

Statistical study of tau-decay data

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In the measured decay properties of the tau there is a discrepancy between the total branching fraction for the one-charged-particle decay modes and the sum of the branching fractions for the known individual modes. This discrepancy is derived from about 60 different measurements of branching fractions and some use of weak-interaction theory. Our statistical study of these 60 measurements shows there are problems in some of the measurements in the estimation of experimental bias or systematic error. But there is no evidence that the discrepancy derives from experimental bias or from incorrect estimation of systematic error.

I. INTRODUCTION

At present, the decay modes of the tau lepton containing one charged particle are not completely understood.¹⁻⁴ By definition the total branching fraction for those modes B_1 is the sum of the separate branching fractions B_α for the individual modes containing one charged particle, such as

$$B_e \text{ for } \tau^- \rightarrow \nu_\tau + e^- + \bar{\nu}_e, \quad (1a)$$

$$B_\mu \text{ for } \tau^- \rightarrow \nu_\tau + \mu^- + \bar{\nu}_\mu, \quad (1b)$$

$$B_\pi \text{ for } \tau^- \rightarrow \nu_\tau + \pi^-, \quad (1c)$$

$$B_\rho \text{ for } \tau^- \rightarrow \nu_\tau + \rho^-, \quad (1d)$$

and

$$B_{\pi_2\pi^0} \text{ for } \tau^- \rightarrow \nu_\tau + \pi^- + 2\pi^0. \quad (1e)$$

As emphasized by Gilman^{1,2} the sum $\sum_\alpha B_\alpha$ of present measurements of the individual branching fractions combined with theoretical constraints on unmeasured branching fractions does not fully explain the present measured value of B_1 . A question raised by this problem is whether the errors σ_α given for the measured branching fractions by the experimenters are correct, whether the appearance of a discrepancy between $\sum_\alpha B_\alpha$ and B_1 is caused by an underestimate of the size of one or more σ_α 's or of σ_1 .

We have examined this question by comparing the given errors σ_α with the scatter of the measurements about the mean for each measurement set. We do this for B_1 and for the B_α 's of the modes in Eqs. (1a)–(1d). Normal error distributions are used. We find that on the whole the errors estimated from the scatter are equal to or smaller than the given errors σ_α according to this test.

In other words, some sets of measurements are overconsistent. By using just the statistical contribution to the measured errors, we can test in some cases whether the overconsistency is caused by overestimation of systematic errors or bias in the measurements.

As an aid to researchers in this area we present tables of the data we used. This is all the data published in journals, in catalogs, or in Ph.D. theses, the authors being the experimenters themselves. We also present a comparison of the measured τ lifetime τ_τ with the leptonic branching fractions B_e and B_μ for the decays in Eqs. (1a) and (1b).

The nature of the present apparent discrepancy^{3,4} between B_1 and $\sum_\alpha B_\alpha$ is diagrammed in Table I. There is no discrepancy if considerations of the various B_α 's is limited to direct measurements. This is because there are no reliable measurements⁴ for some modes contributing to the signature

$$\tau^- \rightarrow \nu_\tau + x^- + n\gamma, \quad n > 4, \quad (2)$$

where x is a charged particle. Also there are no comprehensive and sufficiently small experimental limits on unconventional one-charged-particle decay modes such as

$$\tau^- \rightarrow N^0 + x^-, \quad (3)$$

where N^0 is an unknown, massive, stable neutral particle.

The discrepancy appears when unconventional modes are excluded, and when conventional theory and other data are used^{1,2} to set limits on the modes which could contribute to the event type in Eq. (2). Then B_1 is larger than $\sum_\alpha B_\alpha$ by about 6% (Ref. 4). Table I demonstrates the importance of the measurements and quoted errors on the branching fractions for B_e , B_μ , B_π , B_ρ , and B_1 . This motivated our study.

TABLE I. (a) Summary of measured branching fractions of modes with one charged particle. (b) Summary of one-charged-particle branching fractions in percent.

Type of measurement	Row	(a) Decay mode	Branching fraction (%)	Reference
Exclusive measurements of modes with 0 or 1 π^0	A	$e^- \bar{\nu}_e \nu_\tau$	17.6 ± 0.4	Table VIII
	B	$\mu^- \bar{\nu}_\mu \nu_\tau$	17.7 ± 0.4	Table VIII
	C	$\pi^- \nu_\tau$	10.8 ± 0.6	Table VIII
	D	$\rho^- \nu_\tau$	22.5 ± 0.9	Table VIII
	E	$K^- \nu_\tau$	0.7 ± 0.2	Ref. 3
	F	$K^{*-} \nu_\tau$	1.4 ± 0.1	Ref. 3
Sum of rows A–F called $B_{1e\mu\pi\rho K}$	G		70.7 ± 1.2	
Sum of modes with > 1 π^0 or η 's called $B_{1 \text{ mult neut}}$	H	$\pi^- n \pi^0 \nu_\tau, n > 1$	8–16	Ref. 4
		$\pi^- n \eta \nu_\tau, n > 0$		
		$\pi^- m \pi^0 n \eta \nu_\tau, m + n > 1$		
		$K^- n \pi^0 \nu_\tau, n > 1$		
		\vdots		
		(b)		
Decay-mode category		Branching fraction (%) and origin		
$B_{1e\mu\pi\rho K}$		70.8 ± 1.2 from measurement [Table I(a)]		
$B_{1 \text{ mult neut}}$		8–16	≤ 9.8	
		From measurement (Ref. 4)	From theory and other data (Refs. 1, 2, and 4)	
$B_{1e\mu\pi\rho K} + B_{1 \text{ mult neut}}$		79–87	$\leq 80.5 \pm 1.2$	
B_1		86.6 ± 0.3 from measurement (Table VIII)		

TABLE II. τ topological branching fractions in percent. The statistical error is given first, the systematic error second. We list all measurements provided the measurement is described in a report, journal article, or Ph.D. thesis authored by the experimenters, and the authors have not stated the measurement is superseded by a more recent measurement.

Measurement	B_1		B_3		Energy (GeV)	Experimental group	Reference
	Combined error	Weight	Measurement	Combined error			
70 ^a	± 10		30 ^{a,b}	± 10	3.6–5.0	PLUTO	J. Burmester <i>et al.</i> , Phys. Lett. 68B , 297 (1977)
68 ^{a,b}	± 5		32 ^a	± 5	3.1–7.4	DELCO	W. Bacino <i>et al.</i> , Phys. Rev. Lett. 41 , 13 (1978)
65 ^{a,b}	± 11		35 ^a	± 11	3.9–5.2	DASP	R. Brandelik <i>et al.</i> , Phys. Lett. 73B , 109 (1978)
82 ^{a,b}	± 6.5		18 ^a	± 6.5	6–7.4	Mark I	J. Jaros <i>et al.</i> , Phys. Rev. Lett. 40 , 1120 (1978)
76 ^a	± 6		24 ^a	± 6	12–31.6	TASSO	R. Brandelik <i>et al.</i> , Phys. Lett. 92B , 199 (1980)
84.0	± 2.0	0.019	15.0	± 2.0	32.0–36.8	CELLO	H. J. Behrend <i>et al.</i> , Phys. Lett. 114B , 282 (1982)
$86.0 \pm 2.0 \pm 1.0^a$	± 2.2		$14.0 \pm 2.0 \pm 1.0^a$	± 2.2	29.0	Mark II	C. A. Blocker <i>et al.</i> , Phys. Rev. Lett. 49 , 1369 (1982)
$85.2 \pm 0.9 \pm 1.5^a$	± 1.7		$14.8 \pm 0.9 \pm 1.5^a$	± 1.7	29.0	TPC	H. Aihara <i>et al.</i> , Phys. Rev. D 30 , 2436 (1984)

TABLE II. (Continued).

