Spontaneous CP violation in theories with low-energy supersymmetry

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(Received 16 February 1988)

The problem of strong CP violation in theories with low-energy supersymmetry is studied. Special emphasis is given to theories with spontaneous CP violation. It is found that supersymmetric versions of the Nelson models only receive contributions to $\bar{\Theta}$ of order $(m_g/M_{\rm GUT})$ or at the three-loop level. The resulting Θ is of order 10^{-13} or less. This improves on the nonsupersymmetric Nelson models where Θ arises at the one-loop level. Constraints on the mechanism for spontaneously breaking CP invariance in supersymmetric models are found.

I. INTRODUCTION

There are good theoretical reasons to suspect that physics down to "low energies" (i.e., down to the Fermi scale) may be described by a supersymmetric theory. It therefore becomes important to ask in the case of a particular model or mechanism whether it survives "supersymmetrization" or, more generally, whether supersymmetry has something new to say on any central issue of the standard-model phenomenology. The question we wish to address here is whether various ideas for solving the strong CP problem are compatible with supersymmetry.

Theories with low-energy supersymmetry have much richer possibilities for CP violation. Indeed, in addition to the two sources of weak and strong CP violation of the standard model, $\delta_{\rm KM}$ and $\bar{\Theta}$, respectively, where KM denotes Kobayashi and Maskawa, four more CP-violating phases are present. Two of them can really be reabsorbed by legitimate redefinitions of the fields, so that we are actually left with two genuine new CP-violating phases to deal with. Even allowing for the so-far simplest way of breaking local supersymmetry (the Polonyi superpotential) still one extra phase survives in the low-energy limit.[1]

There are two main approaches to the strong CP problem which have been implemented in realistic "presupersymmetric" models: the Peccei-Quinn (PQ) (axion) idea,[2] and the idea of spontaneous CP violation (SBCP).[3-5] We wish to analyze the impact of the above-mentioned new sources of CP violation on these two main frames of resolution of the strong CP puzzle.

The PQ strategy makes $\bar{\Theta}$ very small, $\bar{\Theta} \ll 10^{-9}$, so that its contribution to the electric dipole moment of the neutron, d_n^e, is negligible. The contributions to d_n^e arising from $\delta_{\rm KM}$ have been shown to be small as well. In the supersymmetric case, however, this is not the whole story. As we shall see below the two new CP-violating phases lead to a value of d_n^e which can exceed the present experimental bound by something between 2 and 3 orders of magnitude if these phases are maximal and the masses of the supersymmetric particles that are exchanged in the one-loop contributions to d_n^e are not much larger than, say, 100 GeV (Ref. 1). This means that (i) the new phase must be somewhat small, less than 10^{-2} to 10^{-3}, or (ii) the supersymmetric masses that enter in the determination of d_n^e (i.e., gluino and squark masses) are rather larger, ~ 1 TeV, or (iii) some extra symmetry must be present to ensure that no new phases are present in the supersymmetric case (or, at least, that they are small).

If one intends to follow the SBCP way, one then needs to ensure that $\bar{\Theta} \leq 10^{-9}$. The presence of CP as an exact symmetry at the beginning implies a vanishing $\Theta_{\rm QCD}$ at the tree level. One must then keep track of all the other phases to make sure that they sneak in to contribute to $\bar{\Theta}$ only at a sufficiently high order in perturbation theory. We shall see that the class of models discovered by Nelson[4] not only survive supersymmetrization, but actually give a smaller $\bar{\Theta}$ in their supersymmetric versions. Indeed, we have found no contributions to $\bar{\Theta}$ at less than the three-loop level in such models (except for some of order $m_g/M_{\rm GUT}$, where m_g is the scale of low-energy supersymmetry breaking, i.e., typically $m_g \sim M_W$). However, if the Nelson idea is to be implemented in supersymmetry (SUSY), it is necessary that certain constraints be satisfied by the sector that does the spontaneous CP breaking. We shall also see that another class of SBCP models—those with approximately real quark mass matrices[5]—are less compatible with SUSY and that $\bar{\Theta}$ ends being uncomfortably close to, or even in excess of, the present bound.

We give now some more detailed justification of the above statements. Throughout this paper we will assume that the world is described by an $N=1$ supergravity model in which local supersymmetry is broken in a hidden sector of the Polonyi type.[6] This is in many ways the most economical method of supersymmetrizing a given model, but we expect our results to have a more general validity. It is well known that the $N=1$ supergravity Lagrangian depends only upon two unknown functions, G and f, of the scalar fields. We shall choose G to be such that the kinetic terms of the scalars are canonical. This occurs if the second derivative of G with respect to the scalars z_i and z_j^* is $G''^i{}_j=\delta^i{}_j$. This assumption (which is sometimes referred to as the condition of "minimality" in $N=1$ supergravity theories) implies strong restrictions on the scalar sector of the soft-breaking terms[7] of the remnant $N=1$ global supersymmetry at low energy. In particular, there are only two new parameters which depend on the details of the local supersymmetry breaking, i.e., on the specific form of the hidden sector. We can identify them with a mass parameter m_g which sets the scale of the low-energy supersymmetry breaking, and a dimensionless parameter A which is ~ 1 and enters the expressions of the trilinear and bilinear scalar terms. In this sector a third dimensionful parameter μ is present; μ appears in the bilinear $\mu H'H$ term of the superpotential, where H' and H denote the superfields whose scalar components are the two Higgs doublets which, in particular, provide the fermionic masses.[8] These three parameters, μ, m_g, and A, are in principle complex. The fourth extra phase that we have previously mentioned appears in the gluino mass. This parameter is determined by the second derivative of the function f and the first and second derivatives of g. The situation in which f is such that no tree-level gluino mass arises is phenomenologically disfavored. Indeed, radiatively induced gluino masses in the context of the minimal supersymmetrization of the standard model (i.e., without adding extra superheavy fields) are not sufficient to respect the present lower bounds given by the UA1 Collaboration.[9] Thus, as we shall explain further in Appendix A, we consider the gluino mass

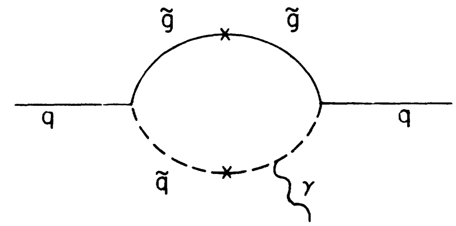

FIG. 1. A contribution to the quark electric dipole moment that can arise in supersymmetric models. $\tilde{q}$ is a scalar quark and $\tilde{g}$ a gluino.

m_3 to be an additional genuine complex parameter in our low-energy Lagrangian. In this same appendix we detail the counting of the phases after all the freedoms in redefining the fields are used and in the context of the simplest (albeit, *ad hoc*) mechanism to break supersymmetry, i.e., using a Polonyi superpotential of the hidden sector.

