdot\epsilon^*)$$
$$+\frac{1}{k_1\cdot p_2}(2p_2\cdot\epsilon^*-\not{\epsilon}^*\not{k}_1)\gamma_\mu[\epsilon_L(1-\gamma_5)+\epsilon_R(1+\gamma_5)]\Big\}u(p_1)$$
$$\times\frac{\left[-g^{\mu\nu}+\frac{k_2^\mu k_2^\nu}{M^2}\right]}{k_2^2-M^2+iM\Gamma}\bar{u}(p_1')\gamma_\nu[\epsilon_L'(1-\gamma_5)+\epsilon_R'(1+\gamma_5)]v(p_2')\ , \tag{37}$$

where $p_1'=(E_1',\mathbf{p}_1')$ and $p_2'=(E_2',\mathbf{p}_2')$ are the momenta of two decay products f and $\bar{f}$, and ϵ_L' and ϵ_R' for the outgoing fermions are defined in Eqs. (3) and (4). The other amplitude contributing to this process is neglected near the Z^0 peak. If the unnormalized Proca vector ϵ_z^μ given in Eq. (1) is used, Eq. (37) can be written as

$$M=\frac{g\epsilon_z^\mu\bar{u}(p_1')\gamma_\mu[\epsilon_L'(1-\gamma_5)+\epsilon_R'(1+\gamma_5)]v(p_2')}{k_2^2-M^2+iM\Gamma}\ . \tag{38}$$

This amplitude can be used to obtain the differential cross section of the reaction as well as the decay process of Z^0. In the latter case, $k_2^2-M^2+iM\Gamma$ is omitted and ϵ_z^μ is replaced by the normalized one, ϵ^μ i.e.,

$$M_{\text{decay}}=g\epsilon^\mu\bar{u}(p_1')\gamma_\mu[\epsilon_L'(1-\gamma_5)+\epsilon_R'(1+\gamma_5)]v(p_2')\ . \tag{39}$$

If the decay products are unpolarized, the absolute square becomes[4]

$$\sum_{f\bar{f}} |M|^2_{\text{decay}} = \frac{2g^2}{3m_f^2}[(|\epsilon_R'|^2+|\epsilon_L'|^2)(M^2-m_f^2)+6m_f^2\epsilon_R'\epsilon_L' + 3(|\epsilon_R'|^2-|\epsilon_L'|^2)Mp_1'\cdot\mathcal{P}+3(|\epsilon_R'|^2+|\epsilon_L'|^2)Q^{\mu\nu}p_{1\mu}'p_{1\nu}'] , \tag{40}$$

where m_f is the mass of decay products and $\mathcal{P}^\mu$ and $Q^{\mu\nu}$ are determined by the density matrix of Z^0 given by Eq. (13) when it is produced is used. Equations (39) and (40) are used in Ref. 4 (there has been a typographical error in sign in the last term of the above equation), but now $\mathcal{P}^\mu$ and $Q^{\mu\nu}$ are different from those in Ref. 4 since we are considering a different process.

The absolute square of the transition amplitude for the whole production and subsequent decay processes corresponding to Eq. (37) or (38) becomes

$$\sum_{f\bar{f}} |M|^2 = \frac{1}{(k_2^2-M^2)^2+M^2\Gamma^2}(-g_{\mu\nu}\langle\epsilon_z^\mu\epsilon_z^{\nu*}\rangle)\sum_{f\bar{f}}|M|^2_{\text{decay}} , \tag{41}$$

where $-g_{\mu\nu}\langle\epsilon_z^\mu\epsilon_z^{\nu*}\rangle$ is given by Eq. (31) for linearly polarized incident beams and by Eq. (14) for unpolarized incident beams.

