Skyrme model and strong-coupling model

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By comparing the Skyrme model with the strong-coupling model of the pion-nucleon interaction in the static limit the collective coordinates in the Skyrme model are shown to be proportional to the square roots of the fields of the pions bound to the bare nucleons in the strong-coupling model.

I. INTRODUCTION

In the large-N limit, QCD becomes equivalent to an effective field theory of mesons,[1] and at low energies this theory reduces to a nonlinear σ model of spontaneously broken chiral symmetry. The simplest choice of the model is the Skyrme model,[2]

$$L=-\tfrac{1}{4}F_\pi^2\mathrm{Tr}(\partial_\mu U\partial_\mu U^\dagger)+(1/32e^2)\mathrm{Tr}[(\partial_\mu U)U^\dagger,(\partial_\nu U)U^\dagger]^2\ , \tag{1.1}$$

where $F_\pi=93$ MeV is the pion decay constant, e is a dimensionless parameter, and U is an SU(2) matrix:

$$U(x)=\exp[i\phi_i(x)\tau_i/F_\pi]\ . \tag{1.2}$$

In the Skyrme model the baryons are viewed as solitons.[2,3] The lowest-energy state with the topologically nontrivial U field configuration can be written as

$$U_0(x)=\exp[iF(r)\hat{\mathbf{r}}\cdot\boldsymbol{\tau}]\ . \tag{1.3}$$

This configuration is transformed by the separate action of the isospin $\mathbf{I}$ and the spin $\mathbf{J}$, but is invariant to the combined action $\mathbf{K}=\mathbf{I}+\mathbf{J}$.

Since the soliton U_0 is not invariant to the action of $\mathbf{I}$ and $\mathbf{J}$, we can obtain a family of soliton solutions,[4] all degenerate with the soliton U_0, by rotating U_0,

$$U=AU_0A^{-1}\ , \tag{1.4}$$

where $A=a_0+i\mathbf{a}\cdot\boldsymbol{\tau}$ with $a_0^2+|\mathbf{a}|^2=1$ is an arbitrary constant SU(2) matrix acting on τ in U_0. A solution of any given A is not an eigenstate of spin and isospin. We need to treat A as a set of collective coordinates. Then, the Lagrangian and all physical variables are expressed in terms of a time-dependent $A(t)$.

When the Lagrangian (1.1) with (1.4) is quantized, the result can be expressed in terms of two commuting angular momenta, the isospin $\mathbf{I}$ and the spin $\mathbf{J}$:

$$\begin{aligned}\mathbf{I}&=\tfrac{1}{2}(\mathbf{a}\times\boldsymbol{\pi}-a_0\boldsymbol{\pi}+\mathbf{a}\pi_0)\ ,\\ \mathbf{J}&=\tfrac{1}{2}(\mathbf{a}\times\boldsymbol{\pi}+a_0\boldsymbol{\pi}-\mathbf{a}\pi_0)\ ,\end{aligned} \tag{1.5}$$

which satisfy the commutation relations

$$\begin{aligned}&[a_j,\pi_k]=i\delta_{jk},\quad [\mathbf{I},A]=-\tfrac{1}{2}\tau A\ ,\\ &[\mathbf{J},A]=\tfrac{1}{2}A\tau,\quad \mathbf{I}^2=\mathbf{J}^2\ .\end{aligned} \tag{1.6}$$

Then, the properly normalized wave function for proton and neutron states of spin up or spin down along the z axis and some of the Δ wave functions are[4]

$$\begin{aligned}&|p\uparrow\rangle=(1/\pi)(a_1+ia_2),\quad |p\downarrow\rangle=-(i/\pi)(a_0-ia_3)\ ,\\ &|n\uparrow\rangle=(i/\pi)(a_0+ia_3),\quad |n\downarrow\rangle=-(1/\pi)(a_1-ia_2)\ ,\\ &|\Delta^{++},s_z=\tfrac{3}{2}\rangle=(\sqrt{2}/\pi)(a_1+ia_2)^3\ ,\\ &|\Delta^{+},s_z=\tfrac{1}{2}\rangle=-(\sqrt{2}/\pi)(a_1+ia_2)[1-3(a_0^2+a_3^2)]\ .\end{aligned} \tag{1.7}$$

