Cosmological aspects of the heterotic string above the Hagedorn temperature

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We consider the cosmology of the heterotic string above the grand-unification scale taking into account the thermodynamical phase structure of an ideal gas of heterotic string excitations above the Hagedorn temperature. We find that the Universe expands up to a critical volume a_c^9 above which the massive (Planckian) excitations should decay into massless excitations (nonequilibrium regime). The Universe then goes on expanding in a radiation-dominated phase with a scale factor $a(t) \sim \beta(t)$. Because the heterotic string is self-dual with respect to temperature, it is argued that there is an induced duality relation for the scale factor $a(t)$.

I. INTRODUCTION

Since the late 1960s and particularly in recent years, much interest has been devoted to the thermodynamical properties of an ideal gas of string excitations. It has long been known that the canonical partition function as well as the energy density of open strings is not analytic above and at the Hagedorn temperature,[1,2] therefore making the latter point a true maximum temperature. For closed strings, this situation is somewhat improved since thermodynamical quantities remain analytic at the Hagedorn temperature, although fluctuations become unacceptably large. This has led to the suggestion that the use of the microcanonical ensemble for closed string is more appropriate.[3,4] In the context of the phenomenologically promising heterotic string, it has been shown that the equilibrium configuration for the massive excitations above the Hagedorn temperature is not thermal. The preferred configuration is for one string (neglecting winding states around some compact dimension[5]) to carry most of the energy. This very inhomogeneous energy distribution is the source of the very large fluctuations one would obtain near the Hagedorn temperature making use of the canonical ensemble. Furthermore the specific heat of the massive excitations has been shown to be negative for such a system. Note that similar physics has been obtained long ago in some particular cases of a statistical model of hadrons.[6,7]

Equilibrium between massive and massless (the heat bath) string excitations above the Hagedorn temperature can be achieved in a finite volume.[3] If the volume of the system is increased above some critical volume V_c, thermodynamical equilibrium is lost. This is a consequence of the fact that massive excitations have negative specific heat. This situation is reminiscent of the physics of black-hole evaporation. Below the Hagedorn temperature, the microcanonical and the canonical ensemble are equivalent. In a very interesting paper, O'Brien and Tan[8] have recently shown, taking advantage of modular invariance and making use of a Poisson summation formula, that the canonical heterotic string partition function is self-dual (in the sense of Kramers-Wannier duality) with respect to temperature, i.e.,

$$\ln Z(\beta, V) = \ln Z(\tilde{\beta}, V) . \tag{1.1}$$

The "dual" inverse temperature $\tilde{\beta}$ is given by

$$\tilde{\beta} = \frac{\pi}{T\beta} , \tag{1.2}$$

in which β is the inverse temperature and T is the string tension:

$$T = \frac{1}{2\pi\alpha'} . \tag{1.3}$$

A relation similar to (1.1) also occurs in the context of the two-dimensional Ising model in which the low-temperature region is mapped into the high-temperature region.[9] Since the heterotic partition function is not analytic above the Hagedorn temperature β_0^{-1}, O'Brien and Tan distinguish essentially three phases for the ideal heterotic string gas:

$$\begin{aligned} &\beta_0 \le \beta < \infty, \quad \text{I (canonical low-temperature phase)} , \\ &0 < \beta \le \frac{\pi}{T\beta_0}, \quad \text{II (dual canonical high-temperature phase)} , \\ &\frac{\pi}{T\beta_0} < \beta < \beta_0, \quad \text{III (microcanonical phase)} , \end{aligned} \tag{1.4}$$

in which β_0 is the inverse Hagedorn temperature.

