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Light-front QCD in the vacuum background

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It is shown that the canonical light-front formulation of quantum chromodynamics is able to incorporate ideas used by Shifman, Vainshtein, and Zakharov to successfully describe many features of the hadronic spectrum in their sum rules. It is pointed out that the new light-front Hamiltonian may lead to a quantitative model for the structure of hadrons.

I. INTRODUCTION

This paper is concerned with the construction of a dynamical formalism for hadrons which combines some of the best features of light-front quantization and sum rules in QCD

The light-front formulation of QCD provides a method to describe the structure and interaction of hadrons in a conceptual framework based upon the Fock-state decomposition of hadronic states which arises naturally in the light-front quantization.[1] It is particularly useful in describing phenomena dominated by short distances, such as, e.g., large-momentum-transfer exclusive processes, where only a few leading Fock components are involved. However, to accomplish a complete description of hadrons one would have to include an infinite number of the Fock components in the eigenvalue problem of the light-front Hamiltonian. Moreover, the light-front formalism has so far ignored subtleties due to the large-scale structure of non-Abelian gauge fields, chiral-symmetry breaking, and the like. Although these do not affect hard processes, they have a profound effect on the structure of the vacuum which in itself may be the very origin of the observed hadronic spectrum.

On the other hand, the sum rules provide a way to describe the spectrum of hadrons.[2] This is done by introducing quark and gluon condensates in diagrammatic QCD calculation of the vacuum polarizations induced by the appropriate currents. Such vacuum polarizations are related to the spectrum of hadrons via dispersion relations. The vacuum condensates are supposed to exist due to nonperturbative effects of QCD. They introduce a new scale and provide the so-called power terms. The basic idea behind the QCD sum rules is that it is the power terms (not higher orders in the strong-coupling series) that limit asymptotic freedom and explain the hadronic spectrum. The sum rules differ greatly from many QCD-like theories such as potential and bag models which often introduce parameters that are not related to the QCD Lagrangian.

The natural question arises as to whether or not it is possible to include the power terms in the light-front approach to QCD. We show that the answer is positive. The resulting dynamical formalism may lead to a quantitative model of hadrons.

In Sec. II we derive the light-front Hamiltonian of QCD including the power terms. They are represented by background quark and gluon fields originating from the vacuum. The quantization and the gauge-covariant expansion of the background fields are included.

Section III is devoted to a test calculation of the vacuum polarization according to rules following from the new Hamiltonian. We reproduce known results for coefficients of the quark and gluon condensates in the x^+-ordered fashion, by simple means of the definite intermediate Fock states of quarks in the vacuum background.

In Sec. IV we draw conclusions and comment on prospects of further applications of the Hamiltonian to the bound states of quarks and gluons. Taking into account the success of the light-front approach in describing high-energy structure of hadrons and the surprisingly good results of sum rules for low-energy hadronic properties we sketch a scenario of how the present possibility to combine both successes in one dynamical scheme would accomplish descriptions of hadrons based on the QCD Lagrangian and properties of the vacuum.

The purpose of this paper is to summarize some formulas and ideas which appear to us more self-consistent than we had expected before we started the preliminary calculations presented below.

II. THE HAMILTONIAN

The main idea is illustrated by the set of substitutions

$$|0\rangle \to |\Omega\rangle, \quad \psi \to \psi + \omega, \quad A \to A + a \tag{2.1}$$

to be done in the standard light-front formulation of QCD (Ref. 1). $|0\rangle$ and $|\Omega\rangle$ are the perturbative and true vacuums, respectively. The quark field ψ and the gluon field A are split into the standard fields ψ and A and into the necessary additional parts w and a. The standard fields ψ and A act in the same way on the true vacuum $|\Omega\rangle$ as they do on the perturbative vacuum $|0\rangle$. The parts w and a detect the nontrivial structure of the physical vacuum $|\Omega\rangle$.

A. The light-front splitting

The splitting of quark and gluon fields mentioned above occurs naturally on the light front. This unique property of the light-front formulation can be seen in an example of the expansion into creators and annihilators of any field ϕ at $x^+=x^0+x^3=0$:

$$\phi(x^+=0,x^-=x^0-x^3,x^\perp) = \int_0^\infty dk^+\int d^2k^\perp(a_k e^{-ikx}+b_k^\dagger e^{ikx}) \ . \quad (2.2)$$

The momentum $k^+=k^0+k^3$ is positive definite. Therefore, the integral starts at the point $k^+=0$. However, this is a singular point of the theory. One should write the field ϕ as

$$\phi(x^+=0,x^-,x^\perp) = \int_\delta^\infty dk^+\int d^2k^\perp(a_k e^{-ikx}+b_k^\dagger e^{ikx})+\int_0^\delta dk^+[?] \ , \quad (2.3)$$

where the parameter δ is an arbitrary small positive lower limit. The question mark exhibits the lack of knowledge about the nature of the singularity at $k^+=0$. The last equation is already the natural splitting occurring in the light-front approach to QCD:

$$\phi=\phi_\delta+\varphi_\delta \ . \quad (2.4)$$

The important point of the light-front splitting is that the part ϕ_δ generates standard looking perturbation theory even if we develop calculations in the true vacuum $|\Omega\rangle$. Only the part φ_δ reports on the difference between $|\Omega\rangle$ and $|0\rangle$. It follows from the fact that the generator P^+ of translations in the x^- direction along the front is positive definite, free from interactions and by definition must annihilate the physical vacuum $|\Omega\rangle$. If $|\Omega\rangle$ would contain any quantum with $k^+>\delta$ created by $a_k^\dagger$ or $b_k^\dagger$ then the P^+ expectation value in $|\Omega\rangle$ would be larger than δ. Therefore, any normal-ordered product of ϕ_δ's has zero expectation value in $|\Omega\rangle$. However, we cannot exclude that the proper limiting procedure yields nonzero expectation values of products of the parts φ_δ in the true vacuum $|\Omega\rangle$. Whenever we find an expectation value of a product such as $\langle\Omega|\varphi_\delta\varphi_\delta|\Omega\rangle$ we replace it by a proper tensor times a number called the condensate. That this might lead to a reasonable approximation in QCD is a nontrivial surprise.[2]

There is no such spectacular splitting in the formulations other than the front form because there are no positive-definite momenta in them and so some even very large opposite momenta may contribute to the vacuum expectation values of normal-ordered products of fields.

B. Details of the Hamiltonian

The QCD Lagrangian is

$$L_t=-\tfrac{1}{2}\mathrm{Tr}F_t^{\mu\nu}F_{t\mu\nu}+\bar\psi_t(i\not{D}_t-m)\psi_t \ , \quad (2.5)$$

where we split fields ψ_t and A_t into parts

$$\psi_t=\psi+\omega, \quad A_t=A+a \quad (2.6)$$

according to Sec. II A. Our notation will be explained more explicitly during further calculations. The background fields ω and a are constrained by their own equations of motion ($\mathrm{Tr}T^aT^b=\frac{1}{2}\delta^{ab}$):

$$d^\mu f_{\mu\nu}=g\bar\omega\gamma_\nu T^a\omega T^a, \quad (\not{d}-m)\omega=0 \ . \quad (2.7)$$

This turns out to be the necessary condition to avoid the problem of inverting the operator $k^+=i\partial^+=i2(\partial/\partial x^-)$ on the background fields themselves in the construction of the Hamiltonian. On the other hand, it means that the background fields are saddle points of the action. Including their vacuum expectation values would correspond to averaging over the saddle points. Thus, the nontrivial structure of the saddle points would be shrunk into the singular vacuum state $|\Omega\rangle$ on the light front. We include the fields ω and a to extract from the singular $|\Omega\rangle$ its influence on the quantum excitations described by the fields ψ and A.

