Particle productivity in pp and pA collisions

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In the framework of the geometrical branching model for pp and pA collisions, we discuss the dependence of the particle-productivity function on impact parameter and give a new description for it without any free parameter. It is then shown that an excellent description of the pp data is obtained.

At the level of parton interaction, the process of multiparticle production in proton-proton (pp) and proton-nucleus (pA) collisions is complicated and at present untractable. However, precisely because the basic interaction that underlies the two types of collisions is common, and because the main difference is in the numbers of bags carrying those partons, one should expect some universal feature between the two. In this paper we explore that feature and give phenomenological evidence for our finding.

In the absence of a many-body theory of soft parton interactions, we need a framework for describing pp and pA collisions. That framework is the geometrical branching model (GBM),[1,2] which successfully describes many features of pp and $\bar{p}p$ collisions over a wide range of energy. The two points of emphasis in the model, the geometrical extension of hadrons and Furry branching as a stochastic process of particle production, are readily extendible to pA collisions. We summarize first the formalism in the pp case and then proceed to the pA case.

In the GBM it is assumed that at each impact parameter b the production process can be well described by Furry branching.[3] The rationale for this central assumption has been discussed at length in Ref. 4. The observed multiplicity distribution is

$$P_n = \int_0^\infty dR^2 g(R) F_n^{k(R)} \equiv \{F_n^k\} , \tag{1}$$

where R is the scaled impact parameter: $R = b/b_0(s)$. The inelasticity function $g(R)$ is

$$g(R) = 1 - e^{-2\Omega(R)} , \tag{2}$$

where $\Omega(R)$ is the eikonal function, whose dependence on R alone, and not on s and b separately, embodies the essence of geometrical scaling. It satisfies the normalization constraint $\{1\}=1$. F_N^k is the multiplicity distribution at R, and is identified with the Furry distribution,[3,4] which is

$$F_n^k(w) = \frac{\Gamma(n)}{\Gamma(k)\Gamma(n-k+1)} \left[\frac{1}{w}\right]^k \left[1-\frac{1}{w}\right]^{n-k} , \tag{3}$$

where k is the number of initial clusters, regarded as a continuous function of R, and w is the evolution parameter $\bar{n}(s,R)/k(s,R)$ that depends only on s, where

$$\bar{n}(s,R) = \sum_n n F_k^{k(R)}(w) . \tag{4}$$

Let the observed average multiplicity be denoted by $\langle n \rangle(s) = \sum_n nP_n$. We define the normalized particle-productivity function $h(s,R)$ by

$$h(s,R) = \bar{n}(s,R)/\langle n \rangle(s) , \tag{5}$$

so that $\{h\}=1$. The assumption that $F_n^k(w)$ depends on R through $k(s,R)$ only and not through w can be effected by requiring that

$$k(s,R) = \langle k \rangle(s) h(s,R) , \tag{6}$$

so that w becomes also $\langle n \rangle / \langle k \rangle$, which is a function of s only. In this way the crux of impact-parameter smearing of the Furry distribution as expressed in (1) is in the R dependence of $h(s,R)$.

In Ref. 1 we made the working hypothesis

$$h(s,R) = h(R) = h_0 \Omega^\gamma(R) \tag{7}$$

on the basis that the particle productivity at each R must depend on the hadron opacity at R in some way, and that in the absence of any knowledge about that dependence a form such as that in (7) seems reasonable, albeit at the expense of a free parameter γ. The normalization factor h_0 is not free because of $\{h\}=1$. We adjusted γ and found an excellent fit of the multiplicity moments $C_p = \langle n^p \rangle / \langle n \rangle^p$, $p \le 5$, of the Koba-Nielsen-Olesen (KNO) scaling function for $\gamma = 0.30 \pm 0.05$ (Ref. 1). While the capability of the model to fit over 15 data points of C_p by one parameter was an encouraging sign that the GBM is a viable model, let alone its ability to predict correctly the forward-backward multiplicity correlation,[2] it nevertheless is blemished by the necessity to employ an adjustable parameter. To remove that defect is therefore of paramount importance in an improvement of the model, as we now proceed to do.