Then, the masses of the baryons are given by

$$M=M_0+(1/2\lambda)J(J+1)\ . \tag{1.8}$$

The mass spectra remind us of the strong-coupling model.[5,6]

In this connection it is interesting to notice that the QCD in the large-N limit has been shown[7,8] to be the same as the static limit in the strong-coupling model by showing that the exact large-N equations are identical to the so-called bootstrap conditions[9] of strong-coupling theory.[10]

The purpose of this paper is to clarify the physical meaning of the collective coordinates A by comparing the strong-coupling model and the Skyrme model.

In the first half of next section we review the strong-coupling model of the pion-nucleon interaction in the static limit which was studied by Pauli and Dancoff[6] in order to make this paper self-contained, and then we clarify the physical meaning of the collective coordinates in the Skyrme model by comparing the Skyrme model and the strong-coupling model. Discussions are given in Sec. III.

II. THE STRONG-COUPLING MODEL AND THE SKYRME MODEL

The Hamiltonian of the pion-nucleon system in the static limit is expressed as

$$H=\tfrac{1}{2}\int[\pi_\alpha^2+(\nabla\phi_\alpha)^2+m^2\phi_\alpha{}^2]d^3x+(f/m)\int\tau_\alpha\phi_\alpha(\boldsymbol{\sigma}\cdot\nabla)K\,d^3x\ . \tag{2.1}$$

The source function $K(x)$ is spherically symmetric and normalized according to

$$\int K(x)d^3x=1\ , \tag{2.2}$$

where we have assumed that a nucleon is at rest at the origin.

The pion fields ϕ_α and their conjugate momenta π_α are decomposed into ϕ_α^0 and ϕ'_α and π_α^0 and π'_α, respectively, where

$$\phi_{\alpha k}^0 = -\int K(x)\frac{\partial\phi_\alpha}{\partial x_k}d^3x = \int \frac{\partial K}{\partial x_k}\phi_\alpha d^3x \ , \tag{2.3}$$

$$\pi_{\alpha k}^0 = \int \frac{\partial \xi}{\partial x_k}\pi_\alpha d^3x \ , \tag{2.4}$$

$$\int \phi'_\alpha \nabla K \, d^3x = \int \pi'_\alpha \nabla \xi \, d^3x = 0 \ , \tag{2.5}$$

$$\xi(x) = X(x)/I \ , \tag{2.6}$$

$$X(x) = \int K(x')(e^{-mr}/4\pi r)d^3x'$$

$$(r = |\mathbf{x}-\mathbf{x}'|), \ \ [(-\nabla^2+m^2)X = K] \ , \tag{2.7}$$

and

$$\delta_{jk}I = \int \frac{\partial X}{\partial x_j}\frac{\partial K}{\partial x_k}d^3x \ . \tag{2.8}$$

They satisfy the commutation relations

$$[\phi_{\alpha j}^0, \pi_{\beta k}^0] = i\delta_{\alpha\beta}\delta_{jk} \ . \tag{2.9}$$

The Hamiltonian, the isospin, and the angular momentum of the system are expressed as

$$H \approx \tfrac{1}{2}N(\pi_{\alpha k}^0)^2 + \frac{1}{2I}(\phi_{\alpha k}^0)^2 + (f/m)\tau_\alpha\sigma_k\phi_{\alpha k}^0 \ , \tag{2.10}$$

$$I_{\alpha\beta} \approx \phi_{\alpha k}^0\pi_{\beta k}^0 - \phi_{\beta k}^0\pi_{\alpha k}^0 + \tfrac{1}{2}\tau_{\alpha\beta} \ , \tag{2.11}$$

$$J_{ij} \approx \tfrac{1}{2}\sigma_{ij} + \phi_{\alpha i}^0\pi_{\alpha j}^0 - \phi_{\alpha j}^0\pi_{\alpha i}^0 \ , \tag{2.12}$$

if we neglect the contributions from ϕ'_α and π'_α, which are negligible in the strong-coupling limit. Here,

$$\delta_{jk}N = \left[\int \frac{\partial \xi}{\partial x_j}\frac{\partial \xi}{\partial x_k}d^3x\right]^{-1} \ . \tag{2.13}$$

