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Graviton emission by a thermal bath of photons

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The rate of emission of gravitons by a thermal bath of protons is calculated. Linearized quantum gravity is used to find the rate of graviton creation by thermal fluctuations of the photons. For the case of photons confined within a sphere of radius R and volume V at temperature T, the power radiated in gravitons is found to be $P=(2.7\times10^{-66}\ \mathrm{erg/s})(V/\mathrm{cm}^3)(T/\mathrm{K})^7\ln(160RT/\mathrm{K\,cm})$. If the thermal bath cools solely by emission of gravitons, the cooling time is of the order of $\tau=(3\times10^{51}\ \mathrm{s})(1\ \mathrm{K}/T)^3\ln(160RT/\mathrm{K\,cm})$. Any region which has been cooling by graviton emission since the early Universe has a maximum temperature of 10 MeV at the present.

The semiclassical theory of gravity is the approximation in which quantized matter fields are coupled to a classical gravitational field through the semiclassical Einstein equation (we use units in which $\hbar=c=G=k_B=1$, where G is Newton's constant and k_B is Boltzmann's constant):

$$G_{\mu\nu}=8\pi\langle T_{\mu\nu}\rangle \ . \tag{1}$$

Here $\langle T_{\mu\nu}\rangle$ is understood to be a suitably renormalized expectation value. The extent to which this theory is a good approximation to a more exact quantum gravity theory has not been fully resolved. It presumably fails at Planck dimensions when the quantum fluctuation of the gravitational field cannot be ignored. It also is expected to fail if the quantum state of the matter field represents a superposition of very different classical configurations, or more generally, if the fluctuations of $T_{\mu\nu}$ are large. The range of validity of the semiclassical approximation has been discussed by several authors recently in the context of cosmology, where one would like to understand better the transition from the quantum to the classical eras.[1] In particular, the calculation of density perturbations in inflationary cosmology requires a matching of quantum and classical behaviors which is not as well understood as one might like.

In a previous paper[2] (I), the limitations of the semiclassical theory were studied in the context of graviton emission by a quantum system in Minkowski spacetime. In the semiclassical theory based upon Eq. (1), the energy and momentum emitted in classical gravity waves are given by the integrated energy-momentum tensor

$$S_{\rm sc}^{\mu\nu}=8\pi\int d^3x\, d^4x' d^4x'' G_r^{;\mu}(x-x')G_r^{;\nu}(x-x'') \times[\langle T_{\alpha\beta}(x')\rangle\langle T^{\alpha\beta}(x'')\rangle -\tfrac{1}{2}\langle T(x')\rangle\langle T(x'')\rangle] \ , \tag{2}$$

where $G_r(x-x')$ is the retarded Green's function, $T=T^{\alpha}_{\alpha}$, and $d^3x=r^2dt\,d\Omega$. Thus, $S_{\rm sc}^{tr}$ is the total energy emitted in gravity waves. In linearized quantum gravity theory, it was shown in I that the corresponding expression is

$$S_q^{\mu\nu}=8\pi\int d^3x\, d^4x' d^4x'' G_r^{;\mu}(x-x')G_r^{;\nu}(x-x'') \times\langle T_{\alpha\beta}(x')T^{\alpha\beta}(x'')-\tfrac{1}{2}T(x')T(x'')\rangle \ . \tag{3}$$

[Note that an overall sign error appears in Eqs. (10), (12), and (18) in I.]

The crucial difference between the semiclassical and quantum expressions is that the former involves products of expectation value whereas the latter involves an expectation value of products. Thus the semiclassical theory is a good approximation when

$$\langle T_{\alpha\beta}(x')T_{\rho\sigma}(x'')\rangle\approx\langle T_{\alpha\beta}(x')\rangle\langle T_{\rho\sigma}(x'')\rangle \ .$$

As was discussed in I, this holds for quantum states which are coherent states, but can fail for other choices of states.