The decay distribution of one of the outgoing fermions can be obtained (in the coordinate system given in Fig. 2) as

$$d\Gamma = \frac{m_f^2}{2(2\pi)^2}\frac{p_1'^2}{\omega_2}\frac{\sum|M|^2_{\text{decay}}}{p_1'\omega_2+\omega_1E_1'\cos\chi}d\Omega_{p_1'} , \tag{42}$$

where χ is the angle between the outgoing fermion and photon directions, with the relation

$$M^2-2E_1'\omega_2-2\omega_1p_1'\cos\chi=0 . \tag{43}$$

In reality the Z^0's can be considered rather as the products of the e^+e^- collision than the normalizable initial state in the decay process since the Z^0 lifetime is expected to be small. So we consider Eq. (41) of the whole process and obtain some results from it. Using Eq. (21), Eq. (41) becomes, in the limit $M\Gamma \ll s$,

$$\sum_{f\bar{f}} |M|^2 = \delta(\omega_1-\omega_0)\frac{-\pi e^2g^4}{2M\Gamma E_1^3\omega_1^2m^2m_f^2\sin^2\theta}\left[\zeta_0\delta_{\lambda\lambda'}+\frac{i}{2}\zeta_1(\lambda-\lambda')+\tfrac{1}{2}\zeta_2(\lambda+\lambda')+\tfrac{1}{2}\zeta_3(\lambda\lambda'-1)\right] , \tag{44}$$

where $\omega_0=E_1-M^2/4E_1$ and ζ's are defined as follows:

$$\zeta_0=m_f^2\epsilon_R'\epsilon_L'F_{01}\lambda_1+(|\epsilon_R'|^2-|\epsilon_L'|^2)F_{02}\lambda_2 + (|\epsilon_R'|^2+|\epsilon_L'|^2)F_{03}\lambda_1 , \tag{45a}$$

$$\zeta_1=(|\epsilon_R'|^2-|\epsilon_L'|^2)F_{12}\lambda_2 + (|\epsilon_R'|^2+|\epsilon_L'|^2)F_{13}\lambda_1 , \tag{45b}$$

$$\zeta_2=m_f^2\epsilon_R'\epsilon_L'F_{21}\lambda_2+(|\epsilon_R'|^2-|\epsilon_L'|^2)F_{22}\lambda_1 + (|\epsilon_R'|^2+|\epsilon_L'|^2)F_{23}\lambda_2 , \tag{45c}$$

$$\zeta_3=m_f^2\epsilon_R'\epsilon_L'F_{31}\lambda_1+(|\epsilon_R'|^2-|\epsilon_L'|^2)F_{32}\lambda_2 + (|\epsilon_R'|^2+|\epsilon_L'|^2)F_{33}\lambda_1 . \tag{45d}$$

Here λ's and F's are defined as

$$\lambda_1=(|\epsilon_R|^2+|\epsilon_L|^2)(1-\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2) + (|\epsilon_R|^2-|\epsilon_L|^2)(\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1-\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2) , \tag{46a}$$

$$\lambda_2=(|\epsilon_R|^2-|\epsilon_L|^2)(1-\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2) - (|\epsilon_R|^2+|\epsilon_L|^2)(\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1-\hat{\mathbf{p}}_2\cdot\hat{\mathbf{s}}_2) , \tag{46b}$$

and

$$F_{01}=-2E_1^2(\omega_1^2\cos^2\theta+\omega_1^2+M^2) , \tag{47a}$$

$$F_{02}=-M^2E_1^2(\omega_1E_2'\cos\theta+\omega_2p_1'\cos\theta') , \tag{47b}$$

$$F_{03}=-\tfrac{1}{2}M^2E_1^2[\omega_1^2\cos^2\theta+2p_1'\cos\theta'(\omega_1\cos\theta+p_1'\cos\theta') + E_1'^2+E_2'^2] , \tag{47c}$$

$$F_{12}=M^2E_1^2\omega_1p_1'S , \tag{47d}$$

$$F_{13}=2E_1^2\omega_1p_1'S(\omega_1E_1'\cos\theta+p_1'\omega_2\cos\theta') , \tag{47e}$$

$$F_{21}=-4E_1^2\omega_1\omega_2\cos\theta , \tag{47f}$$

$$F_{22}=-\tfrac{1}{2}M^2E_1\omega_1[M^2-2\omega_2E_1'+\omega_1\cos\theta\cos\theta' + \omega_1^2(1+\cos^2\theta)] , \tag{47g}$$

$$F_{23}=-E_1^2M^2[\omega_1E_2'\cos\theta+p_1'\cos\theta'(E_2'-E_1')] , \tag{47h}$$

$$F_{31}=2E_1^2M^2 , \tag{47i}$$

$$F_{32}=M^2E_1^2(\omega_1E_1'\cos\theta+p_1'\omega_2\cos\theta') , \tag{47j}$$

$$F_{33}=E_1^2[\omega_1^2(E_1'^2\cos^2\theta-p_1'^2\sin^2\theta)+p_1'^2\cos^2\theta'(2\omega_1^2+M^2) + \omega_1p_1'\cos\theta(4\omega_2E_1'-M^2) + (\tfrac{1}{2}M^2-E_1'\omega_2)^2+\tfrac{1}{4}M^4] , \tag{47k}$$

where

$$S=\sin\theta\sin\theta'\sin(\phi'-\phi) . \tag{47l}$$

From Eq. (44) we can see immediately that the Stokes parameters of the outgoing photon beam are