Measurement	B_1		B_3		Energy (GeV)	Experimental group	Reference
	Combined error	Weight	Measurement	Combined error			
85.2±2.6±1.3	±2.9	0.009	14.8±2.0±1.3	±2.4	14.0	CELLO	H. J. Behrend <i>et al.</i> , Z. Phys. C 23 , 103 (1984)
85.1±2.8±1.3	±3.1	0.008	14.5±2.2±1.3	±2.6	22.0	CELLO	H. J. Behrend <i>et al.</i> , Z. Phys. C 23 , 103 (1984)
87.8±1.3±3.9	±4.1	0.005	12.2±1.3±3.9	±4.1	34.6 average	PLUTO	Ch. Berger <i>et al.</i> , Z. Phys. C 28 , 1 (1985)
84.7±1.1 $^{+1.6}_{-1.3}$	$^{+1.9}_{-1.7}$	0.024	15.3±1.1 $^{+1.3}_{-1.6}$	$^{+1.7}_{-1.9}$	13.9–43.1	TASSO	M. Althoff <i>et al.</i> , Z. Phys. C 26 , 521 (1985)
86.7±0.3±0.6	±0.7	0.157	13.3±0.3±0.6	±0.7	29.0	MAC	E. Fernandez <i>et al.</i> , Phys. Rev. Lett. 54 , 1624 (1985)
86.9±0.2±0.3	±0.4	0.482	13.0±0.2±0.3	±0.4	29.0	HRS	C. Akerlof <i>et al.</i> , Phys. Rev. Lett. 55 , 570 (1985)
86.1±0.5±0.9	±1.0	0.077	13.6±0.5±0.8	±0.9	30.0–46.8	JADE	W. Bartel <i>et al.</i> , Phys. Lett. 161B , 188 (1985)
87.9±0.5±1.2	±1.3	0.046	12.1±0.5±1.2	±1.3	29.0	DELCO	W. Ruckstuhl <i>et al.</i> , Phys. Rev. Lett. 56 , 2132 (1986)
87.2±0.5±0.8	±0.9	0.095	12.8±0.5±0.8	±0.9	29.0	Mark II	W. B. Schmidke <i>et al.</i> , Phys. Rev. Lett. 57 , 527 (1986)
87.1±1.0±0.7 ^{a,b}	±1.2		12.8±1.0±0.7 ^a	±1.2	29.0	Mark II	P. R. Burchat <i>et al.</i> , Phys. Rev. D 35 , 27 (1987)
84.7±0.8±0.6	±1.0	0.077	15.1±0.8±0.6	±1.0	29.0	TPC	H. Aihara <i>et al.</i> , Phys. Rev. D 35 , 1553 (1987)

^aNot included in average.

^bCalculated from B_1 or B_3 measurement using $B_1 + B_3 + B_5 = 1$ with $B_5 = 0.1\%$.

II. DATA USED

We used the branching-fraction data listed in Tables II–V and the lifetime data in Table VI. We have included in the tables all data presented by the experimenters themselves in journal articles, a catalog of unpublished papers, or Ph.D. theses unless the experimenters have stated their measurement is replaced by their own later measurement. We have not included measurements which are reported only through private communication or in reviews. These criteria permit us to work with fixed measurements and permit the reader to examine the details of experiments.

Table II presents B_3 as well as B_1 although most measurements of B_1 and B_3 are strongly correlated. Often only one is measured and the other calculated by $B_1 + B_3 + B_5 = 1$ with $B_5 = 0.1\%$.

To ensure the measurements for a specific branching ratio are statistically independent, we have excluded several measurements from the statistical analysis al-

though they are included in the tables. The Mark II Collaboration has published two measurements^{5,6} of B_1 and two measurements^{5,7} of B_ρ that use the same data set but different analysis techniques. We use only the measurement with the smallest total error. The 1982 Mark II Collaboration and 1984 TPC Collaboration measurements^{8,9} of B_1 are not independent of their more precise recent measurements and thus are also excluded.

III. B_e , B_μ , AND e - μ UNIVERSALITY

The use of the constraint on B_e and B_μ from e - μ universality, $B_\mu = 0.973B_e$, must be carefully considered when comparing or averaging experimental measurements. Four cases occur: (i) the experimenters measure the product branching ratio $B_e B_\mu$ and use the constraint to determine B_e and B_μ ; (ii) the experimenters measure B_e and B_μ independently; (iii) the experimenters measure B_e and B_μ separately but the measurements are strongly

TABLE III. τ leptonic branching fractions in percent. The statistical error is given first, the systematic error second. We list all measurements provided the measurement is described in a report, journal article, or Ph.D. thesis authored by the experimenters, and the authors have not stated the measurement is superseded by a more recent measurement. The first two columns of branching ratios are published measurements. The third column contains values we have determined using the μ - e universality constraint $B_\mu = 0.973B_e$ as described in the text.

Use e - μ universality	$B(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)$		$B(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)$		$B(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)$ assuming $B_\mu = 0.973B_e$		Energy (GeV)	Experimental group	Reference
	Measurement	Weight	Measurement	Weight	Measurement	Weight			
Yes ^a	18.9±1.0±2.8	±3.0	18.3±1.0±2.8	±3.0	18.9±1.0±2.8	±3.0	3.8-7.8	Mark I	M. L. Perl <i>et al.</i> , Phys. Lett. 70B , 487 (1977)
No			17.5±2.7±3.0	±4.0	18.0±2.8±3.1	±4.2	3.8-7.8	Mark I	Same data as above
No			22	$^{10}_{-7}$	22.6	$^{10.3}_{-7.2}$	4.8		M. Cavalli-Sforza <i>et al.</i> , Lett. Nuovo Cimento 20 , 337 (1977)
Yes ^a	22.7	±5.5	22.1	±5.5	22.7	±5.5	4.1-7.4	Lead-glass- wall	A. Barbaro-Galieri <i>et al.</i> , Phys. Rev. Lett. 39 , 1058 (1977)
No			15	±3.0	15.4	±3.1	3.6-5.0	PLUTO	J. Burmester <i>et al.</i> , Phys. Lett. 68B , 297 (1977)
No	16	±1.3	0.116		16.0	±1.3	3.1-7.4	DELCO	W. Bacino <i>et al.</i> , Phys. Rev. Lett. 41 , 13 (1978)
No			22	$^{+7}_{-8}$	22.6	$^{+7.2}_{-8.2}$	6.4-7.4	Iron ball	J. G. Smith <i>et al.</i> , Phys. Rev. D 18 , 1 (1978)
Yes ^a	18.5±2.8±1.4	±3.1	18.0±2.8±1.4	±3.1	18.5±2.8±1.4	±3.1	3.9-5.2	DASP	R. Brandelik <i>et al.</i> , Phys. Lett. 73B , 109 (1978)
No			21±5±3	±6	21.6±5.1±3.1	±6.0	3.6-7.4	DELCO	W. Bacino <i>et al.</i> , Phys. Rev. Lett. 42 , 6 (1979)
No	19	±9.0	0.002	±14	0.0009	±7.6	12-31.6	TASSO	R. Brandelik <i>et al.</i> , Phys. Lett. 92B , 199 (1980)
No			17.8±2.0±1.8	±2.7	18.3±2.1±1.8	±2.8	9.4-31.6	PLUTO	Ch. Berger <i>et al.</i> , Phys. Lett. 99B , 489 (1981)
Yes	17.6±0.6±1.0	±1.3	17.1±0.6±1.0	±1.3	17.6±0.6±1.0	±1.3	3.5-6.7	Mark II	C. A. Blocker <i>et al.</i> , Phys. Lett. 109B , 119 (1982)

TABLE III. (Continued).