A particular case in which the semiclassical theory clearly fails is when the matter field is in a thermal state. Here $\langle T_{\alpha\beta}\rangle$ is constant, so $S_{\rm sc}^{\mu\nu}=0$. However, $S_q^{\mu\nu}$ should be nonzero because thermal fluctuations emit gravitons. The main purpose of this paper is to exhibit an explicit calculation of the flux of energy emitted in gravitons by a thermal bath of electromagnetic radiation using Eq. (3).

We wish to calculate the energy radiated in gravitons by a box filled with photons in thermal equilibrium at temperature T. Our system will be described by the electromagnetic stress-energy tensor $T^{\mu\nu}$, where

$$T^{00}(x)=\tfrac{1}{2}[\mathbf{E}^2(x)+\mathbf{B}^2(x)] \ ,$$
$$T^{0i}(x)=\epsilon^{ijk}E^j(x)B^k(x) \quad (i,j=1,2,3) \ , \tag{4}$$
$$T^{ij}(x)=\tfrac{1}{2}\delta^{ij}T^{00}(x)-[E^i(x)E^j(x)+B^i(x)B^j(x)]$$

are the components expressed in terms of the transverse electromagnetic fields.

Because $T^{\mu\nu}$ is traceless, we only need to calculate the thermal expectation value of the product of $T^{\mu\nu}$, which in terms of the electromagnetic field is

$$\langle T^{\alpha\beta}(x)T_{\alpha\beta}(x')\rangle_\beta=\langle [\mathbf{E}(x)\cdot\mathbf{E}(x')-\mathbf{B}(x)\cdot\mathbf{B}(x')]^2 +[\mathbf{E}(x)\cdot\mathbf{B}(x')+\mathbf{E}(x')\cdot\mathbf{B}(x)]^2\rangle_\beta , \tag{5}$$

where $\langle\ \rangle_\beta$ stands for the thermal expectation value at temperature $T=\beta^{-1}$. In the following we use normal-ordered operators for $T_{\alpha\beta}(x)T^{\alpha\beta}(x')$ in order that the expectation value be finite.[3]

In the Coulomb gauge, the transverse electric and magnetic fields are given by

$$\mathbf{E}(x)=-\frac{\partial \mathbf{A}(x)}{\partial t},\quad \mathbf{B}(x)=\nabla\times\mathbf{A}(x) , \tag{6}$$

where $\mathbf{A}$ is the vector potential, which expanded in terms of creation and annihilation operators becomes

$$\mathbf{A}(x)=\sum_{(\mathbf{k},\lambda)}\mathbf{e}_{\mathbf{k},\lambda}[a_{\mathbf{k},\lambda}\mu_{\mathbf{k}}(x)+a^\dagger_{\mathbf{k},\lambda}\mu^*_{\mathbf{k}}(x)] . \tag{7}$$

Here $\mathbf{e}_{\mathbf{k},\lambda}$ $(\lambda=1,2)$ are two mutually perpendicular real unit vectors which are also orthogonal to $\mathbf{k}$, and $\mu_{\mathbf{k}}(x)$ are mode functions.

In general, these mode functions will be determined by the boundary conditions and the shape of the cavity. However, we will be interested in the case when the temperature is large, $T\gg L^{-1}$, where L is a characteristic dimension of the cavity. The dominant contribution to our result will come from modes whose wavelengths are short compared to L. Consequently, we can neglect the boundaries and assume that the $\mu_{\mathbf{k}}(x)$ are plane waves:

$$\mu_{\mathbf{k}}(x)=\frac{1}{\sqrt{2Vk^0}}e^{-ik\cdot x} , \tag{8}$$

where $k\cdot x=k^0t-\mathbf{k}\cdot\mathbf{x}$, $k^0=\omega_{\mathbf{k}}=|\mathbf{k}|$, and V is the volume of the cavity.