The equations of motion for the fields ψ and A result from subtracting the background equations of motion (2.7) from the full equations of motion for the fields ψ_t and A_t. They are

$$\partial_\mu F^{\mu\nu}+\partial_\mu K^{\mu\nu}=ig[A_\mu,F^{\mu\nu}]+L^\nu+g\bar\psi\gamma^\nu T^a\psi T^a+M^\nu \ ,$$
$$(i\not\partial-m)\psi=g\not{A}\psi+\xi \ , \quad (2.8)$$

where

$$K^{\mu\nu}=ig[A^\mu,a^\nu]+ig[a^\mu,A^\nu] \ ,$$
$$L^\nu=-ig[A_\mu,f^{\mu\nu}+K^{\mu\nu}]-ig[a_\mu,F^{\mu\nu}+K^{\mu\nu}] \ ,$$
$$M^\nu=g\bar\psi\gamma^\nu T^a\omega T^a+g\bar\omega\gamma^\nu T^a\psi T^a \ ,$$
$$\xi=g\not{A}\omega+g\not{a}\psi \quad (2.9)$$

and we use notation defined in the Appendix.

The independent dynamical fields are $\psi_+=\Lambda_+\psi$ and A^i, $i=1,2$, while we use the gauge $A^+=a^+=0$ and $\Lambda_\pm=\frac{1}{2}\gamma^0\gamma^\pm$ together with $\gamma^\pm=\gamma^0\pm\gamma^3$. We use conventions of Bjorken and Drell. The canonical energy-momentum tensor renders the Hamiltonian

$$H_t=\tfrac{1}{2}\int dx^-d^2x^\perp T_t^{+-}=h+H \ , \quad (2.10)$$

where h denotes the background energy and H is

$$H=\int dx^-d^2x^\perp[\mathrm{Tr}(-F^{+\alpha}\partial^-A_\alpha-F^{+\alpha}\partial^-a_\alpha-f^{+\alpha}\partial^-A_\alpha+\tfrac{1}{2}F^{\beta\alpha}F_{\beta\alpha}+F^{\beta\alpha}f_{\beta\alpha}+F^{\beta\alpha}K_{\beta\alpha}+f^{\beta\alpha}K_{\beta\alpha}+\tfrac{1}{2}K^{\beta\alpha}K_{\beta\alpha})$$
$$+\psi_+^\dagger i\partial^-\psi_++\psi_+^\dagger i\partial^-\omega_++\omega_+^\dagger i\partial^-\psi_+] \ . \quad (2.11)$$

The procedure of expressing H by independent degrees of freedom is done essentially the same way as in Ref. 1(b). The only complication stems from the additional terms in the equations of motion. So we only stress the role played by the gauge condition $A^+=a^+=0$ and by the subtraction of pure background terms, which allow inversion of $i\partial^+$ by means of $1/k^+$ in the Fourier representation because k^+ is never smaller than δ. This way we eliminate seagull terms in which $(i\partial^+)^{-1}$ would act on background fields alone. To justify integrations by parts we have to impose vanishing boundary conditions on the independent fields ψ_+ and A^i at spatial light-front infinity. The result for H is

$$H=\int dx^- d^2x^\perp \mathcal{H} , \tag{2.12}$$

where

$$\begin{aligned}\mathcal{H}=&\psi_+^\dagger\sigma\frac{1}{i\partial^+}\sigma\psi_+ +\omega_+^\dagger\sigma\frac{1}{i\partial^+}\sigma\psi_+ +\left[\sigma\frac{1}{i\partial^+}\sigma\psi_+\right]^\dagger\omega_+ +g(\bar{\tilde\psi}+\bar\omega)(\tilde{A\!\!\!/}+a\!\!\!/)(\tilde\psi+\omega)-g\bar\omega a\!\!\!/\omega\\
&+g^2\left[(\bar{\tilde\psi}+\bar\omega)\tilde{A\!\!\!/}\frac{\gamma^+}{2i\partial^+}\tilde{A\!\!\!/}(\tilde\psi+\omega)+\bar{\tilde\psi}a\!\!\!/\frac{\gamma^+}{2i\partial^+}a\!\!\!/\tilde\psi+\bar{\tilde\psi}\tilde{A\!\!\!/}\frac{\gamma^+}{2i\partial^+}a\!\!\!/\tilde\psi+\bar{\tilde\psi}a\!\!\!/\frac{\gamma^+}{2i\partial^+}\tilde{A\!\!\!/}\tilde\psi+\bar\omega\tilde{A\!\!\!/}\frac{\gamma^+}{2i\partial^+}a\!\!\!/\tilde\psi+\bar{\tilde\psi}a\!\!\!/\frac{\gamma^+}{2i\partial^+}\tilde{A\!\!\!/}\omega\right]\\
&+g^2\mathrm{Tr}\Psi\frac{1}{(i\partial^+)^2}\Psi+\mathrm{Tr}\partial^i\tilde A^j\cdot\partial^i\tilde A^j+\mathrm{Tr}\partial^i\tilde A^j\cdot\partial^i a^j+\mathrm{Tr}\partial^i a^j\cdot\partial^i\tilde A^j+2g\,\mathrm{Tr}i\partial^\alpha(\tilde A^\beta+a^\beta)[\tilde A_\alpha+a_\alpha,\tilde A_\beta+a_\beta]\\
&-2g\,\mathrm{Tr}i\partial^\alpha a^\beta[a_\alpha,a_\beta]-\frac{g^2}{2}\mathrm{Tr}[\tilde A^\alpha+a^\alpha,\tilde A^\beta+a^\beta][\tilde A_\alpha+a_\alpha,\tilde A_\beta+a_\beta]+\frac{g^2}{2}\mathrm{Tr}[a^\alpha,a^\beta][a_\alpha,a_\beta]\end{aligned} \tag{2.13}$$

and

$$\Psi=\Psi^aT^a,\quad \Psi^a=(\bar{\tilde\psi}+\bar\omega)\gamma^+T^a(\tilde\psi+\omega)-\bar\omega\gamma^+T^a\omega-[i\partial^+(\tilde A^\alpha+a^\alpha),\tilde A_\alpha+a_\alpha]^a+[i\partial^+a^\alpha,a_\alpha]^a . \tag{2.14}$$

The notation is

$$\tilde\psi=\psi_++\tilde\psi_-,\quad \tilde\psi_-=\frac{1}{i\partial^+}\sigma\psi_+,\quad \sigma=i\partial^i\alpha^i+\beta m,\quad \tilde A^i=A^i,\quad \tilde A^-=\frac{2}{i\partial^+}i\partial^iA^i,\quad \tilde A^+=0 . \tag{2.15}$$