Use $e\text{-}\mu$ universality	$B(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)$		$B(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)$		$B(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)$ assuming $B_\mu = 0.973 B_e$		Energy (GeV)	Experimental group	Reference			
	Measurement	Combined error	Weight	Measurement	Combined error	Weight				Measurement	Combined error	Weight
No	$18.3 \pm 2.4 \pm 1.9$	± 3.1	0.021	$17.6 \pm 2.6 \pm 2.1$	± 3.3	0.016	$18.2 \pm 1.8 \pm 1.4$	± 2.3	0.013	34.0	CELLO	H. J. Behrend <i>et al.</i> , Phys. Lett. 127B , 270 (1983)
No	$20.4 \pm 3.0 \pm 1.4$	$^{+3.3}_{-3.1}$	0.019	$12.9 \pm 1.7 \pm 0.7$	± 1.8	0.052	$15.0 \pm 1.5 \pm 0.3$	± 1.6	0.027	13.9–43.1	TASSO	M. Althoff <i>et al.</i> , Z. Phys. C 26 , 521 (1985)
No	$13.0 \pm 1.9 \pm 2.9$	± 3.5	0.016	$19.4 \pm 1.6 \pm 1.7$	± 2.3	0.032	$17.8 \pm 1.2 \pm 1.5$	± 1.9	0.019	34.6 average	PLUTO	Ch. Berger <i>et al.</i> , Z. Phys. C 28 , 1 (1985)
No	$18.2 \pm 0.7 \pm 0.5$	± 0.9	0.243	$18.0 \pm 1.0 \pm 0.6$	± 1.2	0.117	$18.3 \pm 0.5 \pm 0.4$	± 0.6	0.191	3.8	Mark III	R. M. Baltrusaitis Phys. Rev. Lett. 55 , 1842 (1985)
No	$17.4 \pm 0.8 \pm 0.5$	± 0.9	0.243	$17.7 \pm 0.8 \pm 0.5$	± 0.9	0.208				29.0	MAC	W. W. Ash <i>et al.</i> , Phys. Rev. Lett. 55 , 2118 (1985)
Yes	$17.8 \pm 0.4 \pm 0.3^b$	± 0.5		$17.3 \pm 0.4 \pm 0.3^b$	± 0.5		$17.8 \pm 0.4 \pm 0.3^b$	± 0.5	0.275	29.0	MAC	Same data as above
No				$17.4 \pm 0.6 \pm 0.8$	± 1.0	0.169	$17.9 \pm 0.6 \pm 0.8$	± 1.0	0.069	4.0–46.8	Mark J	B. Adeva <i>et al.</i> , Phys. Lett. B 179 , 177 (1986)
No	$17.0 \pm 0.7 \pm 0.9$	± 1.1	0.163	$18.8 \pm 0.8 \pm 0.7$	± 1.1	0.139	$18.2 \pm 0.5 \pm 0.6$	± 0.8	0.107	34.6 average	JADE	W. Bartel <i>et al.</i> , Phys. Lett. B 182 , 216 (1986)
No	$18.4 \pm 1.2 \pm 1.0$	± 1.6	0.077	$17.7 \pm 1.2 \pm 0.7$	± 1.4	0.086				29.0	TPC	H. Aihara <i>et al.</i> , Phys. Rev. D 35 , 1553 (1987)
Yes	$18.3 \pm 0.7 \pm 0.5$	± 0.9		$17.8 \pm 0.7 \pm 0.5$	± 0.9		$18.3 \pm 0.7 \pm 0.5$	± 0.9	0.085	29.0	TPC	Same data as above
No	$19.1 \pm 0.8 \pm 1.1$	± 1.4	0.100	$18.3 \pm 0.9 \pm 0.8$	± 1.2	0.117	$18.9 \pm 0.6 \pm 0.7$	± 0.9	0.085	29.0	Mark II	P. R. Burchat <i>et al.</i> , Phys. Rev. D 35 , 27 (1987)

^aAdjusted for $B(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau) / B(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) = 0.973$.^bWe have determined the breakdown of statistical and systematic errors.

TABLE IV. $\tau^- \rightarrow \pi^- \nu_\tau$ branching ratio in percent. The statistical error is given first, the systematic error second. We list all measurements provided the measurement is described in a report, journal article, or Ph.D. thesis authored by the experimenters, and the authors have not stated the measurement is superseded by a more recent measurement.

Measurement	Combined error	Weight	Energy (GeV)	Experimental group	Reference
$9.0 \pm 2.9 \pm 2.5$	± 3.8	0.025	4.1–5.0	PLUTO	G. Alexander <i>et al.</i> , Phys. Lett. 78B , 162 (1978)
$8.0 \pm 3.2 \pm 1.3$	± 3.5	0.029	3.6–7.4	DELCO	W. Bacino <i>et al.</i> , Phys. Lett. 42 , 6 (1978)
$11.7 \pm 0.4 \pm 1.8$	± 1.8	0.109	3.5–6.7	Mark II	C. A. Blocker <i>et al.</i> , Phys. Lett. 109B , 119 (1982)
$9.9 \pm 1.7 \pm 1.3$	± 2.1	0.080	34.0	CELLO	H. J. Behrend <i>et al.</i> , Phys. Lett. 127B , 270 (1983)
$11.8 \pm 0.6 \pm 1.1$	± 1.3	0.210	34.6 average	JADE	W. Bartel <i>et al.</i> , Phys. Lett. B 182 , 216 (1986)
$10.7 \pm 0.5 \pm 0.8$	± 0.9	0.438	29.0	MAC	W. T. Ford <i>et al.</i> , Phys. Rev. D 35 , 408 (1987)
$10.0 \pm 1.1 \pm 1.4$	± 1.8	0.109	29.0	Mark II	P. R. Burchat <i>et al.</i> , Phys. Rev. D 35 , 27 (1987)

TABLE V. $\tau^- \rightarrow \rho^- \nu_\tau$ branching ratio in percent. The statistical error is given first, the systematic error second. We list all measurements provided the measurement is described in a report, journal article, or Ph.D. thesis authored by the experimenters, and the authors have not stated the measurement is superseded by a more recent measurement.