The (normal-ordered) thermal expectation value, Eq. (5), becomes

$$\langle T^{\alpha\beta}(x)T_{\alpha\beta}(x')\rangle_\beta=\sum k_1^0k_2^0k_3^0k_4^0(C_{12}C_{34}+D_{12}D_{34}) \times(\langle a_3^\dagger a_4^\dagger a_1a_2\rangle_\beta\mu_1\mu_3^*\mu_2\mu_4^*+\langle a_2^\dagger a_4^\dagger a_1a_3\rangle_\beta\mu_1\mu_3\mu_2^*u_4^*+\langle a_2^\dagger a_3^\dagger a_1a_4\rangle_\beta\mu_1\mu_3^*\mu_2^*\mu_4 +\langle a_1^\dagger a_4^\dagger a_2a_3\rangle_\beta\mu_1^*\mu_3\mu_2\mu_4^*+\langle a_1^\dagger a_3^\dagger a_2a_4\rangle_\beta\mu_1^*\mu_3^*\mu_2\mu_4+\langle a_1^\dagger a_2^\dagger a_3a_4\rangle_\beta\mu_1^*\mu_3\mu_2^*\mu_4) . \tag{9}$$

Here $a_i=a_{\mathbf{k}_i,\lambda_i}$, $\mu_1=\mu_{\mathbf{k}_1}(x)$, etc., μ_1 and μ_3 are evaluated at x, μ_2 and μ_4 at x', and the summation is over $\mathbf{k}_i$ and λ_i, $i=1,\ldots,4$. We have set, e.g.,

$$\begin{aligned}C_{12}&=\mathbf{e}_1\cdot\mathbf{e}_2-(\hat{\mathbf{k}}_1\times\mathbf{e}_1)\cdot(\hat{\mathbf{k}}_2\times\mathbf{e}_2) ,\\ D_{12}&=\mathbf{e}_1\cdot(\hat{\mathbf{k}}_2\times\mathbf{e}_2)+\mathbf{e}_2\cdot(\hat{\mathbf{k}}_1\times\mathbf{e}_1) ,\end{aligned} \tag{10}$$

where $\hat{\mathbf{k}}$ denotes a unit vector.

In order to calculate $\langle a_l^\dagger a_m^\dagger a_na_p\rangle_\beta$, we use the fact that the thermal expectation value for any operator $\hat{O}$ is given by

$$\langle\hat{O}\rangle_\beta=\mathrm{tr}(\rho\hat{O}) , \tag{11}$$

where ρ is the density matrix

$$\rho=\sum_{\{n_\lambda(\mathbf{k})\}}\alpha_{\{n_\lambda(\mathbf{k})\}}|\{n_\lambda(\mathbf{k})\}\rangle\langle\{n_\lambda(\mathbf{k})\}| . \tag{12}$$

Here $|\{n_\lambda(\mathbf{k})\}\rangle$ is an eigenstate of the number operator with $n_\lambda(\mathbf{k})$ particles in mode $(\mathbf{k},\lambda)$. The factor $\alpha_{\{n_\lambda(\mathbf{k})\}}$ is

$$\alpha_{\{n_\lambda(\mathbf{k})\}}=\prod_{(\mathbf{k},\lambda)}[1-\exp(-\beta k^0)]\exp[-\beta k^0 n_\lambda(\mathbf{k})] . \tag{13}$$

Thus

$$\langle a_l^\dagger a_m^\dagger a_na_s\rangle_\beta=(\delta_{ln}\delta_{ms}+\delta_{ls}\delta_{mn})f_l(\beta)f_m(\beta) , \tag{14}$$

where $\delta_{ln}=\delta_{\lambda_l,\lambda_m}\delta_{\mathbf{k}_l,\mathbf{k}_n}$ and $f_l(\beta)=(e^{\beta k_l^0}-1)^{-1}$, the Planck factor.

