Changing coupling "constants" and violation of the equivalence principle

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It is shown that a cosmological evolution of coupling "constants" due to a change in some dilaton field (such as the size of the extra dimensions in Kaluza-Klein theories or superstring theories), if observably large, would imply a long-range force of gravitational strength that would violate the equivalence principle. Our analysis is in the context of superstring theories but should be more general. We also clarify various obscurities and correct mistakes in the recent literature.

I. INTRODUCTION

In recent years there has been much interest in theories with extra "compactified" space dimensions.[1,2] Several papers[3,4] have explored the possibility of a time variation in the size of these dimensions which would be reflected in a time variation of the fundamental "constants." In the last year or so this idea has been carried over to the superstring context.[5–7] Our main point in this paper is that there is a problem with this idea: namely, if the relevant dilaton field associated with the expansion and contraction of the extra dimensions is sufficiently light to allow observable cosmological variation of couplings then it will mediate a long-range gravity strength force that violates the equivalence principle. We discuss this problem in the superstring context because that is of most current interest, but we suspect similar conclusions would apply in general higher-dimensional theories such as Kaluza-Klein theories, and indeed even more generally. A second purpose of our paper, carried out mostly in Appendixes, is to clear up some confusion surrounding recent discussions in the literature. These have to do with differences among various papers in choices of metric (through relative conformal rescalings) that affect the dependence of various couplings on the fields of theory, differences in notation, and differences in physical assumptions. There are also a few errors and oversights that we correct.

II. SUPERSTRING THEORIES

The starting point of our discussion, as of most discussions of this subject, is the bosonic action in the ten-dimensional field-theory limit of superstrings. We assume such a limit makes sense. This action has been written down by many authors:[7]

$$S=\int d^{10}x\sqrt{-g^{(10)}}\left[\frac{1}{2{\kappa_{10}}^2}R-\tfrac{3}{4}{\kappa_{10}}^2\phi^{-3/2}{H_{MNP}}^2-\frac{9}{16}\frac{1}{{\kappa_{10}}^2}(\phi^{-1}\partial_M\phi)^2 -\tfrac{1}{4}\phi^{-3/4}[\tfrac{1}{30}\mathrm{Tr}{F_{MN}}^2-({R_{MNPQ}}^2-4{R_{MN}}^2+R^2)]\right] . \tag{1}$$

F_{MN} is the gauge-field-strength tensor, H_{MNP} is the field strength of the two-index antisymmetric tensor field, and ϕ is a dilaton field. ${\kappa_{10}}^2$ is the ten-dimensional Planck constant. A feature of superstring theories that makes the discussion more complicated than in the Kaluza-Klein case is that there are *two* dilaton fields whose values affect the "fundamental" couplings: ϕ and the size of the extra dimensions. If there is to be a significant time variation of constants some linear combination of these two dilatons must remain nearly massless. But *which* linear combination of them is most plausibly assumed to be light is a matter of conjecture upon which papers have differed.

A convenient and conventional choice of metric in compactifying to four extended space-time dimensions is[7]

$$g_{MN}^{(10)}=\mathrm{diag}(e^{-3\sigma}g_{\mu\nu}^{(4)},e^{\sigma}g_{IJ}^{(6)}) . \tag{2}$$

As is discussed in Appendix A it is possible to choose definitions of the four-metric $g_{\mu\nu}$ that differ by conformal rescalings from that chosen here—indeed Refs. 5 and 6 choose definitions that differ from Eq. (2) and from each other. This is equivalent to choosing definitions of G_N (the four-dimensional Newton's "constant") which, in fact, have different functional dependences on the fields of the theory. These issues are straightened out in Appendix A. For our discussion we will adhere to Eq. (2). With this choice the four-dimensional action takes the form

$$S=K\int d^4x\sqrt{-g^{(4)}}\{\tfrac{1}{2}R^{(4)}-\tfrac{3}{4}\phi^{-3/2}e^{6\sigma}H_{\mu\nu\rho}{}^2-\tfrac{9}{16}(\partial_\mu\phi/\phi)^2-3(\partial_\mu\sigma)^2 -\tfrac{1}{4}\phi^{-3/4}e^{3\sigma}[\tfrac{1}{30}\mathrm{Tr}F_{\mu\nu}{}^2-(R^2\text{ terms})]+\text{other}\}\ . \tag{3}$$

Here we only write those terms that interest us. $K\equiv\int d^6y\sqrt{g^{(6)}}$ and we normalize $g_{IJ}{}^{(6)}$ so that this is one (y^I are the six extra dimensions). Also we have set $\kappa_{10}=1$. From the coefficient of the F^2 term (the factor $\frac{1}{30}$ is due to the trace being in the adjoint representation) we see that $1/g^2_{\rm GUT}=\phi^{-3/4}e^{3\sigma}$. This combination of parameters is usually called $s\equiv\mathrm{Re}S$. The orthogonal combination (note the form of the kinetic terms) is $\phi^{3/4}e^{\sigma}$ and is usually called $t\equiv\mathrm{Re}T$. The variables $\phi_S\equiv(1/\sqrt{2})\ln s$ and $\phi_T\equiv\sqrt{3/2}\ln t$ are also commonly used. In terms of these the kinetic terms for ϕ and σ in Eq. (3) can be rewritten as $-\frac{1}{2}(\partial_\mu\phi_S)^2-\frac{1}{2}(\partial_\mu\phi_T)^2$. Now, it is likely that both ϕ_S and ϕ_T develop vacuum expectation values (VEV's) at the Planck scale (or at least at some superheavy masses); however, until superstring dynamics are fully understood one cannot be sure. In the scenario where there is an E_8' hidden sector in which gaugino condensation occurs to break supersymmetry, it has been argued[8] that indeed ϕ_S develops a superheavy mass and expectation value. Certainly, that the gauge couplings are not very different from one constitutes experimental evidence that $\langle s\rangle\sim M_{\rm Pl}$. The story about the field ϕ_T, or t, is much less clear. At the tree level and even after gaugino condensation breaks supersymmetry its potential remains flat. One-loop calculations in the four-dimensional effective theory[9] have given a nonflat potential for this field. It has been argued also that at the actual potential minimum the theory must be strongly interacting[10] so that a perturbative calculation of the shape of the potential cannot be trusted.

