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Evaluation of the conformal anomaly of $N=1$ superstring theory by the stochastic quantization method

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A generalized Langevin equation for the Majorana fermion field is first derived within the framework of stochastic quantization. Based on it, the conformal anomaly of $N=1$ superstring theory is calculated. As a result, the standard dimension, $D=10$, is correctly reobtained.

Stochastic quantization[1,2] is a relatively new method for the quantization of field theories. Since the original paper by Parisi and Wu,[1] a large number of new results, generalized schemes, and new applications have been found.[2] It is noticeable that some of this progress now sheds some insight into the quantum origin of the anomalies.[3,4] On the contrary the anomaly calculation has been a useful tool to test the newly generalized schemes of stochastic quantization method.[5–10] With respect to the fermion field theory the generalized Langevin equation for the Dirac fermion field[11] is confirmed by the evaluation of the chiral anomaly,[7] parity-violating anomaly,[8] and conformal anomaly.[9] The generalized Langevin equation for the Weyl fermion field is confirmed[10] by the evaluation of the anomaly in the chiral Schwinger model. In this paper, we first show how the Langevin equations must be handled when Majorana fermions are present and illustrate our proposal by evaluating the conformal anomaly in $N=1$ superstring theory. Then, we reobtain the standard dimension concerning its consistency at the quantum level, within the stochastic quantization method.

First, let us find the Langevin equation for the Majorana fermion field in $d=2$ Euclidean space. It is well known that for $d=2$ there exists an antisymmetric matrix C such that the Majorana condition can be satisfied:

$$C^{-1}\gamma^m C=-\gamma^m,\quad {}^tC=-C\ , \tag{1}$$

and the operator $C(\gamma^m\partial_m+m)$ is antisymmetric where ${}^tC$ is the transpose of C. Thus, we can construct the action for a massless Majorana fermion

$$S=\int dx\,\tfrac{1}{2}\,{}^t\psi C(\gamma^m\partial_m)\psi=\int dx\,\tfrac{1}{2}\bar{\psi}(\gamma^m\partial_m)\psi\ . \tag{2}$$

Here, the ψ's are two-component, complex, anticommuting variables and the choice

$$\gamma^1=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\quad \gamma^2=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix},$$
$$\gamma^5=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},\quad C=\begin{bmatrix}0 & -1\\ +1 & 0\end{bmatrix}, \tag{3}$$

satisfies the conditions in (1). Therefore, each component of the Majorana fermion field is simply decoupled and the action becomes

$$S=S_{\psi_1}+S_{\psi_2}\ , \tag{4}$$

where

$$S_{\psi_1}=\int dx\,\tfrac{1}{2}\psi_1(-\partial_1+i\partial_2)\psi_1=\int dx\,\tfrac{1}{2}\psi_1 D(x)\psi_1 \tag{5}$$

and

$$S_{\psi_2}=\int dx\,\tfrac{1}{2}\psi_2(\partial_1+i\partial_2)\psi_2=\int dx\,\tfrac{1}{2}\psi_2 D^\dagger(x)\psi_2\ . \tag{6}$$

The generalized Langevin equation for an arbitrary field $\phi(x)$ is given by

$$\frac{\partial}{\partial\tau}\phi(x,\tau)=-\int dy\,K(x,y)\frac{\delta S_\phi}{\delta\phi(y)}+\xi(x,\tau)\ , \tag{7}$$

where $K(x,y)$ is a heat kernel chosen in such a way to precisely cancel negative eigenvalues from $\delta S_\phi/\delta\phi(y)$ and $\xi(x,\tau)$ is a stochastic noise field which gives the following expectation value (see Ref. 2 for details):

$$\langle\xi(x,\tau)\xi(x',\tau')\rangle_\xi=2K(x,x')a_\Lambda(\tau-\tau')\ .$$

Now, $a_\Lambda(\tau-\tau')$ is a regulator function introduced by Breit, Gupta, and Zaks.[12] Noting that

