Lattice $\lambda\phi^4$ theory with Yukawa couplings to staggered fermions

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An investigation of the $\lambda\phi^4$ theory coupled via Yukawa couplings to fermions has been initiated on the lattice. Dynamical fermions are taken fully into account, using hybrid-molecular-dynamic algorithms. Different ways of transcribing Yukawa couplings onto the lattice are discussed. It is found that the lattice phase diagram is very sensitive to the manner in which this interaction is regularized. Some results for scalar and fermionic correlation functions are also presented.

I. INTRODUCTION AND DESCRIPTION OF THE LATTICE ACTIONS

The Higgs sector of the standard model has more and more frequently become the focus of nonperturbative investigations on the lattice in recent years.[1-10] The lack of understanding of how this sector works and the apparent difficulties with continuum quantum field theories involving nonasymptotically free couplings has motivated people to try to define Higgs models through unconventional means, for instance, by introducing a lattice regularization. The lattice models one ends up with differ from lattice QCD in crucial ways. The latter model has a well-defined continuum limit at the ultraviolet-stable fixed point at $g_0^2 \to 0$. In lattice Higgs models one is forced to look for continuum limits away from the origin in the bare-lattice-coupling-constant space. There has been considerable progress in studies of pure bosonic models (without fermions) using both analytic and numerical methods.[1-5] In most cases the indications are that only trivial noninteracting continuum theories emerge. However, even foregoing taking the cutoff scale to infinity, several authors have been able to obtain interesting bounds on Higgs-boson masses. A very recent phenomenon has been the introduction of fermions into nonasymptotically free models.[6-9] The search for a nontrivial ultraviolet-stable fixed point in Higgs models and also in Abelian gauge theories[11] has hence become more complicated but at the same time more interesting and multifaceted. Fermions bring in new conceptual and technical challenges. In the present paper we summarize our efforts to date in understanding a simple lattice Higgs system coupled to fermions through Yukawa couplings. Some of the issues raised here should also apply to more realistic gauge Higgs systems with fermions.

The model under investigation is the single-component $\lambda\phi^4$ theory coupled to two flavors of staggered fermions (eight continuum species):

$$S = S_B + S_F + S_Y \ . \tag{1.1}$$

Our conventions for the scalar action S_B and the free fermion action S_F are

$$S_B = \sum_n \left[4\phi^2(n) - \sum_\mu \phi(n)\phi(n+\mu) \right] + \frac{m^2}{2}\sum_n \phi^2(n) + \frac{\lambda}{4}\sum_n \phi^4(n) \ , \tag{1.2}$$

$$S_F = \tfrac{1}{2}\sum_n \sum_\mu \sum_{f=1}^{2} \bar{\chi}_f(n)(-1)^{\eta_\mu(n)}[\chi_f(n+\mu) - \chi_f(n-\mu)] + \sum_n \sum_f m_f \bar{\chi}_f(n)\chi_f(n) \ , \tag{1.3}$$

where $\chi_f, \bar{\chi}_f$ are anticommuting variables and $\eta_\mu(n) \equiv n_1 + \cdots + n_{\mu-1}$. In all our studies to date we have set the bare fermion masses m_f equal to zero. For the Yukawa coupling term S_Y between the scalars and the fermions we have considered two possibilities:

$$S_Y^1 = y\sum_n \phi(n)\sum_f [\bar{\chi}_f(n)\chi_f(n)] \tag{1.4}$$

and

$$S_Y^2 = y\sum_n \phi(n)\sum_f \left[\tfrac{1}{16}\sum_{\text{hypercube}} \bar{\chi}_f(n)\chi_f(n) \right] = y\sum_n \Phi(n)\sum_f [\bar{\chi}_f(n)\chi_f(n)] \ , \tag{1.5}$$

with

$$\Phi(n) \equiv \tfrac{1}{16}\sum_{\text{hypercube}} \phi(n) \ , \qquad \sum_{\text{hypercube}} \equiv \text{sum over 16 sites} \ . \tag{1.6}$$

S_Y^2 is the natural expression one would write down if one takes into account

$$\sum_{\text{hypercube}} \bar{\chi}\chi \propto \sum_{\gamma=1}^{4} \bar{\psi}_\gamma \psi_\gamma \tag{1.7}$$

(ψ_γ = four-component spinor of flavor "γ") and insists on retaining all the discrete shift symmetries of free staggered fermions. (In the present case discrete shift transformations on the χ fields must be accompanied by appropriate transformations on the ϕ's.) S_Y^1 also obeys the same symmetries. It is much simpler than S_Y^2 but its interpretation in terms of the continuum ψ_Y fields is not straightforward.