The analysis of Appendix A shows that even imposing a PQ symmetry one is not protected from too large CP violations in strong interactions. Indeed, even though the effect of $\overline{\Theta}$ on d_n^e is made innocuous in this way, the presence of two (or one in the minimal Polonyi scheme) extra phases in the low-energy supersymmetric Lagrangian generate a nonvanishing imaginary part in the diagram of Fig. 1, where gluinos and squarks are exchanged in the one-loop contribution to d_n^e. This contribution has been calculated by several sets of authors:[10,11]

$$d^e_{d\ \mathrm{quark}}/e \sim \frac{\alpha_3}{4\pi}\frac{|m_3 m_q \mu|\,|\langle H'^0\rangle/\langle H^0\rangle|}{(m_{\tilde{d}})^2 \mathrm{Max}(m_{\tilde{d}}^2, |m_3|^2)} \arg[(A-1)m_g m_3^*] + \frac{\alpha_3}{4\pi}\frac{|m_3 m_q|\,|A m_g|}{(m_{\tilde{d}})^2 \mathrm{Max}(m_{\tilde{d}}\,|m_3|^2)} \arg(A m_g m_3^*), \qquad (1)$$

where $m_{\tilde{d}}$ is a down-squark mass.[12] We have explicitly written as $A-1$ the coefficient of the bilinear soft-breaking term $m_g \mu HH'$ (see, however, Ref. 8 for the case when superheavy fields are present[7]). To avoid a too large d_n^e, either $\arg(Am_3)$ and $\arg[(A-1)m_3]$ must be $\lesssim 10^{-3}$ to 10^{-2} or the gluino (or squark) mass must be as high as ~ 1 TeV. In conclusion, the presence of the PQ symmetry certainly alleviates the strong CP problem to a large extent. The phases are required to be only $\sim 10^{-3}-10^{-2}$ instead of $\lesssim 10^{-9}$. Some additional symmetry or constraint may possibly eliminate even this less severe problem.

As we have previously said, the situation remains quite delicate also if spontaneous CP violation is invoked. In this case, Θ_{QCD} and $\arg\mu$ vanish at the tree level. Using the Polonyi superpotential, A turns out to be real at the tree level, too. Finally m_g and m_3 can be kept real (at the tree level) if certain conditions to be spelled out in Appendix B on the functions g and f are met. The point is now that, after the spontaneous breaking of CP [presumably at the grand-unified-theory (GUT) scale[13]] and the subsequent breaking of $SU(2)\times U(1)$ the quark-mass matrix must have a real determinant. In Secs. II and III we consider the supersymmetrization of the Nelson[4] and real quark mass matrix[5] approaches to this problem, respectively. We shall see that the process of supersymmetrization of these two classes of models presents nontrivial implications for the strong CP problem. In particular we shall show that the supersymmetric Nelson-type models are even safer than their original presupersymmetry versions. Another important-to-$\overline{\Theta}$ implication of the presence of supersymmetry (SUSY) is that the superfield whose scalar component breaks CP through its vacuum expectation value (VEV) must be a singlet of the GUT group (the proof is given in Appendix C). Finally, in Sec. IV we summarize our results and state our conclusions.

II. NELSON MODELS AND SUPERSYMMETRY

A. Brief review

Having CP be a symmetry of the Lagrangian (including the $F\tilde{F}$ term) only ensures that $\Theta_{\mathrm{QCD}}=0$ in the basis in which the Yukawa coupling matrices are real. When CP is broken spontaneously by phases of the expectation values of Higgs fields one would generally expect that the tree-level mass matrices of the quarks $M_Q^{(0)}$ would be complex. To ensure that Θ_{QFD} ($\equiv \arg\det M_Q^{(0)}$) vanishes therefore requires something further. By imposing additional symmetries one can arrange that $M_Q^{(0)}$ is complex while $\det M_Q^{(0)}$ is real. To do this with only the usual families of light quarks and leptons is somewhat awkward and leads to difficulties. As Nelson showed,[4] a great simplification is possible if superheavy fermions exist.

Let us suppose a gauge group G under which the quarks and leptons transform as $F+C+\overline{C}$. F consists of the usual family representations and C and $\overline{C}$ are some complex set of representations and the conjugate set. For example, in SO(10) with three light families F would consist of three 16's and $C+\overline{C}$ could consist of, say, $16+\overline{16}$. In order that $\Theta_{\rm QFD}=0$ it is enough to satisfy three conditions[4] (in addition to the CP invariance of L). (1) There should be no tree-level Yukawa couplings of F to C, C to C, or $\overline{C}$ to $\overline{C}$. (2) The tree-level masses of (FF) and $C\overline{C}$ should be CP conserving. (3) There should be CP-violating phases at the tree level in the $F\overline{C}$ masses. Condition (1) can follow from gauge invariance, discrete symmetries, family symmetries, or global symmetries. Condition (2) can follow from a sufficiently economical Higgs content. For instance, if there is only a single SU(2) doublet of Higgs bosons which gives rise to the FF masses, the FF masses will have no CP-violating phases. (Any phase can be absorbed by an anomaly-free global hypercharge rotation.) The quark mass matrix that results has a form

$$L_{\rm qm}=(F\ \ C\ \ \overline{C})\begin{pmatrix} m & 0 & \Omega \\ 0 & 0 & M \\ \Omega & M & 0 \end{pmatrix}\begin{pmatrix} F \\ C \\ \overline{C}\end{pmatrix} . \qquad (2)$$

It is easy to see that $\det M_Q=(\det m)(\det M)^2=\text{real}$. The entries in Ω and M are supposed to be very large—in a unified model typically they are GUT scale. Then upon diagonalizing the superheavy part of M_Q one is left with the same set of light families as if the fermions $C+\overline{C}$ did not exist (decoupling). But the light eigenstates are actually mixtures (with angles of order Ω/M) of the F and C fermions, and the light mass matrix will consequently have CP-violating phases in it coming from the phases in Ω. At low energies then the only remnant of the $C+\overline{C}$ fermions is the fact that the Yukawa couplings of the effective theory are complex. The model is indistinguishable from the KM model at low energies.

B. Breaking *CP* spontaneously in Nelson models

After the spontaneous breaking of CP invariance, a tree-level contribution to $\overline{\Theta}$ can potentially arise from three different sources: (i) a relative phase between the vacuum expectation values of the (at least) two Higgs doublets which are needed in the supersymmetric version of a Nelson model to provide the fermionic masses; (ii) a complex value of the gluino mass; and (iii) complex masses of some colored Higgsinos.

If one supersymmetrizes the Nelson model one must introduce at least two Higgs-doublet superfields, conventionally denoted H and H'. If $\langle H\rangle$ and $\langle H'\rangle^*$ have a relative phase α it will show up in the FF term in violation of condition (2). Then, $\arg(\det M_Q)=\arg\det M=n_F\alpha$, where n_F is the number of light families. The relative phase of $\langle H\rangle$ and $\langle H'\rangle^*$ is controlled by the $\mu HH'$ term at the level of the superpotential (μ may be an explicit mass parameter or may arise from the VEV of some Higgs field) and by the soft SUSY-breaking term $(A-1)\,m_g\mu HH'$ at the level of the soft-breaking terms. The original presence of CP invariance implies that μ is real (at the tree level) if it is an explicit mass parameter. If it arises from some VEV it may still be real at the tree level if that VEV is sufficiently insulated from the sector that breaks CP spontaneously. What is critical then is that A and m_g be real at the tree level and the radiative corrections not give a large phase to $(A-1)\,m_g\mu HH'$. Sufficient conditions for A and m_g to be real at the tree level are discussed in Appendix B. They are simple and easy to satisfy. If we have a Polonyi hidden sector, $f(z)=m\kappa^{-2}(\kappa z+\beta)$, then the parameter A is real at the tree level. As for the reality of m_g particular care must be taken when superheavy fields are present (as indeed they must be in any viable scheme of spontaneous CP breaking). There are two conditions that one must satisfy as shown in Appendix B. Condition I is that

$$\beta_{\rm eff}\equiv\beta_0+m^{-1}\kappa^{-2}f(z^\alpha=0,\langle z^A\rangle_0)=\text{real}\ , \qquad (3)$$

where $f(z^\alpha,z^A)$ is the superpotential of the ordinary (z^α) and superheavy fields (z^A) in the observable sector. Note that β_0 is real since CP is a symmetry of the unbroken Lagrangian. So that condition I amounts to $f(z^\alpha=0,\langle z^A\rangle_0)=\text{real}$. The second condition relates to a certain contribution to m_g which is proportional to $\langle z^A\rangle$ which can arise in some cases [see Eq. (B4) in Appendix B]. This contribution can be rendered harmless by condition II which is that there is no superfield that couples to HH' and also mixes with S. (The field that couples F to $\overline{C}$.) This guarantees that the phase of $\langle S\rangle$ is not transmitted to m_g.