Since the potential for ϕ_T seems less well understood, and since it remains flat even after hidden sector gaugino condensation gives mass to ϕ_S, it is perhaps likely that if *any* combination of ϕ and σ has a flat (or nearly flat) potential it is ϕ_T. That is the assumption we will make for specificity (see Appendix B where the varying assumptions in the literature on this point are compared). In any event it will be seen that our main point does not depend qualitatively on this assumption.

Now, as Maeda has noted,[6] in the four-dimensional action displayed in Eq. (3) the field ϕ_T is a free field which does not couple to light matter. This is seen by rewriting Eq. (3) as

$$S=\int d^4x\sqrt{-g^{(4)}}\{\tfrac{1}{2}R^{(4)}-\tfrac{3}{4}e^{2\sqrt{2}\phi_S}H_{\mu\nu\rho}{}^2-\tfrac{1}{2}(\partial_\mu\phi_T)^2-\tfrac{1}{2}(\partial_\mu\phi_S)^2-\tfrac{1}{4}e^{\sqrt{2}\phi_S}[\tfrac{1}{30}\mathrm{Tr}F_{\mu\nu}{}^2-(R^2\text{ terms})]+\text{other}\}\ . \tag{4}$$

Indeed it is the coupling of ϕ_S to light (i.e., subcompactification-scale mass) matter, in particular gauginos, that can naturally provide a mechanism for giving ϕ_S a potential. If we assume that ϕ_T is the (nearly) massless dilaton then presumably it is the field whose expectation value can vary significantly over cosmological time scales. At first sight Eq. (4) suggests that this variation will have no effect on the values of the couplings. (This is the conclusion actually drawn in Ref. 6 and is the reason the analysis there proceeded to consider only the effects of time variations in ϕ_S.) However, this conclusion is too hasty. The unification scale *does* depend on ϕ_T. Consider the gauge fields, denote them A_μ, which become heavy through compactification (through the flux-line breaking mechanism). The kinetic energy term and mass terms both come from the 10-dimensional kinetic term

$$\begin{aligned}\int d^{10}x\sqrt{-g^{(10)}}[-\tfrac{1}{4}\phi^{-3/4}(\partial_M A_\mu)(\partial_{M'}A_{\mu'})g^{MM'}g^{\mu\mu'}+\cdots]\\ =K\int d^4x\sqrt{-g^{(4)}}\,e^{-3\sigma}[-\tfrac{1}{4}\phi^{-3/4}(\partial_\nu A_\mu\partial^\nu A^\mu e^{6\sigma}+\partial_I A_\mu\partial^I A^\mu e^{2\sigma})+\cdots]\\ =\int d^4x\sqrt{-g^{(4)}}\,s[-\tfrac{1}{4}(\partial_\nu A_\mu\partial^\nu A^\mu+e^{-4\sigma}\lambda^2A_\mu A^\mu)+\cdots]\ .\end{aligned} \tag{5}$$

Here we have used the facts that $\sqrt{-g^{(10)}}=\sqrt{-g^{(4)}}\sqrt{g^{(6)}}e^{-3\sigma}$, $g^{(10)\mu\mu'}=e^{3\sigma}g^{(4)\mu\mu''}$, $g^{(10)II'}=e^{-\sigma}g^{(6)II'}$, and λ^2 is an eigenvalue of the Laplacian on the compact space. So the unification scale which is just the mass of these gauge bosons is given by

$$M^2_{\rm GUT}\sim e^{-4\sigma}M^2_{\rm Pl}=(s^{-1}t^{-1})M^2_{\rm Pl} =e^{-\sqrt{2}\phi_S-\sqrt{2/3}\phi_T}M^2_{\rm Pl}\ . \tag{6}$$

This will affect low-energy physics in various ways. Suppose $\alpha_{\rm GUT}(M_{\rm GUT})$ is assumed to be fixed by the expectation value of s and is independent of ϕ_T. Then to first approximation the effect of shifting $\ln M_{\rm GUT}$ by a certain amount will be to shift the renormalization-group trajectory of α_s versus $\ln\mu$ by the same amount. Then we would find roughly that m_p, $\Lambda_{\rm QCD}$, and $M_{\rm GUT}$ all depend on s and t as $(s^{-1/2}t^{-1/2})$. Things are more complicated since other effects also will depend on ϕ_T, in particular the masses of various particles which will affect the running of the gauge couplings.

In addition to $M_{\rm GUT}$ one expects also that the supersymmetry-breaking scale will depend on ϕ_T. If we consult calculations of the supersymmetry-breaking scale in papers[11] which assume a hidden sector gaugino condensation mechanism we find that $m_{3/2}$

$=s^{-1/2}t^{-3/2}\langle W(s)\rangle$ where $W(s)$ is the part of the superpotential that depends on s. $m_{3/2}$ is the gravitino mass. In any event $m_{3/2}$ will certainly depend on t as well as s. At present there is no reliable standard picture of how the breaking of SU(2)×U(1) occurs in superstring models, though it is probably closely connected to the breaking of supersymmetry. Thus, it is probable that G_F depends on t (through $m_{3/2}$) but we cannot assert at present any precise form of this dependence even if we know how $m_{3/2}$ depends on t. Finally, the masses of the quarks and leptons depend on G_F since they are proportional to SU(2)×U(1) breaking. However, again, we do not have a reliable theory of these fermion masses. If, say, e, u, and d get mass through radiative effects they could have a much different dependence on t than if they arise at the tree level.