S_Y^1 and S_Y^2 are reminiscent of the many ways (more than two) in which people have formulated the (1+1)-dimensional Gross-Neveu (GN) model on a Euclidean lattice.[12] In those formulations most closely related to S_Y^1 (we will call them type-I actions) a phase transition was found at some finite Gross-Neveu coupling $g^2=g_c^2$. For $g^2>g_c^2$ the discrete chiral symmetry was restored. On the other hand, for the analogues of S_Y^2 (we call them type-II actions; they involve multisite couplings) the symmetry was broken for all g^2. The dynamical reason for this difference was that as g^2 increased, different lattice sites became decoupled for type-I actions, whereas for type-II actions the opposite happened and sites became more and more correlated. Using the large-N analyses Ref. 13 showed that the phase transition at g_c^2 for type-I actions was Ising type. Furthermore several authors pointed out that type-I actions typically appear to have terms that break Lorentz invariance in the classical continuum limit. Reference 14, however, showed that Lorentz-invariance-breaking terms become irrelevant in the quantum continuum limit. We note that "spurious" transitions were found also for naive lattice fermions, so they are not just artifacts of staggered lattice fermions.

Returning to the current model, when trying to choose between (1.4) and (1.5) or to argue that continuum physics should not depend on the choice, the situation is vastly more complicated than with the Gross-Neveu model. First of all, one does not know where to take a continuum limit such that interesting continuum physics can be extracted. In the GN case it was easy to distinguish between the infrared-unstable, asymptotically free fixed point at the origin and the Ising critical point at finite coupling. At the origin one is able to define a nontrivial continuum field theory with a nontrivial spectrum. At the Ising critical point presumably only a free fermion theory emerges in the continuum limit. In the present model we have no criteria to distinguish *a priori* between interesting and uninteresting critical points and lengthy, difficult calculations will be required. The most likely scenario is that only the analogue of the Ising transitions (with trivial continuum limit) exists regardless of whether one uses S_Y^1 or S_Y^2. It should be possible to come up with an extended action which in one limit reduces to S_Y^1 and in another to S_Y^2. The critical points of the pure S_Y^1 and the pure S_Y^2 models would then be on the same critical (hyper)surface.

In order to see the difference, at least on the lattice, between the choices S_Y^1 and S_Y^2 it is useful to consider the large-Yukawa-coupling limit. To do this, first rescale the χ fields,

$$\chi_f \to \frac{1}{\sqrt{y}}\chi_f,\quad \bar\chi_f \to \frac{1}{\sqrt{y}}\bar\chi_f \ .$$

Then using S_Y^1 of Eq. (1.4), one obtains, in the $y\to\infty$ limit,

$$S \to S_B + \sum_n \phi(n) \sum_{f=1}^{N_f} \bar\chi_f(n)\chi_f(n)$$

(we work with $N_f=2$) and

$$\int d\bar\chi\, d\chi\, d\phi\, e^{-S} = \int d\phi\, e^{-S_{\rm eff}^1} \tag{1.8}$$

with

$$S_{\rm eff}^1 = S_B - \sum_n \ln[\phi(n)]^{N_f} \ . \tag{1.9}$$

If one uses S_Y^2 of Eq. (1.5) one obtains instead

$$S_{\rm eff}^2 = S_B - \sum_n \ln[\Phi(n)]^{N_f} \ . \tag{1.10}$$

The log terms in (1.10) have a very different ordering effect on the ϕ fields from the log terms in (1.9). For instance, in the Ising limit $\lambda\to\infty$, $m^2\to-\infty$ with $-m^2/\lambda\equiv\beta_H$ fixed, one has $\phi\to\pm1$ (after rescaling) and the $\ln[\phi^2(n)]$ terms in (1.9) become irrelevant. One obtains the same theory as if fermions had never been present. With (1.10), however, the log terms require that