Measurement	Combined error	Weight	Energy (GeV)	Experimental group	Reference
$24 \pm 6 \pm 7^a$	± 9	0.009	3.6–5.2	DASP	R. Brandelik <i>et al.</i> , Z. Phys. C 1 , 233 (1979)
$21.5 \pm 1.7 \pm 3.0^a$	± 3.4	0.063	3.7–6.0	Mark I	C. A. Blocker, thesis, Report No. LBL-10801, 1980
$22.1 \pm 1.9 \pm 1.6$	± 2.5	0.116	14.0–34.0	CELLO	H. J. Behrend <i>et al.</i> , Z. Phys. C 23 , 103 (1984)
$22.3 \pm 0.6 \pm 1.4$	± 1.5	0.323	29.0	Mark II	J. M. Yelton <i>et al.</i> , Phys. Rev. Lett. 56 , 812 (1986)
$23.0 \pm 1.3 \pm 1.7$	± 2.1	0.165	3.8	Mark III	J. Adler <i>et al.</i> , Phys. Rev. Lett. 59 , 1527 (1987)
$25.8 \pm 1.7 \pm 2.5^b$	± 3.0		29.0	Mark II	P. R. Burchat <i>et al.</i> , Phys. Rev. D 35 , 27 (1987)
$22.6 \pm 0.5 \pm 1.4$	± 1.5	0.323	9.4–10.6	Crystal Ball	S. T. Lowe <i>et al.</i> , Report No. SLAC-PUB-4449, 1987 (unpublished)

^aWe have determined the breakdown of statistical and systematic errors.

^bAll $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ included in $\tau^- \rightarrow \rho^- \nu_\tau$. Not included in formal average.

TABLE VI. τ lifetime in units of 10^{-13} sec. The statistical error is given first, the systematic error second. We list all measurements provided the measurement is described in a report, journal article, or Ph.D. thesis authored by the experimenters and the authors have not stated the measurement is superseded by a more recent measurement.

Lifetime	Errors combined in quadrature	Weight	Energy (GeV)	Experimental group	Reference
4.6	± 1.9	0.002	29.0	Mark II	G. Feldman <i>et al.</i> , Phys. Rev. Lett. 48 , 66 (1982)
4.9	± 2.0	0.002	29.0	MAC	W. Ford <i>et al.</i> , Phys. Rev. Lett. 49 , 106 (1982)
4.7	$^{+3.9}_{-2.9}$	0.0006	17.1 average	CELLO	H. J. Behrend <i>et al.</i> , Nucl. Phys. B211 , 369 (1983)
$3.18^{+0.59}_{-0.75} \pm 0.56$	$^{+0.81}_{-0.94}$	0.010	39.8–45.2	TASSO	M. Althoff <i>et al.</i> , Phys. Lett. 141B , 264 (1984)
$3.15 \pm 0.36 \pm 0.40$	± 0.54	0.025	29.0	MAC	E. Fernandez <i>et al.</i> , Phys. Rev. Lett. 54 , 1624 (1985)
$2.63 \pm 0.46 \pm 0.20$	± 0.50	0.029	29.0	DELCO	D. E. Klem <i>et al.</i> , Report No. SLAC-REP-300 1986 (unpublished)
$2.88 \pm 0.16 \pm 0.17$	± 0.23	0.134	29.0	Mark II	D. Amidei <i>et al.</i> , Phys. Rev. D 37 , 1750 (1988)
3.09	± 0.19	0.202	29.0	MAC	H. R. Band <i>et al.</i> , Phys. Rev. Lett. 59 , 415 (1987)
$2.99 \pm 0.15 \pm 0.10$	± 0.18	0.225	29.0	HRS	S. Abachi <i>et al.</i> , Phys. Rev. Lett. 59 , 2519 (1987)
$3.25 \pm 0.14 \pm 0.18$	± 0.23	0.140	10.5	CLEO	C. Bebek <i>et al.</i> , Phys. Rev. D 36 , 690 (1987)
$2.95 \pm 0.14 \pm 0.11$	± 0.18	0.230	9.3–10.6	ARGUS	H. Albrecht <i>et al.</i> , Phys. Lett. B 199 , 580 (1987)

correlated, perhaps because they simultaneously measure the product $B_e B_\mu$; and (iv) only B_e or B_μ is measured. In the measurements listed in Table III, there are four type-(i), six type-(ii), three type-(iii), and eight type-(iv) experiments.

In our analysis of the B_e and B_μ measurements, we first analyze only the subsets of experiments which do not make use of the universality constraint. The experiments in these sets can be equally treated in the analysis, and allow a test of the universality constraint to be made. We then apply, if necessary, the universality constraint to each experiment in Table III and determine a constrained branching ratio B'_e . The third column of branching ratios listed in Table III are the results of this constraint procedure. The statistical analysis is then applied to the full set of constrained measurements.

In the constraint procedure, type-(i) experiments and experiments which measure only B_e are used directly. Experiments which measure only B_μ are scaled: $B'_e = B_\mu / 0.973$. Type-(ii) experiments are constrained using the equations below

$$B'_e = (B_e / \sigma_{B_e}^2 + 0.973 B_\mu / \sigma_{B_\mu}^2) / (1 / \sigma_{B_e}^2 + 0.973^2 / \sigma_{B_\mu}^2) \quad (4)$$

and

$$\sigma_{B'_e} = (1 / \sigma_{B_e}^2 + 0.973^2 / \sigma_{B_\mu}^2)^{-1/2}. \quad (5)$$

The universality constraint can be applied to type-(iii) experiments if the correlation between the B_e and B_μ measurements is known. For the special case where B_e , B_μ , and the product branching ratio $B_{e\mu} = B_e B_\mu$ are measured, then B'_e is determined by minimizing the χ^2 :

$$\begin{aligned} \chi^2 = & (B'_e - B_e)^2 / \sigma_{B_e}^2 + (0.973 B'_e - B_\mu)^2 / \sigma_{B_\mu}^2 \\ & + (0.973 B_e'^2 - B_{e\mu})^2 / \sigma_{B_{e\mu}}^2. \end{aligned} \quad (6)$$

This results in a cubic equation for B'_e and an error given by

$$\sigma_{B_e'} = \left\{ 1/\sigma_{B_e}^2 + 0.973^2/\sigma_{B_\mu}^2 + 2(0.973) \times [3(0.973)B_e'^2 - B_{e\mu}]/\sigma_{B_{e\mu}}^2 \right\}^{-1/2}. \quad (7)$$

Note that these constraint techniques average the systematic errors for B_e , B_μ , and $B_{e\mu}$ within a single experiment. Thus, systematic errors which are common to the B_e , B_μ , and $B_{e\mu}$ measurements will be averaged, resulting, perhaps, in an *underestimate* of the systematic error on B_e' .

IV. ANALYSIS METHOD

Consider a particular branching fraction B_π , for example. As listed in Table IV there are seven different measurements: $B_{\pi 1}, B_{\pi 2}, \dots, B_{\pi i}, \dots$. We want the weighted average $\langle B_\pi \rangle$ and the error on that average, σ_{B_π} . Simplifying the notation, y_i replaces $B_{\pi i}$, y replaces $\langle B_\pi \rangle$, and σ replaces σ_{B_π} . The same notation is used for B_e , B_μ , B_ρ , and B_1 .

Most recent measurements y_i included a statistical error $\sigma_{\text{stat},i}$ and a systematic error $\sigma_{\text{sys},i}$. We follow the Particle Data Group's method¹⁰ of combining these errors in quadrature,

$$\sigma_i = (\sigma_{\text{stat},i}^2 + \sigma_{\text{sys},i}^2)^{1/2}, \quad (8)$$

and we use this combined error unless the experimenters provide a total error. The formal average is

$$y = \frac{\sum_i (y_i/\sigma_i^2)}{\sum_i (1/\sigma_i^2)} \quad (9)$$

and the formal combined error in y is

$$\sigma = \left[\sum_i (1/\sigma_i^2) \right]^{-1/2}. \quad (10)$$

The relative weight of a measurement i is

$$w_i = \sigma^2/\sigma_i^2. \quad (11)$$

The scatter of the individual measurements y_i from y are described by the pulls

$$p_i = (y - y_i)/(\sigma_i^2 - \sigma^2)^{1/2}. \quad (12)$$

For Gaussian distributed errors σ_i , the distribution of pulls is a normal distribution of unit width and zero mean.