For the purposes of this paper we will parametrize the dependence of the low-energy physics "constants" on t as follows:

$$\begin{aligned} &G_N \sim t^0 = \text{const} , \\ &m_p \sim t^a , \\ &G_F \sim t^b , \\ &m_i \sim t^{c_i}, \quad i=e,u,d,\ldots , \\ &\Delta m \equiv m_n - m_p \sim t^d . \end{aligned} \tag{7}$$

[These "constants" need not depend on t exactly as a power; but as we will be dealing with small variations of t from its present value, t_0, we can simply linearize with $\delta m_p/m_p^{(0)} = a\delta t/t_0 + O(\delta t^2)$, etc., and then the forms in Eq. (7) involve no assumption as long as $O(\delta t^2)$ can be neglected.] From the simplistic discussion which gave us $m_p \sim t^{-1/2}$ and $m_{3/2} \sim t^{-3/2}$ we may reasonably guess that a, b, c_i, and d are likely to be numbers of order unity.

At low energy we then expect to have terms in the Lagrangian such as

$$L_{\text{mass}} = m_p^{(0)}(t/t_0)^a(\bar{p}p) + m_n^{(0)}(t/t_0)^{a'}(\bar{n}n) + m_e^{(0)}(t/t_0)^{c_e}(\bar{e}e) + \cdots . \tag{8}$$

Or expressing this in terms of ϕ_T and linearizing:

$$L_{\text{mass}} \sim m_p^{(0)}[1 + a\sqrt{2/3}(\phi_T - \phi_{T_0})]\bar{p}p + m_n^{(0)}\left[1 + \left[a + \frac{\Delta m^{(0)}}{m_n^{(0)}}(d-a)\right]\sqrt{2/3}(\phi_T - \phi_{T_0})\right]\bar{n}n + m_e^{(0)}[1 + c_e\sqrt{2/3}(\phi_T - \phi_{T_0})]\bar{e}e , \tag{9}$$

where we have used the dependence on t of m_p and Δm to get that of m_n. This equation will be important later when we discuss violations of the equivalence principle.

From Eqs. (4) and (9) we have the following equation for $(\phi_T - \phi_{T_0})$:

$$\Box\phi_T + \mu^2(\phi_T - \phi_{T_0}) \simeq \sqrt{2/3}\left[a\rho_p + \left[a + \frac{\Delta m}{m}(d-a)\right]\rho_n + c_e\rho_e\right] , \tag{10}$$

where ρ_i is the energy density in the species i. We can ignore the $\Box\phi$ term at the present epoch compared to the $\mu^2\phi$ term. Suppose we can also neglect the $\Box\phi$ term at the time of helium synthesis (this would be the case, for example, if ϕ_T sits at its potential minimum then; we shall return to this point below). Then, comparing ϕ_T at an early time τ with ϕ_T now we have roughly

$$\phi_T(\tau) - \phi_{T_0} = \sqrt{2/3}\frac{1}{M_{\text{Pl}}^2\mu^2}\left[a\Delta\rho_p + \left[a + \frac{\Delta m}{m}(d-a)\right]\Delta\rho_n + c_e\Delta\rho_e\right] . \tag{11}$$

But since $\Delta\rho_i = \rho_i(\tau) - \rho_i(\text{now}) \simeq \rho_i(\tau)$ for τ an early time, and since m_e/m_p and $\Delta m/m_p$ are small, we have

$$\phi_T(\tau) - \phi_{T_0} \simeq (\tfrac{2}{3})^{1/2}\frac{1}{M_{\text{Pl}}^2\mu^2}a\rho_{\text{nucleon}}(\tau) . \tag{12}$$

We have replaced the factors of M_{Pl} which earlier we set to unity. We may now use Eqs. (8), (9), and (12) to place limits on μ from nucleosynthesis, and other astrophysical arguments.

Let us look first at the primordial ^{4}He abundance. [We give a somewhat oversimplified account of the effects of changing coupling on ^{4}He abundance. To do a better job one must include a variety of effects whose importance was first emphasized by Barrow[4] but which we do not consider here, notably the effects from changes in the strengths of the nuclear forces, such as on the $p+n \to {}^2\text{H}+\gamma$ rate, and the effect of a change in the n lifetime. To estimate the change in the nuclear forces is somewhat tricky since the shape (depth and radius) of the nucleon-nucleon potential well, say, depends indirectly and in a complicated way on the QCD gauge coupling α_s and the u- and d-quark masses. Since tracing this dependence is complicated and our main results will in any case depend only on quite qualitative and crude features of the calculation we content ourselves with the level of the analysis of Ref. 3. We emphasize again, however, that to do a more realistic calculation requires combining the analysis of Ref. 4 with a computation of the dependence of nucleon-nucleon forces on α_s, m_u, and m_d.] The primordial ^{4}He abundance is known to be

$$Y=0.24\pm0.01 \tag{13}$$

and is given by

$$Y=\frac{2}{1+(p/n)_f}, \quad \left[\frac{p}{n}\right]_f=e^{+\Delta m/T_f}, \tag{14}$$

where $(p/n)_f$ is the proton to neutron ratio at freeze-out. The freeze-out temperature T_f is found by equating the cosmic expansion rate and the weak-interaction rate

$$G_F^2T_f^5\sim(G_NT_f^4)^{1/2} . \tag{15}$$

Linearizing around the current values of G_F, G_N, and Δ_m, and denoting by Y_0 the value of Y that would be obtained if these parameters did not evolve in time we have

$$\begin{aligned}\frac{\delta Y}{Y_0}&=\delta(\ln Y)_0\\ &=-\frac{\delta(p/n)_f}{1+(p/n)_f}\bigg|_0\\ &=\frac{(p/n)_f}{1+(p/n)_f}\bigg|_0\,\delta\left[\frac{\Delta m}{T_f}\right]_0\\ &=(1-Y_0/2)\left[\frac{\Delta m}{T_f}\right]\left[\frac{\delta\Delta m}{\Delta m}-\frac{\delta T_f}{T_f}\right],\\ \frac{\delta Y}{Y_0}&=(1-Y_0/2)\ln\left[\frac{2-Y_0}{Y_0}\right]\left[\frac{\delta\Delta m}{\Delta m}-\frac{\delta T_f}{T_f}\right].\end{aligned} \tag{16}$$