We turn now to the second cause of concern for large contributions to $\overline{\Theta}$ at the tree level in the Nelson models: a possibly complex gluino mass. To guarantee the reality of m_3 requires a condition that is given in Eq. (B16) of Appendix B. We stress that this condition is not difficult to implement in a SUSY Nelson scheme. Indeed in Eq. (B17) a very simple way to do this is given.

The final potential difficulty that we must face at the tree level is that there may be complex masses for the colored Higgsinos. We have already taken care of the colored fermionic components of F, C, and $\overline{C}$ when discussing M_Q. We focus on the Higgs supermultiplet(s) that couples F to $\overline{C}$, which we call S. Of course S must contain some $SU(3)_c\times U(1)_{\rm em}$-singlet component(s) whose VEV's contribute to the quark masses. These VEV's are complex according to assumption (3) above; indeed it is this complexity that gives rise to the ϵ parameter in the kaon system. In Appendix C we show that if S contains colored fermion components it is difficult (if not impossible) to prevent them from having complex mass at the tree level. This means that S must be a singlet under any group that contains $SU(3)_c$. This is a very important constraint on model building, but not a severe one. It only says that C must contain some of the same representations as F (so that $F\times\overline{C}$ contains a singlet). This is something that would appear attractive anyway on the grounds of simplicity and economy. (Moreover in certain kinds of models the extra real fermion representations *do* include the F representations and their conjugates, e.g., Kaluza-Klein and some superstring theories.)

For the same reasons, it also appears likely that, in general, the sector which breaks CP spontaneously must consist of gauge-singlet superfields (Appendix C considers the problem only for S fields which couple directly to F and $\bar{C}$ through the $F\bar{C}S$ coupling term).

C. An example of a SUSY Nelson model

For notational simplicity we will discuss an SO(10) example since a whole family is then contained in a single irreducible representation. As will be seen, our results are largely independently of the choice of group since Nelson models all reduce to the KM model in the low-energy limit.

Let F consist of three spinor (**16**) representations of SO(10). C ($\bar{C}$) will consist of some number of **16** ($\overline{\mathbf{16}}$) representations. The $SU(2)_L\times U(1)$ breaking is done by a field $h=\mathbf{10}$. This contains both the H and H' Higgs doublets. Some set of SO(10) singlets, denoted S_i, break CP spontaneously. And the hidden sector is just the Polonyi model. Other Higgs superfields may be required to give a realistic pattern of gauge symmetry breaking, to give split Higgs multiplets, and so forth. We will call them X. The superpotential of the ordinary sector is given by

$$f=\sum_{ij} f_{ij}(F_iF_j)h+\sum_{ijk} g_{ij}^k(F_i\bar{C}_j)S_k +\sum_{ij} M_{ij}(C_i\bar{C}_j)+\mu hh+f_{CPX}(S_i) +f_{\text{Higgs}}(h,X)\ . \quad (4)$$

f_{CPX} is responsible for CP breaking. It must satisfy $f_{CPX}(\langle S_i\rangle)=\text{real}$ (see Appendix B). We have deliberately left out certain terms that SO(10) allows us. We do not have $C\bar{C}S$ terms as they would give phases in $C\bar{C}$ terms violating condition (2). We do not have CCh, $\bar{C}\bar{C}h$, or FCh terms as they would violate condition (1). Finally, we do not allow hhS terms in f_{Higgs} since they would make μ_{eff} complex at the tree level and lead to phases in the FF terms in violation of condition (2). (It would also tend to make $\mu_{\text{eff}}\sim\langle S\rangle\sim M_{\text{GUT}}$ and destroy the hierarchy unless there were a cancellation.)

It should be noted that it is as a consequence of S being a singlet and therefore F and C containing the same representations that SO(10) allows all of these dangerous terms. For, since h must couple to FF, it must also be allowed by SO(10) to couple to FC and CC. In order to forbid these terms, then, one must invoke some other symmetry such as a family, global, or discrete symmetry, or a gauge symmetry not unified with color, under which S is allowed to be nonsinglet. Of course, it is possible just to leave out the unwanted terms since the nonrenormalization theorems will guarantee that this is technically natural, but that is unsatisfying from an explanatory point of view. It is also possible that some topological or other reasons explain their absence.

A simple example of a discrete symmetry that gives the desired structure is Z_3 with $F\to F$, $H\to H$, $C\to zC$, $\bar{C}\to z^*\bar{C}$, $S\to zS$ where $z\equiv e^{2\pi i/3}$. The CP-breaking superpotential can be chosen to be $f_{CPX}(S)=aY(S^3+M^3)$, where the singlet field $Y\to Y$ under Z_3. Then

$$V\supset\left|\frac{df_{CPX}}{dY}\right|^2=a^2\,|S^3+M^3|^2\ .$$

This forces $S=-M$, $-zM$, or $-z^*M$. The last two minima are CP violating. (As always with spontaneous CP violation the Universe would have a domain structure which would necessitate an inflationary cosmology.) We have chosen to use a "nonrenormalizable" f_{CPX} since this is the simplest example we have found using discrete symmetry.

D. Contributions to $\bar{\Theta}$ in supersymmetric Nelson models

1. Radiative correction to μ

The multiplet h is really a "split multiplet," so the term $\mu HH'$ should be written $(\mu_2HH'+\mu_3H_cH_{c'})$, where H,H' are the light doublets and $H_c,H_{c'}$ are their superheavy, colored partners. μ_3 is of order the GUT scale. If μ_2 acquires a phase through radiative corrections, that will give a relative phase to $\langle H\rangle$ and $\langle H'\rangle^*$ which in turn will show up in the FF terms in the quark mass matrix. As noted above μ_2 can be real at tree level because of CP. But we must check whether radiative corrections give it an unacceptably large phase. (Henceforth μ will refer to either μ_2 or μ_3.)

Let us define for the purposes of our analysis a global transformation K which acts on the fields as follows. $K(F)=K(C)=\frac{1}{2}$, $K(\bar{C})=-\frac{1}{2}$, $K(h)=-1$, and $K(S)=0$. Then the terms $\mu HH'$ violate K by 2 units, and $\langle H\rangle$ and $\langle H'\rangle$ each violate it by 1 unit. It is easy to see that radiative corrections to the $\mu_2(HH')$ term must be proportional either to $HH'[\mu\,|\langle H\rangle|^{2m}|\langle H'\rangle|^{2n}]$ or to $HH'[\langle H\rangle^*\langle H'\rangle^*|\langle H\rangle|^{2m}|\langle H'\rangle|^{2n}]$. The latter will not contribute to $\bar{\Theta}$ since its expectation value does not depend on the relative phase of $\langle H\rangle$ and $\langle H'\rangle^*$. We thus need only worry about terms proportional to μ. Such diagrams will have at least two loops. An example is shown in Fig. 2. Let us diagonalize the superheavy quark masses, and for the light quarks go to a basis where the $W^\pm$ couplings are diagonal but the H,H' couplings are not; that is, the "weak basis." Then M_Q has the block-diagonal form

$$\begin{bmatrix} M_L & \\ & M_H \end{bmatrix}$$

where M_L is in the weak basis and M_H is diagonal. Let us write the couplings of the H and H' as $\sum_{ij}f_{Dij}(Q_iD_j^C)H'+\sum_{ij}f_{Uij}(Q_iU_j^C)H$. The product of the Yukawa couplings and mass insertions on the quark (squark) line in Fig. 2 will have the form $\mathrm{Tr}[g_1(h_U)g_2(h_D)g_3(h_U)\cdots]$, where $h_U\equiv f_Uf_U^\dagger$, $h_D\equiv f_Df_D^\dagger$, and the g_i are functions (which are not necessarily simple monomials since there are infrared logarithms). Clearly $\mathrm{Tr}[g_1(h_U)g_2(h_D)]$ is real since g_1 and g_2 are Hermitian. However

