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Multisource model and stochastic properties of multiparticle production in high-energy nucleus-nucleus collisions

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Stochastic properties of multiparticle production processes in high-energy heavy-ion reactions, especially those connected with the multisource model are discussed. The concept of number of effective sources (NES) is deduced from the multisource model; and the existence of a new kind of scaling (NES scaling) is shown. Transverse-energy distributions for different projectile-target combinations at different incident energies are calculated. The obtained results are compared with the recent data. Some new experiments are suggested.

I. INTRODUCTION

Recent high-energy heavy-ion experiments[1–5] have attracted much attention in the nuclear-physics as well as in the particle-physics community. These experiments are of great interest not only because quark-gluon plasma may become feasible[6] in this kind of reaction (provided that such plasma indeed exists in nature); but also because understanding the reaction mechanism of high-energy nucleus-nucleus collisions is itself a challenging problem. In fact, there has been much discussion[7,8] about the possible relationship between the mechanism of multiparticle production processes in hadron-hadron collisions and those in hadron-nucleus and nucleus-nucleus processes. It is expected that these measurements, as well as future experiments of this kind, will yield useful information on the reaction mechanism in general, and will be able to differentiate between the various proposed mechanisms[7,8] in particular.

Multiparticle production in the midrapidity region of high-energy nucleus-nucleus collisions plays a special role. This is because it has been observed in hadron-hadron processes that the "central rapidity plateau" not only becomes broader but also becomes significantly higher—showing that more and more particles are produced in this kinematical region—at higher and higher incident energies. In high-energy nucleus-nucleus collisions it is in the midrapidity region, where most of the exotic phenomena—especially those associated with the existence of quark-gluon plasma—are expected.

This paper is organized as follows. A brief summary of the underlying physical picture of the multisource model is given in Sec. II, where the concept of "number of effective sources" (NES) is also deduced. In Sec. III we discuss the geometrical aspect of the multisource model, and show how to calculate the NES distributions for different projectile-target combinations. In Sec. IV we show the calculated transverse-energy distributions. The incident energies and the projectile-target combinations in the calculated examples are chosen in such a way that the obtained results can be directly compared either with the presently available data or with those expected in the near future. In Sec. V we discuss a method of calculating transverse-energy distributions in other kinematical regions—in particular in limited η and ϕ (pseudorapidity and azimuthal angle) intervals. Some concluding remarks are made in Sec. VI.

II. MULTISOURCE MODEL AND NES (THE NUMBER OF EFFECTIVE SOURCES)

A model—the multisource model (MSM)—has been proposed[8] recently to describe multiparticle production in high-energy hadron-nucleus and nucleus-nucleus collisions. According to this model, hadrons observed in the central rapidity (midrapidity) region in high-energy nucleus-nucleus collisions are decay products of the central systems (C^*'s) formed by the effective-projectile (EP)–effective-target (ET) pairs. The multiplicity of charged hadrons and the transverse-energy flow in the midrapidity region in such a collision event are directly related to the materialization energy (or excitation energy) E_C^* of the system C^* formed in that event. In fact, it is envisaged that every nucleon in the EP-ET pair may donate part of its kinetic energy to form the system C^* when the colliding EP-ET pair "go through" each other (and become the corresponding "leading particles"). In other words, each nucleon in the colliding EP-ET pair may act as an energy source in the formation of the system C^*; and the amount of materialization energy each source contributes is random. Thus the materialization energy E_C^* of the system C^* is the sum of μ random variables, where μ is an integer less or equal to the total number of nucleons in the EP-ET pair. That is, $\mu \le \nu_{\rm EP}+\nu_{\rm ET}$ where $\nu_{\rm EP}$ and $\nu_{\rm ET}$ are the number of nucleons in the EP and ET, respectively.

The multisource model is an extension of the statistical model for nondiffractive hadron-hadron collisions in which the hadrons observed in the central rapidity region are produced via the central system C^* formed by two energy sources.[9] In fact, the multisource model for nucleus-nucleus collisions reduces to the corresponding model for hadron-hadron collisions when we identify the incident hadron with the EP and the target hadron with

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the ET. In this connection, it is useful to recall the following.

(I) A central emitting system C^* is formed in every event of nondiffractive hadron-hadron collision at sufficiently high energies. In such an event, the two colliding hadrons act as two energy sources in the formation process of the C^* system [we shall from now on call it $C^*(2)$ in order to indicate the number of sources]. Each of them may donate a *random* amount of its kinetic energy [in the hadron-hadron c.m.-system (c.m.s) frame] to create the system $C^*(2)$. The materialization energies donated by each of the two energy sources are *not* associated with any specific intrinsic quantum number which could maintain the identity of these two parts after the system $C^*(2)$ is formed. That is, the state of the system $C^*(2)$ after its formation is characterized by the total amount of materialization energy E_C^*, but *not* by the individual contributions from the two sources. In this sense, the system $C^*(2)$ "forgets its history" after it is created. Hence, in the proposed model,[9] the relevant quantities which characterize the particle production process in the midrapidity region of nondiffractive hadron-hadron collisions are the following: first, the integer 2, the number of sources which contribute materialization energy E_C^* to the system $C^*(2)$; second, the quantity $\langle E_C^*|2\rangle$, the mean value of the materialization energy E_C^* in the system $C^*(2)$. As we have shown in Ref. 9 (here, we use the same formulation as that in the second paper—in terms of materialization energy) the probability $P_C(E_C^*|2)$ for the system $C^*(2)$ to be in a state characterized by the materialization energy E_C^* is given by

$$\langle E_C^*|2\rangle P_C(E_C^*|2)=4\frac{E_C^*}{\langle E_C^*|2\rangle}\exp\left[-2\frac{E_C^*}{\langle E_C^*|2\rangle}\right]. \quad (2.1)$$

(II) A high-energy hadron-nucleus collision is a collision between the incident hadron and an effective target which is the group of nucleons along the path of the incident hadron inside the target nucleus. The concept of "effective target" (ET) has been proposed a long time ago;[10] and it is based on the following well-known facts. (a) At sufficiently high incident energies, the overwhelming majority of hadron-hadron collisions are inelastic, where in general many particles are produced. This, taken together with the fact that the average multiplicity of high-energy hadron-nucleus collisions depends only very weakly on the mass number of the nucleus, strongly suggests that the reaction time for multiparticle production in hadron-hadron reactions is very long, much longer than the time interval a high-energy hadron needs to "go through" a nucleon inside the target nucleus. (b) The nuclear force is of short range; and the average binding energy between the nucleons inside the target-nucleus is negligibly small compared to the kinetic energy of the incident hadron. This is why in high-energy hadron-nucleus reactions only the nucleons along the path of the incident hadron inside the nucleus actively take part in the collision process. The group of these nucleons is called the effective target (ET). It should be emphasized that the ET is *not* a "coherent tube of hadronic matter" which has been suggested by other authors.[11] This is because the ET usually acts gently/softly with the incident hadron and in such cases it acts as a group of individual nucleons. It can be considered as a coherent tube only when it collides violently with the incident hadron. This implies in particular that each of the collisions between the incident hadron and the $\nu_{\rm ET}$ nucleons in the ET can be either diffractive or nondiffractive. We are interested only in the nondiffractive collisions between the incident hadron and the nucleons in the ET, because central emitting systems (which contribute to the midrapidity region) are formed only in nondiffractive collisions. If the average chance for a hadron-nucleon collision to be nondiffractive at the given incident energy is p, the chance of having ν nondiffractive collisions among the $\nu_{\rm ET}$ hadron-nucleon collisions in this hadron-ET reaction is given by a binomial distribution

$$B(\nu_{\rm ET},\nu;p)=\begin{pmatrix}\nu_{\rm ET}\\ \nu\end{pmatrix}p^{\nu}(1-p)^{\nu_{\rm ET}-\nu}. \quad (2.2)$$

(III) In a hadron-ET collision event in which a central system $C^*(1+\nu)$ is formed due to nondiffractive interaction between the incident hadron and ν (of the $\nu_{\rm ET}$) nucleons in the ET, the formation process takes place as follows. The incident hadron and the ν nucleons act as $1+\nu$ energy sources. During the interaction, each of the $1+\nu$ sources contributes part of its kinetic energy to form the system $C^*(1+\nu)$; and the amount of materialization energy each of the $1+\nu$ sources donates is random. After its formation, the system forgets its history in the sense that the materialization energy donated by each of the $1+\nu$ sources is *not* associated with any intrinsic quantum number, so that it is not possible to identify the amount of the individual contributions. That is, the state of the system $C^*(1+\nu)$ is characterized by the total amount of materialization energy E_C^* contributed from the $1+\nu$ sources. Note that E_C^* is the sum of $1+\nu$ random variables and hence it is itself a random variable. Therefore, the relevant quantities which characterize the particle production in the midrapidity region in hadron-ET collisions, in which the incident hadron and ν of the nucleons in the ET interact nondiffractively, are first, the integer $1+\nu$, the number of sources which contribute materialization energy E_C^* to the system $C^*(1+\nu)$ and second, the quantity $\langle E_C^*|1+\nu\rangle$ which is the mean value of E_C^* in the system $C^*(1+\nu)$. As we have already pointed out in Ref. 8, the probability $P_C(E_C^*|1+\nu)$ that the system $C^*(1+\nu)$ is in a state characterized by materialization energy E_C^* is

$$\begin{aligned}\langle E_C^*|1+\nu\rangle P_C(E_C^*|1+\nu)\\ &=\frac{(1+\nu)^{1+\nu}}{\Gamma(1+\nu)}\left[\frac{E_C^*}{\langle E_C^*|1+\nu\rangle}\right]^{\nu}\\ &\quad\times\exp\left[-(1+\nu)\frac{E_C^*}{\langle E_C^*|1+\nu\rangle}\right]. \quad (2.3)\end{aligned}$$