Now using Eq. (15) we have

$$2\frac{\delta G_F}{G_F}+3\frac{\delta T_f}{T_f}=\frac{1}{2}\frac{\delta G_N}{G_N} . \tag{17}$$

But we have found that δG_N is zero (that is, G_N is independent of t) while $\delta G_F/G_F=b\delta t/t_0$, and $\delta\Delta m/\Delta m=d(\delta t/t_0)$. So

$$\frac{\delta Y}{Y_0}=(1-Y_0/2)\ln\left[\frac{2-Y}{Y}\right]\left[d+\frac{2}{3}b\right]\left[\frac{\delta t}{t_0}\right] . \tag{18}$$

Thus, if we assume that $\delta Y/Y_0<0.04$ we find

$$(\delta t/t_0)<(0.024)(d+\tfrac{2}{3}b)^{-1} . \tag{19}$$

Now, $(\delta t/t_0)=(\frac{2}{3})^{1/2}(\phi_T-\phi_{T_0})$; so from Eq. (12) we have the variation $\delta t/t_0$ from the helium synthesis era to now has been

$$\begin{aligned}\delta t/t_0&\simeq\frac{2}{3}\frac{1}{M_{\rm Pl}^2\mu^2}a\rho_{\rm nucleon}(T_f)\\ &\simeq\frac{2}{3}\frac{1}{M_{\rm Pl}^2\mu^2}a\left[\left[\frac{n_B}{n_\gamma}\right]m_p\frac{2\zeta(3)}{\pi^2}T_f^3\right]\\ &\simeq a\mu^{-2}(4\times10^{-59})\ {\rm GeV}^2 .\end{aligned} \tag{20}$$

In order, then, to get a $\delta t/t_0$ large enough to be "seen" through its effect on ^{4}He abundance one would need to have according to Eqs. (19) and (20):

$$\mu^{-2}\gtrsim\frac{(d+\frac{2}{3}b)^{-1}}{a}(4\times10^{-29}\ {\rm GeV})^{-2} . \tag{21}$$

This corresponds to a range for the interaction mediated by the $(\phi_T-\phi_{T0})$ field [see Eqs. (9) and (10)] of

$$r\gtrsim\left[\frac{(d+\frac{2}{3}b)^{-1}}{a}\right]^{1/2}(5\times10^9\ {\rm km}) . \tag{22}$$

The consequences of much smaller variations $\delta t/t_0$ than 10^{-2} [see Eq. (19)] could be seen in stellar nucleosynthesis of ^{12}C and ^{16}O, and in the Okla natural reactor.[12] However these probe the behavior of the couplings at times much later than nucleosynthesis when t is changing much more slowly anyway [see Eqs. (12) and (20)]. Thus for our purposes ^{4}He abundance may be the most sensitive test. Let us however be wildly optimistic and assume that in the foreseeable future a $\delta t/t_0$ variation since nucleosynthesis times of even 10^{-9} will be observable. To get a variation even of that size would require

$$\mu^{-2}\gtrsim\frac{1}{a}(2\times10^{-25}\ {\rm GeV})^{-2} , \tag{23}$$

$$r\gtrsim a^{-1/2}10^6\ {\rm km} . \tag{24}$$

Any μ larger than about 10^{-25} GeV will, then, lead to unobservably small variations in the couplings even under very optimistic assumptions. [And we note here that for $\mu\gtrsim10^{-25}$ GeV it will be true that we can ignore $\Box\phi_T$ in Eq. (10) for *any* initial condition; so that our analysis will be valid.] So, an observable variation of couplings implies a μ less than 10^{-25} GeV corresponding to a force of range much larger than the radius of the Earth. If we now look back at Eq. (9) we see that such a force would violate the equivalence principle. In Eq. (9) we set $M_{\rm Pl}$ to unity. Restoring the factor of $M_{\rm Pl}^{-1}$ we see that ϕ_T mediates a force whose strength is comparable to that of gravitation if a and c_e are of order one. But clearly ϕ_T couples differently to p, n, and e^-, in general. In fact the ratio for the ϕ_T force between the Earth and an atom of proton and neutron number (Z,N) and the gravitational force is

$$F_{\phi_T}/F_{\rm grav}\sim a^2\left[1+\frac{\Delta m}{M_{\rm atom}}N\frac{d-a}{a}+\frac{m_e}{M_{\rm atom}}Z\frac{c_e-a}{a}\right] . \tag{25}$$

We expect in the superstring case that a, b, c_i , and d will be of order unity. Equation (25) then tells us that there will be a new long-range scalar force as strong as gravity. This is obviously unacceptable. But even if we assume that a, b, c_i, and d are small compared to unity, which corresponds [Eq. (7)] to a rather weak dependence of low-energy couplings on the value of the dilaton, there will be significant violations of the equivalence principle according to Eq. (25).

The main assumption that has gone into our argument is that for small variations $\delta t/t$ one may ignore the higher-order terms such as $(\delta t/t)^2$ in the equations of motion. That is, we have kept only the mass term of ϕ_T and not higher-order derivatives of the potential at its minimum. However ϕ_T would have to have a strange shape indeed for this assumption to be invalid.