$$\Phi = \sum_{\rm hypercube} \phi \neq 0 \ . \tag{1.11}$$

Hence in $S_{\rm eff}^2$ the contributions from the fermions reinforce symmetry breaking. Even for finite λ and m^2 one expects that, as $y\to\infty$, S_Y^2 will strongly favor symmetry breaking. We have carried out Monte Carlo simulations of the purely bosonic actions $S_{\rm eff}^1$ and $S_{\rm eff}^2$ at $\lambda=1$ and 100. With $S_{\rm eff}^2$ we were not able to find evidence for a symmetric phase in the entire range $-\infty\le m^2<\infty$. Although $\langle\phi\rangle$ becomes very small ($\propto 1/\sqrt{m^2}$ for large m^2) one never saw tunneling between positive and negative $\langle\phi\rangle$ vacua. The situation was different with $S_{\rm eff}^1$ which led to a symmetric phase for m^2 exceeding some m_c^2. We were careful to include "Ising flips" in our Monte Carlo updating procedure, so we do not think the absence of a symmetric phase with $S_{\rm eff}^2$ is a numerical artifact. However more tests should be carried out in the future.

The numerical results indicating the absence of tunneling at $y\to\infty$ in the case of the hypercube lattice Yukawa coupling S_Y^2 are in contradiction with the well-known statement that no dynamical symmetry breaking can happen in finite systems. This apparent contradiction can be resolved in the following manner. First, the usual argument that the ground-state wave function of a finite system can be chosen to be real without nodes is valid only for finite potentials.[15] Furthermore, the variational expression for the ground-state energy indicates that the ground-state wave function must vanish at the point where the potential has a nonintegrable singularity.

To make this point clearer it is instructive to consider the case of a particle with unit mass moving in a one-dimensional x^{-2} potential. The ground-state wave function behaves as x^p, where p is greater than 1 close to the singularity and the derivative vanishes together with the wave function. Thus there is the option to have a symmetric or antisymmetric wave function with respect to space inversion and the ground states form a parity doublet. In other words, the tunneling responsible for the restoration of the symmetry is suppressed by the nonintegrable potential. This holds for any eigenstate with finite energy. Thus one has two degenerate Hilbert spaces corresponding to the states with vanishing wave functions for positive or negative values of the coordinate x. These spaces do not mix during the time evolution. This phenomenon is the toy version of dynamical symmetry breaking.

Note that the usual derivation of the path-integral expression of the time evolution operator is not valid for matrix elements connecting coordinates with different sign. In fact, the trajectories cross the singularity in this case and the phase of the integrand diverges as the cutoff in time is removed. This divergence should lead to the vanishing of these amplitudes in the case of a finite system. This is certainly the case with the quantum-mechanical example discussed above but the subtleties of the convergence when the cutoff in time is removed prevents us from making general statements. Less is known about field theories where the entropy factor may dominate and the tunneling may survive the removal of the cutoff in this way. In any case the actual fate of the symmetry depends on fluctuations on the scale of the ultraviolet cutoff as opposed to the usual spontaneous symmetry breaking where the relevant modes are infrared. In this sense this phenomenon is similar to the usual anomalies except it works in the opposite direction: symmetry-restoring modes which are forbidden in the classical case may become relevant in the presence of the ultraviolet cutoff.

So far we discussed bosonic systems with singular potential in the classical Hamiltonian and argued that there may in fact be dynamical symmetry breaking even in a finite system in this case. The actual field theoretical model is given in the path-integral formalism where the cutoff is included in an unavoidable manner. Thus the transfer matrix extracted from the effective action containing the fermion determinant is already regularized. What is left to inquire is whether the singular potential which appears in the effective action as a result of the fermion zero modes suppresses the tunneling modes as the cutoff is removed. Our numerical results suggest that in the leading-order expansion in the kinetic energy of the fermion the answer depends on the way in which the Yukawa coupling is regularized. The two-phase structure of the single-site Yukawa coupling model can be understood by the analogy with the four-dimensional Ising model: the system skips the singularities when the continuous field variable is approximated by an Ising spin. The same approximation is unable to avoid the singularities in the case of the hypercubic construction since the sum of an even number of spin variables may vanish. Thus the entropy of the symmetry-restoring modes is reduced.

The singularity structure of the effective action depends on the actual value of the Yukawa coupling. The singularities are located for large values of the coupling at configurations where any of the field variables vanish. The singular hypersurface becomes more complicated for intermediate values of the Yukawa coupling since the fermion zero modes become extended when higher orders in the fermion kinetic energy are included. In the case of a soluble toy model with exponentiated Yukawa coupling the symmetry breaking persists for any value of the coupling constant.[16] It would be instructive to visualize the location of the singular hypersurface for an intermediate value of the Yukawa coupling in the field-theoretic case as well.