We propose that $h(R)$ should have the form

$$h(R) = \frac{h_1 \Omega(R)}{1 - e^{-2\Omega(R)}} , \tag{8}$$

where h_1 is a constant fixed by the condition $\{h\}=1$, i.e., $h_1^{-1} = \int dR^2 \Omega(R)$. The origin of this form is rooted in our attempt to apply the GBM to pA collisions. It is found that (8) is the necessary form, if we require a universality in the particle productivity function in pp and pA collisions. The physical basis for the universality is the following. Soft production is difficult to treat because of the many-body interaction of the soft partons. In the GBM it is regarded as a stochastic process initiated by the passing of partons in one hadron through the partons in another at each impact parameter b. Such a process should basically remain the same, if it is generalized to a pA collision: the geometrical consideration is similar, and the branching process is also the same. The essential difference is that there are more initial partons in the pA case, so there should be more initial clusters. Since the basic parton interaction is independent of the host in which the partons reside, the number of initial clusters should depend on the pA thickness function $T(b)$ in the same way that it depends on $\Omega(R)$ in the pp case. This requirement is the embodiment of the universality.

To describe this idea quantitatively, we need a formalism for pA collisions in the GBM. Such a formalism has recently been developed.[5] Since the part relevant to our consideration here is relatively compact, we can describe the essence in brief. Treating the broken hadron that traverses the nucleus as a jet of partons, we nevertheless can have the notion of multiple collisions, where we count by the number of target nucleons ν that suffer collisions. The Glauber description of the probability for ν collisions at b is still valid; it is

$$\pi_\nu(b)=\frac{[\sigma T(b)]^\nu}{\nu!}e^{-\sigma T(b)}\,, \tag{9}$$

where σ is the pN inelastic cross section, even though the proton is broken. (This is a subtle point but is not crucial to our development in the following.) At each collision the multiplicity distribution is $F_{n_i}^{k_i}$, $i=1,\ldots,\nu$. Now, the Furry distribution, due to the factorizability of its generating function, satisfies the property

$$F_n^{K(\nu)}=\sum_{n_1\cdots n_\nu}\delta_{n,\sum_i n_i}\prod_{i=1}^{\nu}F_{n_i}^{k_i}\,, \tag{10}$$

where $K(\nu)=\sum_{i=1}^{\nu}k_i$ is the total number of initial clusters, and $n=\sum_{i=1}^{\nu}n_i$ is the total number of produced particles. Consequently, the overall multiplicity distribution at b for pA collisions is

$$P_n(b)=\sum_{\nu=1}^{A}\pi_\nu(b)F_n^{K(\nu)}\,. \tag{11}$$

Define K' to be the average number of clusters produced per collision so that by definition $K(\nu)=K'\nu$. Since $\pi_\nu(b)$ as expressed in (9) is a narrow Poisson distribution in ν, we can approximate (11) by taking $F_n^{K'\nu}$ out of the summation and replacing the ν in that expression by the average $\bar{\nu}(b)$. Equation (11) now becomes

$$P_n(b)=G(b)F_n^{K'\bar{\nu}(b)}\,. \tag{12}$$

For large A we have

$$G(b)=\sum_{\nu=1}^{\infty}\pi_\nu(b)=1-e^{-\sigma T(b)}\,, \tag{13}$$

$$\bar{\nu}(b)=\sum_{\nu=1}^{\infty}\nu\pi_\nu(b)/G(b)=\frac{\sigma T(b)}{1-e^{-\sigma T(b)}}\,. \tag{14}$$

Upon b smearing of (12) we obtain

$$P_n=\sigma_{in}^{-1}\int d^2b\,G(b)F_n^{K'\bar{\nu}(b)} \tag{15}$$

which is to be compared to (1) for pp collisions. The parallelism between the two cases is striking. Comparing (13) with (2) we have the correspondence $\sigma T(b)\to 2\Omega(R)$. In the context of that correspondence it is therefore natural to expect $K'\bar{\nu}(b)$ in (15) to prescribe $k(R)$ in (1). From (14) and (6) we immediately arrive at (8). Thus, on the basis of the universality requirement we have obtained a parameter-free description of the particle-productivity distribution function $h(R)$.