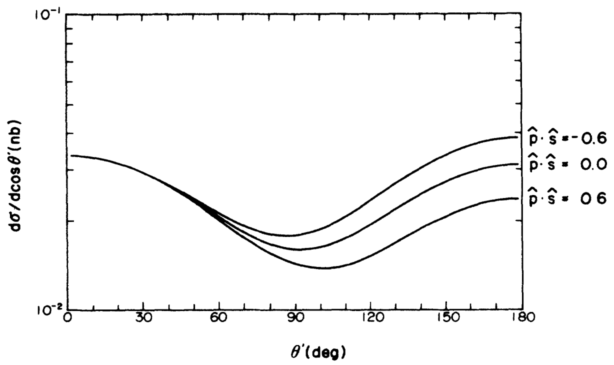

FIG. 4. The differential cross section with respect to the angle θ' for the reaction of $e^+e^-\rightarrow Z^0\gamma\rightarrow\tau\bar{\tau}\gamma$ when the incident electron beam is polarized, such that $\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1=-0.6$, 0, and 0.6, and the polarization as well as the direction of the outgoing photon is not detected.

$$\xi_1=\frac{\zeta_1}{\zeta_0},\quad \xi_2=\frac{\zeta_2}{\zeta_0},\quad \xi_3=\frac{\zeta_3}{\zeta_0}\ . \tag{48}$$

Now these depend on the decay distribution of one fermion of the Z^0 decay products.

When the polarization of outgoing photon beam is detected as specified by Stokes parameters ξ_i^f $(i=1,2,3)$, the differential cross section is obtained by multiplying the matrix form of Eq. (20) by

$$\rho^f=\frac{1}{2}\begin{bmatrix} 1+\xi_2^f & -\xi_3^f+i\xi_1^f \\ -\xi_3^f-i\xi_1^f & 1-\xi_2^f \end{bmatrix} \tag{49}$$

and then taking the trace of it. The result is

$$d\sigma=\frac{1}{2(2\pi)^2}\frac{-\alpha g^4}{E_1^5\omega_1 M\Gamma}\frac{(p_1')^2}{\sin^2\theta}\times\frac{\zeta_0+\zeta_1\xi_1^f+\zeta_2\xi_2^f+\zeta_3\xi_3^f}{p_1'\omega_2+\omega_1 E_1'\cos\chi}d\Omega_{k_1}d\Omega_{p_1'}\ , \tag{50}$$

where ω_1 and ω_2 have the values

$$\omega_1=E_1-\frac{M^2}{4E_1}\ , \tag{51a}$$

$$\omega_2=2E_1-\omega_1=E_1+\frac{M^2}{4E_1}\ , \tag{51b}$$

and Eq. (43) also holds for χ. An angular distribution with respect to the direction of the outgoing τ lepton is shown in Fig. 4 for the simple cases when the polarization as well as the direction of the outgoing photon is not detected and the incoming electron beam is linearly polarized. For this case the forward-backward asymmetry

$$A_{\mathrm{FB}}=\frac{\int_{\cos\theta'=0}^{\cos\theta'=1}d\sigma-\int_{\cos\theta'=-1}^{\cos\theta'=0}d\sigma}{\int_{\cos\theta'=0}^{\cos\theta'=1}d\sigma+\int_{\cos\theta'=-1}^{\cos\theta'=0}d\sigma} \tag{52}$$

becomes -0.052, 0.027, and 0.12 for $\hat{\mathbf{p}}_1\cdot\hat{\mathbf{s}}_1=-0.6$, 0, and 0.6, respectively.

Finally, the effects of transversely polarized incident beam can be considered similarly by using Eqs. (28) and (29). This will be considered further.

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