The standard deviation, σ_{scat} , of the weighted mean is calculated from the average variance s_{scat}^2 for N measurements:

$$s_{\text{scat}}^2 = \sum_i [Nw_i(y_i - y)^2]/(N - 1),$$

which reduces to

$$s_{\text{scat}}^2 = N \left[\sum_i w_i y_i^2 - y^2 \right] / (N - 1).$$

Using $\sigma_{\text{scat}}^2 = s_{\text{scat}}^2/N$, we have

$$\sigma_{\text{scat}} = \left[\left[\sum_i w_i y_i^2 - y^2 \right] / (N - 1) \right]^{1/2}. \quad (13)$$

Observe that the errors σ_i are used in this equation in the

weighting of $(y_i - y)^2$, but are not directly used to calculate σ_{scat} .

Our interest centers on the relative sizes of σ and σ_{scat} . As discussed in Sec. V, if σ is significantly smaller than σ_{scat} , some of the experimenters have given σ_i^2 which are too small. Then the σ used in Table I is too small, and the discrepancy problem is less certain. If σ is significantly larger than σ_{scat} , there are three, not exclusive, explanations. Some experimenters may have overestimated their σ_i^2 . Or some experimenters may have corrected their raw measurements while biased toward a preconceived value for y , the preconception being based on the existing accepted value of y or on theoretical considerations. Finally, systematic errors common to many experiments may exist which the experimenters have accounted for in their determination of σ_i . In this case the correlated contribution to σ_i should be removed before comparing σ and σ_{scat} . There are no sources of accounted correlated systematic errors common to many experiments in these data which are described in the referenced experimental papers. Although with intimate knowledge of all experiments such sources might be found, we have made no attempt here to hunt for them.

We use the ratio

$$r = \sigma_{\text{scat}}/\sigma \quad (14)$$

to measure the relative sizes of σ_{scat} and σ for a set of measurements. We are particularly interested if r is significantly less than 1 or significantly greater than 1. To determine the significance we calculate the probability $P(<r)$ of finding a smaller value of r , and the converse $P(>r)$ of finding a larger value of r . [Since $P(<r) + P(>r) = 1$, only $P(<r)$ need be calculated.] For example, suppose $r = 0.5$ because σ is twice σ_{scat} . If $P(<0.5)$ is 0.10, then $r = 0.5$ has this statistical significance.

The formal average [Eq. (9)] is obtained by minimizing

$$\chi^2 = \sum_i (y_i - y)^2/\sigma_i^2, \quad (15)$$

which has the minimum value

$$\chi_{\text{min}}^2 = \sum_i (w_i y_i^2 - y^2)/\sigma^2. \quad (16)$$

From Eqs. (13) and (16) we see that

$$r = [\chi_{\text{min}}^2/(N - 1)]^{1/2}. \quad (17)$$

Thus the probability $P(>r)$ is identical to the probability of having a larger χ^2 for $N - 1$ degrees of freedom. Figure 1 plots the distribution of r for several values of N .

We apply this analysis method first to each full set of data. However, r can be very sensitive to a particular measurement which has a relatively large σ_i even though that measurement has a small weight w_i and little effect on y . Therefore in each data set we select the minimum number of measurements a, b, \dots, e such that

$$w_a + w_b + \dots + w_e > 0.81.$$

This smaller set of measurements will have a formal error no larger than $1/0.9$ of σ and will contain fewer measure-

ments with relative large σ_i 's. We apply the same method of analysis to these smaller sets of data.

For both the full and small data sets we apply the same method of analysis using just the statistical errors. This tests the effects of systematic errors on the determination of y and σ . We examine the relative importance of statistical and systematic errors in determining the formal error as follows: if σ_{stat} is the formal error obtained using only statistical errors, then we define the contribution to the formal error from systematic errors to be

$$\sigma_{\text{sys}} = (\sigma^2 - \sigma_{\text{stat}}^2)^{1/2}. \quad (18)$$

Here $\sigma_{\text{stat}} = (\sum_i \sigma_{\text{stat},i}^{-2})^{-1/2}$. But note that σ_{sys} is not $(\sum_i \sigma_{\text{sys},i}^{-2})^{-1/2}$.

In a few measurement sets the formal error is asymmetric: $\sigma_+ \neq \sigma_-$. In that event we use the arithmetic average. There is no change in our conclusions if we used the maximum or minimum of σ_+, σ_- because in all cases their difference is relatively small.

V. SYSTEMATIC ERRORS AND r

The combined error σ_i of a measurement y_i is obtained from $\sigma_i = (\sigma_{\text{stat},i}^2 + \sigma_{\text{sys},i}^2)^{1/2}$. The statistical error $\sigma_{\text{stat},i}$ depends on numbers of events and represents a normal error distribution. Hence, our use of $\sigma_{\text{stat},i}$ is straightforward. This is not true for the systematic error $\sigma_{\text{sys},i}$. There are a multitude of uncertainties in the estimate and use of $\sigma_{\text{sys},i}$.

An obvious problem is that $\sigma_{\text{sys},i}$ may not represent a normal error distribution. Suppose it represents an error distribution with tails relatively larger than those of a normal distribution. The use of σ_i to calculate the weighted formal average is still acceptable. But it would be wrong to interpret the formal error σ as representing a normal distribution when one is considering discrepancies which are several σ in magnitude. In a later paper¹¹ we will consider a method of treating errors which does not depend on the normal distribution assumption; in this

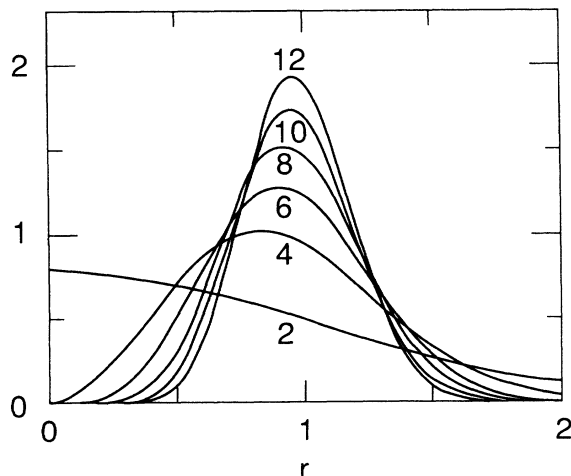


FIG. 1. r distribution for different values of N .

paper we maintain the normal error distribution assumption for $\sigma_{\text{sys},i}$ as well as $\sigma_{\text{stat},i}$.

The determination of a branching fraction requires the counting in a data set of the number of τ decays with that decay mode. This number is then multiplied by factors f, g, h, \dots . These factors include normalization quantities such as total number of τ decays or total luminosity or total cross section, efficiency factors such as detector acceptance, and perhaps other quantities. A few of these factors are obtained by counting events in the data set—the total number of decays, for example—and are assigned a statistical error. But most of the factors are obtained by computation or from other data and are assigned a systematic error. (A few factors may have both types of errors.) Let $\sigma_i(f)$ be the systematic error assigned to the factor f_i by the experimenters who reported branching fraction measurement y_i with errors $\sigma_{\text{stat},i}$ and $\sigma_{\text{sys},i}$.