To prove that this result is phenomenologically acceptable, we show first in Fig. 1 $h(R)$, as determined accord-

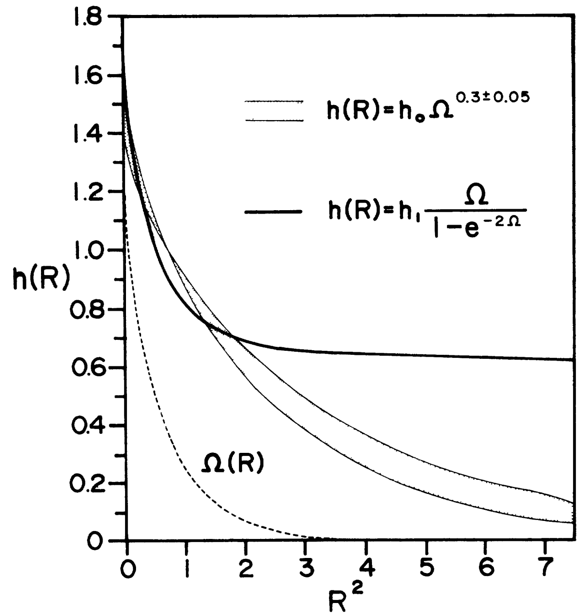

FIG. 1. $h(R)$ vs R^2 according to Eq. (8) (solid line) and Eq. (7) (shaded region). $\Omega(R)$ is shown by the dotted line.

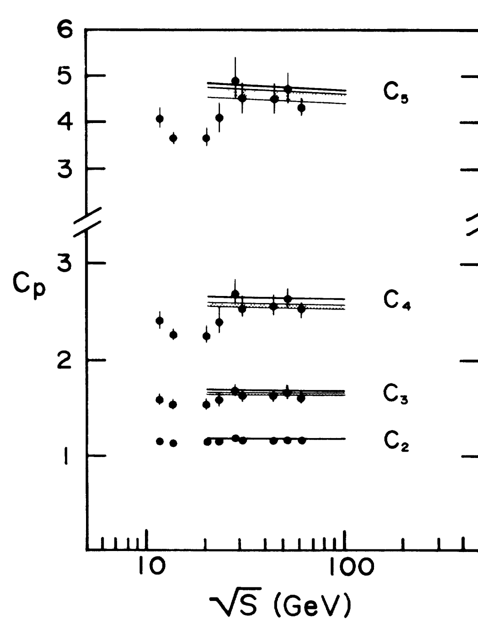

FIG. 2. Normalized multiplicity moments $C_p=\langle n^p\rangle/\langle n\rangle^p$. The solid lines are results from using Eq. (8) and the shaded regions are from using Eq. (7).

ing to (8), by the solid line, which is to be compared to the result obtained by use of the earlier form (7), indicated by the shaded region for $\gamma = 0.30 \pm 0.05$. The input form for $\Omega(R)$ is in accordance to the parametrization[6]

$$e^{-\Omega(R)} = 1 - 0.71 e^{-1.17R^2} \tag{16}$$

that best fits the elastic diffraction peak, and is shown by the dotted line in the same figure. Evidently, Eq. (7) yields a reasonably good approximation of (8) for the important region $R^2 < 3$. The fact that $h(R)$ does not vanish at large R is no cause for concern, since the impact-parameter smearing of powers of $h(R)$ are calculated with the weight factor $g(R)$, which rapidly approaches zero at large R. Nevertheless, it should be recognized that the large R behavior of (8), though unimportant, is unrealistic, and is a consequence of the inaccuracy of our approximation in obtaining (12) when b is large. Since physically the productivity function must vanish at large b or R, the expression in (8) for $h(R)$ should realistically be damped at large R. However, we have chosen not to use such a damping factor in our calculations because the weight factor $g(R)$ always accompanies $h(R)$ in the impact-parameter smearing of any quantity.