$$\frac{\delta S_{\psi_1}}{\delta\psi(y)}=D(y)\psi_1(y)\quad\text{and}\quad\frac{\delta S_{\psi_2}}{\delta\psi_2(y)}=D^\dagger(y)\psi_2(y)$$

and assuming that $DD^\dagger$ and $D^\dagger D$ always have finite positive eigenvalues,[5,7] we choose $K(x,y)$ such that $K(x,y)=-D^\dagger(y)\delta(x-y)$ for ψ_1 and $K(x,y)=-D(y)\delta(x-y)$ for ψ_2. Then, the Langevin equations become

$$\frac{\partial}{\partial t}\psi_1(x,\tau)=-D^\dagger(x)D(x)\psi_1(x,\tau)+\eta_1(x,\tau) \tag{8}$$

and

$$\frac{\partial}{\partial\tau}\psi_2(x,\tau)=-D(x)D^\dagger(x)\psi_2(x,\tau)+\eta_2(x,\tau)\ . \tag{9}$$

Next, using the above set of Langevin equations for the Majorana fermion field we will calculate the conformal anomaly of $N=1$ superstring theory. Since the spacetime is effectively flat in Fujikawa's string path-integral formalism,[13] it is useful to express the conformal anomaly of $N=1$ superstring theory[14] in Fujikawa's string path-integral formalism so as to apply the stochastic quantization method. Using the results of Bouwknegt and van Nieuwenhuizen[14] and neglecting the nonpropagating fields and regarding the conformal gauge field $\rho(x)$ as the background field, the involved partition function of $N=1$ superstring theory becomes (our conventions are those of Ref. 14)

$$Z=\int \mathcal{D}\tilde{X}^a(x)\mathcal{D}\xi(x)\mathcal{D}\tilde{\eta}(x)\mathcal{D}\tilde{\lambda}^a(x)\mathcal{D}\tilde{\bar{C}}(x)\mathcal{D}\tilde{C}(x) \times\exp\left[\int dx[-\tfrac{1}{2}\partial_\mu(\tilde{X}^a\rho^{-1/2})\partial^\mu(\tilde{X}^a\rho^{-1/2})+\xi\rho^{1/2}\not\partial(\rho^{-1}\tilde{\eta})-\tfrac{1}{2}(\tilde{\bar{\lambda}}^a\rho^{-1/4})\not\partial(\tilde{\lambda}^a\rho^{-1/4})+\tilde{\bar{C}}\rho^{1/4}\not\partial(\rho^{-3/4}\tilde{C})]\right] =\ln W[\rho]\ . \tag{10}$$

Now,

$$\begin{aligned}&\tilde{X}^a(x)=\rho^{1/2}(x)X^a(x),\quad \tilde{\eta}(x)=\rho(x)\eta(x)\ ,\\ &\tilde{\lambda}^a(x)=\rho^{1/2}(x)\lambda^a(x),\quad \tilde{\bar{C}}(x)=\bar{C}\ (x),\\ &\tilde{C}(x)=\rho^{1/2}(x)C(x)\end{aligned} \tag{11}$$

and $X^a(x)$ and $\lambda^a(x)$ form the D scalar multiplets (x^a,λ^a,F^a), $a=1,2,\ldots,D$, and $\eta(x)$ and $\xi(x)$ are general coordinate ghosts and antighosts and $C(x)$ and $\bar{C}(x)$ and local supersymmetry ghosts and antighosts, respectively. λ^a is now a Majorana spinor in $d=2$ and $\bar{\lambda}^a={}^t(\lambda^a)C$ and $\not\partial=\gamma^m\partial_m$. The conformal anomaly is defined for the variations

$$\begin{aligned}&\tilde{X}^a(x)\to e^{\alpha(x)/2}\tilde{X}^a(x),\quad \xi(x)\to e^{-\alpha(x)/2}\xi(x)\ ,\\ &\tilde{\eta}(x)\to e^{\alpha(x)}\tilde{\eta}(x),\quad \tilde{\lambda}^a(x)\to e^{\alpha(x)/4}\tilde{\lambda}^a(x)\ ,\\ &\tilde{\bar{C}}(x)\to e^{-\alpha(x)/4}\tilde{\bar{C}}(x),\quad \tilde{C}(x)\to e^{3\alpha(x)/4}\tilde{C}(x)\ ,\\ &\rho(x)\to e^{\alpha(x)}\rho(x)\ .\end{aligned} \tag{12}$$