C. The quantization

We expand fields at $x^+=0$ as

$$\begin{aligned}\tilde\psi(x)&=\int_{k^+>\delta}\frac{dk^+d^2k^\perp}{16\pi^3k^+}\sum_{\lambda,c}(b^c_{k\lambda}u_{k\lambda}u_ce^{-ikx-\epsilon|x|}+d^{c\dagger}_{k\lambda}v_{k\lambda}v_c^*e^{ikx-\epsilon|x|}) ,\\
\tilde A^\mu(x)&=\int_{k^+>\delta}\frac{dk^+d^2k^\perp}{16\pi^3k^+}\sum_{\lambda,c}(a^c_{k\lambda}\epsilon^\mu_{k\lambda}T^ce^{-ikx-\epsilon|x|}+a^{c\dagger}_{k\lambda}\epsilon^{\mu*}_{k\lambda}T^ce^{ikx-\epsilon|x|}) ,\end{aligned} \tag{2.16}$$

where the boundary regulator

$$\epsilon|x|=\epsilon(|x^-|\,|x^1|+|x^2|)$$

is omitted in further calculations. Imposing commutation relations $[\mathbf{k}=(k^+,k^\perp)]$

$$\begin{aligned}\{b_{\mathbf{k}\lambda},b^\dagger_{\mathbf{p}\lambda'}\}&=\{d_{\mathbf{k}\lambda},d^\dagger_{\mathbf{p}\lambda'}\}\\
&=\{a_{\mathbf{k}\lambda},a^\dagger_{\mathbf{p}\lambda'}\}\\
&=16\pi^3k^+\delta^3(\mathbf{k}-\mathbf{p})\delta_{\lambda\lambda'} ,\\
\{b,b\}&=\{d,d\}=\cdots=0\end{aligned} \tag{2.17}$$

implies, in the limit $\delta,\epsilon\to0$ $[\mathbf{x}=(x^-,x^\perp)]$,

$$\begin{aligned}\{\psi_+(\mathbf{x}),\psi_+^\dagger(\mathbf{y})\}_{x^+=y^+=0}&=\Lambda_+\delta^3(\mathbf{x}-\mathbf{y}) ,\\
[A^i(\mathbf{x}),\partial^+A^j(\mathbf{y})]_{x^+=y^+=0}&=i\delta^{ij}\delta^3(\mathbf{x}-\mathbf{y}) .\end{aligned} \tag{2.18}$$

The spinors $u_{k\lambda}$ and $v_{k\lambda}$ are defined as[3]

$$u_{k\lambda}=S(k,m)\left[{}^{\chi_\lambda}_{0}\right],\quad v_{k\lambda}=S(k,m)\left[{}^{\chi_\lambda}_{0}\right]_c , \tag{2.19}$$

where

$$S(k,m)=\left[\frac{2}{k^+}\right]^{1/2}[m\Lambda_-+(k^++\alpha^\perp k^\perp)\Lambda_+] \tag{2.20}$$

is the spinor 4×4 matrix representation of the boost from the rest frame of a particle of mass m to the frame, where it has momentum k. This is an example of boosts which exactly solve the problem of boosting bound states in the light-front form of dynamics. χ_λ is a two-component spinor denoting the spin projection along the z axis and C denotes charge conjugation. Otherwise, we follow the conventions of Ref. 1(b).

Inserting the expansions (2.16) into our Hamiltonian one obtains the desired expression for H_t. It contains the same terms as those given in Ref. 1(b), which lead to the light-front perturbation theory as described there, and many additional terms involving the background fields, which require separate treatment.

D. The vacuum background

The net influence of the vacuum background on quarks and gluons must be Poincaré and gauge invariant. In sum rules it is achieved by using the Fock-Schwinger gauge

$$(x_\mu-\bar x_\mu)a^\mu(x)=0 . \tag{2.21}$$

It allows a useful expansion of the background fields in polynomials of the distance $x_\mu-\bar{x}_\mu$ between the conveniently chosen origin $\bar{x}$ and the actual point of interest. If the background fields are not rapidly varying on the scale of hadronic size it should be sufficient for a reasonably good description of hadrons to include a few terms in the polynomial. Indeed, already the lowest terms in the polynomials lead to quark and gluon condensates which correlate resonance properties[2(a)] and baryon masses.[2(b)] The Poincaré and gauge-invariant results are relatively easily obtained thanks to three facts. The first one is that the choice of the point $\bar{x}$ is equivalent to the choice of gauge and does not contribute to physical quantities. The translation invariance broken by the choice of $\bar{x}$ is restored by gauge invariance. The second fact is that the polynomial coefficients are gauge-covariant quantities, a fact that almost automatically leads to the desired gauge-invariant expressions. The third fact is that the Fock-Schwinger gauge is Lorentz invariant. Altogether the proper choice of gauge provides a convenient method of calculation. On the other hand, the light-front Hamiltonian approach is based on the choice of gauge $A^+=a^+=0$. This seems to ruin the possibility to construct a practical dynamical formalism out of two such different methods. Fortunately, the light-front gauge is effectively not worse than the Fock-Schwinger gauge, as shown in the Appendix. The appropriate expansion in powers of x^μ around $x=0$ starts with

$$\begin{aligned} a^\mu &= \tfrac{1}{2}x_\rho(f_0^{\rho\mu}+\tilde{\eta}^\mu f_0^{+\rho}+\tilde{\eta}^\rho f_0^{+\mu})+O(x^2)\ , \\ \omega &= \omega_0+x_p d^p\omega_0+O(x^2)\ , \end{aligned} \tag{2.22}$$

where $\tilde{\eta}$ is the four-vector described in the Appendix. The Fourier space counterparts are

$$\begin{aligned} a^\mu(x) &= \int d^4k \left[\tfrac{1}{2}(f_0^{\rho\mu}+\tilde{\eta}^\rho f_0^{+\mu}+\tilde{\eta}^\mu f_0^{+\rho}) \times \left[i\frac{\partial}{\partial k^\rho}\delta^4(k)\right]+\cdots\right]e^{ikx}\ , \\ \omega(x) &= \int d^4k \left[\omega_0[\delta^4(k)]+d^\rho\omega_0\left[i\frac{\partial}{\partial k^\rho}\delta^4(k)\right]+\cdots\right]e^{ikx} \end{aligned} \tag{2.23}$$

and we introduce them into the Hamiltonian. If we put $x^+=0$ in the above expansions, then the integral over k^- can be done immediately and the x^+ dependence ("time" dependence) of the background fields is lost. It is properly reconstructed in further calculations in Sec. III on the basis of the following observations.

Let us write the background fields in the form

$$\begin{aligned} a^\mu(x) &= \int d^4k\, a^\mu(k)e^{ikx}\ , \\ \omega(x) &= \int d^4k\, \omega(k)e^{ikx} \end{aligned} \tag{2.24}$$

and observe that we can write their x^+ dependence like

$$\begin{aligned} a^\mu(x) &= e^{ihx^+/2}a^\mu(x^+=0,\mathbf{x})e^{-ihx^+/2}\ , \\ \omega(x) &= e^{ihx^+/2}\omega(x^+=0,\mathbf{x})e^{-ihx^+/2}\ , \end{aligned} \tag{2.25}$$

if the formal conditions

$$\begin{aligned} [h,a^\mu(k)] &= k^-a^\mu(k)\ , \\ [h,\omega(k)] &= k^-\omega(k) \end{aligned} \tag{2.26}$$

hold, where h is the part of the Hamiltonian which counts the background fields. Thus we may consider

$$H_t=H_t(x^+=0)=H+h \tag{2.27}$$

as the conserved Hamiltonian and must include h in a construction of the evolution operator.

If we could introduce δ', a gap value of k^+ almost equal to the parameter δ, and if we limit the background fields not by δ but by the many times smaller difference $\delta-\delta'$, then the Hamiltonian H would not be able to create particles from the vacuum $|\Omega\rangle$. Then the eigenvalue equation $H_t|\Omega\rangle=0$ implies $h|\Omega\rangle=0$. However, we cannot consistently require both that $h|\Omega\rangle=0$ and that the Hamiltonian h counts energies of the background fields. The only possibility is to define the Hamiltonian by subtracting its vacuum eigenvalue λ_0, which is even expected to be infinite.[4] Finally, we obtain

$$H_t=H+h-\lambda_0\ , \tag{2.28}$$

where H is given by Eqs. (2.12)–(2.15) with all fields taken at $x^+=0$ according to the expansions (2.16) and (2.24), h is the background Hamiltonian satisfying the commutation relations (2.26) and λ_0 is the number which shifts the vacuum energy to zero. The Hamiltonian $h-\lambda_0$ must be commuted through all background fields involved in calculations to reconstruct their x^+ dependence.