In the strong-coupling model we determine the eigenvalues of the interaction Hamiltonian

$$H_I = (f/m)\tau_\alpha\sigma_k\phi_{\alpha k}^0 \tag{2.14}$$

at first. For this purpose we diagonalize $\phi_{\alpha k}^0$ regarded as a 3×3 matrix by real orthogonal transformations B and C:

$$(C\phi^0 B)_{rs} = Q_r\delta_{rs} \ . \tag{2.15}$$

Then, the interaction Hamiltonian becomes

$$H_I = (f/m)\tau_r\sigma_r Q_r \tag{2.16}$$

and the minimum of the total potential energy

$$E_{\text{pot}}^0 = \frac{1}{2I}Q_r^2 + (f/m)Q_r\sigma_r\tau_r \tag{2.17}$$

is found to be

$$E_{\text{min}}^0 = -\frac{3f^2I}{2m^2} = -\frac{3D^2}{2I} \ , \tag{2.18}$$

which corresponds to

$$Q_1 = Q_2 = Q_3 = (f/m)I \equiv D \tag{2.19}$$

and to the state

$$|0_N\rangle = (p\downarrow - n\uparrow)/\sqrt{2} \ . \tag{2.20}$$

Since

$$(\boldsymbol{\sigma}+\boldsymbol{\tau})^2|0_N\rangle = 0 \ , \tag{2.21}$$

the state (2.20) corresponds to the classical soliton U_0 with $\mathbf{K}=\mathbf{I}+\mathbf{J}=0$.

According to (2.15) and (2.19), $\phi_{\alpha k}^0$ is expressed as

$$\phi_{\alpha k}^0 = DB_{kr}C_{r\alpha} \equiv De_{\alpha k} \ , \tag{2.22}$$

where $e_{\alpha k}$ satisfies

$$e_{\alpha j}e_{\alpha k} = \delta_{jk} \ \text{ and } \ e_{\alpha k}e_{\beta k} = \delta_{\alpha\beta} \ . \tag{2.23}$$

It is possible to express $e_{\alpha k}$ $(\alpha, k = 1,2,3)$ by means of Euler angles, a, b, and c:

$$\begin{aligned}
e_{1k} &= \cos a\cos b\cos c - \sin b\sin c,\ \cos a\sin b\cos c + \cos b\sin c,\ -\sin a\cos c \ , \\
e_{2k} &= -\cos a\cos b\sin c - \sin b\cos c,\ -\cos a\sin b\sin c + \cos b\cos c,\ \sin a\sin c \ , \\
e_{3k} &= \sin a\cos b,\ \sin a\sin b,\ \cos a \ .
\end{aligned} \tag{2.24}$$

The state (2.20) corresponds to $\phi_{\alpha k}^0 = D\delta_{\alpha k}$, i.e., $e_{\alpha k}$ with $a=b=c=0$. In the following we express (2.20) as $|a=0,b=0,c=0\rangle\,|0_N(0,0,0)\rangle$. The first ket represents the state of the pions bound to the nucleon and the second ket represents the state of the bare nucleon.

There are an infinite number of states,

$$|a,b,c\rangle\,|0_N(a,b,c)\rangle \ , \tag{2.25}$$

all degenerate with the state $|0,0,0\rangle\,|0_N(0,0,0)\rangle$.