The nature of the excitations "living" in the dual phase has been discussed in the literature and can be visualized from the fact that, writing the string partition function (1.1) as the (connected) contribution from the torus to the Polyakov sum over surfaces with Euclidean time (inverse temperature) compactified on the circle,[10] one gets, in addition to the usual Kaluza-Klein (KK) modes (discrete Matsubura frequencies), the winding (solitonic) states around the circle. While the KK modes are quantized in units of $1/\beta^2$ in the string Hamiltonian, the solitonic modes are quantized in units of β^2. Summation over all Matsubara frequencies and winding numbers yields the duality relation (1.1). As in the case of the two-dimensional Ising or Abelian X-Y model, one can construct "order" and "disorder" variables for the closed string. While phase I may be expressed in terms of the "order" variables at inverse temperature β, phase II is expressed in terms of the dual "disorder" variables at inverse temperature $\tilde{\beta}$. However, as discussed by Sathiapalan,[11] there is no way to distinguish physically whether one belongs to a given phase. Transforming "order" variables into "disorder" variables and vice versa can always be done through a general coordinate transformations (diffeomorphism) on the world sheet. In other words high temperature is equivalent to low temperature through the relation (1.2). Other recent studies on phase transition near the Hagedorn temperature include Kogan[12] as well as Atick and Witten.[13]

As in the Ising model, a duality relation such as (1.1) would imply the existence of a critical point at $\tilde{\beta}=\beta$:

$$\beta=\tilde{\beta}=\pi\sqrt{2\alpha'}\equiv\beta_m \ . \tag{1.5}$$

For the heterotic string, however, the critical point β_m lies outside the region of analyticity of the canonical ensemble. Nevertheless, since the thermodynamical properties of the heterotic string excitations can be analyzed through the microcanonical ensemble above the Hagedorn temperature, canonical self-duality implies that a similar analysis can be done for the properties of the dual excitations above the "dual" Hagedorn temperature $\tilde{\beta}_0^{-1}$. The discussion is completely symmetric to the case dealing with the order variables. This suggests an effective splitting in half of the microcanonical region. β_m^{-1} becomes then the true maximum temperature of the theory. The energy density as well as all other relevant thermodynamical quantities can be expressed in terms of either the original or dual excitations, since one has complete degeneracy at that point. The microcanonical region has then the structure

$$\begin{aligned} &\beta_m \le \beta < \beta_0, \quad \text{IIIa} \ , \\ &\frac{\pi}{T\beta_0} < \beta \le \beta_m, \quad \text{IIIb (dual)} \ . \end{aligned} \tag{1.6}$$

The existence of a true maximum temperature β_m^{-1} has cosmological consequences. In particular, taken as an initial condition, it implies, together with the adiabaticity assumption, the existence of a minimum radius (scale factor) for the Universe. There would be no initial singularities. Cosmological scenarios with a maximum temperature have been studied in the context of dual models of hadrons.[2] However, the maximum temperature was usually identified to the Hagedorn temperature and the analysis was carried through entirely in the region of analyticity of the canonical ensemble. More recent studies have also been made but emphasis has been given to the period following localization (zero-slope limit)[14,15]

True string cosmology has recently been investigated in a number of papers for cosmic strings as well as fundamental strings.

In the context of cosmic strings, Mitchell and Turok[16,17] have shown that the statistical properties obtained from numerical simulations of the evolution of a string network can be analytically reproduced through the statistical mechanics of the quantized closed bosonic string in four space-time dimensions (neglecting ghosts and tachyon states). Their study suggests that in an expanding universe, a string-dominated phase should be disregarded in favor of a "scaling solution"[18] where the ratio of the string density to the total density is a fixed fraction ($M_{\mathrm{GUT}}/M_{\mathrm{Pl}}\sim 10^{-3}$) with the right magnitude to account for galaxies and clusters formation. It is interesting to note the equivalence of these authors' "dual" formulation to the numerical simulations of the Higgs condensate.

Fundamental strings have also been argued[19] to play a significant role in the onset of an inflationary period of the expansion of the Universe.