III. THE VACUUM POLARIZATION

The vacuum-polarization tensor we are considering is

$$\Pi^{\mu\nu}(q)=i\int d^4x\, e^{iqx}\langle\Omega|T_+[J_t^\mu(x)J_t^\nu(0)]|\Omega\rangle\ , \tag{3.1}$$

where the quark current J_t^μ is defined as

$$\begin{aligned} J_t^\mu(x) &= :\bar{\psi}_t(x^+,\mathbf{x})\gamma^\mu\psi_t(x^+,\mathbf{x}): \\ &= e^{iH_tx^+/2}:\bar{\psi}_t(x^+=0,\mathbf{x})\gamma^\mu\psi_t(x^+=0,\mathbf{x}):e^{-iH_tx^+/2} \\ &= e^{i(H+h-\lambda_0)x^+/2}[:\bar{\psi}(\mathbf{x})\gamma^\mu\psi(\mathbf{x}):+\bar{\psi}(\mathbf{x})\gamma^\mu\omega(\mathbf{x})+\bar{\omega}(\mathbf{x})\gamma^\mu\psi(\mathbf{x})+\bar{\omega}(\mathbf{x})\gamma^\mu\omega(\mathbf{x})]_{x^+=0}e^{-i(H+h-\lambda_0)x^+/2} \end{aligned} \tag{3.2}$$

and the last normal ordering is understood as setting creators to the left of annihilators. The x^+ ordering operator acts as

$$T_{+}[J(x^{+})J(0)]=\theta(x^{+})J(x^{+})J(0)+\theta(-x^{+})J(0)J(x^{+}) \; . \tag{3.3}$$

We want to check if the light-front Hamiltonian calculation of $\Pi^{\mu\nu}(q)$ agrees with the results of Ref. 2(a). First we illustrate some basically simple light-front techniques to shorten the later discussion of more complicated calculations.

A. Basic calculation tools

After observing that for $q^{+} \gg \delta$ only the first x^{+} ordering survives we act with the leftmost exponential factor to the left vacuum bra and integrate over x^{+} to obtain

$$\Pi^{\mu\nu}(q)=-\int dx^{-}d^{2}x^{\perp}\exp\left[\frac{i}{2}q^{+}x^{-}-iq^{\perp}z^{\perp}\right] \times\langle\Omega\,|\,[:\bar{\psi}_{t}(\mathbf{x})\gamma^{\nu}\psi_{t}(\mathbf{x}):]\,|_{x^{+}=0}[q^{-}-(H+h-\lambda_{0})+i\epsilon]^{-1}[:\bar{\psi}_{t}(0)\gamma^{\nu}\psi_{t}(0):]\,|\,\Omega\rangle \; . \tag{3.4}$$

Consequently, the Hamiltonian must be commuted here to the left vacuum bra to reconstruct the x^{+} dependence of the background quark field.

It is convenient to extract the vacuum polarization $\Pi(q^2)$ from the tensor

$$\Pi^{\mu\nu}(q)=(q^{\mu}q^{\nu}-q^{2}g^{\mu\nu})\Pi(q^{2}) \tag{3.5}$$

by considering the component $\Pi^{++}(q)=q^{+}q^{+}\Pi(q^{2})$. The terms which are independent of ω and of the interaction part H_I in the Hamiltonian $H=H_0+H_I$ are

$$\Pi_{1}^{++}=-\int dx^{-}d^{2}x^{\perp}\exp\left[\frac{i}{2}q^{+}x^{-}-iq^{\perp}x^{\perp}\right]\langle\Omega\,|\,[:\bar{\psi}(\mathbf{x})\gamma^{+}\psi(\mathbf{x}):]\,|_{x^{+}=0}(q^{-}-H_{0})^{-1}[:\bar{\psi}(0)\gamma^{+}\psi(0):]\,|\,\Omega\rangle \; . \tag{3.6}$$

We insert the expansion (2.16) for $\psi(\mathbf{x})\,|_{x^{+}=0}$, use the property of the light-front splitting that $b_k\,|\,\Omega\rangle=d_k\,|\,\Omega\rangle=0$ for $k^{+}>\delta$, execute the commutation relations (2.17) and obtain

$$\Pi_{1}^{++}=-3\int_{\delta}^{q^{+}-\delta}\frac{dk^{+}}{16\pi^{3}}\int d^{2}k^{\perp}8\left[q^{-}-\frac{k^{\perp 2}+m^{2}}{k^{+}}-\frac{(q^{\perp}-k^{\perp})^{2}+m^{2}}{q^{+}-k^{+}}\right]^{-1} . \tag{3.7}$$

The generic substitution

$$x=\frac{k^{+}}{q^{+}},\quad k^{\perp}=xq^{\perp}+l^{\perp} \tag{3.8}$$

results here in the integral ($\kappa=\delta/q^{+}$)

$$\Pi_{1}^{++}=q^{+}q^{+}\frac{3}{2\pi^{3}}\int_{\kappa}^{1-\kappa}dx\int d^{2}l^{\perp}\left[\frac{l^{\perp 2}+m^{2}}{x(1-x)}-q^{2}\right]^{-1} . \tag{3.9}$$

Using rotational symmetry around third axis and changing to the polar coordinates in the transverse plane of $l^{\perp}$ we can substitute $l^{\perp 2}=x(1-x)z$ which gives

$$\Pi_{1}(q^{2})=\frac{3}{2\pi^{2}}\int_{\kappa}^{1-\kappa}dx\,x(1-x)\int_{0}^{\infty}dz\left[z+\frac{m^{2}}{x(1-x)}-q^{2}\right]^{-1} . \tag{3.10}$$

Another often used substitution is $x=\frac{1}{2}(1+u)$ leading here to

$$\Pi_{1}=\frac{3}{16\pi^{2}}\int_{-(1-2\kappa)}^{1-2\kappa}du\,(1-u^{2})\int_{0}^{\infty}dz\left[z+\frac{a-u^{2}}{1-u^{2}}\right]^{-1} , \tag{3.11}$$

where $a=1+4m^2/Q^2$ and $Q^2=-q^2$. For $a=1$ we would obtain, neglecting κ,

$$\Pi_{1}(Q^{2},\mu^{2})=\lim_{\Lambda\to\infty}[\Pi_{1}^{\Lambda}(Q^{2})-\Pi_{1}^{\Lambda}(\mu^{2})]=\lim_{\Lambda\to\infty}\frac{1}{4\pi^{2}}\int_{0}^{\Lambda}dz\left[\frac{1}{z-q^{2}}-\frac{1}{z+\mu^{2}}\right]=-\frac{1}{4\pi^{2}}\ln\frac{Q^{2}}{\mu^{2}} , \tag{3.12}$$

which illustrates the subtraction procedure at $q^2=-\mu^2$ and agrees with Eq. (3.3) of Ref. 2(a). If the quark mass is to be taken into account, we can calculate the imaginary part of $\Pi_1(q^2)$ recalling $i\epsilon$ from Eq. (3.4). Neglecting κ we get

$$\mathrm{Im}\Pi_1(s)=\frac{3}{2\pi^2}\int_0^1 dx\,x(1-x)\int_0^\infty dz\,\pi\delta\left[z+\frac{m^2}{x(1-x)}-s\right]$$
$$=\frac{3}{2\pi}\int_0^1 dx\,x(1-x)\theta\left[-\frac{m^2}{x(1-x)}+s\right]=\frac{1}{4\pi}\frac{\sqrt{a}\,(3-a)}{2}\theta(s-4m^2)\,, \tag{3.13}$$

where $a=1-4m^2/s$. This agrees with Eq. (4.2) of Ref. 2(a), where $\Pi_1(q^2)$ is calculated from $\mathrm{Im}\Pi_1(s)$ via dispersion relation.

