Non-Abelian gauge couplings at finite temperature and density

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With the off-shell one-loop calculation for massless non-Abelian gauge theories in the general covariant gauge we study properties of the gauge coupling renormalized at finite temperature and at finite density, mainly focusing on its dependence on the (finite) baryon-number density ρ_B (or on the chemical potential μ_B). The strong and severe vertex dependence is shown to come out for the μ_B dependence as well as on the temperature (T) dependence: In terms of the parameter $\zeta \equiv \mu_B/T$ the coupling defined through the three-gluon vertex shows a $\ln|\zeta|$ behavior, whereas the one defined through the fermion-gluon vertex shows a strong ζ^2 behavior. This strong vertex dependence survives even at $T=0$. Difficulties appearing in the perturbative calculation of physical quantities, indicated by this disaster, are discussed. We also discuss what insight might be gained from the present analysis for the "(magnetic) screening" of effective charge.

I. INTRODUCTION

In previous papers[1] (hereafter referred to as I) in order to study the perturbative properties of non-Abelian gauge theories at finite temperature we carried out one-loop finite-temperature calculations in the general covariant gauge. The calculation was done in the "magnetic" prescription[2] because it is known[3] that the magnetic field is screened less than the electric field, if at all, and gives dominant contributions at least at long distance. With the result obtained we studied extensively the temperature dependence of the effective coupling renormalized in the "magnetic" prescription at a nonzero finite temperature and also discussed its implications to the "(magnetic) screening" of effective charge. The effective gauge coupling was determined by integrating the coupled renormalization-group equations[4] (RGE's) satisfied by the coupling under changes of the renormalization momentum μ and of the renormalization temperature $\tilde{T}$. The results of our analyses show that the effect of temperature manifests itself as a powerlike behavior of the (inverse of) coupling on the parameter $\xi \equiv \tilde{T}/\mu$, in sharp contrast to the logarithmic dependence on the momentum μ, and that the strong vertex dependence of the coupling is shown to come out irrespective of any choice of gauges. It is also pointed out that "magnetic screening" strongly depends on the vertex through which the effective charge is defined, which has not been recognized by considering only the propagator as in previous works.[5] These results strongly force us to alter, at least in some physically interesting situations, the conventional assumptions[6] about the coupling at finite temperature, and pose a question for the perturbative treatment of field theory at finite temperature (FT^2).

In papers I, however, we only studied the case where effects of the finite baryon-number density were neglected. This is because in the presently interesting physical phenomena, such as the quark-gluon plasma and the evolution of the early Universe, the baryon-number density is negligibly small. On the other hand, recently much attention has also been paid to the study of high-density systems.[7] Taking this fact into account we should say that it is also important to study properties of field theory at finite (especially at high) density. In the present paper we focus our attention on this subject, especially on its perturbative regime.

To be more concrete we carry out under the real-time formalism[8] of FT^2 the one-loop calculation in non-Abelian gauge theories with massless fermions, which are interesting in a theoretical as well as a practical sense, at finite temperature and density in the general covariant gauge. With this result we study how the gauge coupling depends on the density or on the chemical potential that couples to the conserved baryonic charge. As is well known in thermodynamics the baryon-number density ρ_B can be expressed as a function of the reduced chemical potential of the fermion ζ ($\equiv \mu_B/T$, with μ_B the chemical potential) as

$$\rho_B = \sum \frac{(2s+1)n_B}{6\pi^2\beta^2}(\zeta^3+\pi^2\zeta)+\cdots , \tag{1.1}$$

where the ellipsis denotes correction due to interactions, s and n_B are the spin and baryon number, respectively, and $\beta=1/T$. Thus the effect of the finite baryon-number density can be analyzed through the effect of the finite (reduced) chemical potential of the fermion.

Throughout this paper we bear in mind physical processes that take place in a thermoequilibrium system at the temperature T with the finite baryon-number density ρ_B, or with the finite chemical potential μ_B. In the next section we present the coupled renormalization-group equations at finite temperature and density satisfied by the renormalized gauge coupling. Integrating these RGE's we introduce a scale parameter $\Lambda(T,\mu_B)$, in terms

of which the temperature and chemical potential dependences of the coupling is characterized. In Sec. III we perform the one-loop calculation and the result for the scale parameters is presented. The T and μ_B dependences of the coupling are explicitly determined. The conclusions of this paper and several discussions are given in the last section.

II. RENORMALIZATION-GROUP EQUATIONS AT FINITE TEMPERATURE AND DENSITY

Let us consider non-Abelian gauge theories (the gauge group G is assumed to be the compact Lie group, whose structure constant is f^{abd}) with massless fermions in the general covariant gauge. The Lagrangian is

$$\begin{aligned}\mathcal{L}=&-\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}-\frac{1}{2(1-\alpha)}(\partial_\mu A^{a\mu})^2\\&+(\partial_\mu\bar{c}^{\,a})(\partial^\mu c^a+gf^{abd}A^{b\mu}c^d)\\&+\sum_\psi\bar{\psi}(i\partial\!\!\!/+gA\!\!\!/^aT^a)\psi\ ,\end{aligned}\tag{2.1}$$

where all the notations are conventional, e.g., $F^a_{\mu\nu}$ is the covariant curl of the gauge field[9] A^a_μ,