Very recently, Brandenberger and Vafa[20] have studied the thermodynamics and cosmology of fully compactified closed superstring theories, arguing that one must rather explain dynamical "decompactification" of three spatial dimensions rather than the compactification of six "internal" dimensions. These authors have also discussed the physical relevance of duality relations for finite compactification radii (in spatial as well as temperature channels). However, their considerations lack dynamical equations determining the (cosmic) time dependence of the compactification radii.

Other recent articles on string cosmology include the work of Tye[21] and also Kripfganz and Perlt.[22]

In this paper, we wish to study the implications for cosmology, above the grand-unification (GUT) scale, of the thermodynamical phase structures (1.4) and (1.6) of the heterotic string. We shall assume adiabaticity and point out under which circumstances such an assumption breaks down. In the following, we choose the ten-dimensional metric tensor to describe a spherically symmetric homogeneous space-time:[23]

$$ds^2=dt^2-a^2(t)\left[d\mathbf{r}^2+\frac{k(\mathbf{r}\cdot d\mathbf{r})^2}{1-k\mathbf{r}^2}\right] , \tag{1.7}$$

in which $a(t)$ is the time-dependent scale factor, $k=-1,0,1$, and $\mathbf{r}^2\equiv\sum_{i=1}^{9}x_i^2$.

The basic equations are Einstein equations, the energy-momentum, and the entropy conservation law. The energy-momentum tensor is taken to have the perfect-fluid form in a comoving frame:

$$T_{\mu\nu}=pg_{\mu\nu}+(p+\rho)U_\mu U_\nu \ , \tag{1.8}$$

where p is the pressure, ρ is the energy density, and U_μ has the form

$$U_\mu=(1,0,\ldots,0) . \tag{1.9}$$

Both the pressure and energy density are taken to be depending on time alone. Note that the scale factor $a(t)$ in (1.7) is the same for all spatial directions. Since we restrict ourselves to energy scales above the GUT scale ($\sim 10^{15}$ GeV) and consequently prior to inflation (i.e., inflation obtained from standard scenarios), we assume that the size of the Universe is small enough to allow such a symmetric treatment. Inflation is not a solution of Einstein equations at the energy scales consider here. It has been argued by Alvarez[14] that compactification contraction of the extra dimensions should occur after localization. Consequently, inflationary scenarios based on compactification of extra coordinates should be treated within the effective low-energy limit field theory. Here we discuss the string phase prior to localization.

When solving Einstein equations, our initial conditions will be such that the initial state of the Universe is that of an ideal string gas of massive excitations in equilibrium with its radiation in a small volume at very high temperature (of the order of the Planck mass) and high-energy density.

II. THE COSMOLOGICAL MODEL

In the following, we shall make the simplifying assumption that the gravitons condense to create the metric tensor described by Eq. (1.7) with scale factor $a(t)$ and that the relevant dynamical equations are Einstein equations. Alternatively, one could consider condensation of the *dual* gravitons with scale factor $\tilde{a}(t)$. One should also perhaps take into account the string theoretic corrections to general relativity with curvature square terms in the effective action.[24] Nevertheless, we shall ignore such terms for simplicity.

From the metric tensor (1.7) it is straightforward to obtain the Ricci tensor in d dimensions. It is obtained explicitly as[23]

$$R_{00}=(d-1)\frac{\ddot{a}}{a} , \tag{2.1}$$

and

$$R_{ij}=-\frac{g_{ij}}{a^2}[a\ddot{a}+(d-2)(\dot{a}^2+k)] . \tag{2.2}$$

From the Einstein equations,

$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R=-8\pi G T_{\mu\nu} \tag{2.3}$$

and the energy-momentum tensor for a perfect fluid (1.8), one gets the equations

$$\frac{(d-1)(d-2)}{2}\left[\frac{\dot{a}^2+k}{a^2}\right]=8\pi G\rho \tag{2.4}$$

and

$$-(d-2)\left[\frac{1}{2}\left[\frac{\dot{a}^2+k}{a^2}\right](d-3)+\frac{\ddot{a}}{a}\right]=8\pi G p . \tag{2.5}$$

Combining Eqs. (2.4) and (2.5) one gets the energy-conservation law

$$\dot{\rho}+(d-1)\frac{\dot{a}}{a}(p+\rho)=0 , \tag{2.6}$$

which can also be written as

$$a^{d-1}\dot{p}=\frac{d}{dt}[a^{d-1}(\rho+p)] , \tag{2.7}$$

where a^{d-1} can be taken as the spatial volume of the Universe ($d=10$).