It would seem that our argument applies to Kaluza-Klein theories as well, and indeed to any theory where the cosmological evolution of constants occurs because of a variation in the value of a scalar field which is caused by the changing matter density in the Universe.

Whether or not one can find appealing theories to which our argument does not apply, it is interesting that long-range feeble forces that violate the equivalence principle,[13] which have aroused so much recent interest, are intimately connected to the evolution of coupling constants. Clearly our argument can be reversed. If a long-range scalar force that couples to matter exists it corresponds to a very soft potential, and changes in the cosmological matter density will probably lead to shifts in the value of the masses of the particles to which this force couples. Searches for changing couplings and for new feeble forces should be viewed as complementary aspects of the same investigation.

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APPENDIX A

(1) One of the confusing aspects of the whole subject of changing "constants" is that how these constants depend on the fields of the theory depends on how they are defined, and in particular how the metric is defined. Let us suppose that with a metric g in a $(d+1)$-dimensional theory one has an action

$$S=\int d^{d+1}x\sqrt{-g}\left[\frac{1}{2\kappa}R+\cdots\right].$$

Here the Newton "constant" is $G=\kappa/8\pi$. One can rescale the metric by a conformal transformation $g_{\mu\nu}=e^{2\lambda(x)}g'_{\mu\nu}$ where $\lambda(x)$ is some scalar field. Then one finds

$$S=\int d^{d+1}x\sqrt{-g'}\left[\frac{e^{(d-1)\lambda(x)}}{2\kappa}R+\cdots\right],$$

so that the Newton "constant" for this metric is given by $G'_N=e^{-(d-1)\lambda(x)}G_N$. One has to be careful therefore about speaking in absolute terms about how G_N or the other couplings of the theory depend on the fields of the theory or on time. In particular various papers in the literature make general statements such as $(G_N/G_N^0)=(R/R_0)^{-D}$ in Kaluza-Klein theories. Such statements are strictly speaking wrong, or at least only right in a certain definition of the metric. The purpose of this appendix is to clarify this problem and show how certain recent papers' usages are related.

What *can* be given an absolute sense is the value of certain dimensionless physical combinations[14] of parameters at a definite space-time point. For example, suppose there are particles of a certain mass m. One can imagine that a pair of such particles is gravitationally bound in a certain orbit which has definite quantum numbers. The gravitational binding energy as a fraction of m is expressible in terms of $(G_N m^2)$. This is an observable quantity which is invariant under any rescaling of the metric, i.e., conformal change of variables. Thus it is physically unambiguous to ask about the difference between $G_N m^2$ at two definite space-time points. (To ask how it depends on the time coordinate "t" is ambiguous because one can rescale the coordinate t.) Later in this appendix we shall show the consistency of various authors' choice of variables by checking that $G_N M_{\rm GUT}^2$ has the same dependence on the fields of the theory for each choice.

(2) Let us start with a particular form of the ten-dimensional action

$$S=\int d^{10}x\sqrt{-G}\; e^{-2\Phi}\beta_\Phi\;, \tag{A1}$$

where

$$\beta_\Phi=\tfrac{1}{2}R(G)+2(\nabla_\mu\Phi)^2-2\nabla_\mu^2\Phi-\tfrac{1}{4}{\rm tr}F_{\mu\nu}{}^2+\cdots .$$

We are using the notation of Ref. 6. This differs from the form in Eq. (1) of the text in that the coefficients here of $\frac{1}{2}R$ and $-\frac{1}{4}{\rm tr}F_{\mu\nu}^2$ are both $e^{-2\Phi}$ while there the corresponding coefficients are 1 and $\phi^{-3/4}$. To get the form in Eq. (1) from Eq. (A1) one has to do a rescaling of the metric $G_{MN}=e^{1/2\Phi}g_{MN}^{(10)}=\phi^{3/4}g_{MN}^{(10)}$. Thus $\Phi=\frac{3}{2}\ln\phi$. $g_{MN}^{(10)}$ is then the metric appearing implicitly in Eq. (1) [that is, it is used to make the contractions in Eq. (1)]. The usual choice of metric, given in Eq. (2), when compactifying down to four dimensions is

$$g_{MN}^{(10)}={\rm diag}(e^{-3\sigma}g_{\mu\nu}^{(4)},e^{\sigma}g_{IJ}^{(6)})\;. \tag{A2}$$

Thus, to get from Eq. (A1) to the usual four-dimensional form requires that

$$G_{MN}={\rm diag}(\phi^{3/4}e^{-3\sigma}g_{\mu\nu}^{(4)},\phi^{3/4}e^{\sigma}g_{IJ}^{(6)})$$

$$\equiv{\rm diag}\left[\frac{1}{s}g_{\mu\nu}^{(4)},tg_{IJ}^{(6)}\right], \tag{A3}$$

where s and t are the commonly used variables discussed in the text. We can check this explicitly. Let us take $G_{MN}^{(m+n)}={\rm diag}(e^{2\lambda(x)}g_{\mu\nu}^{(m)},e^{2\mu(x)}g_{IJ}^{(n)})$ where we are generalizing to an $(m+n)$-dimensional space-time. Let $\lambda(x)$ and $\mu(x)$ depend only on $x^\mu,\mu=1,\ldots,m$. Then

$$R(G)=e^{-2\lambda}[R^{(m)}(g^{(m)})-2(m-1)\lambda_{;\rho}{}^{\rho}-(m-2)(m-1)\lambda_{;\rho}\lambda^{;\rho}-2n\mu_{;\rho}{}^{\rho}-n(n+1)\mu_{,\rho}\mu^{,\rho}-2n(m-2)\lambda_{,\rho}\mu^{,\rho}]+\cdots . \tag{A4}$$