Furthermore, we have also shown[8] that

$$\frac{\langle E_C^*|1+\nu\rangle}{1+\nu}=\frac{\langle E_C^*|2\rangle}{2} \quad (2.4)$$

is valid, provided that the kinetic energy of the $1+\nu$ sources are the same *where the materialization process takes place.* This condition is exactly satisfied in the hadron-nucleon c.m.s. frame, where the kinetic energy of all $1+\nu$ nucleons are the same. Furthermore, it is expected to be valid also in other Lorentz frames in which the kinetic energies of the $1+\nu$ sources are approximately the same. This is because, compared to its kinetic energy, the amount of energy each of the $1+\nu$ hadrons donates to form the system $C^*(1+\nu)$ is in general an extremely small quantity. By inserting Eq. (2.4) into Eq. (2.3) we obtain

$$P_C(E_C^*|1+\nu)=\frac{1}{\Gamma(1+\nu)}\left[\frac{2}{\langle E_C^*|2\rangle}\right]^{1+\nu}(E_C^*)^{\nu}\times\exp\left[-\frac{2}{\langle E_C^*|2\rangle}E_C^*\right]. \tag{2.5}$$

(IV) The generalization of the multisource model to include nucleus-nucleus collisions is made by taking the "effective projectiles" (EP's) into account. The concept of EP's has been introduced[10] in the same way as that for ET's. (In order to see the similarity, consider the case in which a high-energy heavy ion hits a proton target.) A nucleus-nucleus collision at sufficiently high energies is, according to this picture, the simultaneous collisions of a large set of EP-ET pairs. Since we are interested only in the production processes in the midrapidity region, we consider only the *nondiffractive* processes between the nucleons in the EP-ET pairs. Every EP-ET collision event is associated with a number μ, the number of nucleons that take part in nondiffractive collisions between the ν_{EP} nucleons in the EP and the ν_{ET} nucleons in the corresponding ET. The number μ is obviously either zero (which means that there are no nondiffractive collisions) or an integer between 2 and $\nu_{\mathrm{EP}}+\nu_{\mathrm{ET}}$. For a given EP-ET pair, the probability of obtaining μ nondiffractive nucleon-nucleon collisions is

$$W(\mu|\nu_{\mathrm{EP}},\nu_{\mathrm{ET}})p^{\mu}(1-p)^{\nu_{\mathrm{EP}}+\nu_{\mathrm{ET}}-\mu},$$

where

$$\begin{aligned}W(\mu|\nu_{\mathrm{EP}},\nu_{\mathrm{ET}})&=\sum_{\sigma=1}^{\mu-1}\begin{bmatrix}\nu_{\mathrm{EP}}\\ \sigma\end{bmatrix}\begin{bmatrix}\nu_{\mathrm{ET}}\\ \mu-\sigma\end{bmatrix}\\&=\begin{bmatrix}\nu_{\mathrm{EP}}+\nu_{\mathrm{ET}}\\ \mu\end{bmatrix}-\begin{bmatrix}\nu_{\mathrm{EP}}\\ \mu\end{bmatrix}-\begin{bmatrix}\nu_{\mathrm{ET}}\\ \mu\end{bmatrix}\\&\quad+\delta_{\mu,0}\,.\end{aligned} \tag{2.6}$$

This shows in particular that the system depends only on the number μ, but not on the specific way of composition. (That is, it does not matter how many of the μ nucleons are from the EP and how many of them are from the ET.)

(V) The formation process of the central system $C^*(\mu)$ due to the nondiffractive processes among μ nucleons in a EP-ET pair takes place as follows: The μ nucleons act as μ energy sources. During the interaction, every one of the μ sources donates part of its kinetic energy for the formation of the system $C^*(\mu)$; and the amount of materialization energy each of the μ sources donates is *random.* After it is formed, the system $C^*(\mu)$ forgets its history in the sense that the state is characterized by the total materialization energy E_C^* in $C^*(\mu)$ but not by the μ individual contributions of the μ sources. In fact, the probability $P_C(E_C^*|\mu)$ that the system $C^*(\mu)$ is in a state characterized by E_C^* is again given by Eq. (2.3) where the following replacements should be made: "E_C^* in $C^*(1+\nu)$" by "E_C^* in $C^*(\mu)$," and $1+\nu$ by μ. The relationship corresponding to that in Eq. (2.4) now reads

$$\frac{\langle E_C^*|\mu\rangle}{\mu}=\frac{\langle E_C^*|2\rangle}{2}, \tag{2.7}$$

which is exactly satisfied in the nucleon-nucleon c.m.s. frame in which all the ν participating nucleons have the same kinetic energy; and it should be valid in all reference frames in which the kinetic energies of the above-mentioned nucleons are approximately the same. (See the discussion in the hadron-nucleus case and that in Ref. 8.) Note that the quantity given in Eq. (2.7) $\langle E_C^*|\mu\rangle/\mu \equiv\langle e_C^*\rangle$ [which is identical to that given in Eq. (2.4)] is nothing else but the average materialization energy per energy source. It implies in particular that the average materialization energy per source $\langle e_C^*\rangle$ depends only on $\sqrt{s}$ but not on the number of sources.

By inserting Eq. (2.7) into Eq. (2.5) we obtain the probability $P_C(E_C^*|\mu)$ that the system $C^*(\nu)$ is in a state characterized by the materialization energy $E_C^*(\mu)$:

$$P_C(E_C^*|\mu)=\frac{1}{\Gamma(\mu)}\left[\frac{2}{\langle E_C^*|2\rangle}\right]^{\mu}E_C^{*\mu-1}\times\exp\left[-\frac{2}{\langle E_C^*|2\rangle}E_C^*\right]. \tag{2.8}$$

(VI) It should be emphasized that the multisource model gives a macroscopic, rather than a microscopic, description of the multiparticle production processes in high-energy nuclear reactions. The relevant quantities in our model are the number of sources which contribute in nondiffractive h-ET and EP-ET collisions, and the average amount of materialization energy $(\langle E_c^*|\mu\rangle/\mu)$ each source contributes in the formation of the central system. This is in sharp contrast to the Glauber-type multiple scattering/cascading models in which the number of successive scattering/cascading occurrences is relevant and the experimental hadron-hadron cross sections at corresponding energies enter as dynamical input.

$$m=E_C^*/\langle e_C^*\rangle \tag{2.9}$$

for a given E_C^* in system $C^*(\mu)$ is of particular interest. This is because m is uniquely associated with the materialization energy E_C^* in $C^*(\mu)$, and it shows how many sources there would be if each source contributes the same amount, $\langle e_C^*\rangle$. From now on, we call the quantity m "the number of effective sources" (NES). In terms of m, Eq. (2.8) reads

$$\Omega(m|\mu)=\frac{1}{\Gamma(\mu)}m^{\mu-1}\exp(-m), \tag{2.10}$$

where we have introduced the "NES distribution" $\Omega(m|\mu)$ by

$$\Omega(m|\mu)=\langle e_C^* \rangle P_C(E_C^*|\mu) \ . \tag{2.11}$$

The NES distribution given in Eq. (2.10) has several interesting properties.

(a) For a given μ, $\Omega(m|\mu)$ is a special case of Γ distribution in m. The expected value of $\Omega(m|\mu)$ is μ, and the variance of $\Omega(m|\mu)$ is 2μ. That is, the fluctuation of NES increases twice as fast as the mean values of NES—while the mean value itself is the source number μ.

(b) For a given m, $\Omega(m|\mu)$ is nothing else but a Poisson distribution in μ. This simple relation can, for example, be used to find the source number μ by measuring the NES distribution in hadron-nucleus reactions (see also the discussion in Sec. VI).

(c) The convolution of the two NES distributions, $\Omega(m_1|\mu_1)$ and $\Omega(m_2|\mu_2)$ characterized by the source numbers μ_1 and μ_2, is again a NES distribution. The corresponding source number is $\mu_1+\mu_2$. That is,

$$\int dm_1 dm_2 \delta(m-m_1-m_2)\Omega(m_1|\mu_1)\Omega(m_2|\mu_2) = \Omega(m|\mu_1+\mu_2) \ . \tag{2.12}$$

(d) For a given value of the source number μ, the distribution for m is always $\Omega(m|\mu)$. It is independent of the colliding EP-ET pair, and independent of the colliding energy. This, as we shall see in Secs. III and IV, leads to a new kind of scaling behavior. We shall call this hereafter the "NES scaling." We note that, not only the concept of NES, but also the NES distribution and the existence of NES scaling are natural consequences of the multisource model for hadron-nucleus and nucleus-nucleus collisions.[8]

III. MULTISOURCE MODEL AND GEOMETRICAL AVERAGE OF NES DISTRIBUTION

The geometry of the nuclei plays an important role in high-energy nucleus-nucleus collisions in general, and in the multisource model in particular.