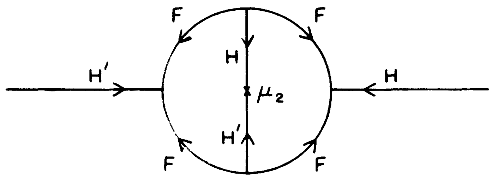

FIG. 2. A radiative correction to the phase of the μ parameter in a supersymmetric Nelson model will have at least two loops since they must have μ insertions.

$$\mathrm{Tr}[g_1(h_U)g_2(h_D)g_3(h_U)g_4(h_D)]$$

can be complex. Since, for such contributions, there are four transitions from U of D or D to U (s)quarks, there must be four *charged* lines coming off the (s)quarks line. This implies that the diagram is *at least* three loops. (This argument is similar in spirit to one found in Ref. 11.) In fact, we believe that the lowest-order graphs are at four loops as shown in Fig. 3, the reason being that one must have an odd numer of μ insertions to violate K by 2 units. This gives (crudely)

$$\delta\mu_2 \sim \frac{\mu_2}{4(2\pi)^8}\frac{m_D^4 m_U^4}{m_g^8} f\left[\frac{\mu_2}{m_g}, \frac{m_{\tilde{q}}}{m_g}\right] . \tag{5}$$

If we take m_g to be roughly 100 GeV then even with $m_D = m_b$ and $m_U = m_t$ we would have $\delta\bar{\Theta} \lesssim 10^{-14}$. Actually $\delta\bar{\Theta}$ will be much smaller than this since the *CP*-violating part of the diagram will depend on the lighter quark masses, so that

$$\delta\bar{\Theta} \ll 10^{-14} . \tag{6}$$

There are also diagrams with virtual (s)leptons and the superheavy color triplets H_c and $H_{c'}$. These diagrams are proportional to μ_3, which is superheavy, rather than μ_2; however, as they also vanish in the limit of unbroken SUSY they are proportional to $m_g \sim \mu_2$. Thus, they should not give a contribution much larger than the ones we have considered. There will also be multiloop diagrams involving the exchange of virtual (superheavy) S fields. These will be at least three loops and be proportional to the Yukawa couplings of the S bosons, which are unknown but could be arbitrarily small. These contributions are also therefore negligible.

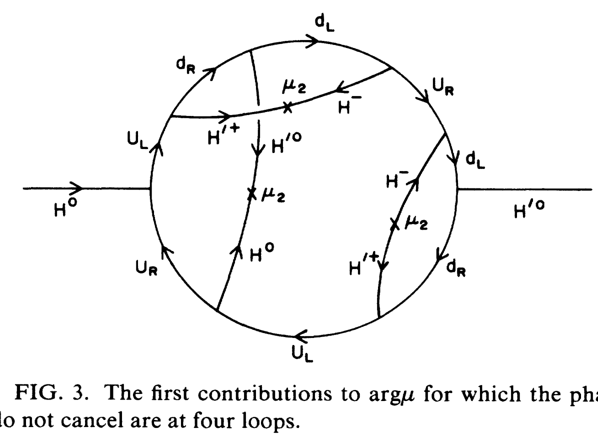

FIG. 3. The first contributions to argμ for which the phases do not cancel are at four loops.

2. *Radiative corrections to the gluino mass, $\tilde{m}_3$*

Similar arguments to the above tell us that the lowest-order contributions to the gluino mass involving only H or H' exchange come at three loops as in Fig. 4. These give contributions roughly of order

$$\delta_{\tilde{m}_3} \sim m_g \frac{(g_3)^2}{4(2\pi)^6}\frac{m_D^4 m_U^4}{m_g^8} f(\text{masses}) . \tag{7}$$

This gives, with $m_g \sim 100$ GeV,

$$\delta\bar{\Theta} \lesssim 10^{-13} . \tag{8}$$

There are also loops involving the exchange of the superheavy S fields. The two-loop diagram of Fig. 5(a) is real as can be seen as follows. Go to the basis in which the whole quark mass matrix is diagonal, and where the (mass)^2 matrix of the S_i is diagonal as well. Then we will have Yukawa couplings as $g_{mn}^k(\bar{q}_{Rm}q_{Ln})S_k$ where g_{mn}^k is complex. Then the diagram in Fig. 5(a) is proportional to $\sum_k g_{mn}^k g_{mn}^{k*} I_k$ which is real. The first complex diagram is the three-loop diagram in Fig. 5(b) which goes roughly as

$$\delta\tilde{m}_g \sim m_g (g_3)^2 \frac{g^4}{4(2\pi)^6} f(\text{masses}) , \tag{9}$$

where g stands for a typical g_{mn}^k. We then have

$$\delta\Theta \lesssim 10^{-6} g^4 . \tag{10}$$

The g_{mn}^k can be arbitrarily small (almost). [This does not affect, necessarily, the size of the Kobayashi-Maskawa (KM) angles in the low-energy effective theory since these arise from the mixing of the C, $\bar{C}$, and F fermions. This mixing goes as $g\langle S\rangle/M$ where M is a typical $C\bar{C}$ mass. g can be small as long as $M/\langle S\rangle$ is correspondingly small.] If $g < 10^{-1}$ then

$$\delta\bar{\Theta} < 10^{-10} . \tag{11}$$

3. *Mass of colored fermions in Higgs supermultiplets*

If these Higgs supermultiplets, which we denoted X in Eq. (4), do not couple to S at the tree level, which may easily be a consequence of the family, discrete, or global symmetry we imposed, then the colored fermions in these supermultiplets will have real tree-level masses by *CP*. The radiative corrections will involve (generally) loops with superheavy virtual particles circulating in them. By

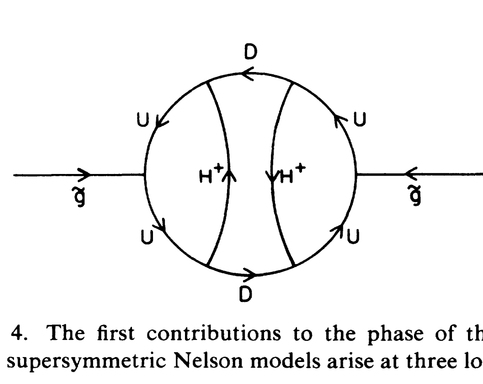

FIG. 4. The first contributions to the phase of the gluino mass in supersymmetric Nelson models arise at three loops.