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+gf^{abd}A^b_\mu A^d_\nu\ ,\tag{2.2}$$

and c is the ghost field, etc. Renormalization constants Z_i's are also defined conventionally, e.g., Z_3 is the gluon wave-function renormalization constant, etc., and are constrained by the Ward-Takahashi identities

$$\frac{Z_1}{Z_3}=\frac{\tilde{Z}_1}{\tilde{Z}_3}=\frac{Z_1^F}{Z_2},\quad \frac{Z_4}{Z_3}=\left[\frac{Z_1}{Z_3}\right]^2,\tag{2.3}$$

and

$$g=(Z_1^{-1}Z_3^{3/2})g_B,\quad 1-\alpha_B=Z_3(1-\alpha)\ ,\tag{2.4}$$

where g_B and α_B are the bare coupling constant and the bare gauge parameter.

Throughout this paper we use the coupling a defined by $a\equiv g^2/4\pi^2$ for convenience, and define the coupling renormalization constant Z_a by $a=Z_a^{-1}a_B$. Then we have three expressions for Z_a's:

$$Z_a(3G)=Z_1^2Z_3^{-3}\ ,\tag{2.5a}$$

$$Z_a(gG)=\tilde{Z}_1^{\,2}Z_3^{-1}\tilde{Z}_3^{\,-2}\ ,\tag{2.5b}$$

$$Z_a(fG)=(Z_1^F)^2Z_3^{-1}Z_2^{-2}\ ,\tag{2.5c}$$

which correspond to couplings defined through the three-gluon ($3G$), the ghost-gluon (gG), and the fermion-gluon (fG) vertices, respectively.

As mentioned in Sec. I, in this paper we consider physical amplitudes representing processes that take place in a thermoequilibrium system at the temperature T with the finite baryon-number density ρ_B, or with the finite chemical potential μ_B. The renormalized gauge parameter α is then to be considered a constant[10] to be fixed at the beginning. In the present finite temperature and density case renormalization of the coupling a is carried out, as a generalization of the prescription by Umezawa and coworkers,[4,11] by introducing as a renormalization point the following three quantities: the renormalization momentum μ, the renormalization temperature $\tilde{T}\equiv\xi\mu$, and the renormalization chemical potential[11] $\tilde{\mu}_B\equiv\zeta\tilde{T}$. $\tilde{T}$ and $\tilde{\mu}_B$ are chosen arbitrarily, as is well known for the renormalization momentum μ, not necessarily to be chosen to be equal to the temperature T and the chemical potential μ_B themselves that characterize the environment in which the process considered takes place. Thus the renormalized coupling a satisfies the following coupled RGE's under changes of μ, $\tilde{T}\,(\equiv\xi\mu)$, and $\tilde{\mu}_B\,(\equiv\zeta\tilde{T})$:

$$\frac{\partial a}{\partial\ln\mu}=\beta_\mu(a,\xi,\zeta)=-ba^2(1+ca+\cdots)\ ,\tag{2.6a}$$

$$\begin{aligned}\frac{\partial a}{\partial\ln\xi}&=\beta_\xi(a,\xi,\zeta)\\&=-\rho_\xi(\xi,\zeta)a^2[1+\eta_{\xi1}(\xi,\zeta)a+\cdots]\ ,\end{aligned}\tag{2.6b}$$

$$\begin{aligned}\frac{\partial a}{\partial\ln\zeta}&=\beta_\zeta(a,\xi,\zeta)\\&=-\rho_\zeta(\xi,\zeta)a^2[1+\eta_{\zeta1}(\xi,\zeta)a+\cdots]\ ,\end{aligned}\tag{2.6c}$$

where $a=a(\mu,\xi,\zeta)$. β functions are calculated through

$$\beta_x(a,\xi,\zeta)\equiv-a\frac{\partial\ln Z_a}{\partial\ln x}\bigg|_{\substack{a_B,\alpha_B,y\,(\neq x)\text{,}\\ \text{cutoff fixed}}},\quad x=\mu,\xi,\zeta\ .\tag{2.7}$$

The leading and the next-to-leading coefficients b and c in Eq. (2.6a) are constants independent of the choice of renormalization schemes and of gauges,

$$b=\frac{11C_2(G)-4T(R)}{6}\ ,\tag{2.8}$$

$$c=\frac{17C_2^2(G)-10C_2(G)T(R)-6C_2(R)T(R)}{2[11C_2(G)-4T(R)]}\ ,\tag{2.9}$$

while other coefficients including the leading ones $\rho_\xi(\xi,\zeta)$ and $\rho_\zeta(\xi,\zeta)$ in Eqs. (2.6b) and (2.6c) are not. For example, in the so-called modified minimal-subtraction ($\overline{\text{MS}}$) scheme ρ's vanish identically. In the above expressions gauge-parameter α dependences are not shown explicitly.