Denoting by S_A and S_B the entropy of the Universe in phases I and IIIa, respectively, one has

$$S_A(a^9,\beta)=\beta a^9[\rho(\beta)+p(\beta)] , \tag{2.8}$$

and

$$S_B(a^9,\beta)=\beta a^9[\rho_r(\beta)+p_r(\beta)]+\ln\Omega(E_s,a^9) , \tag{2.9}$$

in which ρ and p stand for the total energy density and pressure while ρ_r and p_r denote the contribution from the massless modes (radiation) of the heterotic string. The second term in Eq. (2.9) is the entropy of the massive excitations in phase IIIa.

Allowing the scale factor and the temperature to become time dependent one can show that the adiabaticity condition

$$\dot{S}_{A,B}(a^9(t),\beta(t))=0 \tag{2.10}$$

for the evolution of the Universe is equivalent to the energy-conservation law (2.6) or (2.7). Equations (2.4) and (2.8)–(2.10) are the basic equations of our cosmological analysis.

In the next section we will write down explicitly the energy density and the pressure in each of the phases of the heterotic string described in the Introduction.

III. COMPUTATION OF THERMODYNAMICAL QUANTITIES

In this section we proceed to determine explicitly the behavior of the energy density and the pressure of the heterotic string excitations as functions of the inverse temperature β and the volume factor a^{d-1}. Because of duality, we need only consider the cases of phases I and IIIa. The corresponding behavior for the dual excitations can be obtained readily by substituting $\tilde{\beta}$ for β and $\tilde{a}$ for a. From now on, we take the Regge slope $\alpha'=\frac{1}{2}M_{\rm Pl}^{-2}$, where $M_{\rm Pl}$ is the Planck mass.

A. Phase I

In the low-temperature phase I, the canonical partition function for the heterotic string is analytic. Such a partition function has been derived several times in the literature. Here we simply state known results.

Near the Hagedorn temperature β_0^{-1}, the logarithm of the partition function can be expressed as[3]

$$\ln Z(\beta)\simeq\frac{a^9M_{\rm Pl}^{-9}}{(2\pi\beta)^{9/2}}\alpha'^{-9/2}(\beta-\beta_0)^{\alpha-11/2} \times\Gamma(-\alpha+\tfrac{11}{2},\eta(\beta-\beta_0)) \quad (\beta\to\beta_0) , \tag{3.1}$$

where $\Gamma(a,b)$ is the incomplete γ function. The parameters α and β_0 take on the values

$$\alpha=10, \quad \beta_0(1+\sqrt{2})\pi M_{\rm Pl}^{-1} , \tag{3.2}$$

while the usual infrared cutoff η is of the order of the Planck mass. Consequently, the massless modes of the heterotic string are absent from (3.1). We shall treat them separately later on.

Expanding the incomplete γ function in (3.1) about $(\beta-\beta_0)$, one gets

$$\ln Z(\beta)\simeq \frac{a^9 M_{\rm Pl}^{-9}}{(2\pi\beta)^{9/2}} e^{-\eta(\beta-\beta_0)} \times \left[\frac{1}{\eta^{\alpha-11/2}(\alpha-\frac{11}{2})} - \frac{(\beta-\beta_0)}{\eta^{\alpha-13/2}(\alpha-\frac{11}{2})(\alpha-\frac{13}{2})} + \cdots \right] . \tag{3.3}$$