We will see in the next section that when one comes down from the $y \to \infty$ limit, one finds symmetric phases for both S_Y^1 and S_Y^2 above a critical m^2. One can no longer use S_{eff}^1 or S_{eff}^2 and is faced with the task of evaluating a nontrivial fermion determinant. In the next section we describe the algorithms employed to investigate the finite-y region.

II. THE ALGORITHMS AND LATTICE PHASE DIAGRAMS

One objective of the present project was to test existing algorithms for numerical simulations with fermions on models with spontaneous symmetry breaking. We have developed code for the microcanonical[17] and for two variants of the hybrid[18,19] molecular dynamic algorithms. Very briefly, this involves introducing, in addition to the four discrete Euclidean dimensions, a fifth continuous "τ" direction and then solving equations of motion in the τ variable. The Hamiltonian for this (4 + 1)-dimensional system is given by (we will use S_Y^2)

$$H=\frac{1}{2}\sum_n P^2(n)+V(\phi)+\frac{1}{2}\sum_{f=1}^{2}\sum_{n,n'} Q_f(n)(A^\dagger A)^{-1}_{n,n'}Q_f(n') + \frac{\omega^2}{2}\sum_f \sum_n \Pi_f(n) \tag{2.1}$$

with

$$\begin{aligned} V(\phi) &\equiv S_B(\phi) , \\ A_{n,n'} &= \frac{1}{2}\sum_\mu (-1)^{\eta_\mu}(\delta_{n',n+\mu}-\delta_{n',n-\mu}) \\ &\quad +\delta_{n,n'} y\Phi(n) , \end{aligned} \tag{2.2}$$

and Q_f, Π_f, and P are real degrees of freedom. After integrating out the Q_f,Π_f,P degrees of freedom one is left with a functional integral over the ϕ's with

$$\text{weight}=\{\det[A(\phi)]\}^2 e^{-\beta_{\text{eff}} S_B} , \tag{2.3}$$

$$\beta_{\text{eff}}=\frac{L^3T}{2}\Big/\left\langle \frac{1}{2}\sum_n P^2\right\rangle \quad (L^3T=\text{No. of lattice sites}). \tag{2.4}$$

Equation (2.3) tells us that if λ and y are the original couplings in S_B and S_Y^2, then simulating a system with Hamiltonian (2.1) means simulating the model of Eq. (1.1) with effective couplings

$$\lambda_{\text{eff}}=\frac{\lambda}{\beta_{\text{eff}}}, \quad y_{\text{eff}}=\frac{y}{\sqrt{\beta_{\text{eff}}}} . \tag{2.5}$$

In the microcanonical approach one works at fixed total energy and solves Hamilton's equations of motion,

$$\dot{\phi}(n)=P(n)\ , \tag{2.6a}$$

$$\dot{P}(n)=-\frac{\delta V}{\delta\phi}-\frac{1}{2}\frac{\delta}{\delta\phi}\left[\sum_f Q_f(A^\dagger A)^{-1}Q_f\right]\ , \tag{2.6b}$$

$$\dot{Q}_f(n)=\omega^2\Pi_f(n)\ , \tag{2.6c}$$

$$\dot{\Pi}_f(n)=-(A^\dagger A)^{-1}Q_f\ . \tag{2.6d}$$

We have omitted intermediate sums over lattice sites in the right-hand side (RHS) of (2.6b) and (2.6d). Introduce new variables

$$B_f(n)=[(A^\dagger A)^{-1}Q_f]_n \tag{2.7}$$

or

$$Q_f(n)=(A^\dagger A)B_f\ . \tag{2.8}$$

Then (2.6) can be reexpressed as

$$\ddot{\phi}=\dot{p}=-\frac{\delta V}{\delta\phi}+\frac{1}{2}\frac{\delta}{\delta\phi}\left[\sum_f B_f(A^\dagger A)B_f\right]\ , \tag{2.9a}$$

$$\ddot{Q}_f=\omega^2\dot{\Pi}_f=-\omega^2 B_f\ . \tag{2.9b}$$

We have numerically solved Newton's equations (2.9a) and (2.9b) which are of the generic form

$$\ddot{q}=F\quad(q=\phi,Q_1,Q_2)\ . \tag{2.10}$$

We used the Verlet updating scheme

$$q_{\tau+1}=q_\tau+(q_\tau-q_{\tau-1})+\delta\tau^2F(\tau)\ . \tag{2.11}$$

For most of our microcanonical runs we used $\delta\tau=0.01$. However we did check that averages do not change significantly up to $\delta\tau=0.05$ or even larger.