We are about to tabulate some of the problems that can occur in a measurement set $y_1, y_2, \dots, y_i, \dots$ from incorrect evaluation of f_i 's or $\sigma_i(f)$'s. We emphasize two aspects of these incorrect evaluations: (a) we examine whether the formal error σ will be smaller or larger than the *actual error* on y ; (b) we look at the ratio $r = \sigma_{\text{scat}}/\sigma$. If σ_{scat} is significantly less than σ , that is, $r < 1$, then the measurement set is *overconsistent*. If σ_{scat} is significantly greater than σ , then the measure set is *inconsistent*.

(i) *Overestimation of some $\sigma_i(f)$'s.* For the sake of caution and because of the difficulty of evaluating some f 's, experimenters may assign large $\sigma_i(f)$'s. Then, (a) σ is larger than the actual error on y and (b) σ_{scat} is smaller than σ and the measurement set is overconsistent; therefore $r < 1$.

(ii) *Underestimation of some $\sigma_i(f)$'s.* In spite of caution, the history of physics has many examples of underestimation of systematic errors. Then (a) σ is smaller than the actual error on y and (b) σ_{scat} is larger than σ and the measurement set is inconsistent; therefore, $r > 1$.

(iii) *Biasing of y_i 's.* The values of some f_i 's may be set unconsciously so that the resulting y_i tends towards an already published or preconceived value of y . The $\sigma_i(f)$'s may not be set large enough to encompass this bias. Then (a) σ is smaller than the actual error on y and (b) depending on whether different experiments are biased in a similar direction or towards a similar value, σ_{scat} might be smaller than σ and the measurement set may be overconsistent; perhaps $r < 1$.

(iv) *Uncorrelated, unaccounted $\sigma_i(f)$'s.* One experiment i may have a mistake in f_i not encompassed in its $\sigma_i(f)$, another experiment j may have a different mistake in f_j or may have a mistake in another factor g_j , neither may be encompassed in $\sigma_j(f)$ or $\sigma_j(g)$. Then (a) σ is smaller than the actual error on y and (b) σ_{scat} is larger than σ and the measurement set is inconsistent; therefore, $r > 1$.

(v) *Correlated, unaccounted $\sigma_i(f)$'s.* Suppose most measurements in a set use the same factor f , that it is slightly wrong, but the mistake is not encompassed in any of the $\sigma_i(f)$'s. This would shift the value of y . The error

representing this shift would not be in σ , it might show up in σ_{scat} . Then (a) σ is smaller than the actual error on y and (b) σ_{scat} might be larger than σ and the measurement set may be inconsistent; perhaps $r > 1$.

All these factors may simultaneously exist in a specific data set, and competing effects may work together to make the data set appear consistent. For example, experimenters may be tempted to increase poorly understood systematic errors if their result *appears* to be inconsistent with other published results.

However, there is one instance when the existence of problem (iii) (bias) can be demonstrated: i.e., an overconsistent measurement set remains overconsistent when only the statistical errors $\sigma_{\text{stat},i}$ are used. This assumes that experimenters do not overestimate statistical errors. If systematic errors dominate the measurements, the overconsistency may cease when using only $\sigma_{\text{stat},i}$ even if bias is present because the systematic errors exist and contribute to the scatter.

VI. EXAMPLE

We clarify the method and our interpretation by the example of B_1 , summarized in Table VII. We use only the higher-energy measurements as described in Sec. VII A. The average values and errors for the 11 measurements are

$$\begin{aligned} y &= 86.58\%, \quad \sigma = \pm 0.28\%, \quad \sigma_{\text{scat}} = 0.27\% , \\ r &= 0.96, \quad P(< 0.96) = 0.49, \quad P(> 0.96) = 0.51 . \end{aligned} \quad (19)$$

We interpret these values of r and $P(< r)$ to mean that σ_i 's given by the experimenters are the right size as measured by σ_{scat} .

We then analyze this data set using only $\sigma_{\text{stat},i}$ to calculate y and σ . We obtain

$$\begin{aligned} y &= 86.79\%, \quad \sigma_{\text{stat}} = \pm 0.14\%, \quad \sigma_{\text{scat}} = 0.21\% , \\ r &= 1.48, \quad P(< 1.48) = 0.980, \quad P(> 1.48) = 0.020 . \end{aligned} \quad (20)$$

The formal error is now significantly smaller than the error determined from the scatter. This indicates that, as expected, systematic errors are indeed present in the experiments. Using Eq. (18), we obtain $\sigma_{\text{sys}} = \pm 0.24\%$, and the ratio of systematic to statistical errors is $\sigma_{\text{sys}}/\sigma_{\text{stat}} = 1.7$. Thus, the measurement of B_1 is dominated by systematic errors.

We now repeat the analysis using the five measurements with largest weights whose combined weight is greater than 0.81, Table VII. For this small set we find

$$\begin{aligned} y &= 86.64\%, \quad \sigma = \pm 0.29\%, \quad \sigma_{\text{scat}} = 0.33\% , \\ r &= 1.10, \quad P(< 1.10) = 0.70, \quad P(> 1.10) = 0.30 . \end{aligned} \quad (21)$$

The reduction from 11 measurements to 5 does not change y , a desirable feature in a set of measurements. The removal of measurements with small w_i 's and hence relatively large σ_i 's increases r to 1.10. But $P(> 1.10) = 0.30$; therefore, the difference of r from 1 is not significant.

Finally, we analyze the small set using only $\sigma_{\text{stat},i}$. The results are

$$\begin{aligned} y &= 86.73\%, \quad \sigma_{\text{stat}} = \pm 0.15\%, \quad \sigma_{\text{scat}} = 0.23\% , \\ r &= 1.55, \quad P(< 1.55) = 0.953, \quad P(> 1.48) = 0.047 . \end{aligned} \quad (22)$$

The small set, which contains only those experiments with the largest weight, has properties similar to the large set.

The results in Eqs. (19)–(22) show that the measurements used to find the formal average and error for B_1 in this example have reasonable errors attached to them by the experimenters.

VII. RESULTS

This section consists of these parts: the results of the analysis for the individual measurements sets $B_1, B_3, B_e, B_\mu, B_\pi$, and B_ρ ; a combined analysis for B_e, B_μ, B_π , and B_ρ ; and a comparison of B_e and B_μ with τ_τ .

A. Analysis of $B_1, B_3, B_e, B_\mu, B_\pi$, and B_ρ

Table VIII lists quantities found for each measurement set from which a reader can draw conclusions as to the quality of the set. We offer some comments as a guide.

Comments on B_1, B_3 . The set is dominated by the measurement from the HRS Collaboration¹² which contributes half the total weight. Looking at Table II, the three lowest-energy measurements are quite different from the formal average, but only the one from the DELCO Collaboration¹³ is by itself statistically inconsistent. The deviation of the low-energy measurement is usually attributed to insufficient correction for background from the process $e^+e^- \rightarrow \text{hadrons}$. However, we cannot rule out the existence of an energy-dependent, unknown process being confused with the events used to determine B_1 and B_3 at either low or high energy. The *average* of the other low-energy experiments is also inconsistent with the formal average. In order to test the statistical properties of the precise high-energy experiments, we exclude all low-energy experiments from the B_1 and B_3 analyses in Table VIII.

As discussed in Sec. VI, σ_{scat} is consistent with σ , hence a large number of experiments agree on these relatively simple measurements and the formal average seems to be reliable.