From Eqs. (8), (9), and (14), we obtain

$$\langle T^{\alpha\beta}(x)T_{\alpha\beta}(x')\rangle=\left[\frac{1}{2V}\right]^2\sum_{\mathbf{k}_1,\mathbf{k}_2}k_1^0k_2^0f_1(\beta)f_2(\beta)\sum_{\lambda_1,\lambda_2}(C_{12}^2+D_{12}^2)(2+e^{i(k_1-k_2)\cdot(x-x')}+e^{i(k_1+k_2)\cdot(x-x')}+\text{c.c.}) . \tag{15}$$

Using the relation

$$\sum_\lambda e^i_{\mathbf{k},\lambda}e^j_{\mathbf{k},\lambda}=\delta^{ij}-\hat{\mathbf{k}}^i\hat{\mathbf{k}}^j \tag{16}$$

for the Cartesian component of the polarization vectors $(i,j=x,y,z)$, we get

$$\sum_{\lambda_1,\lambda_2}(C_{12}^2+D_{12}^2)=4(1-\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2)^2 . \tag{17}$$

With this expression, Eq. (15) becomes

$$\langle T^{\alpha\beta}(x)T_{\alpha\beta}(x')\rangle_\beta=\left[\frac{1}{V}\right]^2\sum_{\mathbf{k}_1,\mathbf{k}_2}k_1^0k_2^0(1-\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2)^2 f_1(\beta)f_2(\beta)(2+e^{i(k_1-k_2)\cdot(x-x')}+e^{i(k_1+k_2)\cdot(x-x')}+\text{c.c.}) . \tag{18}$$

The retarded Green's function is

$$G_r(x-x')=\frac{\delta(t-t'-|\mathbf{x}-\mathbf{x}'|)}{4\pi|\mathbf{x}-\mathbf{x}'|} . \tag{19}$$

If we use this expression and Eq. (18) in Eq. (3) for the total energy radiated ($S_q^{tr}=S_q^{tt}$), the result would be infinite. However, this is just the result of integrating a constant power over an infinite time. Thus we should drop the integration on t in Eq. (3) and replace S_q^{tt} by P, the radiated power, which may be expressed as

$$P=(2\pi)^{-7}\int d^3k_1d^3k_2F(\mathbf{k}_1,\mathbf{k}_2)\int\frac{r^2d\Omega\, d^3x'd^3x''}{|\mathbf{x}-\mathbf{x}'||\mathbf{x}-\mathbf{x}''|}$$
$$\times(\omega_+^2\exp\{i[\omega_+(|\mathbf{x}-\mathbf{x}''|-|\mathbf{x}-\mathbf{x}'|)-\mathbf{k}_+\cdot(\mathbf{x}'-\mathbf{x}'')]\}$$
$$+\omega_-^2\exp\{i[\omega_-(|\mathbf{x}-\mathbf{x}''|-|\mathbf{x}-\mathbf{x}'|)-\mathbf{k}_-\cdot(\mathbf{x}'-\mathbf{x}'')]\}+\text{c.c.}) . \tag{20}$$

Here we have performed the t' and t'' integrations, replaced the sums on $\mathbf{k}_1$ and $\mathbf{k}_2$ by $V^2(2\pi)^{-6}\int d^3k_1d^3k_2$, and introduced the notation

$$\mathbf{k}_\pm=\mathbf{k}_1\pm\mathbf{k}_2,\quad \omega_\pm=\omega_1\pm\omega_2 , \tag{21}$$

and

$$F(\mathbf{k}_1,\mathbf{k}_2)=\omega_1\omega_2(1-\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2)f_1(\beta)f_2(\beta) . \tag{22}$$