Integrating the above RGE's with a suitable boundary condition,[12,13] we get [by the use of Eq. (2.7)]

$$\begin{aligned}a(\mu,\xi,\zeta)=&\frac{1}{b\ln[\mu/\Lambda(\xi,\zeta)]}\\&-\frac{c\ln\{(b/c)\ln[\mu/\Lambda(\xi,\zeta)]\}}{b^2\ln^2[\mu/\Lambda(\xi,\zeta)]}+\cdots\ ,\end{aligned}\tag{2.10}$$

with the introduction of ξ- and ζ-dependent scale parameter $\Lambda(\xi,\zeta)$ by

$$\begin{aligned}\ln[\Lambda(\xi,\zeta)/\Lambda]&=-\frac{1}{b}\int_{\xi=\zeta=0}^{(\xi,\zeta)}[\rho_\xi(\xi,\zeta)d\ln\xi\\&\qquad+\rho_\zeta(\xi,\zeta)d\ln\zeta]\\&=-\frac{1}{b}[Z_a^{(1)}(\xi,\zeta)-Z_a^{(1)}(\xi=\zeta=0)]\ ,\end{aligned}\tag{2.11}$$

where $\Lambda\equiv\Lambda(\xi=\zeta=0)$, and $Z_a^{(1)}$ denotes the leading or-

der coefficient of the coupling renormalization constant Z_a:

$$Z_a = 1 + Z_a^{(1)} a + \cdots . \tag{2.12}$$

The fact[14] that the ultraviolet divergences in FT^2 appear only in the contributions from the zero-temperature and the zero-chemical potentials guarantees that the divergent parts of $Z_a^{(1)}(\xi,\zeta)$ and of $Z_a^{(1)}(\xi=\zeta=0)$ always cancel out in Eq. (2.11): namely, that the ratio of the scale parameter is always finite. It is to be noted that Eq. (2.11) is nothing but the Celmaster-Gonsalves theorem[15] on the renormalization-scheme dependence (applied to the present situation): Up to the leading one-loop order the ξ ($\equiv \tilde{T}/\mu$) and ζ ($\equiv \tilde{\mu}_B/\tilde{T}$) dependences of the coupling can be completely determined by calculating the ratio of scale parameters through Eq. (2.11).

III. EFFECTIVE COUPLING AT FINITE TEMPERATURE AND DENSITY

A. Renormalization prescription

In massless non-Abelian gauge theories (2.1) we carry out the off-shell one-loop finite-temperature and density calculation in the real-time formalism of FT^2, and evaluate various renormalization constants Z_i's, which allow us to determine the coupling (effective charge) renormalized at finite temperature, as shown above. It is to be noted that, because Lorentz invariance is explicitly broken in FT^2 due to the existence of a heat bath whose rest frame specifies the preferred frame, when defining a renormalized theory besides carefully specifying a subtraction momentum we should also be careful about the prescription for how we determine the subtraction part of each quantity (self-energy, vertex, etc.) by fixing our eyes on what component (temporal and/or spatial) of the quantity we respect. The renormalization prescription used in this paper (to evaluate Z_i's) is the same as those in papers I, and is referred to hereafter as the magnetic momentum-space subtraction ($\overline{\text{MOM}}$) scheme. In this $\overline{\text{MOM}}$ scheme we choose the "static" spacelike subtraction momentum[16] and the "magnetic" prescription[17] to define the subtraction part. (For more details, see I.) It is also to be noted that in the $\overline{\text{MOM}}$ scheme when calculating the three-point vertices, $p_1+p_2\rightarrow -p_3$, we perform the subtraction at the collinear momentum configuration

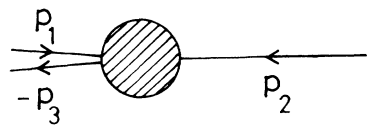

FIG. 1. The three-point vertex, $p_1+p_2\rightarrow -p_3$.

$$p_1=(0,0,0,\mu)=-p_2/2=p_3 , \tag{3.1}$$

with p_2 always attributed to a "gluon," see Fig. 1.

All the Feynman rules except the fermion propagator are completely the same as those in the zero-density (chemical potential) case, and thus are not given. The fermion propagator is[18]

$$S_{\alpha\beta}^{AB}(p)=i\delta^{AB}(\not{p})_{\alpha\beta} \times\left[\frac{1}{p^2}-2\pi i\delta(p^2)\left[\frac{\theta(p_0)}{e^{x}+1}+\frac{\theta(-p_0)}{e^{-x}+1}\right]\right] , \tag{3.2}$$

where

$$x\equiv\beta(p_0-\mu_B)=\beta p_0-\zeta,\quad \beta=1/T . \tag{3.3}$$

Note that in the fermion propagator formula (3.2) we only give the so-called physical component necessary to carry out the leading one-loop calculations (in which we are interested). Then, as is obvious from the above-mentioned facts, all contributions to Z factors without internal fermion lines are exactly the same as those in the zero-density case, which have already been calculated in I. Therefore, in this paper we only present the results coming from contributions *with* internal fermion line(s).