From (3.3) it is straightforward to obtain the energy density ρ_s and pressure p_s of the massive string excitations by making use the thermodynamical relations,

$$\rho_s=\frac{M_{\rm Pl}^9}{a^9}\frac{\partial}{\partial\beta}\ln Z(\beta,a^9) \tag{3.4}$$

and

$$p_s=\beta^{-1}\frac{M_{\rm Pl}^9 \partial}{\partial a^9}\ln Z(\beta,a^9) . \tag{3.5}$$

The results are

$$\rho_s \simeq \frac{\alpha'^{-9/2}}{(2\pi\beta)^{9/2}} e^{-\eta(\beta-\beta_0)} \frac{1}{\eta^{\alpha-13/2}(\alpha-\frac{13}{2})} \quad (\beta\to\beta_0) \tag{3.6}$$

and

$$p_s \simeq \frac{\alpha'^{-9/2}}{(2\pi)^{9/2}} \beta^{-11/2} e^{-\eta(\beta-\beta_0)} \frac{1}{\eta^{\alpha-11/2}(\alpha-\frac{11}{2})} \quad (\beta\to\beta_0) . \tag{3.7}$$

Furthermore, in ten dimensions, the energy density ρ_r and pressure p_r of an ideal gas of massless heterotic string excitations are given by

$$\rho_r=\sigma\beta^{-10}, \quad p_r=\frac{\rho_r}{9} , \tag{3.8}$$

where the proportionality constant σ is given as[3]

$$\sigma=\frac{8\pi^5}{3465}\left[n_b + \left[1-\frac{1}{2^9} \right] n_f \right] , \tag{3.9}$$

with $n_0=n_f=4032$ for the massless modes of the heterotic string. Consequently,

$$\sigma\simeq 5.7\times 10^4 . \tag{3.10}$$

The total energy density and pressure are now given as

$$\rho=\rho_r+\rho_s, \quad p=p_r+p_s . \tag{3.11}$$

One can compare the energy density and pressure of matter (massive modes) and radiation (massless modes) near the Hagedorn temperature. Although we must caution that the size of energy fluctuations becomes rather large near that point, it can be used nevertheless to give a rough idea of the order of magnitude involved. Defining

$$\rho_{0s,r}\equiv\rho_{s,r}(\beta_0), \quad p_{0s,r}\equiv p_{s,r}(\beta_0) , \tag{3.12}$$

one gets

$$\frac{\rho_{0s}}{\rho_{0r}}=\frac{2\pi}{7\sigma}[(1+\sqrt{2})\pi]^{11/2}\sim 1 \tag{3.13}$$

and

$$\frac{p_{0s}}{p_{0r}}=\frac{7}{(1+\sqrt{2})\pi}\frac{\rho_{0s}}{\rho_{0r}}\sim 1 . \tag{3.14}$$

Therefore, as one approaches the Hagedorn temperature from below, matter and radiation contribute equally to the total energy density and pressure. However, as one goes to lower temperatures, the Universe soon becomes radiation dominated because of the exponential decrease of matter contribution in Eqs. (3.6) and (3.7).

B. Phase IIIa

The region described by the phase IIIa lies outside the region of analyticity of the canonical partition function. Making use of a microcanonical ensemble analysis, it has been shown that the microcanonical density of massive states $\Omega(E_s,V)$ has the same functional form as the density of states in mass space for closed strings, thereby meeting the so-called bootstrap condition. Therefore one has[3,6,7]

$$\Omega(E_s,V)\simeq VE_s^{-\alpha}\exp(\beta_0 E_s) \tag{3.15}$$

with the parameters α and β_0 given by Eq. (3.2). The density of states (3.15) is valid for energy densities higher than the critical density ρ_{0s} in Eq. (3.12). Making use of the thermodynamical relations

$$\beta=\frac{\partial \ln\Omega(E_s,V)}{\partial E_s}, \quad p_s=\frac{\partial}{\partial V}\frac{\ln\Omega(E_s,V)}{\beta} , \tag{3.16}$$