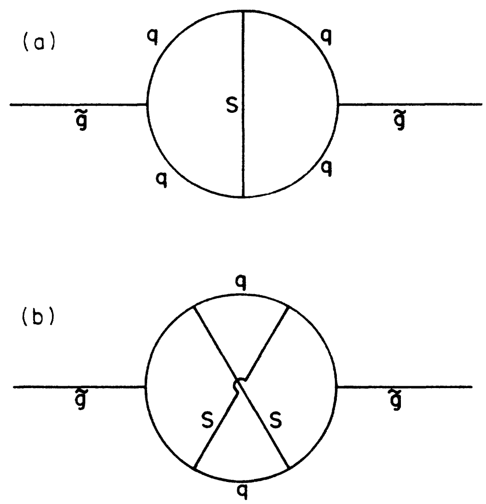

FIG. 5. Contributions to the phase of the gluino mass involving superheavy S fields.

the nonrenormalization theorems of supersymmetry these will be suppressed by $(m_g/M_{\rm GUT})$ with respect to the superheavy tree-level masses. Hence we can neglect the resulting phases which will be $\ll 10^{-13}$.

4. *Radiatively induced "forbidden" terms*

We needed to forbid certain types of terms to get $\overline{\Theta}_{\rm tree}=0$. We will consider the radiative corrections to these in turn. (a) Radiatively induced FC terms, since they break both $SU(2)\times U(1)$ and SUSY, are of order (at n loops) $(g^2/16\pi^2)^n G_F^{-1/2} m_g/M_{\rm GUT}$. (The $M_{\rm GUT}^{-1}$ reflects the mass of particles running around the loops.) The g are coupling constants appearing in those loops. From Eq. (2) one sees that this contributes to $\det M_Q$ a term of order

$$\left[\frac{g^2}{16\pi^2}\right]^n (G_F^{-1/2} m^{-1})\frac{m_g}{M_{\rm GUT}} \ll \frac{m_g}{M_{\rm GUT}} \lesssim 10^{-13}$$

times the tree-level value. Thus the corresponding $\delta\overline{\Theta} \ll 10^{-13}$. (b) The same reasoning and result applies to the CC and $\overline{C}\,\overline{C}$ terms. (c) The complex radiative corrections to the $C\overline{C}$ terms are proportional to SUSY breaking. Thus compared to the $O(M_{\rm GUT})$ tree-level values they are suppressed by $(g^2/16\pi^2)^n m_g/M_{\rm GUT}$, where again the g are couplings and the $(16\pi^2)^{-1}$ are loop factors. So $\delta\overline{\Theta} \ll m_g/M_{\rm GUT} \lesssim 10^{-13}$. (d) Finally, there are the radiative contributions to the FF terms. If these involve virtual superheavy particles then the argument based on the nonrenormalization theorems again will give a result of order $m_g/M_{\rm GUT}$. However, in this case there are also diagrams involving only light particles which we will discuss next.

5. *Radiative corrections to quark mass in the low-energy effective theory*

These effects are essentially the same as those considered by Dugan, Grinstein, and Hall,[11] since the low-energy effective theory is just the SUSY standard model with complex Yukawa couplings, i.e., the supersymmetric version of the KM model. These authors find that the first complex contribution to the quark masses arises at four loops and gives a $\overline{\Theta}$ of order

$$\delta\overline{\Theta} \lesssim \left[\frac{\alpha_3}{4\pi}\right]^2 \left[\frac{\alpha_2}{4\pi}\right]^2 \frac{m_t^2 m_b^2}{M_W^4} s_1^2 s_2 s_3 s_\delta \approx 10^{-13} . \qquad (12)$$

In fact this may be the largest contribution to $\overline{\Theta}$ in this class of models. [The contribution in Eq. (10) could be larger if $g > 10^{-2}$.]

E. Conclusions about Nelson models

On the one hand, SUSY constrains Nelson-type models to the extent that the Higgs fields that break CP and communicate that breaking to the quark fields must be singlets under any group that contains $SU(3)_c$. This means that some of the representations that appear in C must appear in F (so that $F\times\overline{C}\supset 1$). However, these constraints are not onerous since simplicity and economy might suggest this anyway. On the other hand, SUSY greatly helps the Nelson models in that $\overline{\Theta}$ only gets contributions that are of order $(m_g/M_{\rm GUT})$ or that arise at three loops or higher. This is to be compared to the non-SUSY case where $\overline{\Theta}$ receives contributions at one-loop level. These one-loop contributions are of order $g^2/16\pi^2$ in the notation of Eq. (9), and constrain these g to be $\lesssim 10^{-3}$, which seems uncomfortably small. In the SUSY version, by contrast, the first contribution depending on these couplings is the three-loop one given in Eqs. (9) and (10), which only constrains $g \lesssim 10^{-1}$. Thus in spite of a host of potentially worrisome new contributions to $\overline{\Theta}$ arising from supersymmetry (complex contributions to μ, $\tilde{m}_3$, and colored Higgsino masses) it appears that the Nelson models actually give a smaller $\overline{\Theta}$ in their SUSY than in their non-SUSY versions.

III. MODELS WITH REAL QUARK MASSES

Here the story is not quite as hopeful. There actually exists a supersymmetric model of this type in the literature.[14] The idea is that at the tree level the whole quark mass matrix is real (not just its determinant). Thus $\overline{\Theta}_{\rm tree}=0$. This requires that CP violation in the kaon system be mediated by a new force. In Ref. 15 this was a superweak force mediated by a light colored Higgs boson. In Ref. 14 it was a milliweak force mediated by scalar quarks. In the SUSY model of Ref. 14 it was found that the gluino mass gets complex contributions at two loops that tend to give a value for $\overline{\Theta} > 10^{-7}$. The true value can be much smaller than this "naive" estimate because various ratios of unknown parameters are involved. However, the tendency does appear to be that $\overline{\Theta}$ comes out larger (and possibly too large) in the supersymmetric versions of this kind of model than in the nonsupersymmetric ones. Moreover, the communication of the CP violation from the singlet fields that give rise to it to the light fields needs to be more indirect in the SUSY models of this type than in the SUSY Nelson models. For example, in Ref. 14 the fields that get superlarge CP-breaking

VEV's must couple to *other* superheavy gauge-singlet superfields (that do not acquire VEV's) to give them complex masses, and these in turn must couple to the squarks to give *them* (after the superheavy fields are integrated out) complex masses. Thus a kind of "middleman" field is required.

A new possibility for the real quark mass matrix models arises in SUSY, however. The field mediating the milli or superweak force could be a light doublet superfield such as H or H' except that it does not acquire a VEV (or else M_Q would be complex at the tree level). If we call this field X then one can invoke the nonrenormalization theorems of supersymmetry to forbid the dangerous $(H'X)$ couplings that would give $\langle X\rangle \neq 0$. However, such coupling will arise at one loop from diagrams such as that in Fig. 6. Unfortunately, it would seem that these contributions to $\overline{\Theta}$ tend to be too large.

The upshot is that, while models with real quark mass matrices can be (and have been) constructed in the context of supersymmetry, supersymmetry is a complicating factor that makes it much more difficult to keep $\overline{\Theta}$ within bounds.

IV. CONCLUSIONS

The presence of low-energy supersymmetry does not give rise to any new approach to the old problem of large *CP* violation in strong interactions. In fact one might say that the problem becomes even more acute in the SUSY context. In the standard model, once one gets rid of the $\overline{\Theta}$ contribution to d_n^e, the remaining contribution due to the presence of the only *CP*-violating phase, $\delta_{\rm KM}$, is negligibly small. In SUSY apart from $\overline{\Theta}$ and $\delta_{\rm KM}$ one has in general two additional *CP*-violating phases and, thus, additional care must be taken that they do not induce a too large d_n^e. As for the $\overline{\Theta}$ contribution itself, it is true that in a theory with exact or spontaneously broken global SUSY no infinite renormalization to $\overline{\Theta}$ may arise due to the nonrenormalization theorems.[16] However, finite contributions can still be large and, in any case, in the attractive $N=1$ supergravity models global SUSY is explicitly broken at low energy through soft-breaking terms. Then $\overline{\Theta}$ can undergo infinite renormalization since the gluino mass receives logarithmically divergent contributions. Given that there is no specific SUSY way to solve the strong *CP* problem, it behooves us to reconsider the two standard approaches, PQ symmetry and spontaneous *CP* violation, in the SUSY context. Our conclusions can be summarized as follows.