Our subtraction method is fully explained in I. As for the $3G$ vertex, it is worth mentioning here concretely how we determine the subtraction part. Our subtraction procedures are as follows. First, we parametrize the $3G$ vertex as

$$f^{a_1a_2a_3}(\{g_{\mu_1\mu_2}[(p_1-p_2)_{\mu_3}A(p_1^2,p_2^2,p_3^2)+q_{\mu_3}^{(12)}B(p_1^2,p_2^2,p_3^2)]+(\text{cyclic permutations})\} +\{n_{\mu_1}n_{\mu_2}[(p_1-p_2)_{\mu_3}C(p_1^2,p_2^2,p_3^2)+q_{\mu_3}^{(12)}D(p_1^2,p_2^2,p_3^2)]+(\text{cyclic permutations})\}+\cdots) , \tag{3.4}$$

where a_i and μ_i are internal and Lorentz indices of the gluon with momentum p_i ($p_i^0=0$, Fig. 1), and $q^{(12)}$ is a momentum constructed out of p_1 and p_2 so as to be orthogonal to p_1-p_2. At our collinear momentum subtraction point (3.1) $q^{(12)}$ and $q^{(23)}$ automatically vanish and also $B(p_3^2,p_1^2,p_2^2)$ vanishes. Furthermore, the terms indicated by the ellipsis in Eq. (3.4) add up to zero. Next, we subtract out at the subtraction point (3.1) the terms in the first set of curly brackets in Eq. (3.4). This is possible because at our collinear momentum configuration p_3-p_1 vanishes and $A(p_1^2,p_2^2,p_3^2)=A(p_2^2,p_3^2,p_1^2)$ and thus the terms in the first curly brackets are completely subtracted. Those terms in the second curly brackets, which are not included in the subtraction part, are the only contributions that remain after subtraction.

Finally, it is worth mentioning the fact[15,19] that the spirit of the momentum-space-subtraction schemes is to perform the subtraction by setting (or choosing) the sub-

traction point $\tilde{T}$, $\tilde{\mu}_B$, and μ to be equal to the temperature T, the chemical potential μ_B, and the energy scale Q, respectively, which characterize the process considered. The coupling $a(Q,T,\mu_B)$ thus defined is nothing but the effective or (anti-)screened charge that incorporates all the short-wavelength modes satisfying $\lambda \lesssim Q^{-1}$, and should work, on a physical ground, as a "good" expansion parameter for perturbative analysis of the process considered.

B. Effective coupling, or the ratio of scale parameters

Now we present the final result for the ratio of scale parameters $\Lambda(\xi,\zeta)$, Eq. (2.11), or the coupling renormalized at finite temperature and chemical potential $a(\mu,\xi,\zeta)$, Eq. (2.10). Let us express the ratio as

$$b\ln[\Lambda(\xi,\zeta)/\Lambda(\xi,\zeta=0)]_{\overline{\rm MOM}} = \sum_{j=0}^{2} \alpha^j[\Phi^{(j)}(\xi,\zeta)-\Phi^{(j)}(\xi,\zeta=0)] \equiv \Phi \ , \tag{3.5}$$

where α is the renormalized gauge parameter, ξ and ζ are the scaled temperature and chemical potential variables, $\xi \equiv \tilde{T}/\mu$, and $\zeta \equiv \tilde{\mu}_B/\tilde{T}$. $\Phi^{(j)}(\xi,\zeta)$ and Φ depend explicitly on the vertex through which the coupling is defined.

We give at first the exact expressions of $\Phi^{(j)}$'s for the three vertices $3G$, gG, and fG.

1. Exact expressions

(i) $3G$ vertex:

$$\Phi_{3G}^{(0)}(\xi,\zeta)=\tfrac{1}{6}T(R)[8F_2(2\xi,\zeta)+8F_0(2\xi,\zeta) -5F_2(\xi,\zeta)-5F_0(\xi,\zeta)] \ , \tag{3.6a}$$

$$\Phi_{3G}^{(1)}(\xi,\zeta)=\Phi_{3G}^{(2)}(\xi,\zeta)=0 \ . \tag{3.6b}$$

(ii) gG vertex:

$$\Phi_{gG}^{(0)}(\xi,\zeta)=\tfrac{1}{2}T(R)[F_2(\xi,\zeta)+F_0(\xi,\zeta)] \ , \tag{3.7a}$$

$$\Phi_{gG}^{(1)}(\xi,\zeta)=\Phi_{gG}^{(2)}(\xi,\zeta)=0 \ . \tag{3.7b}$$

(iii) fG vertex:

$$\Phi_{fG}^{(0)}(\xi,\zeta)=\tfrac{1}{4}C_2(G)[-F_2(2\xi,\zeta)-F_0(2\xi,\zeta)+4F_2(\xi,\zeta)+4F_0(\xi,\zeta)] -\tfrac{1}{4}C_2(R)[-F_2(2\xi,\zeta)-5F_0(2\xi,\zeta)+8F_2(\xi,\zeta)+8F_0(\xi,\zeta)+8(\pi^2/3+\zeta^2)\xi^2] +\tfrac{1}{2}T(R)[F_2(\xi,\zeta)+F_0(\xi,\zeta)] \ , \tag{3.8a}$$

$$\Phi_{fG}^{(1)}(\xi,\zeta)=\tfrac{1}{32}C_2(G)[F_2(2\xi,\zeta)-7F_0(2\xi,\zeta)-16F_2(\xi,\zeta)-16F_0(\xi,\zeta)-16G(2\xi,\zeta)] -\tfrac{1}{8}C_2(R)[F_2(2\xi,\zeta)+3F_0(2\xi,\zeta)-8F_2(\xi,\zeta)-8F_0(\xi,\zeta)+4G(2\xi,\zeta)] \ , \tag{3.8b}$$

$$\Phi_{fG}^{(2)}(\xi,\zeta)=\tfrac{1}{16}C_2(G)[F_2(2\xi,\zeta)+F_0(2\xi,\zeta)] \ . \tag{3.8c}$$