The second algorithm that we considered was the hybrid method of Duane and Kogut.[18] In this approach one brings the system (2.1) from time to time in contact with a heat bath, usually adjusted so that β_{eff} of (2.4) equals 1. This can be implemented by replacing with some probability P_ζ, Eq. (2.11) by

$$q_{\tau+1}=q_\tau+\delta\tau\zeta+\tfrac{1}{2}(\delta\tau)^2F(\tau)\ , \tag{2.12}$$

where the ζ's are Gaussian distributed random variables obeying,

$$\overline{\zeta(n)\zeta(n')}=\delta_{n,n'}\times\begin{cases}1, & q=\phi\ ,\\ \omega^2, & q=Q_f\end{cases}$$

and (2.13)

$$\bar{\zeta}(n)=0\ .$$

We have worked with P_ζ ranging between 0.02 and 0.05. $P_\zeta=0$ corresponds to the previous pure microcanonical algorithm. $P_\zeta=1$ gives the Langevin limit. We have found that the $\delta\tau$ sensitivity of physical results in the region $0.01\le\delta\tau\le0.1$ deteriorates considerably once P_ζ exceeds $\sim10\%$.

The third algorithm we have employed is the variant of the hybrid method advocated by Gottlieb *et al.*[19] We will call this the "noisy fermion" (NF) method. Instead of (2.1) the starting point is

$$H_{\text{NF}}=\tfrac{1}{2}\sum_n P^2(n)+V(\phi)+\tfrac{1}{2}\sum_f Q_f(A^\dagger A)^{-1}Q_f\ . \tag{2.14}$$

The equation of motion for ϕ is again given by (2.9a) with B_f related to Q_f via (2.7). There is no equation of motion for the Q_f fields. Instead one notes that

$$R_f\equiv A^{\dagger-1}Q_f \tag{2.15}$$

are Gaussian distributed variables. So at each τ slice one needs to generate such R_f's and then use

$$B_f=(A^\dagger A)^{-1}Q_f=A^{-1}R_f \tag{2.16}$$

to obtain new B_f's.

We now discuss the advantages and disadvantages of the three algorithms. The microcanonical method has the virtue that energy conservation provides a convenient test of our codes, the accuracy of the conjugate-gradient inversions [necessary to solve (2.16) and (2.7)] and the size of $\delta\tau$. It is however known from experience with other models that convergence to equilibrium is slow. It is also much more difficult to simulate the system at precisely the couplings one is interested in. So we have used the microcanonical method only in the initial stages of the project, mainly to test the code.

The hybrid method allows more flexibility in working at given values for the couplings. We have "refreshed" [i.e., occasionally replaced (2.11) by (2.12)] both the ϕ-field and Q_f-field velocities. The first and the last terms in (2.1) then obey equipartition by construction. We note that the third term in (2.1) is also quadratic and hence should obey, for each f,

$$\frac{1}{L^3T}\left\langle\tfrac{1}{2}\sum_{n,n'}Q_f(n)(A^\dagger A)^{-1}_{n,n'}Q_f(n')\right\rangle=0.5\ . \tag{2.17}$$

We have used (2.17) as one of the criteria for sufficient equilibration. For small λ and y ($\lesssim5$) equipartition sets in fairly rapidly, usually after a few thousand τ steps, using $\Delta\tau=0.01$, $P_\zeta=2\%$, and $\omega^2\sim1$. For larger y we discovered that one needed to increase ω^2 in order to converge to (2.17). We do not know whether for small ω^2

TABLE I. Comparison of different algorithms at $\lambda=1$ and $m^2=3$ on an 8^4 sized lattice.