All the above agreement reflects the equivalence of the light-front formulation of perturbation theory and the Feynman rules. The interesting fact is that we have only one x^+ ordered diagram from Fig. 1, while in other time-ordered formulations (instant forms) we would have two time-ordered diagrams, equivalent to the one Feynman-loop diagram.

B. The quark condensate

To check the coefficient of the matrix element $\langle\Omega\,|\,\bar\omega\omega\,|\,\Omega\rangle$ in $\Pi(q^2)$ we restrict ourselves to the zero-order term in the coupling constant. Then the relevant terms are

$$\Pi^{++}_{\langle\bar\omega\omega\rangle}=-\int dx^-d^2x^\perp\exp\left[\frac{i}{2}q^+x^--iq^\perp x^\perp\right]\langle\Omega\,|\,[\bar\omega(\mathbf{x})\gamma^+\psi(\mathbf{x})+\bar\psi(\mathbf{x})\gamma^+\omega(\mathbf{x})]\,|_{x^+=0}[q^--(H_0+h-\lambda_0)]^{-1}$$
$$\times[\bar\omega(0)\gamma^+\psi(0)+\bar\psi(0)\gamma^+\omega(0)]\,|\,\Omega\rangle\,. \tag{3.14}$$

Inserting the expansions from Eq. (2.16) and (2.24) we see that only two terms

$$\langle\Omega\,|\,[\bar\omega(\mathbf{x})\gamma^+\psi_{(-)}(\mathbf{x})]\,|_{x^+=0}[q^--(H_0+h-\lambda_0)]^{-1}\bar\psi_{(-)}(0)\gamma^+\omega(0)\,|\,\Omega\rangle$$
$$+\langle\Omega\,|\,[\bar\psi_{(-)}(\mathbf{x})\gamma^+\omega(\mathbf{x})]\,|_{x^+=0}[q^--(H_0+h-\lambda_0)]^{-1}\bar\omega(0)\gamma^+\psi_{(+)}(0)\,|\,\Omega\rangle \tag{3.15}$$

are contributing. Commuting the denominator to the left, executing all commutation relations, using the light-front splitting properties and after summing indices and integrating over momenta we obtain the expression

$$\Pi^{++}_{\langle\bar\omega\omega\rangle}=\int d^4k\left\{-\left\langle\Omega\left|\left[\left[\omega_0+d^\rho\omega_0 i\frac{\partial}{\partial k^\rho}+\cdots\right]\delta^4(k)\right]^\dagger\gamma^0\frac{\gamma^+(\not q-\not k+m)\gamma^+}{(q-k)^2-m^2}\omega_0\right|\Omega\right\rangle\right.$$
$$\left.+\left\langle\Omega\left|\bar\omega_0\frac{\gamma^+(\not q+\not k-m)\gamma^+}{(q+k)^2-m^2}\left[\left[\omega_0+d^\rho\omega_0 i\frac{\partial}{\partial k^\rho}+\cdots\right]\delta^4(k)\right]\right|\Omega\right\rangle\right\} \tag{3.16}$$

illustrated by diagrams from Fig. 2. Noting that

$$\langle\Omega\,|\,\bar\omega^a_{0\alpha}\omega^b_{0\beta}\,|\,\Omega\rangle=\tfrac{1}{3}\delta^{ab}\tfrac{1}{4}\delta_{\alpha\beta}\langle\Omega\,|\,\bar\omega_0\omega_0\,|\,\Omega\rangle \tag{3.17}$$

we see that only the terms with covariant derivatives give nonzero results. Thus we arrive at

$$\Pi^{++}_{\langle\bar\omega\omega\rangle}=-4i\frac{\partial}{\partial k^\rho}\frac{(q+k)^+}{(q+k)^2-m^2}\bigg|_{k=0}\langle\Omega\,|\,\bar\omega_0\gamma^+d^\rho\omega_0\,|\,\Omega\rangle \tag{3.18}$$

and using the fact that

$$\langle\Omega\,|\,\bar\omega^a_{0\mu}d^\alpha\omega^b_{0\nu}\,|\,\Omega\rangle=\frac{1}{48i}\delta^{ab}(\gamma^\alpha)_{\nu\mu}m\langle\Omega\,|\,\bar\omega_0\omega_0\,|\,\Omega\rangle \tag{3.19}$$

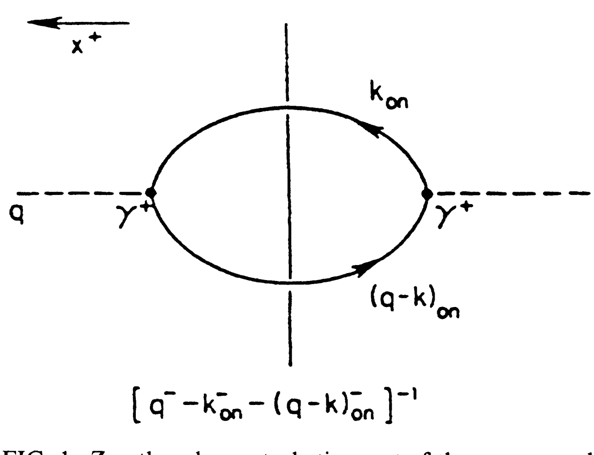

FIG. 1. Zeroth-order perturbative part of the vacuum polarization. The vertical line denotes the state of a quark and an antiquark in the vacuum $|\,\Omega\rangle$. The subscript "on" reminds us that $k^-_{on}=(k^{\perp 2}+m^2)/k^+$.

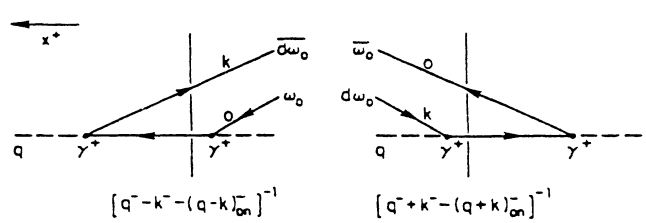

FIG. 2. Diagrammatic representation of the quark condensate coefficient to the lowest order of perturbation theory.

we finally obtain

$$\Pi^{++}_{\langle\bar{\omega}\omega\rangle}=-\frac{\partial}{\partial k^-}\frac{q^+}{(q+k)^2-m^2}\bigg|_{k=0}2m\langle\Omega\,|\,\bar{\omega}_0\omega_0\,|\,\Omega\rangle \tag{3.20}$$

or

$$\Pi_{\langle\bar{\omega}\omega\rangle}=\frac{2m}{Q^4}\langle\Omega\,|\,\bar{\omega}_0\omega_0\,|\,\Omega\rangle \tag{3.21}$$

in complete agreement with Ref. 2(b).