In the above expressions (3.6)–(3.8) various functions are defined as follows:

$$F_0(\xi,\zeta)\equiv\frac{1}{2}\xi\int_0^\infty dx\left[\frac{1}{e^{x+\zeta}+1}+\frac{1}{e^{x-\zeta}+1}\right]\ln\left|\frac{1+x\xi}{1-x\xi}\right| , \quad \xi>0 \ , \tag{3.9a}$$

$$F_2(\xi,\zeta)\equiv\frac{1}{2}\xi^3\int_0^\infty dx\left[\frac{x^2}{e^{x+\zeta}+1}+\frac{x^2}{e^{x-\zeta}+1}\right]\ln\left|\frac{1+x\xi}{1-x\xi}\right|-\frac{1}{2}\left[\frac{\pi^2}{3}+\zeta^2\right]\xi^2, \quad \xi>0 \ , \tag{3.9b}$$

$$G(\xi,\zeta)\equiv\frac{1}{2}\xi^2\int_0^\infty dx\left[\frac{x}{e^{x+\zeta}+1}+\frac{x}{e^{x-\zeta}+1}\right]\frac{\mathcal{P}}{x^2\xi^2-1} \ , \tag{3.9c}$$

where $\mathcal{P}$ denotes the principal part.

It is to be noted that for $3G$ and gG vertices the ratio $\Phi(\xi,\zeta)$, thus the chemical potential dependence of the coupling $a(\xi,\zeta)$, is *gauge independent.*

Next we give various limiting behavior of the ratio Φ, Eq. (3.5), in which we are interested in the actual physical situations.

2. Behavior at $\xi \simeq 0$ with ζ kept fixed

In this limit $|\tilde{\mu}_B| \sim \tilde{T} \ll \mu$. The ratio Φ corresponding to each vertex behaves as

$$\Phi_{3G} \simeq \tfrac{123}{18}T(R)(2\pi^2\zeta^2+\zeta^4)\xi^4+O(\xi^6) \ , \tag{3.10}$$

$$\Phi_{gG} \simeq \tfrac{1}{6}T(R)(2\pi^2\zeta^2+\zeta^4)\xi^4+O(\xi^6) \ , \tag{3.11}$$

and

$$\Phi_{fG} \simeq \left[[-C_2(G)+2C_2(R)+\tfrac{1}{6}T(R)] +\frac{\alpha}{3}[2C_2(G)+C_2(R)] +\frac{\alpha^2}{3}C_2(G)\right](2\pi^2\zeta^2+\zeta^4)\xi^4+O(\xi^6) \ . \tag{3.12}$$

When we set the renormalization point μ, $\tilde{T}$, and $\tilde{\mu}_B$ to be equal to Q, the momentum that characterizes the pro-

cess considered, T, the temperature of the environment, and μ_B, the chemical potential of the fermion that participates in the process, respectively, then the coupling in this limit may be suitable to describe the high-energy process, in which fermions with chemical potential μ_B participate, taking place in the thermal reservoir.

3. *Behavior in the limit* $|\zeta| \to \infty$ *with* ξ *kept fixed*

In this limit $\mu \sim \tilde{T} \ll \tilde{\mu}_B$. The behavior of Φ's is as follows:

$$\Phi_{3G} \simeq \Phi_{gG} \simeq \tfrac{2}{3} T(R) \ln|\zeta| + O(1) \tag{3.13}$$

and

$$\Phi_{fG} \simeq -2C_2(R)\xi^2\zeta^2 + \{[C_2(G) - \tfrac{4}{3}C_2(R) + \tfrac{2}{3}T(R)] - \tfrac{9}{8}\alpha[C_2(G) - \tfrac{16}{27}C_2(R)] + \tfrac{1}{12}\alpha^2 C_2(G)\}\ln|\zeta| + O(1) \tag{3.14}$$

The coupling in this limit may well describe processes in which high-density fermions participate.

It is to be noted that the two limiting procedures $\xi \to \infty$ and $|\zeta| \to \infty$ are mutually commutable, and that in the limit of both ξ and $|\zeta|$ becoming large the behavior of Φ's are again given by the above Eqs. (3.13) and (3.14).