one obtains the following expressions for the energy density and pressure of the massive string excitations:

$$\rho_s=\frac{\alpha M_{\rm Pl}^9}{(\beta_0-\beta)\alpha^9}, \quad p_s=\frac{M_{\rm Pl}^9}{\beta a^9} , \tag{3.17}$$

in which we replaced the volume factor V by $a^9 M_{\rm Pl}^{-9}$. The contribution to the energy density and pressure from the massless excitations is the same as in phase I and is given explicitly by Eq. (3.8). An important offshoot of (3.17) is that the massive modes have negative microcanonical specific heat above the Hagedorn temperature. Consequently, one can have equilibrium between massive and massless modes only in a finite volume. The condition for equilibrium between massive and massless modes has been given by Bowick and Wijewardhana.[3] In terms of the volume of the system the condition is stated as

$$a^9 < a_c^9 = \frac{\beta_c^{10}}{\sigma}\left[E_c - \frac{\alpha}{\beta_0-\beta_c}\right] M_{\rm Pl}^9 , \tag{3.18}$$

in which E_c is the total energy of the system at inverse temperature β_c. This critical temperature is related to E_c through the formula

$$\beta_0 E_c = \alpha\left[1+\frac{11}{10}\frac{\beta_c}{(\beta_0-\beta_c)}+\frac{1}{10}\frac{\beta_c^2}{(\beta_0-\beta_c)^2}\right] . \tag{3.19}$$

For a volume larger than the critical volume a_c^9, thermodynamical equilibrium breaks down.

In the next section, we will take a closer look at the adiabaticity condition.

IV. THE ADIABATICITY CONDITION

The adiabaticity assumption for the evolution of the Universe has been formulated through Eqs. (2.8)–(2.10).

If we denote by S_A and S_B the initial entropy content of phases I and IIIa, respectively, then inserting the corresponding energy density and pressure into Eqs. (2.8) and (2.9), one finds an expression relating the scale factor to the temperature in each phase.

In the canonical phase-I region, such an expression take a rather simple form for a temperature lower than the Hagedorn temperature, when matter contribution is effectively damped down and the Universe becomes radiation dominated. Insertion of Eq. (3.8) into (2.8) leads to

$$a(t) \simeq \left[\frac{9S_A}{10\sigma}\right]^{1/9} \beta(t) M_{\rm Pl} . \tag{4.1}$$

In the microcanonical phase-IIIa region, the situation is somewhat less trivial. Insertion of (3.8) and (3.17) into (2.9) yields the transcendental equation

$$a^9 = \exp\left[S_B' - \alpha \ln(\beta_0-\beta) M_{\rm Pl} - \frac{\alpha\beta_0}{\beta_0-\beta} - \frac{10\sigma\beta^{-9}a^9}{9} M_{\rm Pl}^{-9}\right] , \tag{4.2}$$

where S_B' differs from S_B by an irrelevant constant term. It is easy to show that there is a maximum volume $\bar{a}^9$ at inverse temperature $\bar{\beta}$ satisfying the relation

$$\bar{a}^9 = \frac{\alpha\bar{\beta}^{11} M_{\rm Pl}^9}{10\sigma(\beta_0-\bar{\beta})^2} . \tag{4.3}$$