(i) It has been observed experimentally[1,5] that the square root of the total inelastic cross section of a high-energy nucleus-nucleus collision is approximately proportional to $A_P^{1/3}+A_T^{1/3}$, where A_P and A_T are the mass numbers of the projectile and target nucleus, respectively. This shows it is a good approximation to consider such reactions as two colliding spheres of radii $A_P^{1/3}r_0$ and $A_T^{1/3}r_0$ where r_0 is the nuclear unit radius.[12]

(ii) Effective target (ET) and effective projectile (EP) have been introduced[10] to describe high-energy nucleus-nucleus collisions in a physical picture which is based on the following empirical and/or semiempirical knowledge. The nuclear force is of short range, and the average binding energy between two nucleons is only 8 MeV. The reaction time for multiparticle production processes is much longer than the time needed for a hadron at high incident energies (say ≥ 10 GeV/A) to travel a few fermis in the nucleus. Geometrically, EP's and ET's which interact gently/softly with each other are groups of individual nucleons; and the nucleons in each group stand more or less in a row, so that all EP's and ET's can be considered as nucleons inside cylinder-form envelopes. The axis of the cylinders are parallel to the incident axis. The cross section of the cylinders are approximately πr_0^2 and the length is typically a few times r_0, depending of course on the size of the nucleus and the position of the cylinders in the nucleus. The positions of the interacting EP-ET pairs, and the number of nucleons in each EP or ET is determined by the geometry of the nuclei in collision. (See Appendix A for details.)

We consider the plane in which the straight line joining the centers of the two colliding nuclei lies in the plane perpendicular to the collision axis. We call the distance b between the two centers in this plane the impact parameter of the corresponding nucleus-nucleus collision events. Since the impact parameter b characterizes the part of the projectile and that of the target which go through each other, we first estimate the number of interacting EP-ET pairs, λ as well as the number of nucleons in each EP and each ET ($\nu_{\mathrm{EP}i}, \nu_{\mathrm{ET}i}$; $i=1,\ldots,\lambda$) for fixed b (note that all these quantities depend on the impact parameter b). In doing so, we assume for the sake of simplicity that the nucleons are homogeneously distributed in nuclei and that every EP-ET collision is a head-on collision. The positions of the λ EP-ET pairs are randomly chosen; and this is performed by using a Monte Carlo program. The details of this calculation are described in Appendix A.

The NES (number of effective sources) distribution $\Omega(m,b|A_P,A_T)$ for a given impact parameter b in collision processes between nuclei of mass numbers A_P and A_T is

$$\Omega(m;b|A_P,A_T)=N \sum_{m_1,\ldots,m_\lambda} \delta\left[m, \sum_{i=1}^{\lambda} m_i\right] \prod_{i=1}^{\lambda} \sideset{}{'}\sum_{\mu_i=0}^{\nu_{\mathrm{EP}i}+\nu_{\mathrm{ET}i}} W(\mu_i|\nu_{\mathrm{EP}i},\nu_{\mathrm{ET}i}) p^{\mu_i}(1-p)^{\nu_{\mathrm{EP}i}+\nu_{\mathrm{ET}i}-\mu_i}\Omega(m_i|\mu_i) \ . \tag{3.1}$$

Here, m_i is the NES in the ith EP-ET pair ($\nu_{\mathrm{EP}i}$ nucleons in the EP and $\nu_{\mathrm{ET}i}$ nucleons in the ET); $\lambda=\lambda(b)$ is the number of EP-ET pairs for the given impact parameter b; $\delta(m,\sum m_i)=1$ for $m=\sum m_i$, and 0 otherwise; and the functions $W(\mu_i|\nu_{\mathrm{EP}i},\nu_{\mathrm{ET}i})$ and $\Omega(m_i|\mu_i)$ are defined in Eqs. (2.6) and (2.10), respectively; N is a normalization constant. The prime on the summation sign means that all those terms in which one (or more) of the μ_i's ($i=1,\ldots,\lambda$) is equal to unity, and the term in which $\mu_1=\cdots=\mu_\lambda=0$ should be excluded. For a nondiffractive nucleon-nucleon process in a colliding EP-ET pair to take place, there should be at least *two* nucleons and hence there should be at least *two* energy sources. Furthermore, the $\mu_1 = \cdots = \mu_\lambda = 0$ term corresponds to a process with *no* central emitting system C^*, and thus the process is *not* a nondiffractive one.

By using the simple properties of $\Omega(m_i|\mu_i)$ mentioned in Sec. II, we can rewrite Eq. (3.1) in the form

$$\Omega(m;b|A_P,A_T) = N \sum_{l=1}^{\lambda}{}' \left\{ \sum_{\mu_1=2}^{\nu_{EP1}+\nu_{ET1}} \cdots \sum_{\mu_l=2}^{\nu_{EPl}+\nu_{ETl}} \prod_{i=1}^{l} W(\mu|\nu_{EPi},\nu_{ETi}) p^{\mu}(1-p)^{\nu-\mu}\Omega(m|\mu) \right\}, \tag{3.2}$$

where μ and ν on the right-hand side of this equation are, respectively,

$$\mu = \sum_{i=1}^{l} \mu_i \tag{3.3}$$

and

$$\nu = \sum_{i=1}^{l} (\nu_{EPi} + \nu_{ETi}) . \tag{3.4}$$

The prime on the summation sign in Eq. (3.2) means the following. Choose any l out of the λ EP-ET pairs; calculate the expression inside the large parentheses; sum over all the possibilities in choosing l from the λ pairs; and then sum over l. The right-hand side of Eq. (3.2) is calculated numerically. Details on this are discussed in Appendix B. We now take the NES distribution $\Omega(m;b|A_P,A_T)$ for a fixed impact parameter b from Eq. (3.1) [or from Eq. (B1)], average it over all possible b values, and thus obtain the corresponding NES distribution $\Omega(m|A_P,A_T)$ for colliding nuclei of mass numbers A_P and A_T:

$$\Omega(m|A_P,A_T) = \int_0^{b_{\max}} \Omega(m;b|A_P,A_T) b\, db \Big/ \int_0^{b_{\max}} b\, db , \tag{3.5}$$

where $b_{\max}$ is chosen to be $(A_P^{1/3} + A_T^{1/3} - 1)r_0$, because only the nonperipheral collisions are taken into account. [See (i) in this section and (d) in Sec. IV for further details.] As examples, we have calculated $\Omega(m|A_P,A_T)$ for the following projectile-target combinations: ^{16}O ion on Au, Ag, Cu, and Al targets; and ^{32}S ion on Au, Ag, RbBr, Cu, Sc, and S targets.[1] The results are shown in Figs. 1 and 2, respectively.

We note that, while the NES distribution $\Omega(m|\mu)$ for a fixed source number μ has a simple analytic form, given in Eq. (2.10), the NES distribution $\Omega(m|A_P,A_T)$ for a fixed projectile-target (of mass numbers A_P and A_T, respectively) combination is more useful in practice. In fact, as we shall see in Sec. IV, $\Omega(m|\mu)$ and $\Omega(m|A_P,A_T)$ have a number of interesting properties.

IV. MULTISOURCE MODEL AND MIDRAPIDITY TRANSVERSE-ENERGY DISTRIBUTIONS

In this section we show how to apply the multisource model (MSM) to calculate transverse-energy distributions, and why the NES concept is useful.

According to the MSM, transverse energies produced in the midrapidity region are due to the central systems which are formed when the nucleons in the effective projectiles (EP's) and the effective target (ET's) go through one another in nondiffractive EP-ET collision events. Such an event is characterized by the number of nucleons which take part in the nondiffractive process. That is, in an event which μ nucleons interact nondiffractively, μ energy sources take part in the formation process of the corresponding emitting central system $C^*(\mu)$. In this sense, the system $C^*(\mu)$ remembers its history. The system $C^*(\mu)$ consists of a group of (in the midrapidity region randomly distributed) clusters which subsequently decay. The properties of such hadronic clusters are assumed to be exactly the same as those in hadron-hadron collisions. In particular, the clusters are assumed to be charge neutral, distributed with equal probability in rapididty space, and to decay isotropically on the average into two or three hadrons (details can be found in the

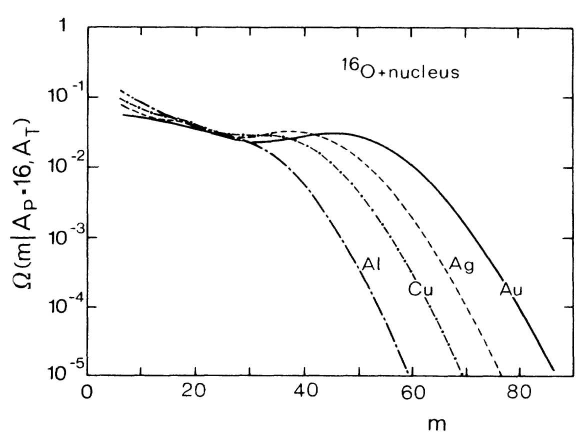

FIG. 1. NES distributions for ^{16}O ion on different nuclear targets.