y	Noisy fermion $\langle\phi\rangle$	Hybrid $\langle\phi\rangle$	Monte Carlo $\langle\phi\rangle$
1	~0		
1.5	~0		
1.6	~0		
1.7	0.125(0.019)		
1.8	0.246(0.012)		
2.0	0.367(0.007)	0.367(0.009)	
5.0	0.683(0.002)	0.684(0.002)	
10.0	0.721(0.002)	0.722(0.002)	
15.0	0.728(0.001)	0.735(0.001)	
20.0	0.730(0.002)		
100.0	0.733(0.002)		
200.0	0.733(0.002)		
∞			0.733

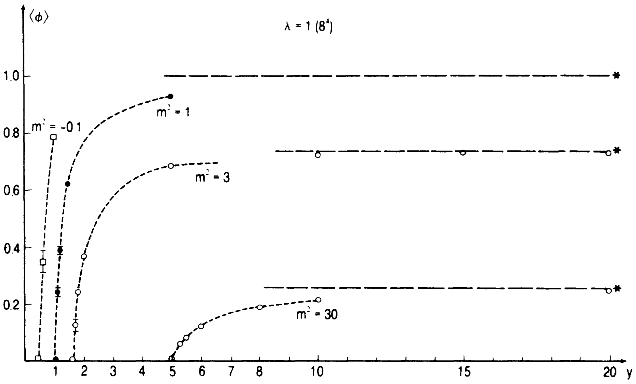

FIG. 1. $\langle \phi \rangle$ vs y at $\lambda=1$ for several m^2 values. The stars to the right show the Monte Carlo results for the infinite-y effective action $S^2_{\rm eff}$ of Eq. (1.10).

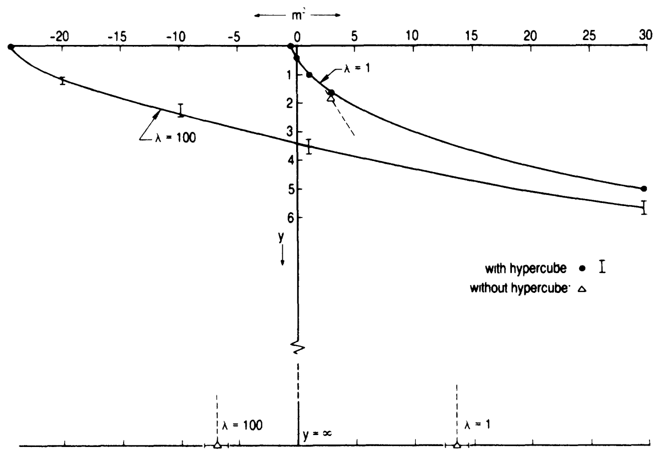

FIG. 3. The phase diagram in the (y,m^2) parameter space for $\lambda=1$ and 100. The lines separate the symmetric (large-m^2) from the broken (small-m^2) regions.

and large y equipartition would never have emerged or only extremely slowly (we only went up to a few tens of thousands of τ steps). Rather than continue to increase ω^2 we then switched to the third, the noisy fermion (NF), algorithm. With this algorithm (2.17) is satisfied automatically and cannot be used as a check. We tested our NF code by comparing with the hybrid code for small y and with the Monte Carlo results using $S^2_{\rm eff}$ of (1.10) for large y. Table I gives a comparison of different methods at $\lambda=1$, $m^2=3$ on 8^4 lattices. After this exercise we decided to use the NF algorithm for most of our subsequent runs.

Figure 1 shows $\langle \phi \rangle$ as a function of y for several m^2 values and $\lambda=1$. One sees that even for positive m^2 one can have spontaneous symmetry breaking if y exceeds some critical value. One also observes rapid saturation to the infinite y value for $\langle \phi \rangle$. The stars in Fig. 1 indicate the Monte Carlo results using $S^2_{\rm eff}$.

We have also developed a code for the Yukawa coupling S^1_Y. Figure 2 compares the two actions at $\lambda=1$, $m^2=3$. In the y range shown there does not seem to be any significant qualitative difference. We mention however that the number of conjugate-gradient iterations increases by approximately a factor of 2 when one goes from S^2_Y to S^1_Y. Furthermore once $y \gtrsim 3.5$ it becomes more and more difficult to attain equilibrium using S^1_Y. In order to keep $\beta_{\rm eff} \sim 1$ many more refreshings of velocities were required (P_ζ had to be increased). Occasionally we found the number of conjugate-gradient iterations shoot up to a couple of hundreds. In short, using S^1_Y our algorithms do not seem stable enough to handle the large-y region. So unfortunately it has not been possible to carry out comparisons between S^1_Y and S^2_Y except in the small-y region or at $y \to \infty$.