Further comparison with Eqs. (3.18) and (3.19) yields

$$\bar{\beta} = \beta_c, \quad \bar{a}^9 = a_c^9 , \tag{4.4}$$

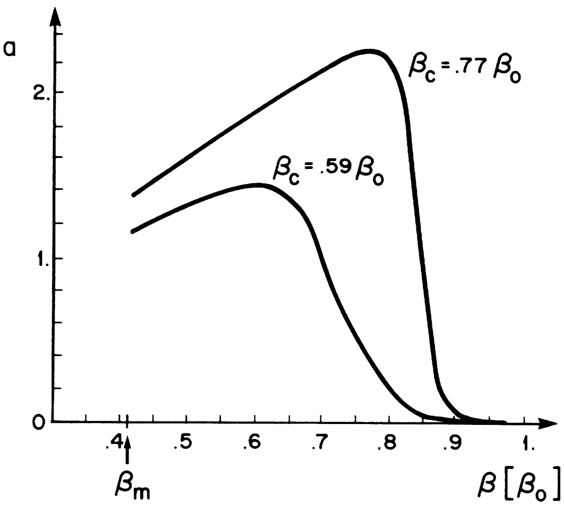

FIG. 1. Behavior of the scale factor a as a function of the inverse temperature β for two different values of the critical temperature β_c^{-1}.

as expected. It is now possible, making use of Eqs. (4.2)–(4.4) to express the entropy S_B' as a function of β_c alone. One then obtains a one-parameter family of curves $a(\beta;\beta_c)$. Figure 1 displays numerical solutions to (4.2) for two different values of the critical temperature β_c^{-1}.

In the next section, we will analyze the solutions to Einstein equations (2.4) for various choices of initial conditions at temperatures above the Hagedorn temperature.

V. INITIAL CONDITIONS AND EXPANDING SOLUTIONS

At initial time, we shall assume that the Universe was described by a mixture of matter and radiation in thermodynamical equilibrium at a temperature above the Hagedorn temperature.

In phase IIIa, the formal solution to Einstein equation (2.4) is given by

$$t = \int \frac{da\, a^4}{\left[\frac{2\pi G}{9}\left[\sigma\beta^{-9}a^{-9} + \frac{\alpha\beta M_p^9}{\beta_0-\beta}\right]\frac{a}{\beta} - k a^8\right]^{1/2}} , \tag{5.1}$$

in which we used expressions (3.8) and (3.17) for the energy density of the massless and massive excitations, respectively. However, as is readily seen from Fig. 1, $\beta(a)$ is not single valued and one must specify the integration path in temperature space. Converting (5.1) into an integration over the inverse temperature β, one gets

$$t = \int_{\beta(0)}^{\beta(t)} \frac{d\beta}{\beta} \frac{a^5\left[10\sigma a^9\beta^{-9}M_{\rm Pl}^{-9} - \frac{\alpha\beta^2}{(\beta_0-\beta)^2}\right]}{(10\sigma a^9\beta^{-9}M_{\rm Pl}^{-9}+9)\left[\frac{2\pi G}{9}\left[\sigma\beta^{-9}a^9 + \frac{\alpha\beta M_{\rm Pl}^9}{\beta_0-\beta}\right]\frac{a}{\beta} - k a^8\right]^{1/2}} , \tag{5.2}$$