C. The gluon condensate

For massive quarks and in leading order of perturbation theory the gluon condensate coefficient follows from

$$\Pi^{++}_{\langle ff\rangle}=-\int dx^-d^2x^\perp\exp\left[\frac{i}{2}q^+x^--iq^\perp x^\perp\right]\langle\Omega\,|\,[:\bar{\psi}(\mathbf{x})\gamma^+\psi(\mathbf{x}):]\,|_{x^+=0}[q^--(H+h-\lambda_0)]^{-1}[:\bar{\psi}(0)\gamma^+\psi(0):]\,|\,\Omega\rangle\ , \tag{3.22}$$

where only the terms

$$H_I^{(1)}+H_I^{(2)}=\int dx^-d^2x^\perp:\left[g\bar{\tilde{\psi}}\not{a}\tilde{\psi}+g^2\bar{\tilde{\psi}}\not{a}\frac{\gamma^+}{2i\partial^+}\not{a}\tilde{\psi}\right]\bigg|_{x^+=0} . \tag{3.23}$$

from the interaction Hamiltonian $H_I=H-H_0$ contribute. An expansion up to terms bilinear in the background fields gives

$$[q^-(H+h-\lambda_0)]^{-1}\Longrightarrow\frac{1}{q^--H_0-(h-\lambda_0)}\left[H_I^{(1)}\frac{1}{q^--H_0-(h-\lambda_0)}H_I^{(1)}+H_I^{(2)}\right]\frac{1}{q^--H_0-(h-\lambda_0)} . \tag{3.24}$$

The already familiar procedure generates six expressions illustrated by diagrams in Fig. 3. Such a small number of diagrams is a remarkable feature because in formulations other than the light-front form (if it would be possible) one would expect $3\times 4!=72$ time-ordered diagrams. According to the equalities observed in Fig. 3 the complete answer is [we neglect $\delta\ll q^+(m^2/Q^2)$]

$$\begin{aligned}\Pi^{++}_{\langle ff\rangle}=&-\frac{1}{16\pi^3}\int_0^{q^+}\frac{d^3p}{p^+k^+}\frac{1}{q^--p^--k^-}\int d^4k_1d^4k_2\langle\Omega\,|\,g^2a^c_{\mu_2}(k_2)a^c_{\mu_1}(k_1)\,|\,\Omega\rangle\\&\times\left[\frac{\mathrm{Tr}(\not{p}+m)\gamma^{\mu_2}(\not{q}-\not{k}+\not{k}_2+m)\gamma^{\mu_1}(\not{p}+\not{k}_1+\not{k}_2+m)\gamma^+\not{k}\gamma^+}{[(q-k+k_2)^2-m^2][(q-k+k_1+k_2)^2-m^2]}\right.\\&\left.-\frac{\mathrm{Tr}(\not{p}+m)\gamma^{\mu_2}(\not{p}+\not{k}_2+m)\gamma^+(\not{k}+\not{k}_1-m)\gamma^{\mu_1}(\not{k}-m)\gamma^+}{[(q-k+k_2)^2-m^2]\{[q-(p+k_2)_{\mathrm{on}}+k_1+k_2]^2-m^2\}}\right] ,\end{aligned} \tag{3.25}$$

where

$$p^-=\frac{p^{\perp 2}+m^2}{p^+},\quad k^+=q^+-p^+,\quad k^\perp=q^\perp-p^\perp,\quad k^-=\frac{k^{\perp 2}+m^2}{k^+} . \tag{3.26}$$

The vacuum expectation value of the background gluon fields is

$$\langle\Omega\,|\,g^2a^c_{\mu_2}(k_2)a^c_{\mu_1}(k_1)\,|\,\Omega\rangle=-\tfrac{1}{48}\langle\Omega\,|\,g^2f^c_{0\mu\nu}f^{c\mu\nu}_0\,|\,\Omega\rangle A^{\rho_2\rho_1}_{\mu_2\mu_1}\left[\frac{\partial}{\partial k_1^{\rho_1}}\delta^4(k_1)\right]\left[\frac{\partial}{\partial k_2^{\rho_2}}\delta^4(k_2)\right] . \tag{3.27}$$

It follows from Eq. (2.23) and the vacuum property that

$$\langle\Omega\,|\,g^2f_0^{\alpha\beta a}f_0^{\gamma\delta b}\,|\,\Omega\rangle=\tfrac{1}{96}\delta^{ab}(g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma})\langle\Omega\,|\,g^2f^c_{0\mu\nu}f^{c\mu\nu}_0\,|\,\Omega\rangle . \tag{3.28}$$

The tensor A may be written as

$$A^{\rho_2\rho_1}_{\mu_2\mu_1}=(g^{\rho_2\rho_1}+\eta^{\rho_2}\tilde{\eta}^{\rho_1}+\eta^{\rho_1}\tilde{\eta}^{\rho_2})(g_{\mu_1\mu_2}-\eta_{\mu_2}\tilde{\eta}_{\mu_1}-\eta_{\mu_1}\tilde{\eta}_{\mu_2})-(g^{\rho_2}_{\mu_1}+\tilde{\eta}^{\rho_2}\eta_{\mu_1}-\eta^{\rho_2}\tilde{\eta}_{\mu_1})(g^{\rho_1}_{\mu_2}+\tilde{\eta}^{\rho_1}\eta_{\mu_2}-\eta^{\rho_1}\tilde{\eta}_{\mu_2}) , \tag{3.29}$$

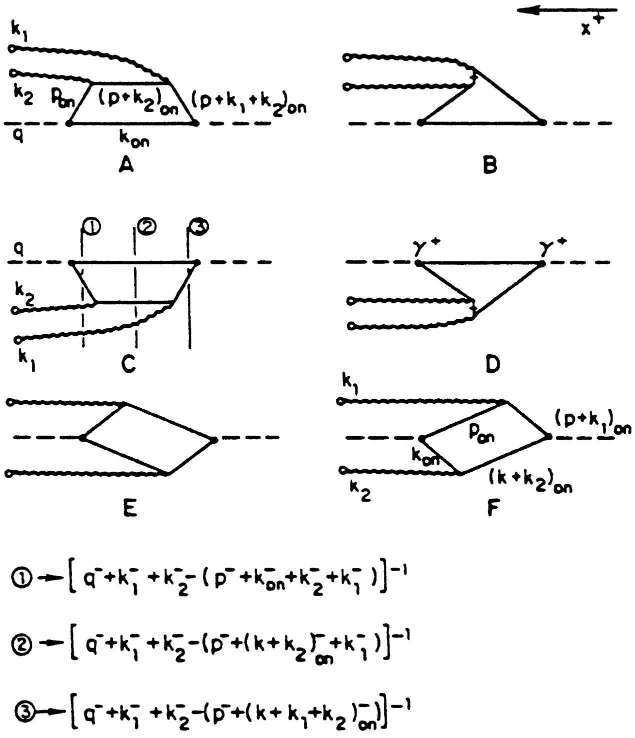

FIG. 3. All diagrams contributing to the gluon condensate coefficient in the lowest order of perturbation theory. The notation examples are given. The sum of the diagrams is equal to $2(X+Y)$, where $X=A+B=C+D$ and $Y=E=F$.

where the light-front vectors η and $\tilde{\eta}$ are described in the Appendix.

The most ergonomic way to evaluate (3.27) is to use the fact that only 13 from 256 elements of the tensor A are different from zero. One sums up the corresponding contributions to (3.25), calculating them separately. Some algebra and generic substitutions described in Sec. III A allow the elementary derivation of the answer

$$\Pi^{++}_{\langle ff\rangle}=q^+q^+\frac{\langle \Omega\,|\,g^2 f^a_{0\mu\nu} f^{a\mu\nu}_0\,|\,\Omega\rangle}{48\pi^2 q^4}c\ , \qquad (3.30)$$

where

$$c=\frac{3(a+1)(a-1)^2}{4a^2}\frac{1}{2\sqrt{a}}\ln\frac{\sqrt{a}+1}{\sqrt{a}-1}-\frac{3a^2-2a+3}{4a^2} \qquad (3.31)$$

and $a=1-4m^2/q^2$, in agreement with Eq. (4.5) of Ref. 2(b).

IV. CONCLUSION

The light-front Hamiltonian formulation of QCD contains singularities which can be removed at a price of introducing vacuum expectation values of different fields. The most important vacuum expectation values are quark and gluon condensates. The canonical light-front calculation of their coefficients in the vacuum polarization induced by the quark vector current reproduces formulas used in the QCD sum rules.[2] The sum rules show that the quark and gluon condensates are universal numbers which explain many features of the hadronic spectrum. The light-front Hamiltonian, which contains terms introducing condensates, may then have a chance to describe the structure of hadrons to a good approximation. The following additional reasons support this hope.