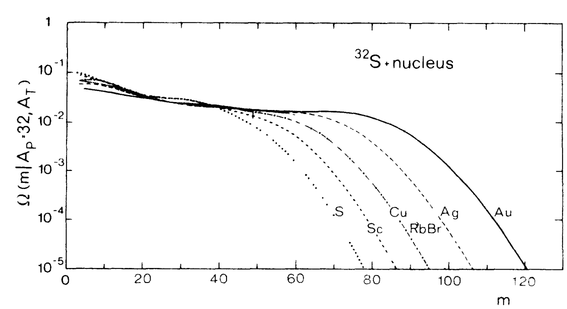

FIG. 2. NES distributions for ^{32}S ion on different nuclear targets.

third paper of Ref. 9). This simple dynamical assumption is based in particular on the empirical fact that the short-range rapidity correlations between the observed hadrons in nuclear reactions are more or less the same as those in hadron-hadron collisions at comparable incident energies. (See, e.g., Ref. 8 and the references given therein.) The sum of the cluster masses is called the materialization energy E_C^* of the system $C^*(\mu)$. The amount of materialization energy each of the μ sources contributes to form $C^*(\mu)$ is random, so that E_C^* is the sum of μ random variables. Viewed from the nucleon-nucleon c.m.s. frame, the average amount of energy each one of the μ sources contributes is the same; it is $\langle e_C^* \rangle = \langle E_C^* | \mu \rangle / \mu$. Furthermore, in processes in which the kinetic energy of each of the μ nucleons is the same (this is clearly the case in the nucleon-nucleon c.m.s. frame of EP-ET collisions), $\langle e_C^* \rangle$ is independent of μ. Hence, the number of effective sources (NES) m defined as $E_C^* / \langle e_C^* \rangle$ is a useful quantity in characterizing the nondiffractive EP-ET collision events.

Now, since in this picture,[8] the transverse energy E_t produced in the midrapidity region is due to the decay of clusters formed in $C^*(\mu)$, and the corresponding materialization energy E_C^* is nothing else but the sum of the masses of such clusters, it is expected that E_t and E_C^* are directly proportional to each other. That is,

$$\frac{E_t}{\langle E_t | \mu \rangle} = \frac{E_C^*}{\langle E_C^* | \mu \rangle} . \tag{4.1}$$

From Eqs. (2.7), (2.9), and (4.1), we have

$$\frac{m}{\mu} = \frac{E_t}{\langle E_t | \mu \rangle} , \tag{4.2}$$

$$m = E_t / \langle e_t \rangle . \tag{4.3}$$

Here, $\langle e_t \rangle = \langle E_t | \mu \rangle / \mu = \langle E_t(hh) \rangle / 2$ is the average transverse energy each energy source contributes, where $\langle E_t(hh) \rangle$ is the average transverse energy in hadron-hadron collisions at the same (hadron-hadron c.m.s.) colliding energy. Hence, the corresponding E_t distribution can be obtained by inserting Eq. (4.3) into Eqs. (2.9) and (2.10):

$$P_C(E_t | \mu) = \frac{1}{\langle e_t \rangle} \Omega \left[\frac{E_t}{\langle e_t \rangle} \middle| \mu \right] . \tag{4.4}$$

Furthermore, since the clusters formed in the central system $C^*(\mu)$ are assumed to have approximately the same size, which is in good agreement with the empirical fact that the multiplicity n of charged hadrons is approximately proportional to the transverse energy E_t in the midrapidity region, we have

$$\frac{n}{\langle n | \mu \rangle} = \frac{E_C^*}{\langle E_C^* | \mu \rangle} . \tag{4.5}$$

Taken together with the fact that the electromagnetic part of the transverse energy is predominantly due to π^0 decay and most of the produced particles are pions, isospin invariance implies that a similar relationship should be valid also for the electromagnetic part of the transverse energy E_t^{em} in the midrapidity region:

$$\frac{E_t^{\text{em}}}{\langle E_t^{\text{em}} | \mu \rangle} = \frac{E_C^*}{\langle E_C^* | \mu \rangle} . \tag{4.6}$$

Hence, it follows from Eqs. (2.7), (2.9), (4.5), and (4.6) that the NES m can also be written as

$$m = n / \langle n_u \rangle = E_t^{\text{em}} / \langle e_t^{\text{em}} \rangle , \tag{4.7}$$

where $\langle n_u \rangle = \langle n | \mu \rangle / \mu = \langle n(hh) \rangle / 2$ and

$$\langle e_t^{\text{em}} \rangle = \langle E_t^{\text{em}} | \mu \rangle / \mu = \langle E_t^{\text{em}}(hh) \rangle / 2$$

are the average multiplicity of charged hadrons and the average value of the electromagnetic part of the transverse energy per energy source at the same incident energy (per nucleon for nuclear reactions), respectively. This means, the corresponding n and the E_t^{em} distribution $P_C(n|\mu)$ and $P_C(E_t^{\text{em}}|\mu)$ are nothing else but

$$P_C(n | \mu) = \frac{1}{\langle n_u \rangle} \Omega \left[\frac{n}{\langle n_u \rangle} \middle| \mu \right] \tag{4.8}$$

and

$$P_C(E_t^{\text{em}} | \mu) = \frac{1}{\langle e_t^{\text{em}} \rangle} \Omega \left[\frac{E_t^{\text{em}}}{\langle e_t^{\text{em}} \rangle} \middle| \mu \right] , \tag{4.9}$$

respectively. That is to say, once we know the NES distribution, Eq. (2.10), the corresponding distributions for E_t n, and E_t^{em} can be immediately obtained, provided that the corresponding average values $\langle e_t \rangle$, $\langle n_u \rangle$, and $\langle e_t^{\text{em}} \rangle$ are known.

It should be pointed out that, since the average values $\langle e_t \rangle$, $\langle n_u \rangle$, and $\langle e_t^{\text{em}} \rangle$ do not depend on μ, they are *independent* of the geometrical average mentioned in Sec. III. This means that for a given projectile-target combination characterized by the mass numbers A_P and A_T, we have a *unique* NES distribution $\Omega(m | A_P, A_T)$. The corresponding E_t, n, and E_t^{em} distributions for this projectile-target combination can be calculated from $\Omega(m | A_P, A_T)$ simply by inserting the average values $\langle e_t \rangle$, $\langle n_u \rangle$, and $\langle e_t^{\text{em}} \rangle$ in $\Omega(m | A_P, A_T)$ as given in Eq. (3.5).

Examples of such NES distributions for different projectile-target combinations have been shown in Figs. 1 and 2. By using the above-mentioned procedures, E_t and E_t^{em} distributions for the projectile-target combinations O-Au, O-Ag, O-Cu, and O-Al at 200 GeV/nucleon and those at 60 GeV/nucleon have been calculated. The results are shown in Figs. 3–6 together with the new NA35 data.[1]

In these calculations, the values $\langle e_t \rangle$ and $\langle e_t^{\text{em}} \rangle$ for the two incident energies 200 and 60 GeV/nucleon, have been determined by fitting the ^{16}O on Au data at these two energies, respectively. The values are as follows:

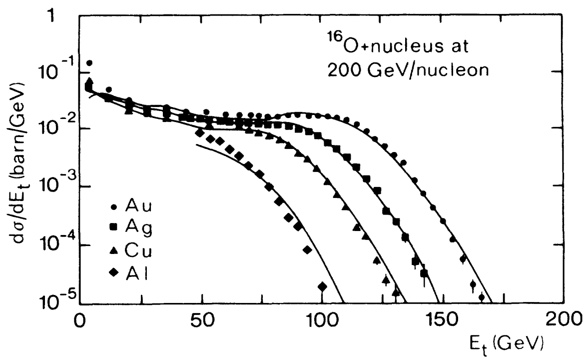

FIG. 3. E_t (total transverse energy) distributions for ^{16}O ion at 200 GeV/nucleon on different nuclear targets. The data are taken from Ref. 1.

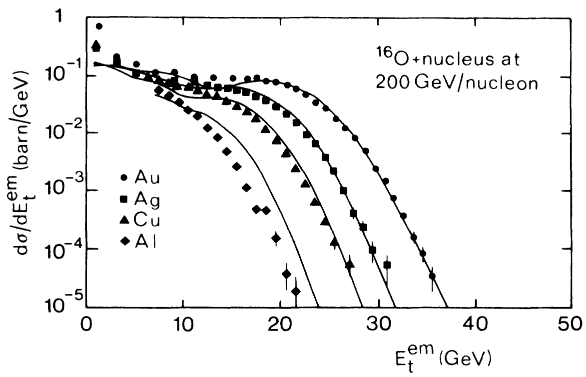

FIG. 5. E_t^{em} (the electromagnetic part of the measured transverse energy) distributions for ^{16}O ion at 200 GeV/nucleon on different nuclear targets. The data are taken from Ref. 1.

Incident energy (GeV/nucleon)	$\langle e_t \rangle$ (GeV)	$\langle e_t^{\text{em}} \rangle$ (GeV)
200	2.00	0.41
60	1.23	0.28

These values are then applied to all other reactions. It is interesting to note the following.

(a) The ratio between $\langle e_t \rangle$ at 200 GeV/nucleon and that at 60 GeV/nucleon is approximately the same as the $\langle e_t^{\text{em}} \rangle$ ratio between these two energies. This is expected to be the case, provided that the composition of the produced particles does not change very much when the incident energy is increased from 60 to 200 GeV/nucleon.

(b) The ratio between $\langle e_t^{\text{em}} \rangle$ and $\langle e_t \rangle$ at a given incident energy is less than $\frac{1}{3}$, which would be the case, if only pions are produced. This result seems to show that quite a number of baryon-antibaryon pairs are created at these incident energies.

(c) Because of the close relationship[13] between the impact parameter and the multiplicity in high-energy hadron-hadron collisions, and the fact that crude approximations have been made in carrying out the geometrical averaging process of our calculation, it is expected that $\langle e_t \rangle$ and $\langle e_t^{\text{em}} \rangle$ determined from nucleus-nucleus collision data should be higher than those in hadron-hadron collisions at the same energy. We recall (see Secs. II and III) that in determining the number of nucleons in the EP and in the ET, we have assumed that the nucleons are homogeneously distributed in the nuclei, and that the EP-ET collisions are always head-on collisions. This means, we have included in our calculation only those events associated with small impact parameters in nucleon-nucleon collisions; and hence the average multiplicity and the average transverse energy should be higher than those in the corresponding minimum-bias nucleon-nucleon collision events.