Since

$$S(abc)e_{\alpha k}(abc)\sigma_k S^{-1}(abc) = \sigma_\alpha \tag{2.26}$$

and

$$S(abc)\tau_\alpha S^{-1}(abc) = \tau_\alpha \ ,$$

we find

$$|0_N(a,b,c)\rangle = S^{-1}(abc)\,|0_N(0,0,0)\rangle \ , \tag{2.27}$$

where

$$S(abc)=\exp(i\sigma_3 c/2)\exp(i\sigma_2 a/2)\exp(i\sigma_3 b/2) . \quad (2.28)$$

Any state for a given (a,b,c) is not an eigenstate of spin,

$$\mathbf{J}=\mathbf{L}+\tfrac{1}{2}\boldsymbol{\sigma} ,$$

$$L_\pm = L_1 \pm iL_2 = \exp(\pm ib)\left[\pm\frac{\partial}{\partial a}+\left[-i\frac{\partial}{\partial c}+i\cos a\frac{\partial}{\partial b}\right]\Big/\sin a\right] , \quad (2.29)$$

$$L_3=-i\frac{\partial}{\partial b} ,$$

not that of isospin,

$$\mathbf{I}=\mathbf{T}+\tfrac{1}{2}\boldsymbol{\tau} ,$$

$$T_\pm = T_1 \pm iT_2 = -\exp(\mp ic)\left[\pm\frac{\partial}{\partial a}+\left[i\frac{\partial}{\partial b}-i\cos a\frac{\partial}{\partial c}\right]\Big/\sin a\right] , \quad (2.30)$$

$$T_3=i\frac{\partial}{\partial c} ,$$

nor that of the Hamiltonian ($\mathbf{L}^2=\mathbf{T}^2$),

$$H=(N/4D^2)\mathbf{L}^2+(Df/m)\tau_\alpha\sigma_k e_{\alpha k}(abc)+\text{const} . \quad (2.31)$$

Their eigenstates can be constructed as linear combinations of (2.25):

$$|I,I_3,J,J_3\rangle = \int \sin a\, da\, db\, dc\, A^{I=J}_{I_3,J_3}(abc)\,|abc\rangle\,|0_N(a,b,c)\rangle . \quad (2.32)$$

The wave functions $A^{I=J}_{I_3,J_3}(abc)$ can be obtained by solving the Schrödinger equation with the S-transformed Hamiltonian

$$H'=SHS^{-1}=(N/4D^2)S\mathbf{T}^2S^{-1}+(Df/m)\sigma_r\tau_r+\text{const} , \quad (2.33)$$

where

$$S\mathbf{T}^2S^{-1}=\mathbf{T}^2+\boldsymbol{\sigma}\cdot\mathbf{T}+\tfrac{3}{4} . \quad (2.34)$$

Since

$$\langle 0_N(0,0,0)\,|\,\sigma_r\,|\,0_N(0,0,0)\rangle=0$$

and (2.35)

$$\langle 0_N(0,0,0)\,|\,\tau_\alpha\,|\,0_N(0,0,0)\rangle=0 ,$$

we can use the following approximation for the Hamiltonian:

$$H'=(N/4D^2)\mathbf{L}^2+\text{const}=(N/4D^2)\mathbf{T}^2+\text{const} \quad (2.36)$$

in strong-coupling theory.

Thus, we obtain the rotational mass spectra

$$E=(N/4D^2)I(I+1)+\text{const},\quad I=J . \quad (2.37)$$

The above results were obtained by Pauli and Dancoff in 1942.[6]

It is well known that eigenfunctions of the operator

$$\mathbf{L}^2=\mathbf{T}^2 = -\frac{1}{\sin a}\frac{\partial}{\partial a}\left[\sin a\frac{\partial}{\partial a}\right] -\frac{1}{\sin^2 a}\left[\frac{\partial^2}{\partial b^2}+\frac{\partial^2}{\partial c^2}-2\cos a\frac{\partial^2}{\partial b\,\partial c}\right] \quad (2.38)$$

are the rotation matrices[11]

$$D^j_{Mm}(bac)=e^{-iMb}d^j_{Mm}(a)e^{-imc} . \quad (2.39)$$

The eigenvalues are

$$j(j+1),\quad j=0,\tfrac{1}{2},1,\tfrac{3}{2},2,\ldots . \quad (2.40)$$

As the wave functions for a symmetric top, the eigenfunctions with half-integer j are discarded since they are double valued in the Euler angles θ,ϕ,ψ. In our case eigenfunctions with half-integer j are allowed since the Euler angles a,b,c are angles in the internal space.