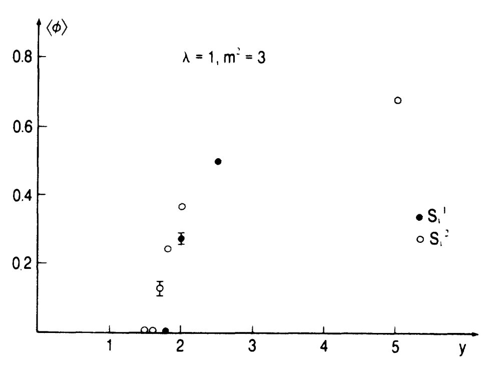

FIG. 2. Comparison of S^1_Y and S^2_Y at $\lambda=1$ and $m^2=3$.

Figure 3 summarizes the data on the boundary between symmetric and broken phases in the (y,m^2) plane for $\lambda=1$ and 100.

III. MASSES AND WAVE-FUNCTION RENORMALIZATION CONSTANTS

It should be clear from the preceding two sections that many conceptional issues still need to be resolved before our lattice model can be used to extract interesting continuum physics. Nevertheless we have initiated studies of correlation functions in this model. We present some results for the numerically more stable action S^2_Y at $\lambda=1$ and $y=2$. We hope that our calculations will be useful to researchers intending to investigate Higgs systems with dynamical fermions and Yukawa couplings.

For the scalar correlations we evaluate

$$GB(t)=\langle OPB(t)OPB(0)\rangle - \langle OPB(t)\rangle\langle OPB(0)\rangle \tag{3.1}$$

with

$$OPB(t) \equiv \frac{1}{L^3}\sum_{\mathbf{n}} \phi(\mathbf{n},t) \tag{3.2}$$

and for the fermions we look at

$$GF(t)=\left\langle \sum_{\mathbf{n}} \chi(\mathbf{n},t)\bar{\chi}(0)\right\rangle . \tag{3.3}$$

We discuss results at $\lambda=1$, $y=2$, and $m^2=4.2$. These values leave us in the broken phase with $\langle\phi\rangle=0.152\pm0.004$. We worked on $8^3\times16$ ($\sim$90 000 τ steps at $\Delta\tau=0.01$) and $8^3\times12$ ($\sim$140 000 steps) sized lattices and also accumulated some data on an $8^3\times24$ ($\sim$20 000 steps) lattice. Bosonic correlations were measured every fifth step and fermionic correlations every 200th step (with 12 source points). For the fermions the renormalized mass M_{FR} and the wave-function renormalization constant Z_F were determined from

$$\tilde{G}F(p)=\sum_{t=0}^{T-1} e^{ipt}GF(t)=Z_F\frac{M_{FR}+i\sin p}{\sin^2 p+M_{FR}^2}\ . \quad (3.4)$$

For antiperiodic boundary conditions in time,

$$p=\frac{(2l+1)\pi}{T}\ , \quad (3.5)$$

and we used the minimal p with $l=0$.

From (3.4) one has

$$M_{FR}=\frac{\text{Re}\{\tilde{G}F(p)\}}{\text{Im}\{\tilde{G}F(p)\}}\sin p\ , \quad (3.6)$$

$$Z_F=\text{Re}\{\tilde{G}F(p)\}\frac{\sin^2 p+M_{FR}^2}{M_{FR}}\ . \quad (3.6a)$$

We find

$$8^3\times12:\quad M_{FR}=0.289\pm0.010\ ,\quad Z_F=0.984\pm0.002\ , \quad (3.7)$$

$$8^3\times16:\quad M_{FR}=0.301\pm0.003\ ,\quad Z_F=0.985\pm0.003\ . \quad (3.8)$$

The errors quoted were obtained by dividing the measurements on the $8^3\times12$ ($8^3\times16$) lattices into 7 (5) sets and then calculating the standard deviation among the sets. We have also analyzed the $8^3\times12$ data by fitting $GF(t)$ to

$$GF(t)=A[e^{-E_Ft}-(-1)^t e^{-(T-t)E_F}]\ . \quad (3.9)$$

One then finds for $M_F\equiv\sinh(E_F)$,

$$M_F=0.298\pm0.010 \quad (3.10)$$

which agrees well with (3.7).