4. *Behavior in the* $\tilde{T} \to 0$ *limit with nonzero* $\tilde{\mu}_B$

In this limit $|\zeta| \to \infty$ and $\xi \to 0$, and the functions $F_i(\xi,\zeta)$ $(i=0,2)$ and $G(\xi,\zeta)$ defined in Eqs. (3.9) smoothly approach the following compact forms:

$$F_0(\xi,\zeta) \to \tfrac{1}{2}[(1+\phi)\ln(1+\phi) + (1-\phi)\ln|1-\phi|], \tag{3.15}$$

$$F_2(\xi,\zeta) \to -\tfrac{1}{3}\phi^2 + \tfrac{1}{6}[(1+\phi^3)\ln(1+\phi) + (1-\phi^3)\ln|1-\phi|], \tag{3.16}$$

$$G(\xi,\zeta) \to \tfrac{1}{4}\ln|1-\phi^2|, \tag{3.17}$$

where $\phi \equiv |\tilde{\mu}_B|/\mu = \xi|\zeta|$. With these results we easily get the expressions of Φ's in this limit. It is worth noticing that the behavior of Φ's in the limit $\phi \to 0$ coincides with that obtained by taking the limit $\tilde{T} \to 0$ in the $\xi \simeq 0$ formula, Eqs. (3.10)–(3.12), studied in Sec. III B 2, and that those in the limit $\phi \to \infty$ also coincide with those in the limit $|\zeta| \to \infty$ and subsequently $\xi \to 0$ with $\xi|\zeta| = \phi$.

Taking this limit leads us to the zero-temperature field theory in which fermions have a finite nonzero chemical potential. Such a theory may be suited to study the effect of a nonzero baryon density in the cold environment.

C. Numerical analysis

In order to illustrate how the coupling depends on the chemical potential as well as on the temperature we present here the result of numerical analysis on the ratio of scale parameters. For brevity in this section we consider QCD with four flavors. For the SU(5) grand unified theory (GUT) in a symmetric phase there are little changes from this case, and for any other non-Abelian gauge theories essential features do survive.

At first, let us see the chemical potential dependence. In Fig. 2 we show the $\zeta(\equiv \mu_B/T)$ dependence of the ratio of scale parameters $\ln[\Lambda(\xi,\zeta)/\Lambda(\xi,\zeta=0)]$ with $\xi(\equiv T/\mu)$ kept fixed[20] at $\xi = 0.5$. Because the ratio calculated through the fG vertex depends on the choice of gauges, we give here results of two typical gauge choices, the Feynman gauge ($\alpha=0$) and the Landau gauge ($\alpha=1$). For $3G$ and gG vertices ratios are gauge independent. This figure shows the μ_B dependence of $a^{-1}(\mu,\xi,\zeta)$ to be used for processes characterized by the energy scale μ, taking place in a thermal reservoir at temperature T [see Eq. (2.10)]. At a glance we can see that the vertex dependence is significant and severe. For the $3G$ and gG vertices the ratio of scale parameters, thus the coupling, *increases* monotonically and shows a logarithmic $|\zeta|$ dependence as $|\zeta|$ becomes large, whereas it *decreases* monotonically and shows a quadratic $|\zeta|$ dependence at large $|\zeta|$ for the fG vertex irrespective of choice of gauges [see Eqs. (3.14) and (3.15)]. Figure 3 shows the $\phi(\equiv \xi|\zeta| = |\mu_B|/\mu)$ dependence of $\ln[\Lambda(\xi,\zeta)/\Lambda(\xi=\zeta=0)]$ in the limit $T \to 0$. From this figure we can see the μ_B dependence of $a(\mu, T=0, \mu_B)$ to be used for processes taking place in a cold environment. In this case al-

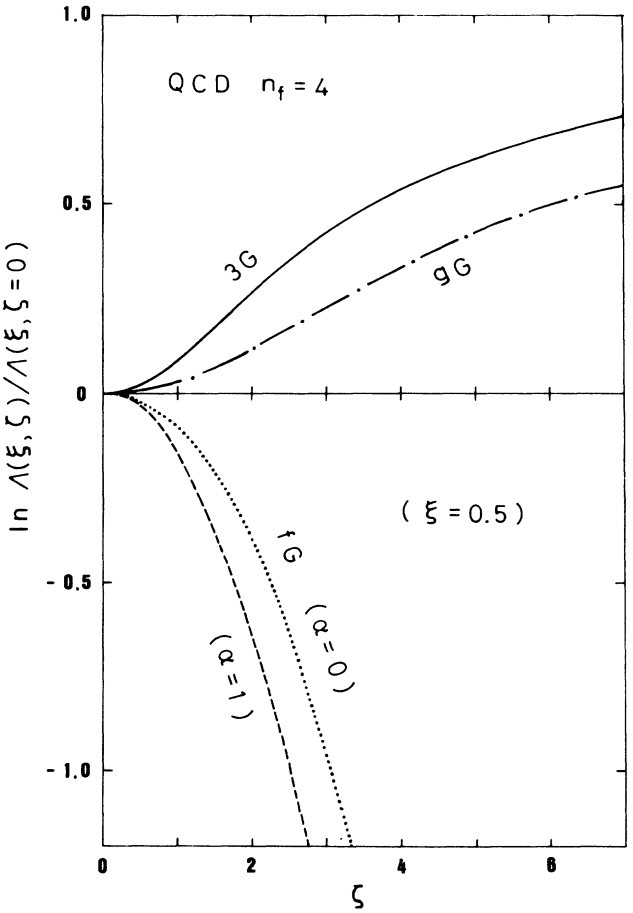

FIG. 2. The $\zeta(\equiv \mu_B/T)$ dependence of the ratio of scale parameters $\ln[\Lambda(\xi,\zeta)/\Lambda(\xi,\zeta=0)]$ with $\xi(\equiv T/\mu)$ fixed at $\xi = 0.5$.