We can show that the wave functions which appear in (2.32) are expressed as

$$\begin{aligned} A^{I=J}_{I_3,J_3}(abc) &= (-1)^{I_3}[(2I+1)/8\pi^2]^{1/2}D_{I_3,-J_3}(cab) \\ &= (-1)^{I_3}[(2I+1)/8\pi^2]^{1/2} \\ &\quad \times \exp(-iI_3c)d^I_{I_3,-J_3}(a)\exp(iJ_3b) \end{aligned} \quad (2.41)$$

since they satisfy the relations

$$\begin{aligned} L_\pm A^{I=J}_{I_3,J_3} &= [(J\mp J_3)(J\pm J_3+1)]^{1/2}A^{I=J}_{I_3,J_3\pm1} , \\ L_3 A^{I=J}_{I_3,J_3} &= J_3 A^{I=J}_{I_3,J_3} , \end{aligned} \quad (2.42)$$

$$\begin{aligned} T_\pm A^{I=J}_{I_3,J_3} &= [(I\mp I_3)(I\pm I_3+1)]^{1/2}A^{I=J}_{I_3\pm1,J_3} , \\ T_3 A^{I=J}_{I_3,J_3} &= I_3 A^{I=J}_{I_3,J_3} . \end{aligned} \quad (2.43)$$

Here we have neglected $\sigma/2$ and $\tau/2$ because of (2.35). In deriving (2.42) and (2.43) we have used the relations[12]

$$d^j_{a,b\pm1}=[(j\pm b+1)(j\mp b)]^{-1/2} \times[-a/\sin\theta+b\cot\theta\mp\partial/\partial\theta]d^j_{ab}(\theta)$$

and (2.44)

$$d^j_{ab}(a)=d^j_{-b,-a}(a) .$$

If we introduce the bound pion fields, which are eigenstates of T_3 and L_3,

$$\phi^0_{I_3,J_3}=U_{I_3\alpha}U_{J_3k}\phi^0_{\alpha k}\quad(I_3,J_3=1,0,-1) \quad (2.45)$$

with

$$U=\frac{1}{\sqrt{2}}\begin{bmatrix}-1 & -i & 0\\ 0 & 0 & \sqrt{2}\\ 1 & -i & 0\end{bmatrix}, \tag{2.46}$$

we find

$$\phi^0_{I_3,J_3}(abc)=(8\pi^2/3)^{1/2}DA^1_{I_3,J_3}(abc)\ . \tag{2.47}$$

It is easy to show that wave functions for proton and neutron states of spin up or down along the z axis $A^{1/2}_{I_3,J_3}$ are related to $A^{\frac{1}{2}}_{2I_3,2J_3}$ through

$$A^{1/2}_{I_3,J_3}(abc)=\pm(6\pi^2)^{-1/4}[A^{\frac{1}{2}}_{2I_3,2J_3}(abc)]^{1/2}\ . \tag{2.48}$$

Hence, we have found that collective coordinates in the Skyrme model $A=a_0+i\mathbf{a}\cdot\boldsymbol{\tau}$ correspond to the square roots of the fields of the pions bound to the bare nucleon in the strong-coupling model through

$$A^{1/2}_{I_3,J_3}(abc)=\text{const}\times[\phi^0_{2I_3,2J_3}(abc)]^{1/2}\ . \tag{2.49}$$

III. DISCUSSIONS

In Sec. II we have found that the collective coordinates in the Skyrme model are proportional to the square roots of the fields of the pions bound to the bare nucleons [Eq. (2.49)].