(d) For reasons similar to those mentioned in paragraph (c), especially because we only considered the nonperipheral collisions, it is not difficult to see that the values of $\langle e_t \rangle$ and $\langle e_t^{\text{em}} \rangle$ for heavier nuclei should be slightly higher.

The absolute normalization for $d\sigma/dE_t$ and for $d\sigma/dE_t^{\text{em}}$ at different incident energies (E_{inc}) and for different projectile-target (of mass numbers A_P and A_T, respectively) combinations are determined once and for all by the following conditions:

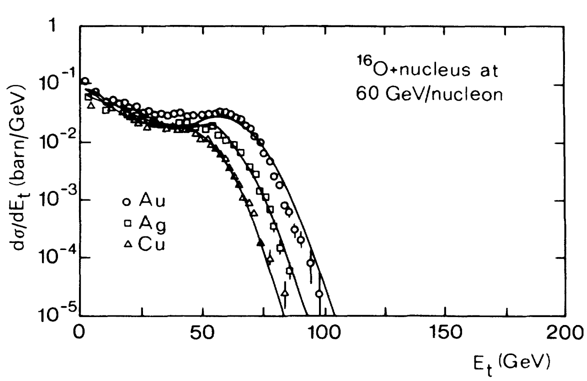

FIG. 4. E_t (total transverse energy) distributions for ^{16}O ion at 60 GeV/nucleon on different nuclear targets. The data are taken from Ref. 1.

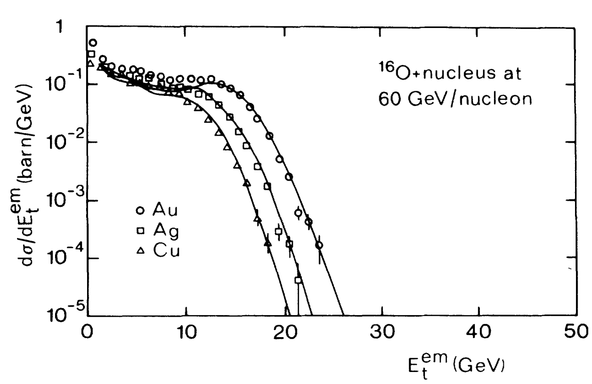

FIG. 6. E_t^{em} (the electromagnetic part of the measured transverse energy) distributions for ^{16}O ion at 60 GeV/nucleon on different nuclear targets. The data are taken from Ref. 1.

$$\int dE_t \frac{d\sigma}{dE_t}(E_{\rm inc};A_P,A_T)=\int dE_t^{\rm em}\frac{d\sigma}{dE_t^{\rm em}}(E_{\rm inc};A_P,A_T)$$
$$=\sigma_{\rm nonperi}(E_{\rm inc};A_P,A_T)$$
$$=\pi r_0^2(A_P^{1/3}+A_T^{1/3}-1)^2\ , \tag{4.10}$$

where $r_0=1.2$ fm is the unit nuclear radius;[12] and $\sigma_{\rm nonperi}$ stands for the inelastic cross section for nonperipheral collisions. The term -1 in the parentheses on the last line of this equation is due to the fact that peripheral collision events are excluded.

Note that the equations in (4.10) imply the following. (i) The integrated cross sections for E_t and $E_t^{\rm em}$ distributions are the same for nonperipheral nucleus-nucleus collisions. (ii) This part of the inelastic cross section (denoted by $\sigma_{\rm nonperi}$) is independent of the incident energy. (iii) It is the geometrical area of a circle of radius $(A_P^{1/3}+A_T^{1/3}-1)r_0$. (iv) Nothing has been said about the peripheral nucleus-nucleus collisions which contribute much to the total inelastic cross section.

In Figs. 7 and 8 we show the effect of the geometrical averaging.

The existence of a unique NES distribution for each given projectile-target nuclei pair, and the fact that the E_t and $E_t^{\rm em}$ distributions calculated from the NES distribution agree with the data[1] suggest that there is indeed a new kind of scaling—NES scaling. In order to check the validity of NES scaling in practice, one can plot the quantities at different incident energies shown in the following:

$$\langle e_t\rangle \frac{1}{\sigma}\frac{d\sigma}{dE_t} \quad \text{vs} \quad \frac{E_t}{\langle e_t\rangle}\ ,$$

$$\langle e_t^{\rm em}\rangle \frac{1}{\sigma}\frac{d\sigma}{dE_t^{\rm em}} \quad \text{vs} \quad \frac{E_t^{\rm em}}{\langle e_t^{\rm em}\rangle}\ ,$$

$$\langle n\rangle \frac{1}{\sigma}\frac{d\sigma}{dn} \quad \text{vs} \quad \frac{n}{\langle n_u\rangle}\ ,$$

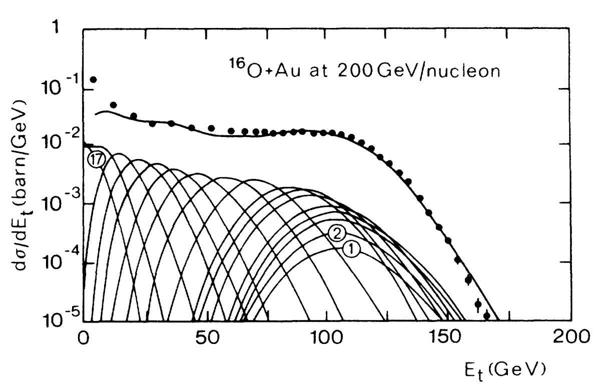

FIG. 7. Dependence of E_t distributions on impact parameter b is demonstrated on the example: 200 GeV/nucleon ^{16}O ion on Au target (already shown in Fig. 3). The curve marked by 1 is the contribution from $b=0.5$ fm, the one marked by 2 is that from $b=1.0$ fm, and the one marked by 17 is that from $b=9.0$ fm. In fact, the difference in b for any two adjacent curves is 0.5 fm.

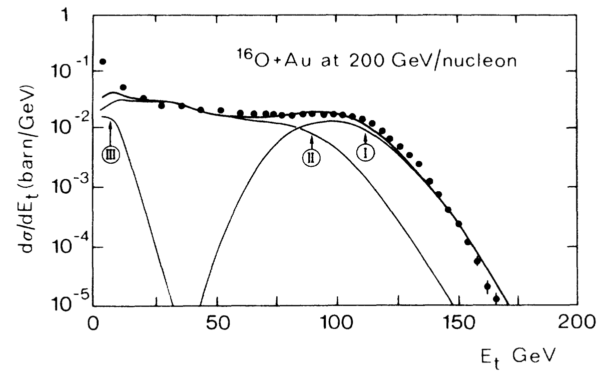

FIG. 8. Dependence of E_t distributions on impact parameter b is demonstrated on the example: 200 GeV/nucleon ^{16}O ion on Au target (already shown in Figs. 3 and 7). The curves marked by I, II, and III are contributions from the b regions: $0\le b<4.0$ fm, $4.0\le b<8.0$ fm, and $8.0\le b<9.0$ fm, respectively.

for a given projectile-target combination. One will find that not only does each of the data set for the same quantity at different incident energies (for example that at 200 GeV/nucleon and that at 60 GeV/nucleon) lie on one curve, but also all three sets (that is, data for different quantities) lie on one and the same curve. The reason why this should be the case is that the three sets of variables on the right-hand sides of the above are only different expressions of the *same* quantity—the number of effective sources (NES)—and the curve is nothing else but the NES distribution for the given projectile-target combination. As an illustration example, we show in Fig. 9 such a plot for O-Au collisions at 200 and 60 GeV/nucleon, where the new NA35 data[1] for E_t and $E_t^{\rm em}$ distributions are used (for details, see figure caption).

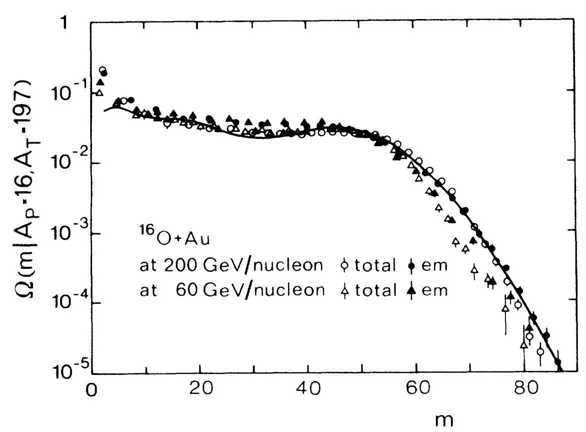

FIG. 9. Test for NES scaling is demonstrated by plotting the E_t distribution and $E_t^{\rm em}$ distribution data for 200- and 60-GeV/nucleon ^{16}O ion on Au target in the following way:

$$\langle e_t\rangle \frac{1}{\sigma}\frac{d\sigma}{dE_t} \quad \text{vs} \quad \frac{E_t}{\langle e_t\rangle},\quad \langle e_t^{\rm em}\rangle \frac{1}{\sigma}\frac{d\sigma}{dE_t^{\rm em}} \quad \text{vs} \quad \frac{E_t^{\rm em}}{\langle e_t^{\rm em}\rangle}\ .$$

Here $d\sigma/dE_t$ and $d\sigma/dE_t^{\rm em}$ are taken from Ref. 1. The values for $\langle e_t\rangle$ and $\langle e_t^{\rm em}\rangle$ are taken from the table given in Sec. IV; σ stands for the integrated cross section which can either be obtained from the data or from Eq. (4.10).