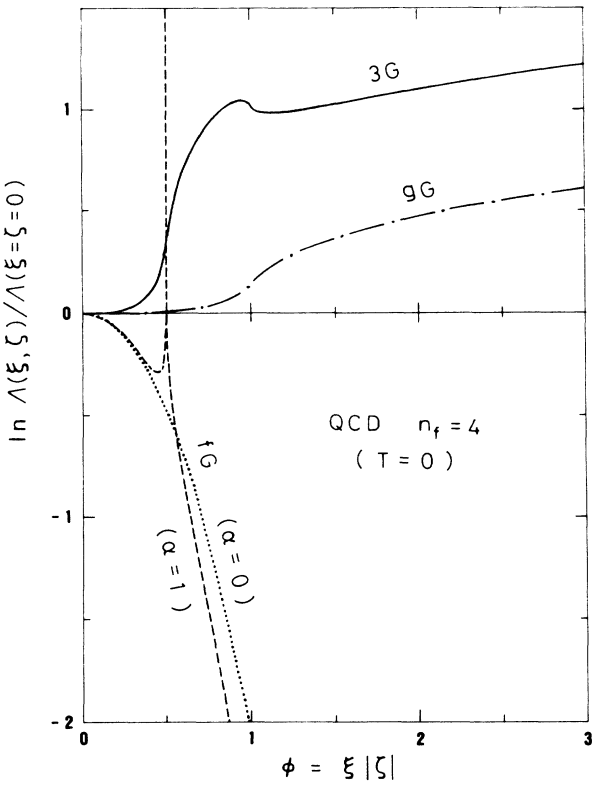

FIG. 3. The $\phi(\equiv|\mu_B|/\mu)$ dependence of the ratio of scale parameters in the limit $T\to 0$. For the fG-vertex case it shows a singular behavior at $\phi=\frac{1}{2}$ as $\ln|1-2\phi|$ in the general covariant gauge except in the Feynman gauge, see text.

though two couplings defined through the $3G$ and gG vertices asymptotically coincide ($\ln\phi$ increases at large ϕ), the coupling defined through the fG vertex again shows completely different behavior (ϕ^2 decreases at large ϕ) from the above two cases. It is to be noted that except in the Feynman gauge ($\alpha=0$) the coupling defined through the fG vertex a_{fG} has a logarithmic singularity at $\phi=\frac{1}{2}$ due to the function $G(\xi,\zeta)$ [see Eq. (3.17)]. This singularity appears only at the exact zero temperature ($T=0$), it is smeared at finite temperature, and eventually disappears as T increases. The gauge dependence of this singularity indicates that in the calculation of physical quantities the singularity of the coupling a_{fG} at $\phi=\frac{1}{2}$ should be canceled out with the should-be singularity to appear in the perturbative coefficient: namely, that the perturbative calculation near $\phi=\frac{1}{2}$ becomes almost meaningless.

From these results we understand that we *cannot* define a coupling that can absorb simultaneously the large μ_B-dependent radiative corrections to the three (i.e., $3G$, gG, and fG) vertices. This fact indicates that with any choice of coupling we always face the problem of large higher-order corrections that may invalidate the results of low-order analyses.

IV. CONCLUSIONS AND DISCUSSION

In this paper in order to study properties of gauge field theory at finite temperature and density, especially in its perturbative regime, we carried out off-shell one-loop calculations in non-Abelian gauge theories with massless fermions in the general covariant gauge. Along extensions of previous works[1,3] calculation was done in the finite-temperature generalization of the momentum-space-subtraction scheme supplemented with the "magnetic" prescription. With the results we studied the temperature and the chemical-potential dependences of the coupling $a(\mu,\xi,\zeta)$ (renormalized at finite temperature and density), in terms of which perturbative calculation is carried out. The coupling $a(\mu,\xi,\zeta)$ can be expressed in the leading one-loop approximation as

$$a(\mu,\xi,\zeta)=\frac{1}{b\ln[\mu/\Lambda(\xi,\zeta)]}\ , \tag{4.1}$$

and thus its $\xi=T/\mu$ and $\zeta=\mu_B/T$ dependences are completely characterized by the scale parameter $\Lambda(\xi,\zeta)$.

Main observations of the present paper are as follows.

(1) The chemical potential dependence of the coupling $a(\mu,\xi,\zeta)$ through the parameters $\zeta\equiv\mu_B/T$ at fixed $\xi\equiv T/\mu$ shows a severe vertex dependence. The couplings defined through the $3G$ and gG vertices show $\ln|\zeta|$ increases, while for the fG-vertex case the coupling shows a strong ζ^2 decrease.

(2) Even at $T=0$ the strong vertex dependence still survives exactly the same as above. The μ_B dependence of $a(\mu,\xi,\zeta)$ through the parameter $\phi\equiv|\mu_B|/\mu$ shows a logarithmic increase for the $3G$ and gG vertices, while it shows a quadratic decrease for the fG vertex. The coupling defined through the fG vertex, a_{fG}, develops a gauge-dependent logarithmic singularity at $\phi=\xi|\zeta|=\frac{1}{2}$, which might cause trouble in the perturbative calculation of physical quantities near $\phi\simeq\frac{1}{2}$.