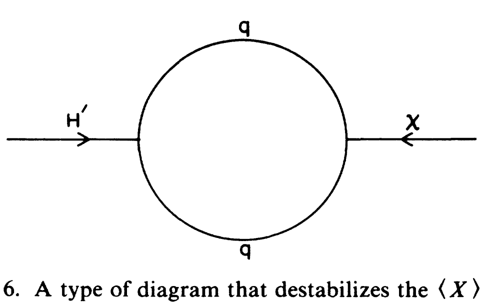

FIG. 6. A type of diagram that destabilizes the $\langle X\rangle=0$ vacuum needed in the real-quark-mass matrix models.

(i) PQ solution. $\overline{\Theta}$ is made inoffensively small; however, the two additional SUSY *CP*-violating phases give rise to a too large d_n^e unless they are $\lesssim 10^{-2}$ to 10^{-3} and/or gluino and squark masses are rather high (~ 1 TeV). Even using the minimal Polonyi way of breaking local SUSY one is left with an extra phase at low energy and so the same problem is faced. This extra phase can be avoided only if no tree-level gluino mass is present, but this seems untenable in view of the recent experimental lower bounds on gluino masses. Finally, we should remind the reader of a well-known difficulty in implementing the PQ mechanism: the scale of PQ symmetry breakdown should lie between 10^9 and 10^{12} GeV for cosmological and astrophysical reasons,[17] a situation which results in rather complicated SUSY GUT schemes.

(ii) SBCP solution. We have analyzed the SUSY versions of two SBCP classes of models, the Nelson and the real quark-mass types of models. The presence of SUSY leads to an improvement in particular in the former case. Indeed the radiative contributions to $\overline{\Theta}$ arise only at the three-loop level, or are suppressed by $m_g/M_{\rm GUT}$ factors. Thus SUSY makes things easier with respect to the original non-SUSY Nelson proposal without needing any particular additional complication. Indeed, it is true that in the SUSY context there are additional potential dangers of a tree-level contribution to $\overline{\Theta}$ after *CP* is spontaneously broken; we have, however, shown that these risks can be relatively easily averted at least in the Polonyi realization of the hidden sector.

In conclusion, we think that with or without SUSY the strong *CP* problem remains open. The PQ alternative does not benefit from the presence of low-energy SUSY and, in fact, at least for sufficiently light (~ 100 GeV) supersymmetric masses the new SUSY phases must be rather small ($\sim 10^{-3}-10^{-2}$). On the other hand, the SBCP alternative, at least in its Nelson implementation, becomes, in the SUSY context, safer than in the original non-SUSY version. Clearly the presence of extra sources of *CP* violation in SUSY leads, as we have seen, to values of d_n^e which may lie just around the corner of the present experimental bounds. An increasing effort in this experimental direction should be even more eagerly awaited to the extent one believes in low-energy supersymmetry.

ACKNOWLEDGMENTS

S.M.B. would like to thank the Aspen Center for Physics where much of this work was carried out.

APPENDIX A

We assume a superpotential of the form

$$F(z^a,z)=f(z^a)+\tilde{f}(\tilde{z}) \ , \tag{A1}$$

where z^a is the ordinary sector superfields and $\tilde{z}$ the hidden sector superfield. We take f to have the Polonyi form

$$\tilde{f}(\tilde{z})=m\kappa^{-2}(\kappa\tilde{z}+\beta) \ , \tag{A2}$$

where $\kappa^2=8\pi/M_{\rm Pl}^2$. Then the ordinary Higgs potential will be

$$V=\exp\left[\kappa^2\left[\sum_a|z^a|^2+|\bar{z}|^2\right]\right]\left[\sum_a|\partial f/\partial z^a+\kappa^2 z^{a*}(f+\bar{f})|^2+|\partial f/\partial z+\kappa^2\bar{z}^*(f+\bar{f})|^2-3\kappa^2|f+\bar{f}|^2\right]$$
$$+D \text{ terms} . \tag{A3}$$

Following Hall, Lykken, and Weinberg[7] we assume that

$$\partial f/\partial z^a=0,\quad f(z^a)=0,\quad D \text{ terms}=0 \tag{A4}$$

at the minimum of V. Let us define $h=\kappa\bar{z}$ then

$$V=\exp\left[\kappa^2\sum_a|z^a|^2\right]e^{|h|^2}\left[\sum_a\left|\frac{\partial f}{\partial z^a}+\kappa^2 z^a*[f+m\kappa^{-2}(h+\beta)]\right|^2+|m\kappa^{-1}+\kappa h^*[f+m\kappa^{-2}(h+\beta)]|^2\right.$$
$$\left.-3\kappa^2|f+m\kappa^{-2}(h+\beta)|^2\right] . \tag{A5}$$

The constant term in V, i.e., the cosmological constant, is

$$\Lambda=e^{|h|^2}m^2\kappa^{-2}(|1+|h|^2+\beta h^*|^2-3|h+\beta|^2) . \tag{A6}$$

For this to vanish requires that

$$|1+|h|^2+\beta h^*|^2=3|h+\beta|^2 . \tag{A7}$$

The quadratic term in z^a is

$$V_2=\sum_a|z^a|^2m^2e^{|h|^2}(|1+|h|^2+\beta h^*|^2-2|h+\beta|^2)=\sum_a|z^a|^2m^2e^{|h|^2}|h+\beta|^2=|m_g|^2\sum_a|z^a|^2 , \tag{A8}$$

where

$$m_g\equiv\kappa^2e^{|h|^2/2}f^* . \tag{A9}$$

The cubic terms in z^a are

$$V_3=e^{|h|^2}\left[\sum_a\left[\frac{\partial f}{\partial z^a}z^a\right]m(h^*+\beta^*)\right.$$
$$+m(1+|h|^2+\beta h^*)hf^*$$
$$\left.-3m(h^*+\beta^*)f\right]+\text{H.c.} \tag{A10}$$

If we denote the terms of order $(z^a)^n$ in f by f_n, so that $f=f_3+f_2+f_1$ then the coefficient of f_3 in the low-energy effective SUSY-breaking terms is

$$C_3=e^{|h|^2}mh^*(1+|h|^2+\beta^*h) , \tag{A11}$$

while that of f_2 is

$$C_2=e^{|h|^2}m(h^*|h|^2+\beta^*|h|^2+\beta^*) , \tag{A12}$$

C_3 and C_2 are proportional, respectively, to the conventional parameters A and B.

Now we want $V(h,\beta)=0$ (i.e., $\Lambda=0$) at the minimum $\partial V(h,\beta)/\partial h=0$, where

$$V(h,\beta)=e^{|h|^2}m^2\kappa^{-2}(|1+|h|^2+\beta h^*|^2$$
$$-3|h+\beta|^2) . \tag{A13}$$

This requires a tuning of the β parameter to take some special value $\hat{\beta}$. For $\beta=\hat{\beta}$ then there is an $\hat{h}$ such that

$$V(\hat{h},\hat{\beta})=\left.\frac{\partial V}{\partial\beta}\right|_{\hat{h},\hat{\beta}}=0 . \tag{A14}$$

But from the form of V we see that it is invariant under the transformation $\beta\to e^{i\theta}\beta, h\to e^{i\theta}h$. Thus at $\beta=\hat{\beta}e^{i\theta}\equiv\hat{\hat{\beta}}$ and $h=\hat{h}e^{i\theta}\equiv\hat{\hat{h}}$ it will also be true that

$$V(\hat{\hat{h}},\hat{\hat{\beta}})=\left.\frac{\partial V}{\partial\beta}\right|_{\hat{\hat{h}},\hat{\hat{\beta}}}=0 . \tag{A15}$$

So for this value of β it also is true that Λ vanishes at the minimum. Thus $\hat{\beta}e^{i\theta}$ for *any* θ is an equally good value to tune β to. However, if we look at Eqs. (A11) and (A12) we see that C_2 and C_3 acquire phases $e^{-i\theta}$. Since C_2 and C_3 get the same phase this is equivalent to m_g getting a phase and the parameter A (and B) remaining real.