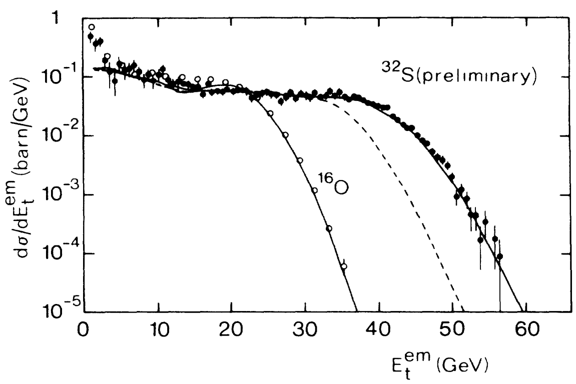

FIG. 10. E_t^{em} distribution for 200-GeV/nucleon ^{16}O ion and ^{32}S ion on Au target. The solid curve for ^{32}S is the calculated result for $\langle e_t^{em} \rangle = 0.48$ GeV while the dashed curve is that for $\langle e_t^{em} \rangle = 0.41$ GeV. The (solid) curve for ^{16}O is identical to that shown in Fig. 4. The points for ^{32}S are the preliminary data of NA35 Collaboration (Ref. 1).

It is clear that transverse-energy distributions for other projectile-target combinations can be obtained in a similar way. As an example, we calculated the E_t^{em} distribution for S on Au at 200 GeV/nucleon and compare the obtained result with the preliminary data of NA35 Collaboration[1] (see Fig. 10). Here, we used the corresponding NES distribution for S-Au from Fig. 2.

We see that the E_t^{em} distribution for S on Au can indeed be reproduced when we set $\langle e_t^{em} \rangle = 0.48$ GeV, where the value for σ_{nonperi} is again calculated from Eq. (4.10). As we have already pointed out, the slight increase in $\langle e_t^{em} \rangle$ is due to the dependence of E_t^{em} on the impact parameter for nucleon-nucleon collisions[13] and the fact that a rather crude approximation has been made in our geometrical calculations. A more detailed study of this and other related problems is now in progress;[14] the result will be published elsewhere.

V. MULTISOURCE MODEL AND TRANSVERSE-ENERGY DISTRIBUTIONS IN DIFFERENT RAPIDITY REGIONS

Until now, we have only considered the kinematical region which is completely covered by the decay products of the clusters in the central system C^*. This kinematical range has been called the midrapidity region, and it corresponds to the rapidity interval in which transverse distributions have been measured by NA35 Collaboration.

In the present section we discuss the following. (a) Is it possible to describe in the framework of the multisource model transverse-energy distributions outside the midrapidity region? (b) How do transverse-energy distributions depend on the size of the rapidity window and/or azimuthal-angle window in which the measurements are made?

Question (a) can be answered as follows: Transverse-energy distributions outside the midrapidity interval for multiparticle production in nucleus-nucleus collisions can also be obtained by generalizing the multisource model for hadron-nucleus collisions. We recall that according to the proposed model,[8] where the basic processes in hadron-nucleus reactions are hadron-ET collisions, and in a hadron-ET collision event in which ν of the ν_{ET} nucleons in the ET take part in nondiffractive processes, the following metastable systems are formed. One C^* system formed by $1+\nu$ energy sources, one P^* system formed by two energy sources, and ν T^* systems, each of which is formed by two energy sources. These systems are independent of one another, so that the transverse-energy distribution for the entire rapidity range can be expressed as the product of the distributions of the system. To be more precise, the transverse-energy distribution in the entire rapidity range can be obtained by noting that the probability that a total amount E^* of energy turns into materialization energy is

$$P(E^*|1+\nu) = \sum{}' P_C(E_C^*|1+\nu) P_P(E_P^*|2) \times P_{T1}(E_{T1}^*|2) \cdots P_{T\nu}(E_{T\nu}^*|2) \ . \qquad (5.1)$$

Here, the prime on the summation sign means that the sum over $E_C^*, E_P^*, E_{T\nu}^*$ should be taken such that the condition

$$E^* = E_C^* + E_P^* + E_{T1}^* + \cdots + E_{T\nu}^* \qquad (5.2)$$

is satisfied. $P_C(E_C^*|1+\nu)$ is the probability that the system $C^*(1+\nu)$ is in a state characterized by the materialization energy E_C^*, where the analytic expression for $P_C(E_C^*|1+\nu)$ is given in Eq. (2.3). $P_C(E_P^*|2)$ and $P_{Ti}(E_{Ti}^*|2)$ $(i=1,\ldots,\nu)$ are, respectively, the probabilities for the systems $P^*(2)$ and $T_i^*(2)$. The corresponding expressions have the same form as $P_C^*(E_C^*|2)$ given in Eq. (2.1).

Hence, $P(E^*|1+\nu)$ can be calculated from Eq. (5.1) provided that the average values $\langle E_C^*|1+\nu \rangle$, $\langle E_P^*|2 \rangle$, $\langle E_{Ti}^*|2 \rangle$ $(i=1,\ldots,\nu)$ are known. We note that $\langle E_C^*|1+\nu \rangle$ is related to $\langle E_C^*|2 \rangle$ through Eq. (2.4), and that $\langle E_P^*|2 \rangle = \langle E_{Ti}^*|2 \rangle$ $(i=1,\ldots,\nu)$ are assumed to be equal to the corresponding values for hadron-hadron collisions at the same incident energy. This means $\langle E_C^*|1+\nu \rangle$ is determined by the integer $1+\nu$, $\langle e_C^* \rangle \equiv \langle E_C^*|2 \rangle /2$, and

$$\langle e_F^* \rangle = \langle E_P^*|2 \rangle /2 = \langle E_{Ti}^*|2 \rangle /2 \quad \text{for } i=1,\ldots,\nu \ . \qquad (5.3)$$

Taken together with the fact that the transverse energy is directly proportional to the materialization energy in the multisource model, transverse-energy distributions in hadron-ET collisions in which the incident hadron and ν of the nucleons in the ET interact nondiffractively, can be readily obtained from Eq. (5.1).

The generalization to EP-ET collisions is made analogously to that for the midrapidity region in which only the C^* system is taken into account. Here, we again consider an EP-ET collision event in which μ among the $\nu_{\text{EP}} + \nu_{\text{ET}}$ nucleons in the EP-ET collision event interact nondiffractively.

The probability that in this process a total amount E^* of energy turns into materialization energy is

$$P(E^*|\mu) = \sum{}' P_C(E_C^*|\mu) Q_F(E_F^*|\mu) \ , \qquad (5.4)$$

where $P_C(E_C^*|\mu)$ is the probability that the system $C^*(\mu)$

is in a state characterized by the materialization energy E_C^*; where the analytic expression for $P_C(E_C^*|\mu)$ is given in Eq. (4.4). $Q_F(E_F^*|\mu)$ is the probability that a total amount of E_F^* energy is materialized in the P^* and T^* systems in the projectile and target fragmentation regions. That is,

$$Q_F(E_F^*|\mu)=P_F(E_{F1}^*|2)\cdots P_F(E_{F\mu}^*|2)\ . \tag{5.5}$$

Here, the index F (fragmentation region) in P_F and in E_{Fi}^* $(i=1,\ldots,\mu)$ may be either P or T, which means it may be either a contribution from a P^* or a contribution from a T^* system. Both $P_P(E_{Pj}^*|2)$ and $P_T(E_{Tk}^*|2)$ (E_{Pj}^* and E_{Tk}^* stand for any two of the $E_{F1}^*,\ldots,E_{F\mu}^*$) have the same analytic form as $P_C(E_C^*|2)$ shown in Eq. (2.1). The average value $\langle E_F^*|2\rangle$ is again $2\langle e_C^*\rangle$ while $\langle E_C^*|2\rangle$ is $2\langle e_C^*\rangle$. The prime on the summation sign on the right-hand side of Eq. (5.4) means that the condition

$$E^*=E_C^*+E_{F1}^*+\cdots+E_{F\mu}^* \tag{5.6}$$

should be satisfied. Hence, $P^*(E^*|\mu)$ is determined by the integer μ, the average materialization energy per energy souce in the fragmentation region $\langle e_F^*\rangle$ and that in the central (midrapidity) region $\langle e_C^*\rangle$.

The transverse-energy distributions and multiplicity distributions for fixed μ are obtained again by taking into account that transverse energies and multiplicities are directly proportional to the corresponding materialization energies.

Transverse-energy distributions and multipicity distributions for given projectile-target (of mass numbers A_P and A_T, respectively) combinations are obtained by first calculating the combinatorial factor for a given EP-ET pair with $\nu_{\rm EP}+\nu_{\rm ET}$ nucleons and then carrying out a geometrical average in analogy to that discussed in Sec. III.