The light-front scheme offers a relativistic formulation of the few-body problem free from typical difficulties[1(c)] such as boosting the bound states. The most transparent description of the short-distance structure of hadrons is achieved just using light-front dynamics.[1] The new terms in the Hamiltonian containing background fields act at distances on the order of the hadronic size. The gluon condensate provides forces looking like the harmonic oscillator force confining color. A choice of the origin of the harmonic potential does not violate translation invariance because it is equivalent to a choice of gauge. The harmonic constant of the potential is proportional to the gluon condensate. There is a chance to explain in QCD the existence of the bag pressure by the value of the gluon condensate. There is no problem with relativistic motion of bags. The quark condensate is a signal of chiral-symmetry breaking. Both quarks and gluons are expected to obtain effective masses. The notion of the mass of a confined object such as a quark or a gluon can be introduced by means of the effective parameter which plays the role of a mass in the eigenvalue equation of the Hamiltonian.

The light-front Hamiltonian suggests the following picture of the nucleon:

$$|N\rangle=\psi^{|\Omega\rangle}_{3q}q^\dagger_1 q^\dagger_2 q^\dagger_3\,|\,\Omega\rangle+\psi^{|\Omega\rangle}_{3qg}q^\dagger_1 q^\dagger_2 q^\dagger_3 g^\dagger\,|\,\Omega\rangle$$
$$+\psi^{|\Omega\rangle}_{3q\bar{q}q}q^\dagger_1 q^\dagger_2 q^\dagger_3 \bar{q}^\dagger q^\dagger\,|\,\Omega\rangle+(\text{less important terms})\ . \qquad (4.1)$$

The light-front Fock wave functions describe pointlike quarks and gluons by effective parameters resulting from the interaction of these particles with the vacuum $|\,\Omega\rangle$. The truncation of "less important terms" may be a good approximation if at small virtualities the effective constituent masses are large enough and the interactions are such that the additional components would have to raise the eigenvalue of the Hamiltonian. The first wave function $\psi^{|\Omega\rangle}_{3q}$ would provide a direct connection between notions of constituent and current quarks. A phenomenological model based on this idea gives good agreement with static and deep-inelastic nucleon data.[5] The wave function $\psi^{|\Omega\rangle}_{3qg}$ would explain the origin of counting rules in exclusive processes or the half of the nucleon momentum carried by gluons in the deep-inelastic structure function F_2. The third wave function would explain the asymptotic behavior of the first one[6] and the presence of a meson cloud around the nucleon core. Perturbative effects would be superposed on this leading approximation. The eigenvalue problem for the new Hamiltonian deserves investigation.

Further tests of the Hamiltonian have to be carried out. A careful treatment of the parameter δ splitting fields is not completed yet. Consequently, the mixing of gluons and massless quarks still awaits explanation. The delicate point of renormalization in the presence of δ is not clear. Physical results must be independent of the actual value of the parameter δ. This may lead to a set of δ-invariance conditions. The flow of probability through the δ splitting might lead to the low-energy theorems.[7] Although it is difficult to draw firm conclusions at this stage, we hope that the approach we have outlined will lead the way to a description of hadronic structure.

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APPENDIX

Let us denote fields as

$$A_t = A + a, \quad \psi_t = \psi + \omega, \quad iD^\mu = i\partial^\mu - gA^\mu ,$$

$$id^\mu = i\partial^\mu - ga^\mu, \quad F^{\mu\nu} = ig[D^\mu, D^\nu], \quad f^{\mu\nu} = ig[d^\mu, d^\nu] \tag{A1}$$

and introduce $\mathbf{A}$ and $\boldsymbol{\psi}$, the background fields in the Fock-Schwinger gauge $x_\mu \mathbf{A}^\mu = 0$:

$$i\mathbf{D}^\mu = i\partial^\mu - g\,\mathbf{A}^\mu, \quad \mathbf{F}^{\mu\nu} = ig[\mathbf{D}^\mu, \mathbf{D}^\nu] . \tag{A2}$$

The fields $\mathbf{A}$ and $\boldsymbol{\psi}$ can be expanded around $x=0$ as[2]

$$\mathbf{A}^\mu = \frac{1}{2\times 0!} x_\rho \mathbf{F}_0^{\rho\mu} + \frac{1}{3\times 1!} x_\alpha x_\rho \mathbf{D}^\alpha F_0^{\rho\mu} + \frac{1}{4\times 2!} x_\alpha x_\beta x_\rho \mathbf{D}^\alpha \mathbf{D}^\beta \mathbf{F}_0^{\rho\mu} + \cdots ,$$

$$\boldsymbol{\psi} = \boldsymbol{\psi}_0 + x_\alpha \mathbf{D}^\alpha \boldsymbol{\psi}_0 + \tfrac{1}{2} x_\alpha x_\beta \mathbf{D}^\alpha \mathbf{D}^\beta \boldsymbol{\psi}_0 + \cdots . \tag{A3}$$

Suppose the background fields a and ω are related to the background fields $\mathbf{A}$ and $\boldsymbol{\psi}$ by the gauge transformation h:

$$\mathbf{A}^u = h a^\mu h^{-1} + \frac{i}{g} \partial^\mu h h^{-1}, \quad \boldsymbol{\psi} = h\omega . \tag{A4}$$

Then the covariant derivatives are connected by

$$i\mathbf{D}^{\alpha_1} \cdots i\mathbf{D}^{\alpha_n} \mathbf{F}^{\mu\nu} = h i d^{\alpha_1} \cdots i d^{\alpha_n} f^{\mu\nu} h^{-1} ,$$

$$i\mathbf{D}^{\alpha_1} \cdots i\mathbf{D}^{\alpha_n} \boldsymbol{\psi} = h i d^{\alpha_1} \cdots i d^{\alpha_n} \omega . \tag{A5}$$

Therefore, if we impose the condition

$$h(0) = 1 , \tag{A6}$$

then the expansions (A3) can be rewritten as

$$\mathbf{A}^\mu = \frac{1}{2\times 0!} x_\rho f_0^{\rho\mu} + \frac{1}{3\times 1!} x_\alpha x_\rho d^\alpha f_0^{\rho\mu} + \frac{1}{4\times 2!} x_\alpha x_\beta x_\rho d^\alpha d^\beta f_0^{\rho\mu} + \cdots ,$$

$$\boldsymbol{\psi} = \omega_0 + x_\alpha d^\alpha \omega_0 + \tfrac{1}{2} x_\alpha x_\beta d^\alpha d^\beta \omega_0 + \cdots , \tag{A7}$$

i.e., gauge-invariant expressions containing gauge-covariant coefficients in the $a^+=0$ gauge. The expansion we are looking for can be found from the inverse relation to (A4),

$$a^\mu = h^{-1} \mathbf{A}^\mu h - \frac{i}{g} h^{-1} \partial^\mu h, \quad \omega = h^{-1} \boldsymbol{\psi} , \tag{A8}$$

where A^μ and ψ are given by (A7) and the gauge transformation h is a solution to the condition $a^+=0$:

$$\partial^+ h = \frac{g}{i} \mathbf{A}^+ h \tag{A9}$$

with the constraint condition (A6).