Combining Eqs. (A4), (A3), and (A1) we find

$$S=\int d^4x\sqrt{-g^{(4)}}(s^{-2}t^3)(st^{-3})$$
$$\times(\tfrac{1}{2}s^{+1}R(g^{(4)})-\tfrac{1}{2}s\{6(-\tfrac{1}{2}\ln s)_{;\rho}{}^{\rho}-6[(-\tfrac{1}{2}\ln s)_{,\rho}]^2-12(\tfrac{1}{2}\ln t)_{;\rho}{}^{\rho}-42[(\tfrac{1}{2}\ln t)_{,\rho}]^2$$
$$-24(-\tfrac{1}{2}\ln s)_{,\rho}(\tfrac{1}{2}\ln t)^{,\rho}\}+2s[\nabla_\mu\tfrac{1}{2}\ln(s^{-1}t^3)]^2-2s\nabla_\mu^2[\tfrac{1}{2}\ln(s^{-1}t^3)]-\tfrac{1}{4}s^2\mathrm{tr}F_{\mu\nu}{}^2)+\cdots \tag{A5}$$

or

$$S=\int d^4x\sqrt{-g^{(4)}}\left\{\tfrac{1}{2}R(g^{(4)})-\left[\frac{-5}{2}\frac{\nabla_\mu^2 s}{s}+\frac{11}{4}\left[\frac{\nabla_\mu s}{s}\right]^2+6\frac{\nabla_\mu^2 t}{t}-\frac{21}{4}\left[\frac{\nabla_\mu t}{t}\right]^2\right]-\tfrac{1}{4}s\,\mathrm{tr}F_{\mu\nu}{}^2\right\}+\cdots . \tag{A6}$$

We have used $\Phi=\frac{3}{2}\ln\phi=\frac{1}{2}\ln[(1/s)t^3]$ and $\sqrt{G}=s^{-2}t^3\sqrt{g^{(4)}}\sqrt{g^{(6)}}$. Now integrating by parts we find

$$S=\int d^4x\sqrt{-g^{(4)}}\left[\tfrac{1}{2}R(g^{(4)})-\frac{1}{4}\left[\frac{\nabla_\mu s}{s}\right]^2-\frac{3}{4}\left[\frac{\nabla_\mu t}{t}\right]^2-\tfrac{1}{4}s\,\mathrm{tr}F_{\mu\nu}{}^2\right]+\cdots$$
$$=\int d^4x\sqrt{-g^{(4)}}[\tfrac{1}{2}R(g^{(4)})-\tfrac{1}{2}(\nabla_\mu\phi_S)^2-\tfrac{1}{2}(\nabla_\mu\phi_T)^2-\tfrac{1}{4}e^{\sqrt{2}\phi_S}\mathrm{tr}(F_{\mu\nu})^2]+\cdots \tag{A7}$$

which is to be compared with Eq. (4) of the text.

(3) Now, the authors of Refs. 5 and 6 do not in fact use the same definition of the four-dimensional metric as in Eq. (A3) and that we have used in the text. Maeda [see Eq. (3) of Ref. 6] uses $G_{MN}=\mathrm{diag}(g_{\mu\nu}^{(4)},R^2g_{IJ}^{(6)})$. Thus his R is given by

$$R=t^{1/2} . \tag{A8}$$

And Maeda's metric in four dimensions is related to ours, which we will denote simply as $g_{\mu\nu}^{(4)}$, by

$$g_{\mu\nu}^{(4)}(\text{Maeda})=\frac{1}{s}g_{\mu\nu}^{(4)} . \tag{A9}$$

If we look at Eq. (10) of Ref. 5 we see that Wu and Wang use $g_{MN}^{(10)}=(g_{\mu\nu}^{(4)},R_6^2\tilde{g}_{mn})$. So we have

$$g_{\mu\nu}^{(4)}(\text{Wu and Wang})=e^{-3\sigma}g_{\mu\nu}^{(4)}$$
$$=(st)^{-3/4}g_{\mu\nu}^{(4)} \tag{A10}$$

and their variable R_6 is given by

$$R_6=e^{1/2\sigma}=(st)^{1/8} . \tag{A11}$$

Now let us see how G_N and $(g_4)^{-2}$ depend on s and t in our variables, in those of Maeda, and in Wu and Wang's. In our variables

$$G_N\sim s^0t^0\sim\text{const},\quad (g_4)^{-2}\sim s^1 , \tag{A12}$$

as can be seen from Eq. (3) of the test, or Eq. (A6). To get to the other definitions requires just a conformal rescaling of $g_{\mu\nu}^{(4)}$. From Eq. (A9) we have

$$G_N(\text{Maeda})\sim s^{-1},\quad (g_4)^{-2}(\text{Maeda})\sim s^1 . \tag{A13}$$

[This is to be compared with Eqs. (4) and (5) of Ref. 6. It should be noted that Maeda uses the symbol ϕ for what we call s. Also his Eq. (5) has a mistake as g_4^{+2} should go as $1/s$ or in his notation $1/\phi$ not $1/\phi^2$.] From Eq. (A10) we have

$$G_N(\text{Wu and Wang})\sim s^{-3/4}t^{-3/4} ,$$
$$(g_4)^{-2}(\text{Wu and Wang})\sim s . \tag{A14}$$

To calculate M_{GUT}^2 in the various metrics we must look at the coefficients of the terms $(\partial_I A_\mu)^2$. In particular we have from the text M_{GUT}^2 in our metric given by

$$M_{\text{GUT}}^2\sim s^{-1}t^{-1} . \tag{A15}$$

While

$$M_{\text{GUT}}^2(\text{Maeda})\sim R^{-2}\sim t^{-1} ,$$
$$M_{\text{GUT}}^2(\text{Wu and Wang})\sim R_6^{-2}\sim e^{-\sigma}\sim s^{-1/4}t^{-1/4} . \tag{A16}$$

We can combine all these results in a table.