Let us now turn to question (b) mentioned at the beginning of this section. Similar to that[15] in multiparticle production processes in hadron-hadron and hadron-nucleus collisions, we expect to see the following for each independent emitting system. In minimum-bias events at a given (c.m.s.) energy $\sqrt{s}$, the transverse-energy distribution $P_w(E_{tw};s)$ inside a given window w of a given kinematical region is

$$P_w(E_{tw};s)=\int dE_t/\epsilon P(E_t;s)B(E_t,E_{tw};q_w;\epsilon)\ . \tag{5.7}$$

Here, $P(E_t;s)$ is the E_t distribution of the entire system—in this case the central system C^*; $q_w=\langle E_{tw}\rangle/\langle E_t\rangle$ is the average probability for a unit of transverse energy to be inside w; $\langle E_{tw}\rangle$ and $\langle E_t\rangle$ are, respectively, the average value of E_{tw} and E_t. (The subscript w in P_w, E_{tw}, and q_w indicates that they refer to quantities inside the window w.) $B(E_t,E_{tw};q_w;\epsilon)$ is the "generalized binomial distribution" for the continuous random variables E_t/ϵ and E_{tw}/ϵ (the energy unit ϵ is in general an s-dependent parameter):

$$B(E_t,E_{tw};q_w;\epsilon)=\frac{\Gamma(E_t/\epsilon+1)}{\Gamma(E_{tw}/\epsilon+1)\Gamma(E_t/\epsilon-E_{tw}/\epsilon+1)}\times q_w^{E_{tw}/\epsilon}(1-q_w)^{E_t/\epsilon-E_{tw}/\epsilon}\ . \tag{5.8}$$

This means the transverse-energy distribution $P_w(E_{tw}/s)$ can be obtained from Eq. (6.1) provided that $P(E_{t,s})$ and q_w are known. The generalization to several independent emitting systems can be carried out in a straightforward manner similar to that in hadron-hadron collisions.[15]

VI. CONCLUDING REMARKS

The agreement between the existing data[1] and the multisource model strongly suggests that multiparticle production processes in high-energy hadron-nucleus and nucleus-nucleus collisions are closely related to such production processes in hadron-hadron reactions at comparable energies. It is shown in particular that the NES (number of effective sources) concept deduced from the multisource model[8] can be used to organize the transverse-energy and multiplicity-distribution data in high-energy hadron-nucleus and nucleus-nucleus collisions in an extremely simple manner. The result of our analysis leads us to the conclusion that the geometrical and the stochastical aspects of multiparticle production play a dominating role in such processes.

In this connection, experimental studies of the following kind would be very interesting.

(a) In high-energy hadron-nucleus collisions, one measures the E_t (total transverse energy), the $E_t^{\rm em}$ (electromagnetic part of the transverse energy), and the n^- (multiplicity of negatively charged hadrons) distributions in the midrapidity region for different sets of events characterized by the number (N_P) of identified fast protons in the forward angles. Since N_P is proportional to $\nu_{\rm ET}$ (the number of nucleons in the effective target in such collisions), such measurements can be used to check the validity of NES scaling in hadron-ET collisions, where the corresponding NES distribution is

$$\Omega(m|1,\nu_{\rm ET})=\sum_\nu \binom{\nu_{\rm ET}}{\nu} p^\nu(1-p)^{\nu_{\rm ET}-\nu}\Omega(m|\nu)\ . \tag{6.1}$$

We recall that m is related to E_t, $E_t^{\rm em}$, and n by Eqs. (4.2) and (4.7); and that the explicit expression for $\Omega(m|\nu)$ is given in Eq. (2.10). It is a more direct check of NES scaling, because the collision geometry is much simpler than those for nucleus-nucleus reactions.

(b) In high-energy hadron-nucleus and nucleus-nucleus collisions one measures the long-range transverse-energy correlations in given rapidity windows w. To be more precise, one chooses a rapidity window (large or small, symmetric or asymmetric with respect to the center of the nucleon-nucleon c.m.s. frame or any other frame) in the entire rapidity range. Divide it into two parts (for example, a "forward" and a "backward" region), measure the E_t or $E_t^{\rm em}$ in these two regions, and make a contour plot similar to that performed by UA5 Collaboration[16] for the multiplicities in the entire rapidity region. The existence or nonexistence of long-range correlations for a given rapidity window size can be used to study clustering phenomena (whether, if yet, how large, how many clusters are formed, etc.) in general, and the size of the emitting systems in particular.

ACKNOWLEDGMENTS

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APPENDIX A

We consider the collision between a projectile nucleus of mass number A_P and a target nucleus of mass number A_T, where $A_P \le A_T$; and define the impact parameter b as the distance between the centers of the colliding nuclei in the plane P perpendicular to the colliding axis. We denote the overlapping region of the colliding nuclei in this plane by $\Sigma(b)$, and the area of $\Sigma(b)$ by $S(b)$.

In order to take the fact that nuclear forces are of short range into account, we randomly divide the above-mentioned overlapping region $\Sigma(b)$ into subdomains σ_i ($i=1,\ldots,\lambda$); the area of each σ_i is approximately equal to πr_0^2 where r_0 is the unit nuclear radius. The area σ_i in the plane P is the common cross section of two cylinders which represent the ith interacting EP-ET pair. The volume of the ith EP (V_{Pi}) and that of the ith ET (V_{Ti}) can be calculated provided that the position of σ_i in the above-mentioned plane is known. The σ_i's ($i=1,\ldots,\lambda$) are randomly chosen by using a simple Monte Carlo program. In this program σ_i are taken as circles of radius r_0. The center of each circle is determined by a pair of random numbers, and the circles are allowed to overlap partially. In practice, the process of random sampling is carried out under the following conditions.

(1) The distance between the centers of any two circles should not be less than $D_{\min}$.

(2) The center of a circle is allowed to be outside the domain $\Sigma(b)$, provided that its distance to the nearest boarder of $\Sigma(b)$ does not exceed a number $a_{\max}$.

(3) Let us denote by s_i the area of σ_i inside the domain $\Sigma(b)$. The value s_i is determined in such a way that the overlapping region between any two circles (say, σ_i and σ_j) is counted only once, the area of the overlapping region is always counted to the circle which appears first (that is, the area $\sigma_i \cap \sigma_j$ is considered to be a part of s_i if s_i occurs first in the random sampling process). The sampling process terminates when the sum of all s_i has reached 95% of $s(b)$. In order to estimate $(\nu_{\mathrm{EP}i},\nu_{\mathrm{ET}i})$, the number of nucleons in the ith EP and that of the ith ET, we calculate the volumes of this EP-ET pair (V_{Pi},V_{Ti}), assume that the nucleons are homogeneously distributed inside the nucleus, denote the density $3/(4\pi r_0^3)$ by ρ, and set $\nu_{\mathrm{EP}i}=N(\rho V_{Pi})$ and $\nu_{\mathrm{ET}i}=N(\rho V_{Ti})$. Here $N(x)$ means "take the integer nearest to the number x."

The number of EP-ET pairs determined by the above-mentioned method is denoted by λ, which depends, of course, on b.

The parameters we used in our calculation are $r_0=1.2$ fm, $D_{\min}=1.5$ fm, and $a_{\max}=0.4$ fm. While the value for r_0 is taken from the literature,[12] the values for $D_{\min}$ and $a_{\max}$ are obtained by fitting the data. Calculations carried out by using different sets of $D_{\min}$ and $a_{\max}$ show that the result does not depend sensitively on the choice of these values.

APPENDIX B

In this appendix we show that $\Omega(m,b\,|\,A_P,A_T)$ given in Eq. (3.2) can be rewritten in a form which is much easier to evaluate numerically.

First, we rewrite Eq. (3.2) as

$$\Omega(m,b\,|\,A_P,A_T)=N\sum_{l=1}^{\lambda}{}'\sum_{\mu=2}C(\mu,l\,|\,\nu_{\mathrm{EP}1}+\nu_{\mathrm{ET}1},\ldots,\nu_{\mathrm{EP}l}+\nu_{\mathrm{ET}l})p^{\mu}(1-p)^{\nu-\mu}\Omega(m\,|\,\mu)\;, \tag{B1}$$

where

$$C(\mu,l\,|\,\nu_{\mathrm{EP}1}+\nu_{\mathrm{ET}1},\ldots,\nu_{\mathrm{EP}l}+\nu_{\mathrm{ET}l})=\sum_{\mu_1=2}^{\nu_{\mathrm{EP}1}+\nu_{\mathrm{ET}1}}\cdots\sum_{\mu_l=2}^{\nu_{\mathrm{EP}l}+\nu_{\mathrm{ET}L}}\delta\left[\mu,\sum_{\alpha=1}^{l}\mu_\alpha\right]\prod_{i=1}^{l}W(\mu_i\,|\,\nu_{\mathrm{EP}i},\nu_{\mathrm{ET}i})\;. \tag{B2}$$

Note that μ and ν in Eq. (B1) are, respectively, the sum of all μ_i and that of all $\nu_{\mathrm{EP}i}+\nu_{\mathrm{ET}i}$ for $i=1,\ldots,l$ [see Eqs. (3.3) and (3.4)]; and the Kronecker delta $\delta(\mu,\sum_{\alpha=1}^{l}\mu_\alpha)$ in Eq. (B2) guarantees that the condition $\mu=\sum_{\alpha=1}^{l}\mu_\alpha$ [see Eq. (3.3)] is satisfied.