Comment on B_e, B_μ . As discussed in Sec. III, we first analyze the 10 unconstrained measurements of B_e and the 16 unconstrained measurements of B_μ listed in Table III. The results are given in Table VIII. The measured ratio of B_μ/B_e is

$$B_\mu/B_e = 1.005 \pm 0.034 , \quad (23)$$

which is consistent with the expected value of 0.973. Systematic and statistical errors are about equal: $\sigma_{\text{sys}}/\sigma_{\text{stat}} = 0.9$. When the full sets of measurements are used, the sets are consistent as defined in Sec. V. However, the small set of B_μ measurements tends to be overconsistent:

TABLE VII. Example of the calculation of statistical quantities using the topological branching fraction B_1 in percent.

Measurement	Combined error	Weight	Pull	Used in largest weights analysis
84	± 2.0	0.019	1.27	
85.2	± 2.9	0.009	0.46	
85.1	± 3.1	0.008	0.46	
87.8	± 4.1	0.005	-0.31	
84.7	$^{+1.9}_{-1.7}$	0.024	1.02	
86.7	± 0.7	0.157	-0.28	Yes
86.9	± 0.4	0.482	-1.32	Yes
86.1	± 1.0	0.077	0.44	Yes
87.9	± 1.3	0.046	-1.09	
87.2	± 0.9	0.095	-0.79	Yes
84.7	± 1.0	0.075	1.89	Yes

B_μ , small set: $r=0.47$, $P(<r)=0.045$.

If only statistical errors are used, a hint of overconsistency remains:

B_μ , small set, $\sigma_{stat,i}$: $r=0.67$, $P(<r)=0.187$.

The B'_e data set is the largest set, and due to the univer-

sality constraint, the formal errors are much smaller than for the B_e or B_μ measurements. Both the full and small set tend to be overconsistent:

B'_e , full set: $r=0.73$, $P(<0.73)=0.046$,

B'_e , small set: $r=0.52$, $P(<0.52)=0.069$.

Either the experiments may have overestimated their er-

TABLE VIII. Calculated values of y , $|\sigma|$, $|\sigma_{scat}|$, r , $P(<r)$, $\sigma_{sys}/\sigma_{stat}$, and number of measurements for B_1, B_3 , unconstrained B_e and B_μ, B'_e, B_π , and B_ρ . Values of y , σ , and σ_{scat} are in percent.

Branching fraction	Measurement selection	Number of measurements	y Formal average	σ Formal error	σ_{scat}	r	P(<r)	$\frac{\sigma_{sys}}{\sigma_{stat}}$
								σ_{stat}
B_1	Full set	11	86.58	0.28	0.27	0.96	0.491	1.7
	σ_{stat} only	10	86.79	0.14	0.21	1.48	0.980	
	small set	5	86.64	0.29	0.33	1.10	0.699	1.7
B_3	σ_{stat} only	5	86.73	0.15	0.23	1.55	0.953	
	Full set	11	13.32	0.28	0.24	0.87	0.313	1.7
	σ_{stat} only	10	13.13	0.14	0.19	1.40	0.957	
B_e	small set	5	13.27	0.29	0.30	1.03	0.618	1.7
	σ_{stat} only	5	13.18	0.15	0.21	1.42	0.908	
	Full set	10	17.62	0.44	0.37	0.83	0.297	0.9
B_μ	σ_{stat} only	8	17.81	0.34	0.45	1.31	0.898	
	small set	5	17.56	0.48	0.44	0.93	0.514	0.9
	σ_{stat} only	4	17.88	0.37	0.45	1.22	0.784	
B'_e	Full set	16	17.71	0.41	0.37	0.91	0.345	0.9
	σ_{stat} only	12	17.80	0.31	0.33	1.05	0.645	
	small set	6	17.95	0.45	0.21	0.47	0.045	0.9
B'_c	σ_{stat} only	6	17.92	0.34	0.23	0.67	0.187	
	Full set	21	17.96	0.26	0.19	0.73	0.047	1.0
	σ_{stat} only	15	18.07	0.19	0.15	0.79	0.158	
B_π	small set	6	18.13	0.29	0.15	0.52	0.069	0.9
	σ_{stat} only	6	18.16	0.21	0.16	0.73	0.245	
	Full set	7	10.78	0.60	0.35	0.59	0.083	2.1
B_ρ	σ_{stat} only	7	11.25	0.26	0.28	1.07	0.671	
	small set	4	11.00	0.64	0.36	0.56	0.190	2.2
	σ_{stat} only	4	11.33	0.27	0.33	1.23	0.789	
B_ρ	Full set	6	22.45	0.85	0.18	0.21	0.001	2.1
	σ_{stat} only	6	22.47	0.35	0.14	0.39	0.020	
	small set	3	22.56	0.95	0.18	0.19	0.035	2.4
σ_{stat} only	3	22.52	0.37	0.14	0.38	0.138		

rors, in which case the formal error is too large, or else there may be bias in the measurements in which case the formal error is too small. When using just the statistical errors, a hint of bias remains.

Comment on B_π . Systematic errors dominate these measurements: $\sigma_{\text{sys}}/\sigma_{\text{stat}}=2.1$. Here again the full set of measurements tend toward overconsistency as defined in Sec. V:

$$B_\pi, \text{ full set: } r=0.59, P(<r)=0.083 .$$

This overconsistency remains when the smaller sets are used, although the statistical significance is weaker since there are only four measurements in the small set:

$$B_\pi, \text{ small set: } r=0.56, P(<r)=0.19 .$$

The systematic errors are so much larger than the statistical errors, that when only the statistical errors are used, no hint of overconsistency remains.

Comment on B_ρ . Like the B_π measurements, systematic errors dominate the measurements: $\sigma_{\text{sys}}/\sigma_{\text{stat}}=2.1$. Both the full and small set are very overconsistent:

$$B_\rho, \text{ full set: } r=0.21, P(<r)=0.001 ,$$

$$B_\rho, \text{ small set: } r=0.19, P(<r)=0.035 .$$

The overconsistency is so strong that even though the systematic errors are more than twice as large as the statistical ones, the data sets remain overconsistent when only statistical errors are used:

$$B_\rho, \text{ full set, } \sigma_{\text{stat},i}: r=0.39, P(<r)=0.020 ,$$

$$B_\rho, \text{ small set, } \sigma_{\text{stat},i}: r=0.38, P(<r)=0.138 .$$

There is statistical evidence for bias in these measurements. The formal error on the average is *too small* since this bias is not included in the systematic errors.

B. Combined analysis of B_e , B_μ , B_π , and B_ρ

The three data sets B_e' , B_π , and B_ρ , show evidence of overconsistency as measured by r . However, r is most sensitive to points which are furthest from the mean and can change considerably if one measurement is far from the mean. Another indicator of the consistency of a data set is the distribution of pulls [Eq. (12)], which should be a normal distribution of unit width and zero mean for a data set with Gaussian errors. r is very nearly equal to the rms deviation of the pull distribution. Figure 2 shows the sum of the pull distributions for the three data sets B_e' , B_π , and B_ρ along with the expected distribution. Here also there is clear evidence of the overconsistency of the data sets. Figure 3 shows the same distribution for the small sets. Of the 13 measurements in the three small sets, none is more than one σ away from the small set mean.