So our experience has been that fermionic correlations are fairly easy to evaluate and that one does not require a huge amount of statistics to find stable results. We will see in a moment that one is not that lucky with bosonic correlations.

We have tried to analyze our data for bosonic correlations in a variety of ways. The most straightforward way to extract masses is to fit $GB(t)$ to

$$GB(t)=A+\sum_j B_j(e^{-E_jt}+e^{-(T-t)E_j})\ . \quad (3.11)$$

When working in the broken phase on lattices with finite extent in the time direction one cannot in general neglect the t-independent constant A. This has made it very difficult to obtain stable fits. A further complication arises because the sum over j in (3.11) should include the two-fermion threshold. If there is in addition a single particle pole below this threshold it looks like one needs at the very least a five-parameter fit to $GB(t)$. Unfortunately our data for $GB(t)$ is not good enough to allow us to attempt such a fit.

There is another way in which a mass scale can be extracted from our data for $GB(t)$. Namely, one can define a renormalized (or effective) mass M_R as follows:

$$\tilde{G}B(p)=L^3\sum_{t=0}^{T-1}e^{ipt}GB(t)=\frac{Z(p)}{2(1-\cos p)+M_R^2},\quad p=\frac{2\pi n}{T}\ . \quad (3.12)$$

There is no reason why the renormalized mass M_R should be related in any simple way to the E_j's of Eq. (3.11). Furthermore the constant A in (3.11) may mess things up in $\tilde{G}B(p=0)$. In general $Z(p)$ need not be independent of p. One is actually interested in the limit $Z(p\to0)\equiv Z_\phi$. M_R should not be viewed as a physical mass. It is analogous to the mass scale set by the second derivative of the effective potential at the minimum.

We have used our $GB(t)$ data to evaluate $\tilde{G}B(p)$. By plotting $\tilde{G}B(p)^{-1}$ vs $2(1-\cos p)$ one can read off Z_ϕ^{-1} and m_R^2 from the slope and the intercept at $p=0$. Our results are shown in Fig. 4. The solid line through the $8^3\times16$ (and also $8^3\times24$) data leads to $M_R=0.67$ and $Z_\phi=0.46$. The dashed line gives $M_R=0.46$ and $Z_\phi=0.30$. We note that only $\tilde{G}B(p)$ at $p=0$ is affected by the constant A of Eq. (3.11) and the subtraction term in (3.1). So in Fig. 4 the $p\neq0$ points should have smaller systematic errors. By drawing straight lines through the data points we have assumed that $Z(p)$ is independent of p at least in the p range shown. We feel that at the present stage our numbers for M_R and Z_ϕ should be taken as very tentative. The statistical errors in Fig. 4 are large. A much more detailed finite spatial and temporal size analysis should be undertaken. And we also need to understand better threshold effects. On the other hand, in whatever

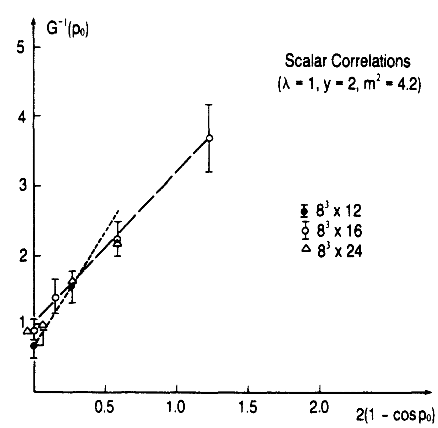

FIG. 4. The inverse propagator $\tilde{G}B(p_0)^{-1}$ vs $2(1-\cos p_0)$ at $\lambda=1$, $y=2$, and $m^2=4.2$. p_0 is referred to as plain p in the text.

way we tried to massage the data, we have found it difficult to get Z_ϕ up to a value closer to 1. This contrasts with our findings for Z_F and with findings by others for Z_ϕ in scalar models without fermions. We are planning extensive studies of correlation functions in the present model in both the symmetric and broken phases. This will enable us to investigate the behavior of interesting ratios such as M_F/M_R, $M_F/\langle \phi_R \rangle$, and $M_R/\langle \phi_R \rangle$ as one approaches an appropriate critical point of the lattice model. In the meantime, as mentioned already in the beginning of this section, more work should be devoted to clarifying the relationship between different ways of transcribing Yukawa couplings onto the lattice.

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