In the Skyrme model, the large-distance behavior of the pion fields are expressed as[4]

$$\phi_\alpha=(BF_\pi/4)(x_k/r^3)\mathrm{Tr}(\tau_k A^\dagger\tau_\alpha A)$$
$$(4\pi F_\pi M_N B\leftrightarrow 3g_{\pi NN})\ . \tag{3.1}$$

The isovector parts of the magnetic moments and the axial-vector couplings of the nucleons (N) and the resonances (Δ) are expressed as the matrix elements of

$$\boldsymbol{\mu}=b\,\mathrm{Tr}(\boldsymbol{\tau}A^\dagger\tau_3 A)\quad [b\leftrightarrow -3(\mu_p-\mu_n)/4] \tag{3.2}$$

and

$$\int d^3x\, A^\alpha_k(x)=g\,\mathrm{Tr}(\tau_k A^\dagger\tau_\alpha A)\quad (g\leftrightarrow -g_A) \tag{3.3}$$

between corresponding soliton states, respectively.[4] The parameters b and g are integrals involving the function $F(r)$, which appears in (1.3), and the parameter B is obtained from the large-distance ($r\rightarrow\infty$) behavior of $F(r)\rightarrow B/r^2$.

In the strong-coupling model the large-distance behavior of the pion fields are given by[6]

$$\phi_\alpha(x)=\phi^0_{\alpha k}\partial\xi(x)/\partial x_k+\phi'_\alpha(x)\rightarrow -(f/4\pi m)(x_k/r^3)e_{\alpha k}\ . \tag{3.4}$$

The S-transformed magnetic moment operator is expressed as[6]

$$S\mu_k S^{-1}=-(ef^2/3m^2)e_{3k}\times\int(dX/dr)^2d^3x-(e/4M_N)e_{3k}\ , \tag{3.5}$$

where

$$\boldsymbol{\mu}=\int(\mathbf{x}\times\mathbf{j}_\pi)d^3x/2+(e/2M_N)(1+\tau_3)\boldsymbol{\sigma}/2\ . \tag{3.6}$$

[However, there is an ambiguity in the definition of (3.6) since the continuity equation for the charge-current density is not satisfied everywhere inside the source in the strong-coupling model.]

From (3.1)–(3.5) we find that the dependence of various operators on α and k in two models are related to each other through the relation

$$\mathrm{Tr}(\tau_k A^\dagger\tau_\alpha A)=2e_{\alpha k}=2\phi^0_{\alpha k}/D\ . \tag{3.7}$$

This relation can be derived by making use of the relation (2.49) as well as (1.7), (2.22), and (2.24).

The expectation values of the operators (3.1)–(3.5) between the nucleonic states N and N' are obtained by making use of the relation

$$\langle N'|\mathrm{Tr}(\tau_k A^\dagger\tau_\alpha A)|N\rangle=2\langle N'|e_{\alpha k}|N\rangle$$
$$=-\tfrac{2}{3}\langle N'|\tau_\alpha\sigma_k|N\rangle\ . \tag{3.8}$$

There are similar relations among matrix elements between N and Δ and Δ' and Δ.

Hence, we have found that in the Skyrme-model calculations fermionic operators A and $A^\dagger$ always appear in bosonic combinations, $\mathrm{Tr}(\tau_k A^\dagger\tau_\alpha A)$, which correspond to the bound-pion field operators in the strong-coupling model, $\phi^0_{\alpha k}$. Therefore, the collective coordinates A have been found to be operators for pionic excitations of the nucleons effectively. This may be a natural consequence of the fact that the U field is a nonlinear functional of the pion fields ϕ_α.

Both the Skyrme model with (1.1)–(1.4) and the strong-coupling limit of the πN interaction model (2.1) give the same energy levels with $I=J$, (1.8) and (2.37).

Since resonances with the same internal quantum numbers are associated with a Regge trajectory ($\Delta J=2$) and since no exotic resonances, such as resonances with $(I,J)=(\frac{5}{2},\frac{5}{2}),(\frac{7}{2},\frac{7}{2})$ etc., have been observed, apparently the collective coordinates $A=a_0+i\boldsymbol{\tau}\cdot\mathbf{a}$ or $A^{1/2}_{I_3,J_3}$ are not good ones for higher nucleon resonances. The collective coordinates A are excellent operators only for transitions among $N(939)$ and $\Delta(1232)$. The correspondence (2.49) strongly suggests us to look for collective coordinates, bilinear forms of which are with $\Delta I=0$. For this purpose inclusion of $I=0$ mesons into the Skyrme model is suggested by the correspondence.