We have used (8) to calculate C_p as in Ref. 1 and obtained the result shown by the solid lines in Fig. 2. The shaded regions represent the result obtained previously on the basis of the parametrized form (7) for $h(R)$. Evidently, the new result is totally satisfactory: with the geometrical-scaling eikonal function $\Omega(R)$ as input, we have obtained KNO scaling with the correct scaling function without the use of any adjustable parameters. The s dependence (or the lack of it) is determined through $\langle n \rangle(s)$ which we take from experiment, and the evolution parameter w is determined by C_2, as explained in Ref. 1.

We mention that a form similar to (8) has been considered in Ref. 7, and that (14) is used in Ref. 8 in connection with pA collisions. We have related the two in the framework of the GBM under the requirement of universal behavior among pp and pA collisions. Extension to include AA collisions is obviously of interest, and is under investigation. We add that the phenomenology of pA collisions based on the model briefly sketched between (9) and (15) has been carried out and succeeds in describing the data extremely well.[5]

It should be emphasized that the self-reproducing property of the Furry distribution expressed in (10) is essential in the derivation of (12). In that sense the particle productivity function as given in (8) is a consequence of the GBM, when the pp and pA universality is imposed. Since the result is a parameter-free expression for $h(R)$ that gives an excellent description of the KNO-scaling data, we believe that we have significantly improved the GBM.

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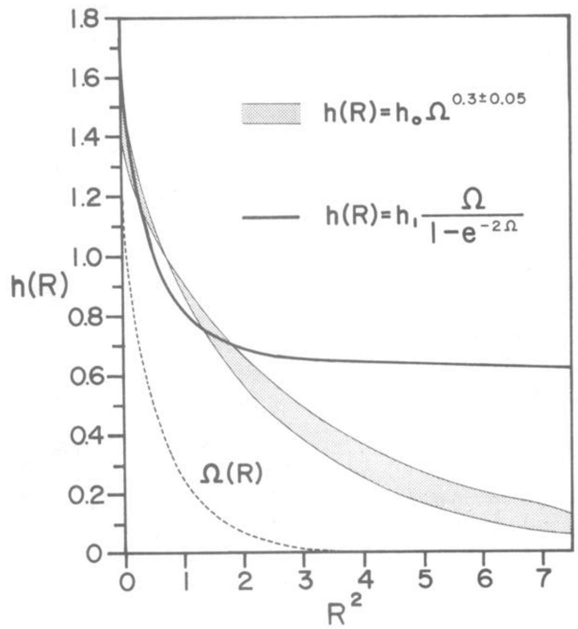

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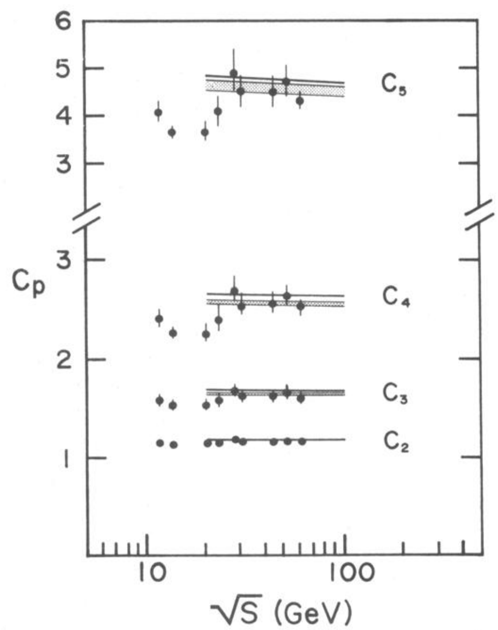

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