Because the power may always be measured at a distance which is large compared to the dimensions of the system, we may assume that $r\gg r',r''$ ($r=|\mathbf{x}|$, etc.) and write

$$|\mathbf{x}-\mathbf{x}'||\mathbf{x}-\mathbf{x}''|\sim r^2 \tag{23}$$

in the denominator and

$$|\mathbf{x}-\mathbf{x}''|-|\mathbf{x}-\mathbf{x}'|\sim r'\cos\theta'-r''\cos\theta'' \tag{24}$$

inside the exponentials. Now the $d\Omega$ integration becomes trivial and we have

$$P=2(2\pi)^{-6}\int d^3k_1d^3k_2F(\mathbf{k}_1,\mathbf{k}_2)\int d^3x'd^3x''(\omega_+^2\exp\{i[\omega_+(r'\cos\theta'-r''\cos\theta'')-\mathbf{k}_+\cdot(\mathbf{x}'-\mathbf{x}'')]\}$$
$$+\omega_-^2\exp\{i[\omega_-(r'\cos\theta'-r''\cos\theta'')-\mathbf{k}_-\cdot(\mathbf{x}'-\mathbf{x}'')]\}+\text{c.c.}) . \tag{25}$$

If we choose to perform first the integrations on $\mathbf{k}_1$ and $\mathbf{k}_2$, then we find, after some calculation, that

$$P=\tfrac{1}{4}T^4\int d^3\xi'd^3\xi''g(\xi) , \tag{26}$$

where $\xi=|\xi'-\xi''|$ and

$$g(\xi)=\frac{1}{15\xi^{10}}\{10\xi^4(12\xi\coth\xi-\xi^2+6)\operatorname{csch}^4\xi$$
$$+2\xi^2[30\xi^3\coth^3\xi+10\xi^2(60-\xi^2)\coth^2\xi-\xi(\xi^4+15\xi^2+45)\coth\xi-\xi^4-225]\operatorname{csch}^2\xi$$
$$-2\xi(\xi^4-30\xi^2+225)\coth\xi-4\xi^4+630\} . \tag{27}$$

Here $\xi'=2\pi T\mathbf{x}'$ and $\xi''=2\pi T\mathbf{x}''$, and the integrations on these variables are over the volume of the cavity expressed in these units. The function $g(\xi)$ is plotted in Fig. 1. Because $g(\xi)>0$ for $\xi\lesssim 3$ and $g(\xi)<0$ for $\xi\gtrsim 3$, pairs of points $(\mathbf{x}',\mathbf{x}'')$ which are separated by less than a few thermal wavelengths yield a positive contribution to P, whereas those with larger separations yield a negative contribution.

Note that if $g(\xi)$ were constant for all ξ, then we would have $P\propto V^2T^{10}$. However, the fact that points for which $\xi\gtrsim 3$ give a negative contribution indicates that P should increase less rapidly with increasing V or T. To evaluate P, it would be necessary to explicitly evaluate the integral in Eq. (26) for a particular choice of the cavity geometry. Because of the complicated form of $g(\xi)$, this is difficult.[4]

Instead, we will return to Eq. (25) and calculate P for the case of a spherical cavity by first performing the integrations on $\mathbf{x}'$ and $\mathbf{x}''$. Performing the integrations on $\mathbf{x}',\mathbf{x}''$ and the directions of $\mathbf{k}_1$ and $\mathbf{k}_2$ yields

$$P=8\pi^{-2}\int_0^\infty dk_1 dk_2 (k_1k_2)^3 f_1(\beta)f_2(\beta)[y_+(k_1,k_2)+y_-(k_1,k_2)]\ , \tag{28}$$

where

$$\begin{aligned} y_\pm &= \int_{-1}^{1} dx(1-x)^2\omega_\pm^2\delta_\pm^{-6}[\sin(R\delta_\pm)-R\delta_\pm\cos(R\delta_\pm)]^2\ ,\\ \delta_\pm &= (k_1\pm k_2)-(k_1^2+k_2^2\pm 2k_1k_2x)^{1/2}\ , \end{aligned} \tag{29}$$

and $k_i=|\mathbf{k}_i|$. Here R is the radius of the sphere. The integral in Eq. (29) may be evaluated in terms of sine and cosine integral functions and of elementary functions. Because we are interested in the limit that $R \gg \beta$, we may take the large-R limit of $y_\pm$, in which case we can write