Note that $G_N M_{\text{GUT}}^2$ which should be invariant under rescalings of the metric indeed comes out to be proportional to $s^{-1}t^{-1}$ in all metrics. [We can check that we agree with Wu and Wang. They say that $G_N\sim R_6^{-6}$. But $R_6^{-6}=e^{-3\sigma}=s^{-3/4}t^{-3/4}$ which agrees with Table I. And, according to their Eq. (23), $(g_4)^{-2}\sim\phi^{-3/4}R_6^6$. But $\phi^{-3/4}R_6^6=\phi^{-3/4}e^{3\sigma}=s$ which again agrees with Table I.]

TABLE I. A comparison of notation, conventions, and definitions of various papers on this subject. Maeda refers to Ref. 6; Wu and Wang to Ref. 5.

		This paper	Maeda	Wu and Wang
Variables used		$s \equiv \phi^{-3/4} e^{3\sigma}$	"ϕ"$(=s)$	ϕ
		$t \equiv \phi^{3/4} e^{\sigma}$	$R^2(=t)$	$R_6^2(=e^{\sigma})$
	or	$\phi_S = \frac{1}{\sqrt{2}} \ln s$		
		$\phi_T = (\frac{3}{2})^{1/2} \ln t$		
Four-metric		$g_{\mu\nu}^{(4)}$	$\frac{1}{s} g_{\mu\nu}^{(4)}$	$(st)^{-3/4} g_{\mu\nu}^{(4)}$
G_N		$\sim$const	$\sim s^{-1}$	$\sim s^{-3/4} t^{-3/4}$
$(g_4)^{-2}$		$\sim s$	$\sim s$	$\sim s$
$M_{\rm GUT}^2$		$\sim s^{-1} t^{-1}$	$\sim t^{-1}$	$\sim s^{-1/4} t^{-1/4}$
$(G_N M_{\rm GUT}^2)$		$\sim s^{-1} t^{-1}$	$\sim s^{-1} t^{-1}$	$\sim s^{-1} t^{-1}$

APPENDIX B

Not only do the choice of metric and the notations vary among papers but they make different assumptions about which fields remain fixed and which evolve cosmologically. Maeda, noting that G_N and $(g_4)^{-2}$ depend only on s (in his metric) decided that interesting evolution of "constants" would only result if s were allowed to evolve (or equivalently ϕ_S). (In his notation he denotes s by ϕ.)

In the text we argue that the *low energy* "constants" *do* depend on t as well as s, and that $\langle s \rangle$ is likely to be fixed by the supersymmetry-breaking mechanism in such as gaugino condensation while t is not. Thus we regard t (or equivalently ϕ_T) as being the most likely to have a (nearly) flat potential and hence to evolve. Wu and Wang fix ϕ and assume that what they call R_6 has a (nearly) flat potential and evolves. This is equivalent to saying $(\sqrt{3}\,\phi_T - \phi_S)$ is fixed (massive) while the orthogonal combination $(\phi_T + \sqrt{3}\,\phi_S)$ has a (nearly) flat potential and evolves. So the three papers we are comparing make three different assumptions about what the flat direction is in the (ϕ_S, ϕ_T) plane.

From the point of view of the main qualitative conclusion of the text *which* linear combination of ϕ_S and ϕ_T evolves is not very significant. We assume ϕ_T is light and examine the equation [see Eq. (10) of the text]

$$\ddot{\phi}_T + 3H\dot{\phi}_T + \mu^2(\phi_T - \phi_{T_0}) = (\sqrt{\tfrac{2}{3}}\,p)\, e^{-\sqrt{2/3}\,p(\phi_T - \phi_{T_0})} \rho \,.$$

Maeda assumes ϕ_S is light and examines

$$\ddot{\phi}_S + 3H\dot{\phi}_S + \mu^2(\phi_S - \phi_{S_0}) = \frac{1}{\sqrt{2}} e^{-\sqrt{2}\,\phi_S} \rho \,.$$

An intermediate choice (such as Wu and Wang's) would give a very similar equation.

APPENDIX C

If the dilaton responsible for the evolving couplings is exactly massless then there are two problems: an infinite range force and the instability of the classical solutions for the dilaton. To take a concrete example of the later problem one can consider the solutions found in Ref. 5. In that paper (see Appendix B) it was assumed that ϕ developed a mass and remained constant while R_6 had a flat potential. The full equations of motion in $d=10$ are just the Einstein equations. (Many papers derive the four-dimensional equations of motion from a four-dimensional action derived by substituting an ansatz for the metric into the ten-dimensional action. One thereby gets the same equations for the dilaton, to the order in $M_{\rm Pl}^{-2}$ that one cares about, as one would if one deduced the four-dimensional equations of motion from the full ten-dimensional ones.) For a matter-dominated Universe the equations are [see Eqs. (14)–(16) of Ref. 5]

$$\ddot{R}_3/R_3 + 2\ddot{R}_6/R_6 = -\frac{7}{24}\frac{\kappa_{10}^2 \rho_0}{R_6}[R_3(t_0)/R_3(t)]^3 \,,$$

$$\frac{2k}{R_3^2} + \ddot{R}_3/R_3 + 2\dot{R}_3^2/R_3^2 + 6(\dot{R}_3/R_3)(\dot{R}_6/R_6) = \frac{1}{8}\frac{\kappa_{10}^2 \rho_0}{R_6^6}[R_3(t_0)/R_3(t)]^3 \,, \qquad \text{(C1)}$$

$$\frac{\ddot{R}_6}{R_6} + 5\dot{R}_6^2/R_6^2 + 3(\dot{R}_3/R_3)(\dot{R}_6/R_6) = \frac{1}{8}\frac{\kappa_{10}^2 \rho_0}{R_6^6}[R_3(t_0)/R_3(t)]^3 \,.$$