We must also look into the phases of the gaugino mass. The gaugino mass matrix is given by

$$M_{\alpha\beta}=\tfrac{1}{4}f'^{k}_{\alpha\beta}e^{-G/2}G'_iG'^{-1i}{}_k , \tag{A16}$$

where α,β are indices in the adjoint representation of the gauge group. G is given by

$$G=-\tfrac{1}{4}\sum_i z_iz^{i*}-\ln f/2-\ln f^*/2 , \tag{A17}$$

if we choose a "canonical" form of the Kahler metric

$$G'_i\equiv\partial G/\partial z^{i*},\quad G''\equiv\partial G/\partial z^i .$$

f is the full superpotential. $f_{\alpha\beta}$ is an analytic function of the z^i. There are two cases to consider: $m_3=0$ at the tree level or $m_3\neq0$ at the tree level. The first case can arise if one chooses the "canonical" form $f_{\alpha\beta}=\delta_{\alpha\beta}$.

Then $f'_{\alpha\beta}k=0$. In that case, m_3 comes from loops and its phase is computable in terms of $\arg A$, $\arg\mu$, and $\arg m_g$. (The phases of the quark masses and Yukawa couplings cancel in these loops.) Thus, we have only *three* "new" phases to worry about: those of A, μ and m_g. But as discussed in the text two phases can be rotated away using R rotations and rotations of the H superfield. This leaves one "new" phase. If the hidden sector is Polonyi, then as we showed A is real. This reduces the number of "new" phases to zero. Thus the one-loop contribution to d_q involving the squarks and gluino vanishes.

The more realistic case is that $m_3 \neq 0$ are the tree level. In this case $f'^{k}_{\alpha\beta} \neq 0$ $[\alpha,\beta \subset SU(3)]$ for some z^k. It would seem that in general in a theory with explicit CP violation not only would m_3 be complex, but its phase would be unrelated to the phase we found for m_g above. In this case one would expect there to be a one-loop contribution to d_q involving the squarks and gluino. In order for this contribution to be sufficiently small then one must either fine-tune the CP-violating phases (by about 10^{-2} or 10^{-3}) or one must have m_3, etc., be sufficiently large to make the diagram small. In the next appendix we will deal with the case of models with spontaneous CP violation.

APPENDIX B

In models with spontaneous CP breaking the parameters μ and β (of the Polonyi sector) start out to be real at the tree level. However, CP is broken by the complex superlarge VEV's of some Higgs fields. The question is what effect this has on the CP properties of the low-energy soft SUSY-breaking terms.

We take the superpotential to be as before $F(z^a,\tilde{z})=f_0(z^a)+\tilde{f}_0(\tilde{z})$. At the tree level all the parameters of F are CP conserving. Let us distinguish the fields z^a into z^α and z^A. The z^α have Fermi-scale VEV's that can be neglected relative to $M_{\rm Pl}$, while the z^A are the Higgs bosons that have GUT-scale VEV's, which are needed to break unified gauge symmetries and CP. In general $\langle f_0(z^\alpha,z^A)\rangle_{\rm tree}=f_0(0,\langle z^A\rangle)\neq 0$. Moreover, if $\langle z^A\rangle$ breaks CP one would expect typically that $\langle f_0\rangle$ would be complex. Let us shift

$$f(z^a)\equiv f_0(z^a)-\langle f_0\rangle \; , \qquad \tilde{f}(\tilde{z})\equiv \tilde{f}_0(\tilde{z})+\langle f_0\rangle \; . \tag{B1}$$

Then

$$\tilde{f}(\tilde{z})=m\kappa^{-2}(\kappa\tilde{z}+\beta_0)+\langle f_0\rangle = m\kappa^{-2}(\kappa\tilde{z}+\beta_{\rm eff}) \; , \tag{B2}$$

where

$$\beta_{\rm eff}\equiv\beta_0+m^{-1}\kappa^2\langle f_0\rangle \; . \tag{B3}$$

This shift makes $\langle f(z^a)\rangle=0$ at the potential minimum as assumed in Appendix A, so that the β of Appendix A is what we call $\beta_{\rm eff}$ here. Thus if $\beta_{\rm eff}$ has a phase coming from $\langle f_0\rangle$ it will show up in the soft term $(A-1)m_g\mu(HH')$.

Another source of CP violation in the soft SUSY-breaking terms is the following. Suppose there is some singlet field J that couples to (HH') in the superpotential. Then in the soft term $\tilde{m}_g(HH')$ there is a contribution to $\tilde{m}_g$ given by

$$\delta\tilde{m}_g=\sum_A f^{-1}_{AJ}\langle z^A\rangle^* f_{JHH'} \; . \tag{B4}$$

Here $f_{JHH'}\equiv(\partial^3/\partial J\partial H\partial H')f$ and $f_{AJ}\equiv(\partial^2/\partial z^A\partial J)f$. Since CP is spontaneously broken this can be complex.

There is a further *apparent* source of low-energy CP violation which turns out not to be genuine. In Eq. (A3) one sees the terms

$$\sum_A T_A \equiv \sum_A \left| \frac{\partial f}{\partial z^A}+\kappa^2 z^{A*}(f+\tilde{f}) \right|^2 \; . \tag{B5}$$

These would appear to contribute cross terms of the form

$$\sum_A T_A \supset \sum_A \left[\left[\frac{\partial f}{\partial z^A}+\kappa^2 z^{A*}\tilde{f} \right] \kappa^2 z^A \right]^* f \; . \tag{B6}$$

It would seem that the coefficients of the quadratic and cubic soft terms which we denoted C_2 and C_3 would pick up a contribution of

$$\delta C_n = \left\langle \left[\frac{\partial f}{\partial z^A}+\kappa^2 z^{A*}\tilde{f} \right] \kappa^2 z^A \right\rangle^* \; . \tag{B7}$$

However, the minimization of the potential V leads to the conditions $(\partial/\partial z^B)(\sum_A T_A)=0$ for all B. This implies that $\langle \partial f/\partial z^A+\kappa^2 z^{A*}(f+\tilde{f})\rangle_0=0$ for all A. Since, after shifting, $\langle f\rangle=0$, this makes the contributions δC_n *vanish*.

There is one more genuine CP-violating effect in the soft SUSY-breaking terms that we must worry about: the phase of the gluino (and generally gaugino) mass. The gaugino mass matrix is given by [see Eqs. (A16) and (A17)]

$$M_{\alpha\beta}=\tfrac{1}{4}(f'^{k}_{\alpha\beta}e^{-G/2}G'_i G''^{-1i}{}_k) \tag{B8}$$

with

$$G=-\tfrac{1}{4}\sum_i z_i z'^* - \ln f/2 - f^*/2 \tag{B9}$$

for canonical Kahler metric.