The action in (10) is invariant under (12). Therefore, if the integration measure of (10) is also invariant under (12), we should obtain

$$W[e^\alpha\rho]=W[\rho]\ . \tag{13}$$

The existence of a conformal anomaly means that this relation is not valid and the functional $W[\rho(x)]$ is not conformally invariant. Therefore, we intend to calculate a variation of W by the infinitesimal conformal transformation

$$\delta W=\int dx_1\alpha(x_1)\rho(x_1)\frac{\delta W[\rho]}{\delta[\alpha(x_1)\rho(x_1)]} =\delta W_1+\delta W_2+\delta W_3+\delta W_4\ , \tag{14}$$

where

$$\delta W_1=\frac{\int\mathcal{D}\tilde{X}^a(x)\int dx\left[-\tfrac{1}{2}\partial_\mu\partial_\mu\left[\frac{\tilde{X}^a}{\sqrt{\rho}}\right]\frac{\tilde{X}^a}{\sqrt{\rho}}\right]\exp\left\{\int dx\left[-\tfrac{1}{2}\partial_\mu\left[\frac{\tilde{X}^a}{\sqrt{\rho}}\right]\partial_\mu\left[\frac{\tilde{X}^a}{\sqrt{\rho}}\right]\right]\right\}}{\int\mathcal{D}\tilde{X}^a(x)\exp\left\{\int dx\left[-\tfrac{1}{2}\partial_\mu\left[\frac{\tilde{X}^a}{\sqrt{\rho}}\right]\partial_\mu\left[\frac{\tilde{X}^a}{\sqrt{\rho}}\right]\right]\right\}}\ , \tag{15}$$

$$\delta W_2=\frac{\int\mathcal{D}\xi(x)\mathcal{D}\tilde{\eta}(x)\int dx\left[\tfrac{1}{2}\xi\left[\sqrt{\rho}\not\partial\frac{1}{\rho}\right]+\xi\sqrt{\rho}\overleftarrow{\not\partial}\frac{1}{\rho}\tilde{\eta}\right]\exp\left[\int dx\left[\xi\sqrt{\rho}\not\partial\frac{1}{\rho}\tilde{\eta}\right]\right]}{\int\mathcal{D}\xi(x)\mathcal{D}\tilde{\eta}(x)\exp\left[\int dx\left[\xi\sqrt{\rho}\not\partial\frac{1}{\rho}\tilde{\eta}\right]\right]}\ , \tag{16}$$

$$\delta W_3=\frac{\int\mathcal{D}\tilde{\lambda}^a(x)\int dx[\tfrac{1}{8}\tilde{\bar{\lambda}}^a\rho^{-1/4}\not\partial(\tilde{\lambda}^a\rho^{-1/4})-\tfrac{1}{8}\tilde{\bar{\lambda}}^a\rho^{-1/4}\overleftarrow{\not\partial}\rho^{-1/4}\tilde{\lambda}^a]\exp\left[\int dx[-\tfrac{1}{2}\tilde{\bar{\lambda}}^a\rho^{-1/4}\not\partial(\rho^{-1/4}\tilde{\lambda}^a)]\right]}{\int\mathcal{D}\tilde{\lambda}^a(x)\exp\left[\int dx[-\tfrac{1}{2}\tilde{\bar{\lambda}}^a\rho^{-1/4}\not\partial(\rho^{-1/4}\tilde{\lambda}^a)]\right]}\ , \tag{17}$$