$$P=\frac{8R^3}{3\pi^2}\int_0^\infty dk_1 f_1\left[\int_0^{k_1} dk_2(\omega_+^5+\omega_-^5)f_2\mathrm{Si}(4k_2R)+\int_{k_1}^\infty dk_2 f_2\{\omega_+^5\mathrm{Si}(4k_1R)+\omega_-^5[\mathrm{Si}(4k_2R)-\mathrm{Si}(4(k_1-k_2)R)]\}\right]\ , \tag{30}$$

where $\mathrm{Si}(z)=\int_0^z dt\, t^{-1}\sin t$ is the sine integral function. If we expand the $\omega_\pm^5$ factors, in most of the resulting integrals we may replace the sine integral functions by $\pi/2$ [$\mathrm{Si}(z)\to\pi/2$ as $z\to\infty$]; the exception is when this would lead to an integral that diverges at the lower limit of integration. The result of doing this is

$$\begin{aligned} P=\pi^{-2}VT^7[\tfrac{16}{21}\pi^5 M(4R\beta^{-1})+20N_{3,2}+10N_{1,4}+5\tilde N_{4,1}\\ +10\tilde N_{3,2}+10\tilde N_{2,3}+5\tilde N_{1,5}-\tilde N_{5,0}-N_{0,5}]\ , \end{aligned} \tag{31}$$

where $V=\frac{4}{3}\pi R^3$ is the volume of the sphere and where

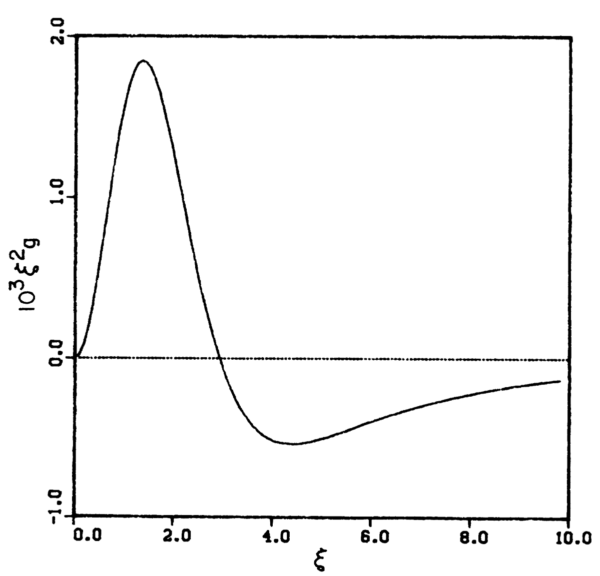

FIG. 1. The function $g(\xi)$ is plotted in units of $10^{-3}\xi^{-2}$ as a function of $\xi=|\mathbf{x}'-\mathbf{x}''|$. The integral of $g(\xi)$ over d^3x' and d^3x'' yields the radiated power.

$$\begin{aligned} N_{n,m} &= \int_0^\infty \frac{dx\, x^n}{e^x-1}\int_0^x \frac{dy\, y^m}{e^y-1}\ ,\\ \tilde N_{n,m} &= \int_0^\infty \frac{dx\, x^n}{e^x=1}\int_x^\infty \frac{dy\, y^m}{e^y-1}\ , \end{aligned} \tag{32}$$

and

$$M(\alpha)=\int_0^\infty \frac{\mathrm{Si}(\alpha x)}{e^x-1}dx\ .$$

The $M(\alpha)$ term is the part in which we cannot replace $\mathrm{Si}(\alpha x)$ by $\pi/2$. The asymptotic form of $M(\alpha)$ for large α is

$$M(\alpha)\sim\tfrac{1}{2}\pi\ln\alpha+0.890+O(\alpha^{-1})\ . \tag{33}$$

Numerical evaluation of the $N_{n,m}$ and $\tilde N_{n,m}$ integrals yields the result (in Planck units)

$$P=10VT^7[3.7\ln(4RT)+8.1]\ . \tag{34}$$

This can be expressed as

$$P=(2.7\times10^{-66}\ \mathrm{erg/s})VT^7\ln(160RT)\ , \tag{35}$$

where T is in K and lengths in centimeters.