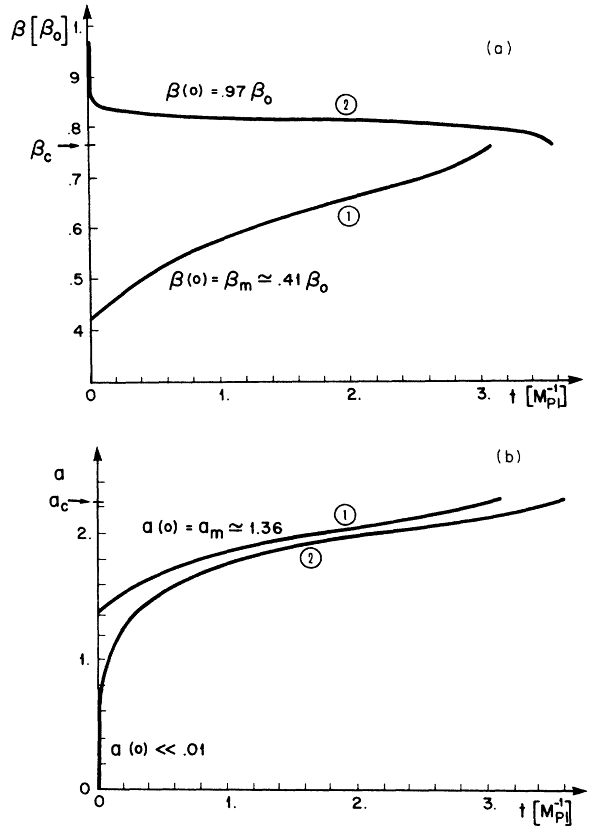

FIG. 2. (a) Behavior of the inverse temperature β as a function of time for two different initial conditions $\beta(0)$. Note that $\beta_c = 0.77\beta_0$ and $k=0$. (b) Behavior of the scale factor a as a function of time for two different initial conditions $a(0)$ [corresponding to the initial conditions $\beta(0)$ on (a)]. $a_c = 2.24$ and $k=0$.

in which $a(\beta)$ is a single-valued function of temperature. Figure 2 displays numerical solutions to Eq. (5.2).

Clearly, the results indicate that for initial inverse temperature $\beta(0) \neq \beta_c$, the Universe expands adiabatically up to the critical volume a_c^9. If the initial temperature is such that $\beta(0) < \beta_c$, expansion is accompanied with cooling. On the other hand, if $\beta_c < \beta < \beta_0$, the Universe actually becomes hotter as it expands toward a_c^9. That this can be possible should not be too surprising if one recalls that the massive string excitations have negative microcanonical specific heat in that region.

For volumes larger than a_c^9, the present model does not provide answers. Rather, thermal equilibrium between massive and massless modes is lost above the latter critical value. A complete discussion would necessarily involve the effects of the string interactions. One could then argue that the massive modes decay into radiation and that, subsequently, a radiation-dominated universe in thermal equilibrium finally emerges. In such a situation, the relation between the scale factor and the temperature takes the simple form described by Eq. (4.1). Insertion of (4.1) and the relation (3.8) for the energy density of radiation into Eq. (2.4) yields the following solution for large time ($k=0$):

$$\beta(t) \sim t^{1/5} . \tag{5.3}$$

Recalling (4.1) one gets

$$a(t) \sim t^{1/5} . \tag{5.4}$$

At later times, standard cosmological scenarios may proceed with possible symmetry-breaking mechanisms giving rise to mass generation for some of the massless heterotic string excitations (perhaps through contraction—compactification of the internal coordinates).

VI. CONCLUSION

In this paper we discussed an evolutionary scenario based on the condition that the Universe was initially filled by an ideal gas of heterotic string excitations in ten dimensions, with thermodynamical equilibrium between massive and massless modes at a temperature above the Hagedorn temperature.

We then showed that irrespective of the initial value of the temperature, the volume expands and actually reaches a critical value above which a nonequilibrium regime sets in. Taking into account the string interaction, the massive modes should decay into radiation. When the new equilibrium regime is achieved, the early Universe keeps expanding in a radiation-dominated phase with the power law:

$$a(t) \sim t^{1/5} . \tag{6.1}$$

It is interesting to note that the duality relation (1.2) for temperature induces a duality relation for the scale factor:

$$\tilde{a} = \tilde{a}(a) . \tag{6.2}$$

In radiation-dominated periods of the expanding Universe, one has usually

$$a \sim \beta . \tag{6.3}$$

Correspondingly, in the dual phase, one should have

$$\tilde{a} \sim \tilde{\beta} . \tag{6.4}$$

Equation (1.2) then implies

$$\tilde{a} \sim \frac{1}{a} . \tag{6.5}$$

Although the above duality relation for the scale factor (radius) of the Universe is an induced one in the present context, such a relation naturally occurs if one truly compactifies from the start all the heterotic string coordinates.[25] Cosmological studies follow a different route in that case (see Brandenberger and Vafa[20]). Our choice (1.7) for the metric mimics this compactification of space. It is therefore not too surprising to obtain a duality relation in the present context.

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