Now let us see what conditions are required to ensure that the strong CP problem is solved in theories with spontaneous CP violation. First, there is the effect that $\beta_{\rm eff}$ given by Eq. (B3) as

$$\beta_{\rm eff}=\beta_0+m^{-1}\kappa^2\langle f_0\rangle \tag{B3}$$

could be complex. As shown in Appendix A, and above, this would lead to a phase in the coefficients C_2 and C_3. (In fact it is effectively a phase in m_g.) This in turn would give a phase in the soft term $\tilde{m}_g HH'$. This would finally show up as a relative phase of $\langle H\rangle$ and $\langle H'\rangle^*$ and hence as a contribution to $\Theta_{\rm QFD}\equiv\arg\det M_Q$. CP guarantees that β_0 and $m^{-1}\kappa^2$ in (B3) are real. But when CP breaks spontaneously it could be that $\langle f_0\rangle$ is complex. To solve the strong CP problem in supersymmetric SBCP models then we must ensure that

$$\text{(condition I)} \quad f(z^a=0,\langle z^A\rangle_0)=\text{real} \tag{B10}$$

is satisfied (here z^A are the superfields with superlarge VEV's, while the z^a are those with VEV's of order m_g). It is easy to find superpotentials that satisfy condition I. For example, consider a form given in Ref. 14:

$$f(z^a)=f_r(z^a)+f_{CPX}(\phi,X)\ , \tag{B11}$$

where $f_r(\langle z^a\rangle_0)=\text{real}$ and f_{CPX} is the part of the superpotential that spontaneously breaks *CP*, given by

$$f_{CPX}=X(\phi^2+M^2)\ . \tag{B12}$$

Then

$$V(\phi)=\left|\frac{\partial f}{\partial X}\right|^2+\left|\frac{\partial f}{\partial \phi}\right|^2 = |\phi^2+M^2|^2+4|X\phi|^2\ . \tag{B13}$$

This is minimized by $\langle X\rangle=0$ and $\langle\phi\rangle=\pm iM$. The phase of $\langle\phi\rangle$ breaks *CP* but at its minimum

$$f_{CPX}(\langle\phi\rangle,\langle X\rangle)=0=\text{real}\ . \tag{B14}$$

In the case of the Nelson models the field ϕ corresponds to what we have called S in the text; and $f_r(z^a)$ contains terms that involve S, namely, $(F\bar{C})S$ Yukawa terms. Since normally the quark and lepton superfields F and C will not acquire VEV's, however, we have assumed that $f_r(\langle z^a\rangle)=\text{real}$.

The second source of trouble that we identified was the terms shown in Eq. (B4). These would also lead to a relative phase between $\langle H\rangle$ and $\langle H'\rangle^*$ and thus a contribution to $\arg\det M_Q\equiv\Theta_{\text{QFD}}$. The second condition we must impose is thus

$$\text{(condition II)} \quad \sum_A f^{-1}_{AJ}\langle z^A\rangle^* f'_{JHH}=\text{real}\ . \tag{B15}$$

In Nelson models this is also easily satisfied. The only $\langle z^A\rangle$ that are complex in Nelson models are $\langle S\rangle$. We already have forbidden $HH'S$ terms in the construction of Nelson models [see discussion after Eq. (4) of text]. That means $J\neq S$. Condition II is automatically satisfied then as long as there is no other superfield J that couples to HH' and also mixes with S. But there is no reason to introduce such superfields.

Third, and finally, we must worry about the gluino mass. The phase of this directly contributes to $\bar{\Theta}$. Using Eq. (B9) we get $G''^{i}_k=-\frac{1}{4}\delta^i_k$. Moreover

$$G'_S=\partial G\partial S^*=-\frac{1}{4}S-\left[(F\bar{C})+\frac{\partial f_{CPX}(S)}{\partial S}\right]^*\bigg/ f\ .$$

At the minimum this is $(G'_S)_{\min}=-\frac{1}{4}\langle S\rangle-\langle(\partial f_{CPX}/\partial S)^*/f\rangle$. This could be complex, and in general is. The other terms such as G'_F and $G'_{\bar{C}}$ which involve s vanish at the minimum and so are not dangerous. The dangerous term is then

$$M_{\alpha\beta}=-f'^S_{\alpha\beta}G'_S e^{-G/2}\ .$$

To ensure that the gluino mass is real at the tree level we must require that

$$\text{(condition III)} \quad -f'^S_{33}G'Se^{-G/2}=\text{real}\ . \tag{B16}$$

An easy way to do this is to require that

$$f'^S_{33}=0 \tag{B17}$$

[f_{33} stands for $f_{\alpha\beta}$ with $\alpha,\beta\in\text{SU}(3)_c$].

Altogether, then, we have seen that there are three additional constraints on the construction of supersymmetric models with spontaneous *CP* violation that must be satisfied if the strong *CP* problem is to be solved. And we have also seen that these constraints are easily satisfied in Nelson-type models.

APPENDIX C

We will show that in the SUSY versions of the Nelson models the superfields S must be singlets under any simple group, G, containing $\text{SU}(3)_C$ as a subgroup. Let us suppose S is not a singlet under G. Then it will contain colored components. Moreover, since it is a superfield, it will contain colored fermions. The basic problem is to find a means by which to give the color-singlet, scalar components in S complex VEV's without giving the colored, fermionic components of S complex masses.

To see the problem let us try a particular example. Let us consider a superpotential that contains among other terms

$$f\supset a\zeta(SS')+\zeta g(\eta_i)+MSS'\ . \tag{C1}$$

The superfields η_i are G singlets whose role it is to break *CP* spontaneously. $g(\eta_i)$ is a function suitably designed to do this. The field ζ is a G singlet whose role is to communicate the *CP* breaking to S. The term $|\partial f/\partial\zeta|^2$ in the potential contains cross terms proportional to $SS'g(\eta_i)^\dagger$. This introduces *CP* violation into the potential for S. (S' is just a superfield in the representation of G conjugate to S needed to make a G-invariant term in f.) The term MSS' is required to give mass to the colored fermions in S. M may be a VEV of a scalar field or it may be an explicit (real) mass parameter in f. Now, note the problem, which is that $V\supset|\partial f/\partial S'|^2\supset|a\zeta S+MS|^2$. This contains a term linear in ζ that forces it to have a VEV. (Actually even if $M=0$ the soft, SUSY-breaking term $Am_g[\zeta g(\eta_i)]$ will be linear in ζ and ensure that $\langle\zeta\rangle\neq 0$.) Because of its coupling to η_i, the VEV of ζ will inevitably be complex. The result is that the colored fermions in S get complex masses through the $a\zeta SS'$ term. We chose ζ to be G singlet. If we had chosen ζ to be nonsinglet then $\langle S\rangle$ (which must be complex to give *CP* violation in the kaon system) would have led directly to complex masses for the colored fermions in ζ through the same $a\zeta SS'$ term.

Through arguments of this sort it is possible to show that, for any reasonably simple superpotential, if S is a G nonsinglet the colored fermions in S will acquire complex masses. It may be possible to avoid this problem, but to do so would probably require a complicated and artificial form for the superpotential, which is certainly undesirable in a solution to a "naturalness problem."

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[8]The trilinear soft-breaking terms are $Am_g\hat{g}_s$, where $\hat{g}_s$ is obtained from the trilinear terms in the superpotential g_3 by taking the scalar component of each superfield. Neglecting the contribution due to the VEV's of possible superheavy fields, the bilinear soft-breaking term is $(A-1)m_g\mu HH'$. The effect of the superheavy fields on this bilinear term and, more generally, for our discussion of the strong *CP* problem is dealt with in Appendix B (Ref. 7).

[9]We understand that the tiny window of small gluino masses, of the order of some GeV, although not completely ruled out yet, is however very unlikely.

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