We can represent

$$h = e^{i\lambda} \tag{A10}$$

and find, order by order in powers of x^μ,

$$\lambda = \lambda_0 + \lambda_\mu x^\mu + \lambda_{\mu\nu} x^\mu x^\nu + \cdots . \tag{A11}$$

To construct the solution we need two vectors η and $\tilde{\eta}$ equal to

$$\eta = (\eta^- = 2, \eta^+ = 0, \eta^\perp = 0) ,$$

$$\tilde{\eta} = (\tilde{\eta}^- = 0, \tilde{\eta}^+ = 1, \tilde{\eta}^\perp = 0) . \tag{A12}$$

The only difficulty is to find solutions of the equations such as

$$\lambda^{+\alpha_1 \cdots \alpha_n} = u^{\alpha_1 \cdots \alpha_n} , \tag{A13}$$

where $u^{\alpha_1 \cdots \alpha_n}$ and $\lambda^{\alpha_0 \alpha_1 \cdots \alpha_n}$ are symmetric tensors. This is done by writing

$$\tilde{\lambda}^{\mu\alpha_1 \cdots \alpha_n} = \tilde{\eta}^\mu u^{\alpha_1 \cdots \alpha_n} , \tag{A14}$$

symmetrizing in indices μ and α_i, $i=1,\ldots,n$, subtracting unwanted walls, cubes, ..., and adding lost edges of the desired tensor $\lambda^{\alpha_0 \alpha_1 \cdots \alpha_n}$.

We have calculated the resulting expansion up to third order in x^μ. It reads

$$a^{\mu}=\tfrac{1}{2}x_{\rho}\left[f_0^{\rho\mu}+\frac{4}{g}\lambda^{\rho\mu}\right]+\tfrac{1}{3}x_{\alpha}x_{\rho}\left[d^{\alpha}f_0^{\rho\mu}+\frac{9}{g}\lambda^{\alpha\rho\mu}\right]+\tfrac{1}{8}x_{\alpha}x_{\beta}x_{\rho}\left[d^{\alpha}d^{\beta}f_0^{\rho\mu}-4i\left[\lambda^{\alpha\beta},f_0^{\rho\mu}+\frac{2}{g}\lambda^{\rho\mu}\right]+\frac{32}{g}\lambda^{\alpha\beta\rho\mu}\right]+O(x^4)\ , \tag{A15}$$

where

$$\begin{aligned}
&\lambda^{\alpha\beta}=\tilde{\eta}^{\alpha}u^{\beta}+\tilde{\eta}^{\beta}u^{\alpha},\quad u^{\alpha}=\frac{g}{4}\eta_{\mu}f_0^{\mu\alpha}\ ,\\
&\lambda^{\alpha\beta\gamma}=\tilde{\eta}^{\alpha}u^{\beta\gamma}+\tilde{\eta}^{\beta}u^{\alpha\gamma}+\tilde{\eta}^{\gamma}u^{\alpha\beta}-(\tilde{\eta}^{\alpha}\tilde{\eta}^{\beta}u^{\mu\gamma}+\tilde{\eta}^{\alpha}\tilde{\eta}^{\gamma}u^{\mu\beta}+\tilde{\eta}^{\beta}\tilde{\eta}^{\gamma}u^{\mu\alpha})\eta_{\mu}\ ,\\
&u^{\alpha\beta}=\frac{g}{18}(d^{\alpha}f_0^{\mu\alpha}+d^{\beta}f_0^{\mu\alpha})\eta_{\mu}\ ,\\
&\lambda^{\alpha\beta\gamma\delta}=\tilde{\eta}^{\alpha}u^{\beta\gamma\delta}+\tilde{\eta}^{\beta}u^{\alpha\gamma\delta}+\tilde{\eta}^{\gamma}u^{\mu\alpha\beta\delta}+\tilde{\eta}^{\delta}u^{\alpha\beta\gamma}\\
&\qquad-(\tilde{\eta}^{\alpha}\tilde{\eta}^{\beta}u^{\mu\gamma\delta}+\tilde{\eta}^{\alpha}\tilde{\eta}^{\gamma}u^{\mu\beta\delta}+\tilde{\eta}^{\alpha}\tilde{\eta}^{\delta}u^{\mu\beta\gamma}+\tilde{\eta}^{\beta}\tilde{\eta}^{\gamma}u^{\mu\alpha\delta}+\tilde{\eta}^{\beta}\tilde{\eta}^{\delta}u^{\mu\alpha\gamma}+\tilde{\eta}^{\gamma}\tilde{\eta}^{\delta}u^{\mu\alpha\beta})\eta_{\mu}\\
&\qquad+(\tilde{\eta}^{\alpha}\tilde{\eta}^{\beta}\tilde{\eta}^{\gamma}u^{\mu\nu\delta}+\tilde{\eta}^{\alpha}\tilde{\eta}^{\beta}\tilde{\eta}^{\delta}u^{\mu\nu\gamma}+\tilde{\eta}^{\alpha}\tilde{\eta}^{\gamma}\tilde{\eta}^{\delta}u^{\mu\nu\beta}+\tilde{\eta}^{\beta}\tilde{\eta}^{\gamma}\tilde{\eta}^{\delta}u^{\mu\nu\alpha})\eta_{\mu}\eta_{\nu}\ ,\\
&u^{\alpha\beta\gamma}=\frac{g}{192}(d^{\alpha}d^{\beta}f_0^{\mu\gamma}+d^{\gamma}d^{\alpha}f_0^{\mu\beta}+d^{\beta}d^{\gamma}f_0^{\mu\alpha}+d^{\beta}d^{\alpha}f_0^{\mu\gamma}+d^{\gamma}d^{\beta}f_0^{\mu\alpha}+d^{\alpha}d^{\gamma}f_0^{\mu\beta})\eta_{\mu}\ .
\end{aligned} \tag{A16}$$

Each succeeding-order correction does not modify the previous result. The field tensor $f^{\mu\nu}$ and the quark field look like

$$\begin{aligned}
&f^{\mu\nu}=f_0^{\mu\nu}+x_{\alpha}d^{\alpha}f_0^{\mu\nu}+\tfrac{1}{2}x_{\alpha}x_{\beta}(d^{\alpha}d^{\beta}f_0^{\mu\nu}-2i[\lambda^{\alpha\beta},f_0^{\mu\nu}])+O(x^3)\ ,\\
&\omega=\omega_0+x_{\alpha}d^{\alpha}\omega_0+\tfrac{1}{2}x_{\alpha}x_{\beta}(d^{\alpha}d^{\beta}\omega_0-2i\lambda^{\alpha\beta}\omega_0)+\tfrac{1}{6}x_{\alpha}x_{\beta}x_{\gamma}(d^{\alpha}d^{\beta}d^{\gamma}\omega_0-6i\lambda^{\alpha\beta}d^{\gamma}\omega_0-6i\lambda^{\alpha\beta\gamma}\omega_0)\\
&\qquad+\tfrac{1}{24}x_{\alpha}x_{\beta}x_{\gamma}x_{\delta}(d^{\alpha}d^{\beta}d^{\gamma}d^{\delta}\omega_0-12i\lambda^{\alpha\beta}d^{\gamma}d^{\delta}\omega_0-24i\lambda^{\alpha\beta\gamma}d^{\delta}\omega_0-24i\lambda^{\alpha\beta\gamma\delta}\omega_0-12\lambda^{\alpha\beta}\lambda^{\gamma\delta}\omega_0)+O(x^5)\ .
\end{aligned} \tag{A17}$$

Similar expansions exist in axial-vector gauges $n_{\mu}A^{\mu}=0$ when $n^2=\pm 1$. It remains to substitute $\tilde{\eta}=n$ and $\eta=n$, roughly speaking. The choice of the origin of the expansion is equivalent to a choice of gauge because it is so in the Fock-Schwinger gauge and we obtain the $a^+=0$ expansion by a gauge transform from the Fock-Schwinger gauge. The gauge condition $a^+=0$ is invariant under three independent Lorentz boosts under which the whole light-front scheme is invariant.

The expansion (A16) is not, in fact, complicated if we can restrict it to the lowest terms.

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