and

$$W_4=\frac{\int\mathcal{D}\tilde{\bar{C}}(x)\mathcal{D}\tilde{C}(x)\int dx[\tfrac{1}{4}\tilde{\bar{C}}\rho^{1/4}\not\partial(\rho^{-3/4}\tilde{C})+\tfrac{3}{4}\tilde{\bar{C}}\rho^{1/4}\overleftarrow{\not\partial}\rho^{-3/4}\tilde{C}]\exp\left[\int dx(\tilde{\bar{C}}\rho^{1/4}\not\partial\rho^{-3/4}\tilde{C})\right]}{\int\mathcal{D}\tilde{\bar{C}}(x)\mathcal{D}\tilde{C}(x)\exp\left[\int dx(\tilde{\bar{C}}\rho^{1/4}\not\partial\rho^{-3/4}\tilde{C})\right]}\ . \tag{18}$$

δW_1 and δW_2 were calculated in our previous paper.[9] δW_3 will be calculated using the generalized Langevin equations (8) and (9) for the Majorana fermion fields and the stochastic quantization method similar to that for the Fermi fields[11] will be used for δW_4. In these calculations, we also use the ultraviolet regularization scheme introduced by Breit, Gupta, and Zaks[12] and the infrared regularization scheme[5,7] which assume finite, nonzero eigenvalues for the involved operators.

We calculate δW_3 first. Taking into account each component of the Majorana fermion fields separately, δW_3 becomes

$$\delta W_3=\frac{D}{4}[\delta W_3(\tilde{\lambda}_1)+\delta W_3(\tilde{\lambda}_2)] \ , \tag{19}$$

where

$$\delta W_3(\tilde{\lambda}_1)=\frac{\int \mathcal{D}\tilde{\lambda}_1 \int dx\, \tilde{\lambda}_1[\rho^{-1/4}(-\partial_1+i\partial_2)\rho^{-1/4}]\tilde{\lambda}_1 \exp\left[\int dx(-\frac{1}{2})\tilde{\lambda}_1[\rho^{-1/4}(-\partial_1+i\partial_2)\rho^{-1/4}]\tilde{\lambda}_1\right]}{\int \mathcal{D}\tilde{\lambda}_1 \exp\left[\int dx(-\frac{1}{2})\tilde{\lambda}_1[\rho^{-1/4}(-\partial_1+i\partial_2)\rho^{-1/4}]\tilde{\lambda}_1\right]} \tag{20}$$

and

$$\delta W_3(\tilde{\lambda}_2)\frac{\int \mathcal{D}\tilde{\lambda}_2 \int dx\, \tilde{\lambda}_2[\rho^{-1/4}(\partial_1+i\partial_2)\rho^{-1/4}]\tilde{\lambda}_2 \exp\left[\int dx(-\frac{1}{2})\tilde{\lambda}_2[\rho^{-1/4}(\partial_1+i\partial_2)\rho^{-1/4}]\tilde{\lambda}_2\right]}{\int \mathcal{D}\tilde{\lambda}_2 \exp\left[\int dx(-\frac{1}{2})\tilde{\lambda}_2[\rho^{-1/4}(\partial_1+i\partial_2)\rho^{-1/4}]\tilde{\lambda}_2\right]} \ . \tag{21}$$

Our actions for $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ are

$$\begin{aligned} S_{\tilde{\lambda}_1}&=\int dx\, \tfrac{1}{2}\tilde{\lambda}_1[\rho^{-1/4}(-\partial_1+i\partial_2)\rho^{-1/4}]\tilde{\lambda}_1 \\ &=\int dx\, \tfrac{1}{2}\tilde{\lambda}_1 D_\rho(x)\tilde{\lambda}_1 \end{aligned} \tag{22}$$

and

$$\begin{aligned} S_{\tilde{\lambda}_2}&=\int dx \tfrac{1}{2}\tilde{\lambda}_2[\rho^{-1/4}(\partial_1+i\partial_2)\rho^{-1/4}]\tilde{\lambda}_2 \\ &=\int dx\, \tfrac{1}{2}\tilde{\lambda}_2 D_\rho^\dagger(x)\tilde{\lambda}_2 \ . \end{aligned} \tag{23}$$