Second, in order to calculate the coefficient $C(\mu,l\,|\,\nu_{\mathrm{EP}1}+\nu_{\mathrm{ET}1},\ldots,\nu_{\mathrm{EP}l}+\nu_{\mathrm{ET}l})$ we rewrite $\delta(\mu,\sum_{\alpha=1}^{l}\mu_\alpha)$, so that the right-hand side of Eq. (B2) can be factorized. We note that

$$\delta\left[\mu,\sum_{\alpha=1}^{l}\mu_\alpha\right]=\frac{1}{\pi}\int_{-\pi}^{\pi}dt\,\cos\left[\sum_{\alpha=1}^{l}\mu_\alpha\right]t\,\cos\mu t \tag{B3}$$

is valid for all positive integers μ; and with the help of this identity, we obtain

$$C(\mu,l|\nu_{\mathrm{EP}1}+\nu_{\mathrm{ET}1},\ldots,\nu_{\mathrm{EP}l}+\nu_{\mathrm{ET}l})=\frac{1}{2\pi}\int_{-\pi}^{\pi}dt\left\{\sum_{\mu_1=2}^{\nu_{\mathrm{EP}1}+\nu_{\mathrm{ET}1}}\cdots\sum_{\mu_l=2}^{\nu_{\mathrm{EP}1}+\nu_{\mathrm{ET}l}}\left[\exp\left[i\sum_{\alpha=1}^{l}\mu_\alpha\right]t+\exp\left[-i\sum_{\alpha=1}^{l}\mu_\alpha\right]t\right]\right.$$

$$\left.\times\prod_{\alpha=1}^{l}W(\mu_\alpha|\nu_{\mathrm{EP}\alpha},\nu_{\mathrm{ET}\alpha})\right\}\cos\mu t\ . \tag{B4}$$

Now, since

$$\sum_{\mu_1=2}^{\nu_{\mathrm{EP}_1}+\nu_{\mathrm{ET}_1}}\cdots\sum_{\mu_l=2}^{\nu_{\mathrm{EP}l}+\nu_{\mathrm{ET}l}}\exp\left[\pm i\sum_{\alpha=1}^{l}\mu_\alpha\right]t\prod_{\alpha=1}^{l}W(\mu_\alpha|\nu_{\mathrm{EP}\alpha},\nu_{\mathrm{EP}\alpha})=\prod_{\alpha=1}^{l}\left[\sum_{\mu_\alpha=2}^{\nu_{\mathrm{EP}\alpha}+\nu_{\mathrm{ET}\alpha}}\exp(\pm i\mu_\alpha t)W(\mu_\alpha|\nu_{\mathrm{EP}\alpha},\nu_{\mathrm{ET}\alpha})\right],\tag{B5}$$

Eq. (B4) can be written as a sum of two terms:

$$C(\nu,l|\nu_{\mathrm{EP}1}+\nu_{\mathrm{ET}1},\ldots,\nu_{\mathrm{ET}l}+\nu_{\mathrm{ET}l})=C^{(+)}(\mu,l|\nu_{\mathrm{EP}1}+\nu_{\mathrm{ET}1},\ldots,\nu_{\mathrm{EP}l}+\nu_{\mathrm{ET}l})$$

$$+C^{(-)}(\mu,l|\nu_{\mathrm{EP}1}+\nu_{\mathrm{ET}1},\ldots,\nu_{\mathrm{EP}l}+\nu_{\mathrm{ET}l})\ ,\tag{B6}$$

where $C^{(+)}$ and $C^{(-)}$ stand for

$$C^{\pm}(\nu,l|\nu_{\mathrm{EP}1}+\nu_{\mathrm{ET}1},\ldots,\nu_{\mathrm{ET}l}+\nu_{\mathrm{ET}l})=\frac{1}{2\pi}\int_{-\pi}^{\pi}dt\prod_{\alpha=1}^{l}\left\{\sum_{\mu_\alpha=2}^{\nu_{\mathrm{EP}\alpha}+\nu_{\mathrm{ET}\alpha}}\left[\begin{bmatrix}\nu_{\mathrm{EP}\alpha}+\nu_{\mathrm{ET}\alpha}\\ \mu_\alpha\end{bmatrix}-\begin{bmatrix}\nu_{\mathrm{EP}\alpha}\\ \mu_\alpha\end{bmatrix}\right]\exp(\pm i\mu_\alpha t)\right\}\cos\mu t\ .\tag{B7}$$

While combining Eqs. (B4) and (B5), we made use of the fact that $W(\mu_\alpha|\nu_{\mathrm{ET}\alpha},\nu_{\mathrm{ET}\alpha})$ is defined [see Eq. (2.6)] as

$$W(\mu_\alpha|\nu_{\mathrm{EP}\alpha},\nu_{\mathrm{ET}\alpha})=\begin{bmatrix}\nu_{\mathrm{EP}\alpha}+\nu_{\mathrm{ET}\alpha}\\ \mu_\alpha\end{bmatrix}-\begin{bmatrix}\nu_{\mathrm{EP}\alpha}\\ \mu_\alpha\end{bmatrix}-\begin{bmatrix}\nu_{\mathrm{ET}\alpha}\\ \mu_\alpha\end{bmatrix}+\delta_{\mu\alpha,0}\ .\tag{B8}$$

Third, the expression in large curly brackets on the right-hand side of Eq. (B7) can be simplified by taking the following trivial identities into account:

$$\begin{bmatrix}M\\ N\end{bmatrix}=0\quad\text{for } N>M\ ,\tag{B9}$$

$$\sum_{N=2}^{M}\begin{bmatrix}M\\ N\end{bmatrix}e^{iNT}=(1+e^{it})^M-1-Me^{it}\ .\tag{B10}$$

In fact, it can be readily seen that

$$\sum_{\mu_\alpha=2}^{\nu_{\mathrm{EP}\alpha}+\nu_{\mathrm{ET}\alpha}}\left[\begin{bmatrix}\nu_{\mathrm{EP}\alpha}+\nu_{\mathrm{ET}\alpha}\\ \mu_\alpha\end{bmatrix}-\begin{bmatrix}\nu_{\mathrm{EP}\alpha}\\ \mu_\alpha\end{bmatrix}-\begin{bmatrix}\nu_{\mathrm{ET}\alpha}\\ \mu_\alpha\end{bmatrix}\right]\exp(\pm i\mu_\alpha t)=\begin{cases}(1-z^{\nu_{\mathrm{EP}\alpha}})(1-z^{\nu_{\mathrm{ET}\alpha}})\ ,\\ [1-(z^*)^{\nu_{\mathrm{EP}\alpha}}][1-(z^*)^{\nu_{\mathrm{ET}\alpha}}]\ ,\end{cases}\tag{B11}$$

where the first and the second line on the right-hand side of Eq. (B11) correspond to the $(+)$ and the $(-)$ in $\exp(\pm i\mu_\alpha t)$, respectively. Here, z is defined as

$$z=1+\exp(it)\ ,\tag{B12}$$

and z^* is the complex conjugate of z. Hence, we have

$$C(\mu,l|\nu_{\mathrm{EP}1}+\nu_{\mathrm{ET}1},\ldots,\nu_{\mathrm{EP}l}+\nu_{\mathrm{ET}l})=\frac{1}{2\pi}\int_{-\pi}^{\pi}dt\left[\prod_{\alpha=1}^{l}(1-z^{\nu_{\mathrm{EP}\alpha}})(1-z^{\nu_{\mathrm{ET}\alpha}})+\prod_{\alpha=1}^{l}[1-(z^*)^{\nu_{\mathrm{ET}\alpha}}][1-(z^*)^{\nu_{\mathrm{ET}\alpha}}]\right]\cos\mu t\ .\tag{B13}$$

Fourth and last, since the expression in large parentheses on the right-hand side of Eq. (B13) is nothing else but a polynomial in z and a polynomial in z^* with the same coefficient β_n where the highest power in z (and that in z^*) is

$$n_{\max}=\sum_{\alpha=1}^{l}(\nu_{\mathrm{EP}\alpha}+\nu_{\mathrm{ET}\alpha}) , \tag{B14}$$

we have

$$C(\mu,l\,|\,\nu_{\mathrm{EP1}}+\nu_{\mathrm{ET2}},\ldots,\nu_{\mathrm{EP}l}+\nu_{\mathrm{ET}l})$$
$$=\frac{1}{2\pi}\int_{-\pi}^{\pi}dt\left[\sum_{n=0}^{n_{\max}}\beta_n[z^n+(z^*)^n]\right] . \tag{B15}$$

Taken together with the fact that [see Eq. (B12)]

$$\sum_{n=0}^{n_{\max}}\beta_n(z+z^*)^n=2\sum_{n=0}^{n_{\max}}\beta_n 2^n\cos^n\frac{t}{2}\cos\frac{n}{2}t \tag{B16}$$

and that (the identity is obviously valid for all positive integer n)

$$\cos^n\frac{t}{2}\cos\frac{n}{2}t=2^{-2}\sum_{r=0}^{n}\begin{bmatrix}n\\ r\end{bmatrix}\cos(n-r)t , \tag{B17}$$

we obtain, from Eq. (B15),

$$C(\mu,l\,|\,\nu_{\mathrm{EP1}}+\nu_{\mathrm{ET1}},\ldots,\nu_{\mathrm{EP}l}+\nu_{\mathrm{ET}l})$$
$$=\frac{1}{\pi}\sum_{n=0}^{n_{\max}}\beta_n\sum_{r=0}^{n}\begin{bmatrix}n\\ r\end{bmatrix}\int_{-\pi}^{\pi}dt\cos(n-r)t\cos\mu t$$
$$=\sum_{n=0}^{n_{\max}}\beta_n\begin{bmatrix}n\\ \mu\end{bmatrix} . \tag{B18}$$

Here, the dependence of $n_{\max}$ on l and $\nu_{\mathrm{EP1}}+\nu_{\mathrm{ET1}},\ldots,\nu_{\mathrm{EP}l}+\nu_{\mathrm{ET}l}$, is given by Eq. (B14). The coefficients β_n ($n=0,1,\ldots,n_{\max}$) can be readily calculated in a straightforward way on a computer. On inserting Eqs. (B18) and (2.10) into Eq. (B1) we obtain $\Omega(m,b\,|\,A_P,A_T)$ for a fixed impact parameter b.

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