Apart from the logarithmic factor, the power radiated is proportional to the volume of the cavity. This result can be understood as due to the fact that regions of the cavity whose separation is large compared to the thermal wavelength $[\hbar c/(k_B T)]$ radiate independently of one another, and the gravitons produced escape without further interactions. The power is slightly enhanced by the logarithmic factor, which suggests that there is a weak correlation between different regions. This is possibly the effect of stimulated emission of gravitons.

We can estimate the power radiated in gravitons by the core of the Sun.[5] If the core temperature is $T=10^7$ K and the core radius is $R=1.7\times10^{10}$ cm, then

$$P_{\odot}=2\times 10^{16}\ \text{erg/s}\ . \quad (36)$$

On the other hand, a newly formed neutron star[6] has a temperature of about $T=10^{11}$ K. Taking $R=10^{6}$ cm yields

$$P_{\text{NS}}\simeq 10^{31}\ \text{erg/s}\ . \quad (37)$$

This is still too small to contribute noticeably to the cooling of the neutron star, which is dominated by neutrino emission.

We can also estimate the cooling time for an object which cools by graviton emission. If the energy content is primarily the thermal energy of the photons, then the time scale τ for this energy to be radiated is

$$\tau=\frac{3\times 10^{51}\ \text{s}}{T^{3}\ln(160RT)}\ . \quad (38)$$

(Again, T is in K and R is in centimeters.) Thus an object formed in the early Universe ($\tau\simeq 10^{10}$ yr) can have a temperature of no more than $T_m\simeq 10^{11}$ K $\simeq 10$ MeV at present. Various authors[7] have speculated on the possible existence of confined regions of an exotic phase of matter created in the early Universe and persisting at present. Assuming these objects are transparent to gravitons, we have a bound on their present interior temperature.

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[1]S. Wada, Prog. Theor. Phys. **75**, 1365 (1986); J. J. Halliwell, Phys. Lett. B **196**, 444 (1987); Phys. Rev. D **36**, 3626 (1987); W. Boucher and J. Traschen, *ibid.* **37**, 3522 (1988).

[2]L. H. Ford, Ann. Phys. (N.Y.) **144**, 238 (1982), hereafter referred to as I.

[3]Note that for an operator such as $T_{\alpha\beta}(x)T^{\alpha\beta}(x')$ which is quartic in the fundamental fields, normal ordering is not simply subtracting the vacuum expectation value, as it is for quadratic operators.

[4]When $RT\gg 1$, one might expect that we could replace the integration on $d^3\xi'$ by one on $4\pi\xi^2 d\xi$ and the integration on $d^3\xi'$ by an overall factor of $(2\pi T)^3 V$, obtaining

$$P\approx 8\pi^4 T^7 V\int_0^{\infty}\xi^2 g(\xi)d\xi\ ,$$

i.e., a power proportional to the cavity's volume. However, the above integral vanishes. This apparently reflects the fact that the power is actually proportional to $V\ln RT$ [see Eq. (35)] rather than to V. We have not found any simple way to derive this logarithmic factor from Eq. (26).

[5]S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), p. 266. Here an estimate is given of the power radiated in gravitons produced in electron-electron and electron-proton collisions in the Sun. The result is about 1 order of magnitude smaller than our estimate, Eq. (36). It is presumably a coincidence that the two effects are comparable for the Sun as the effect considered by Weinberg yields a power $\propto T^{3/2}$.

[6]G. Schäfer and H. Dehner [Phys. Rev. D **27**, 2684 (1983)] have discussed the radiation of gravitons due to particle collisions in a degenerate Fermi fluid.

[7]See, e.g., T. D. Lee and G. C. Wick, Phys. Rev. D **8**, 2291 (1974); E. Witten, *ibid.* **30**, 272 (1984).