Using Eqs. (8) and (9), the Langevin equations read

$$\frac{\partial}{\partial\tau}\tilde{\lambda}_1(x,\tau)=-D_\rho^\dagger(x)D_\rho(x)\tilde{\lambda}_1(x,\tau)+\eta_1(x,\tau) \tag{24}$$

and

$$\frac{\partial}{\partial\tau}\tilde{\lambda}_2(x,\tau)=-D_\rho(x)D_\rho^\dagger(x)\tilde{\lambda}_2(x,\tau)+\eta_2(x,\tau) \ . \tag{25}$$

According to the stochastic quantization prescription,

$$\begin{aligned}\delta W_3=\frac{D}{4}\Bigg[&\lim_{\tau\to\infty}\left\langle\int dx\, \tilde{\lambda}_1(x,\tau)D_\rho(x)\tilde{\lambda}_1(x,\tau)\right\rangle_{\eta_1} \\ &+\lim_{\tau\to\infty}\left\langle\int dx\, \tilde{\lambda}_2(x,\tau)D_\rho^\dagger(x)\tilde{\lambda}_2(x\tau)\right\rangle_{\eta_2}\Bigg] \ . \end{aligned} \tag{26}$$

Substituting the solutions of (24) and (25) into (26) and following the same calculation procedure used in our previous paper,[9] δW_3 becomes

$$\begin{aligned}\delta W_3=\frac{D}{4}\lim_{\Lambda\to\infty}\int dx\, \mathrm{tr}\int \frac{d^2k}{(2\pi)^2}e^{-ikx} \\ \times e^{\rho^{-1/4}\not\partial\rho^{-1/2}\not\partial\rho^{-1/4}(1/\Lambda^2)}e^{ikx} \ . \end{aligned} \tag{27}$$

Recalling the well-known formula[13]

$$\begin{aligned}\lim_{\Lambda\to\infty}\mathrm{tr}\int \frac{d^2k}{(2\pi)^2}e^{-ikx}e^{-H/\Lambda^2}e^{ikx} \\ =\lim_{\Lambda\to\infty}2\left[\frac{3n+1}{24\pi}(-\partial_\mu\partial_\mu \ln\rho)+\frac{\rho}{4\pi}\Lambda^2\right] \ , \end{aligned} \tag{28}$$

where tr denotes the trace in spinor space and

$$H=-\rho^{-(n+1)/2}\not\partial\rho^n\not\partial\rho^{-(n+1)/2} \tag{29}$$

we obtain, with $n=-\frac{1}{2}$,

$$\delta W_3=D\lim_{\Lambda\to\infty}\int dx\left[\frac{-\frac{1}{4}}{24\pi}\partial_\mu\partial_\mu \ln\rho-\frac{\rho}{8\pi}\Lambda^2\right] \ . \tag{30}$$

Next, we calculate δW_4. From the form of δW_4, our action for $\bar{\tilde{C}}(x)$ and $\tilde{C}(x)$ reads

$$S=\int dx\, \bar{\tilde{C}}\not{D}\tilde{C} \ , \tag{31}$$

where

$$\not{D}=-\rho^{1/4}\not\partial\rho^{-3/4} \ . \tag{32}$$

The Langevin equations are

$$\begin{aligned}\frac{\partial}{\partial\tau}\bar{\tilde{C}}(x,\tau)&=-\bar{\tilde{C}}(x,\tau)\overleftarrow{\not{D}}\,\overleftarrow{\not{D}}^\dagger+\bar{\delta}(x,\tau) \ , \\ \frac{\partial}{\partial\tau}\tilde{C}(x,\tau)&=-\not{D}^\dagger\not{D}\tilde{C}(x,\tau)+\not{D}^\dagger\delta(x,\tau) \ . \end{aligned} \tag{33}$$