We quantify the overconsistency of the summed pull distribution by evaluating the rms deviation R_Σ . For the full sets $R_\Sigma=0.636$. The probability that R_Σ is less than or equal to 0.636 for an equivalent set of experiments having Gaussian errors is 0.0014 ± 0.0001 . For the small sets, $R_\Sigma=0.484$. The probability of finding a smaller R_Σ

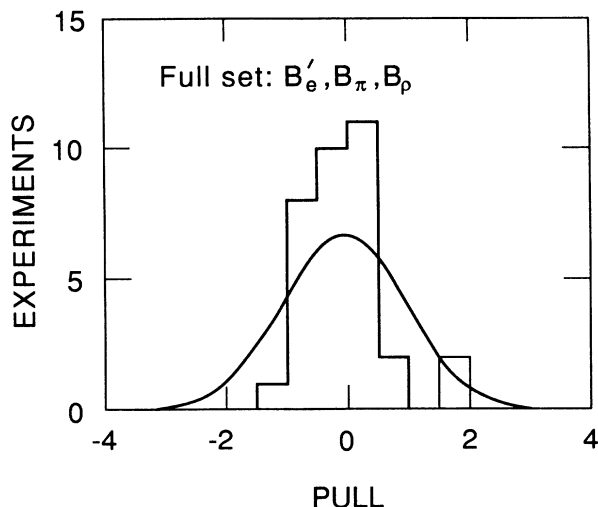


FIG. 2. Sum of the pull distributions for the B_e' , B_π , and B_ρ full data sets.

is 0.0064 ± 0.0004 .

Another method to measure the combined statistical significance of the observed overconsistency is to study the sum of the r values, $\sum r$, for the three data sets. There is no reason to expect the overconsistency to be of the same magnitude in the three different types of measurements. For example, the ratio of systematic to statistical errors is twice as large for B_π and B_ρ as it is for B_e' . The summed pull distribution will not be sensitive to a very overconsistent data set if that data set has relatively few measurements. The value of $\sum r$ for the full sets is 1.53. The probability that $\sum r$ is less than or equal to 1.53 for an equivalent set of experiments having Gaussian errors is 0.00017 ± 0.00005 . For the small sets, $\sum r=1.27$. The probability of a smaller $\sum r$ is 0.0054 ± 0.0003 .

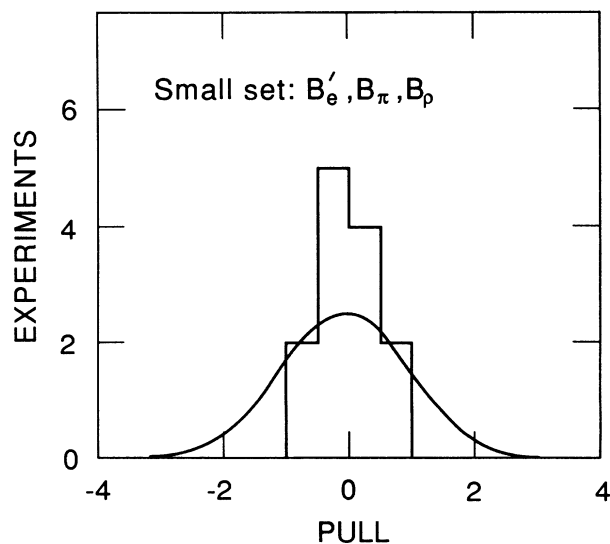


FIG. 3. Sum of the pull distributions for the B_e' , B_π , and B_ρ small data sets.

TABLE IX. Calculated values of y , $|\sigma|$, $|\sigma_{\text{scat}}|$, r , $P(<r)$, $\sigma_{\text{sys}}/\sigma_{\text{stat}}$, and number of measurements for the τ lifetime τ_τ . Values of y , σ , and σ_{scat} are in 10^{-13} sec units.

Measurement selection	Number of measurements	y formal average	$ \sigma $ formal error	$ \sigma_{\text{scat}} $	r	$P(<r)$	$\sigma_{\text{sys}}/\sigma_{\text{stat}}$
Full set	11	3.026	0.085	0.055	0.65	0.062	0.8
σ_{stat} , only	7	3.024	0.070	0.062	0.88	0.411	
Small set	5	3.025	0.089	0.058	0.65	0.208	0.8
σ_{stat} , only	4	3.027	0.073	0.082	1.12	0.710	

C. Comparison of B_e , B_μ , and τ_τ

The analysis of the τ_τ set of measurements, Table IX, shows again some evidence for overestimation of some σ_i or biasing of some y_i :

$$\tau_\tau, \text{ full set: } r=0.65, \quad P(<r)=0.062 .$$

This overconsistency remains when the smaller sets are used, although the statistical significance is weaker since there are only five measurements in the small set:

$$\tau_\tau, \text{ small set: } r=0.65, \quad P(<r)=0.21 .$$

The assumption of e - μ - τ universality leads to the prediction

$$\tau_\tau = \left(\frac{m_\mu}{m_\tau} \right)^5 \tau_\mu B_e , \quad (24)$$

$$B_\mu = 0.973 B_e , \quad (25)$$

when the e mass and all neutrino masses are set to 0. From Table VIII, the full-set formal average for B'_e is

$$B'_e = (17.96 \pm 0.26)\% . \quad (26)$$

Then from Eqs. (24) and (26),

$$\tau_\tau(\text{predicted}) = (2.874 \pm 0.042) \times 10^{-13} \text{ sec} \quad (27)$$

compared with the full-set measured value from Table IX:

$$\tau_\tau(\text{measured}) = (3.026 \pm 0.085) \times 10^{-13} \text{ sec} . \quad (28)$$

The difference is

$$\begin{aligned} \tau_\tau(\text{measured}) - \tau_\tau(\text{predicted}) \\ = (0.152 \pm 0.095) \times 10^{-13} \text{ sec} \quad (29) \end{aligned}$$

and is not statistically significant. We have noted the errors in B_e , B_μ , and τ_τ may be overestimated or there may be biases in B_e , B_μ , and τ_τ . Given these uncertainties, the difference in Eq. (29) cannot be interpreted as requiring larger values of B_e and B_μ .

VIII. CONCLUSIONS

We studied the measurements of various decay branching fractions and the lifetime of the τ lepton for statistical consistency, assuming normal error distributions. There is clear evidence for overestimation of errors or bias in the individual measurements for B'_e , B_π , and particularly for B_ρ . By considering only the statistical errors, there is clear evidence for bias in the B_ρ measurements, and hints of bias in other measurements. Therefore, the formal error on the average of the B_ρ measurements is too small. Since the error on the ρ branching ratio is the largest contribution to the error on the sum of the well-measured one-prong decay modes, the significance of the one-prong discrepancy is reduced.

While we find evidence for bias, there is no evidence that the bias causes the discrepancy in summing the branching fractions. For example, although the B_ρ measurements cluster too much, they may still cluster about the true value of B_ρ . Or the true value of B_ρ may be larger, decreasing the discrepancy; or the true value of B_ρ may be smaller, increasing the discrepancy. We do not know the size or sign of the bias.

There is no evidence for widespread underestimation of systematic errors in the sets of measurements examined here. Hence, the discrepancy should not be ignored simply by claiming that the errors should be set larger.

In summary, our examination of the branching fraction measurements has not resolved the existing problem in understanding the one-charged-particle decay modes of the tau. Resolution of this discrepancy requires new information such as measurements with greatly improved statistical and systematic precision, or explicit measurement of as yet unmeasured or poorly measured modes.

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