It has been found that we cannot improve the situation by including ρ and ω mesons into the Skyrme model since the collective coordinates are still $A=a_0+i\mathbf{a}\cdot\boldsymbol{\tau}$ in these cases.[13]

Next, let us compare theoretical predictions of both models with experimental results.

The relation (3.7) shows that relative ratios among magnetic transition rates, those among axial-vector transition rates and relative magnitudes of the coupling strengths of the pions to $N(939)$ and $\Delta(1232)$ in two models are same and are compatible with experimental results.[4]

However, predicted values on other physical observables by two models are different.

In the strong-coupling model of πN interaction (2.1), there are only two adjustable parameters: one is the πN coupling constant f,

$$f^2/4\pi=9(g_{\pi NN}m/2M_N)^2/4\pi=0.08\times 9=0.72\ , \tag{3.9}$$

and the other is the source function $K(r)$. From (2.13), (2.19), (2.37), and $M_\Delta - M_N = 300$ MeV, we find

$$\int |\nabla X|^2 d^3x = 3m^2/400f^2\ \text{MeV} = 16\ \text{MeV}\ . \tag{3.10}$$

By substituting this result into (3.5), we obtain the following result for the nucleon magnetic moment:

$$\mu_p = -\mu_n = 1.7e/2M_N\ . \tag{3.11}$$

In the strong-coupling model the isoscalar part of the nucleon magnetic moment vanishes.[6] We can also estimate the mean-square radius of the cloud of the charged pions surrounding the bare nucleon,

$$(\langle r^2\rangle)^{1/2} = \int r^2(\nabla K\cdot\nabla X)d^3x \bigg/ \int (\nabla K\cdot \nabla X)d^3x = 0.86\ \text{fm}\ , \tag{3.12}$$

which should be compared with the experimental results on the mean-square radius of the charge distribution of the proton $(\langle r^2\rangle)^{1/2}=0.81$ fm. In deriving the result (3.12), we have assumed that $K(r)=(M^3/8\pi)e^{-Mr}$ and $M=650$ MeV, which is derived from (3.9), (3.10), and $\int (dX/dr)^2 d^3x = 5M/64\pi$.

In the strong-coupling model of πN interaction it is impossible to improve the above results since there are no extra parameters unless extra mesons are introduced into the model.

In the Skyrme model with (1.1) the results, $(\mu_p-\mu_n)/2=1.6e/2M_N$, $(\mu_p+\mu_n)/2=0.3e/2M_N$, $(\langle r^2\rangle)^{1/2}_{M,I=0}=0.92$ fm, and $g^2_{\pi NN}/4\pi=6.3$, have been obtained by using the experimental results on M_N and M_Δ as inputs.[4] These results should be compared with experimental results, $2.35e/2M_N$, $0.44e/2M_N$, 0.81 fm, and 14.4, respectively. The agreement between theoretical and experimental results can be improved by introducing ω and ρ mesons and other contributions into the Skyrme model.[13] In the strong-coupling model we cannot use M_N and M_Δ as inputs.

Finally we comment about the source function $K(x)$. Until the quark structure of the nucleon was made clear, the source function had been considered to be a cutoff factor which indicates the inapplicability of the static model to emission and absorption of pions with large momenta. Now the source function represents the distribution of quarks in the nucleon which emits and absorbs mesons.

Because of this fact the range of two-nucleon interaction mediated by a heavy meson with mass M becomes much longer than $\hbar/Mc$ since the potential

$$(g/r)\exp(-Mcr/\hbar) \tag{3.13}$$

is replaced by

$$(g/r)\int K(x')\exp(-Mc\,|\mathbf{x}''-\mathbf{x}'|/\hbar) \times K(|\mathbf{x}-\mathbf{x}''|)d^3x'd^3x''\ . \tag{3.14}$$

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