Since $\bar{\tilde{C}}(x)$ and $\tilde{C}(x)$ are bosonic supersymmetry antighost and ghost fields, stochastic noise fields $\bar{\delta}(x,\tau)$ and $\delta(x,\tau)$ give the following expectation values:

$$\begin{aligned}\langle\delta_\alpha(x,\tau)\bar{\delta}_\beta(x',\tau')\rangle_\delta&=\langle\bar{\delta}_\beta(x',\tau')\delta_\alpha(x,\tau)\rangle_\delta \\ &=2\delta_{\alpha\beta}\delta(x-x')a_\Lambda(\tau-\tau') \ . \end{aligned} \tag{34}$$

According to the stochastic quantization prescription,

$$\begin{aligned}\delta W_4=\lim_{\tau\to\infty}\Big\langle\int dx[\tfrac{1}{4}\bar{\tilde{C}}(x,\tau)\rho^{1/4}\not\partial\rho^{-3/4}\tilde{C}(x,\tau) \\ +\tfrac{3}{4}\bar{\tilde{C}}(x,\tau)\rho^{1/4}\overleftarrow{\not\partial}\rho^{-3/4}\tilde{C}(x,\tau)]\Big\rangle_\delta \ . \end{aligned} \tag{35}$$

Substituting the solutions of (33) to (35) and also following the same calculation procedure of Ref. 9, δW_4 becomes

$$\delta W_4 = \lim_{\Lambda\to\infty} \int dx \int \frac{d^2k}{(2\pi)^2} e^{-ikx}\,\mathrm{tr}\left(-\tfrac{1}{4} e^{\rho^{1/4}\bar{\partial}\rho^{-3/2}\bar{\partial}\rho^{1/4}(1/\Lambda^2)} + \tfrac{3}{4} e^{\rho^{-3/4}\bar{\partial}\rho^{1/2}\bar{\partial}\rho^{-3/4}(1/\Lambda^2)}\right) e^{ikx} . \tag{36}$$

Using Eq. (28) with $n=-\frac{3}{2}$ and $n=\frac{1}{2}$, we obtain

$$\delta W_4 = \lim_{\Lambda\to\infty} \int dx \left[\frac{-\frac{7}{4}-\frac{15}{4}}{24\pi} (\partial_\mu \partial_\mu \ln\rho) + \frac{\rho}{4\pi}\Lambda^2 \right] . \tag{37}$$

Combining (30) and (37) and the results of our previous paper,[9] we finally obtain

$$\begin{aligned}\delta W &= \delta W_1 + \delta W_2 + \delta W_3 + \delta W_4 \\ &= \lim_{\Lambda\to\infty} \int dx \left[\frac{-\frac{3}{4}D + \frac{30}{4}}{24\pi} \partial_\mu \partial_\mu \ln\rho \right] .\end{aligned} \tag{38}$$

There, for $D=10$ the anomaly contributions cancel.

In conclusion, we first derived the generalized Langevin equation for Majorana fermion field and using this Langevin equation we calculate the conformal anomaly of $N=1$ superstring theory. As a result, we correctly reobtain the standard dimension confirming its consistency. Although the Majorana fermion field is a Fermi field and the action is in bilinear form, we do not use the results of Sakita[11] but choose the new appropriate kernel, since $\tilde{\lambda}(x)$ and $\tilde{\tilde{\lambda}}(x)$ are not independent fields. Since the action of $\tilde{\tilde{C}}$ and $\tilde{C}$ is in bilinear form in $\tilde{\tilde{C}}$ and $\tilde{C}$, we can use the kernel that is used for the Fermi fields by Sakita.[11] However, we used the commuting noise fields since $\tilde{\tilde{C}}$ and $\tilde{C}$ are the bosonic antighost and ghost fields.

Although the regularization scheme[12,5,7] and assumptions taken in our previous paper[9] work very well also in this calculation, a more detailed analysis remains to be performed.

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