From these observations we conclude that not only on the temperature dependence but also on the chemical-potential dependence the vertex dependence of the coupling is so strong and severe that we cannot define a single coupling that can absorb simultaneously the large radiative corrections to the three types of vertices that appear in a given process with definite μ, T, and μ_B. In a present renormalization prescription at the collinear momentum configuration the coupling defined through the fG vertex always shows completely different behavior from those defined through the $3G$ and gG vertices. Together with the analysis in I we conclude that with any choice of coupling we always face the problem of large higher-order corrections that may invalidate the results of low-order perturbative analyses. Namely, we cannot get reliable low-order perturbative results for processes, in which fermions with nonzero chemical potential participate, taking place in a high-temperature environment, except for the case of very low temperature and small baryon-number density.

Finally, we give a few comments on the infrared behavior at $T=0$, $\phi\equiv|\mu_B|/\mu\gg 1$ of the present result in conjunction with the "screening" of effective charge. Because we have already discussed the effect of temperature in papers I, we here mainly discuss the effect of chemical potential.

(i) The coupling $a(\mu,T=0,\mu_B)$ defined in the "magnetic" prescription behaves, as is shown in Sec. III C, in the

infrared limit $\phi \gg 1$ as

$$a^{-1}(\mu, T=0, \mu_B) \sim b[\ln(\mu/\Lambda) - \Phi \mid_{T\to 0,\ \phi=\xi\mid\zeta\mid}] , \quad (4.2)$$

$$\Phi \mid_{T\to 0,\ \phi=\xi\mid\zeta\mid} \underset{\phi\to\infty}{\sim} \begin{cases} \ln\phi \text{ for } 3G \text{ and } gG \text{ vertices} , \\ -\phi^2 \text{ for } fG \text{ vertex} . \end{cases} \quad (4.3)$$

This result (leading behavior does not depend on the choice of gauge[21]) implies that the "magnetic" charge defined through the fG vertex is screened, whereas those defined through the $3G$ and gG vertices are not (nor even show anti-"screened" behavior). "Screening" of the effective charge due to the increase of baryon-number density strongly depends on the choice of vertices through which the effective charge is defined.

(ii) To see in more detail the origin from which the screening (anti-"screening") effect comes from, we present the infrared ($\phi \gg 1$) behavior of chemical-potential-dependent corrections to each self-energy and vertex. They are

$$\delta Z_3 \sim T(R) a \ln\phi + O(1) , \quad (4.4a)$$

$$\delta Z_2 \sim -C_2(R) a \phi^2 + O(\ln\phi) , \quad (4.4b)$$

and

$$\delta Z_1 \sim T(R) a \ln\phi + O(1) , \quad (4.5a)$$

$$\delta Z_1^F \sim C_2(R) a \phi^2 + O(\ln\phi) , \quad (4.5b)$$

where we write $\delta Z = Z - 1$ with $Z = Z(\xi,\zeta) \mid_{T=0} - Z(\xi=\zeta=0)$, and a is a coupling $a \equiv g^2/4\pi^2$. There are no chemical-potential-dependent corrections to the ghost self-energy and to the ghost-gluon vertex. From these types of behavior we can see that chemical-potential-dependent fermion self-energy correction and correction to the fG vertex act to screen the effective charge, correction to the $3G$ vertex acts to screen, while gluon self-energy correction acts to anti-screen the effective charge. The infrared behavior of chemical-potential-dependent corrections to the coupling (effective charge) is

$$\delta Z_a(3G) \sim \delta Z_a(gG) \sim -T(R) a \ln\phi + O(1) , \quad (4.6a)$$

$$\delta Z_a(fG) \sim C_2(R) a \phi^2 + O(\ln\phi) , \quad (4.6b)$$

namely, the effective charge defined through the $3G$ and gG vertices is antiscreened, while that defined through the fG vertex is screened as the baryon-number density becomes high.

(iii) It is to be noted that for any self-energies and vertices, thus for any couplings, leading chemical-potential-dependent contributions in the infrared limit ($\phi \gg 1$) are in fact *gauge independent,* and thus may represent physical effects on the (anti)screening of the effective charge. Next-to-leading $O(\ln\phi)$ contributions to the fG vertex depend on the choice of gauges, while those to the $3G$ and gG vertices *do not.* (As for the temperature-dependent corrections, even the leading contributions depend on gauges, see papers I.)

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[17]By "magnetic" prescription we mean the prescription in which, when determining the subtraction part, Lorentz indices of various propagator and vertices are restricted to spatial indices. In the temporal axial gauge and in the Landau gauge this prescription allows us to single out the magnetic effects in the static limit, thus it is really the magnetic prescription. No such simple interpretation is possible in an arbitrary covariant gauge. See, e.g., Kajantie and Kapusta (Ref. 3).

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[21]For couplings defined through $3G$ and gG vertices the chemical-potential- (ζ-)dependent part is in fact totally gauge independent, see Eqs. (3.6) and (3.7).