Let us assume that $\{\bar{R}_6(t),\bar{R}_3(t)\}$ is an *exact* solution of these equations. Let us perturb them as

$$R_3(t)=\bar{R}_3(t)[1+a(t)] ,\quad R_6(t)=\bar{R}_6(t)[1+b(t)] , \tag{C2}$$

and assume that $R_3(t)$ and $R_6(t)$ are also exact solutions. We can subtract the equations with $\{\bar{R}_3,\bar{R}_6\}$ from those with $\{R_3,R_6\}$ to get equations for the perturbations a and b. If we assume that the fields are changing on a time scale $\bar{\tau}$ much shorter than the age of the Universe t_0, then we can drop all terms nonleading in $\bar{\tau}/t_0$. One finds, from Eq. (C1),

$$\begin{aligned} &\ddot{a}/(1+a)+2\ddot{b}/(1+b)=0 ,\\ &\ddot{a}+2\dot{a}^2/(1+a)+6\dot{a}\dot{b}/(1+b)=0 ,\\ &\ddot{b}+5\dot{b}^2/(1+b)+3\dot{b}\dot{a}/(1+a)=0 . \end{aligned} \tag{C3}$$

The right-hand sides of Eqs. (C3) are simply nonleading in $(\bar{\tau}/t_0)$ and thus set to zero. Now, let us define $y\equiv\dot{b}/(1+b)$ and $w\equiv\dot{a}/(1+a)$. In terms of y and w Eqs. (C3) become

$$\begin{aligned} &2\dot{y}+2y^2+\dot{w}+w^2=0 ,\\ &\dot{w}+3w^2+6yw=0 ,\\ &\dot{y}+6y^2+3yw=0 . \end{aligned} \tag{C4}$$

Let us expand in powers of $\tau=t-t_0$:

$$w=\frac{1}{\bar{\tau}}\sum_{n=0}^{\infty}\alpha_n(\tau/\bar{\tau})^n ,\quad y=\frac{1}{\bar{\tau}}\sum_{n=0}^{\infty}\beta_n(\tau/\bar{\tau})^n . \tag{C5}$$

Setting equal powers of $(\tau/\bar{\tau})$ equal in Eq. (C4) one gets

$$\begin{aligned} &\alpha_1+2\beta_1=-\alpha_0^2-2\beta_0^2 ,\\ &\alpha_1=-3\alpha_0^2-6\alpha_0\beta_0 ,\\ &\beta_1=-6\beta_0^2-3\alpha_0\beta_0 , \end{aligned} \tag{C6}$$

which in turn give

$$\begin{aligned} &(\alpha_0+\beta_0)(\alpha_0+5\beta_0)=0 ,\\ &\therefore \beta_0=-\alpha_0 ,\\ &\text{or } \beta_0=-\tfrac{1}{5}\alpha_0 . \end{aligned} \tag{C7}$$

Taking the first case one can proceed to solve for all the other coefficients α_n, β_n. One finds

$$y=-w . \tag{C8}$$

This in turn gives

$$\dot{w}=3w^2,\quad w=-\frac{1}{3(\tau+c)}=-y , \tag{C9}$$

which implies, by the definitions of w and y,

$$a=\left[1+\frac{\tau}{c}\right]^{-1/3}-1,\quad b=\left[1+\frac{\tau}{c}\right]^{+1/3}-1 , \tag{C10}$$

where we have imposed $a(t_0)=b(t_0)=0$.

Substituting back into Eq. (C2) one has

$$R_3(t)=\bar{R}_3(t)\left[1-\frac{t-t_0}{|c|}\right]^{-1/3} ,\quad R_6(t)=\bar{R}_6(t)\left[1-\frac{t-t_0}{|c|}\right]^{+1/3} . \tag{C11}$$

These solutions start out equal to $\bar{R}_3$, $\bar{R}_6$ at $t=t_0$, but at time $t=t_0+|c|$ we find R_6 has collapsed to zero and R_3 has blown up to infinity. c is a constant of integration that is arbitrary but which we have taken negative, and of order $\bar{\tau}$.

The second solution in Eq. (C7) is $\beta_0=-\frac{1}{5}\alpha_0$. This yields the solution

$$y=-\tfrac{1}{5}w \tag{C12}$$

and this in turn gives, with $a(t_0)=b(t_0)=0$,

$$a=(1+\tau/c)^{5/9}-1 ,\quad b=(1+\tau/c)^{-1/9}-1 . \tag{C13}$$

Substituting back into Eq. (2),

$$R_3(t)=\bar{R}_3(t)\left[1-\frac{t-t_0}{|c|}\right]^{5/9} ,\quad R_6(t)=\bar{R}_6(t)\left[1-\frac{t-t_0}{|c|}\right]^{-1/9} . \tag{C14}$$

Again we see an instability, except here R_6 blows up after some finite time and R_3 collapses to zero.

We emphasize that these solutions are exact up to corrections of order $\bar{\tau}/t_0$.

According to Refs. 5 and 15 (which follow the usual stability analysis of Ref. 16) there are solutions for $k=-1$ that are stable under perturbations. Why the different result here? The key point is that the usual stability analysis linearizes with respect to perturbations a and b. This is all right if a and b remain small. If one follows the linear stability analysis in this case one finds restoring forces for a of order a/t^2 and $\dot{a}/t$, and similarly for b. [See Eq. (18) of Ref. 15.] Now, suppose as we have done that perturbations a and b grow rapidly compared to the expansion rate of the Universe. That is, let $\dot{a}$ and $\dot{b}$ be of order $1/\bar{\tau}$, where $\bar{\tau}\ll t$. Then the time it takes a and b to grow to be of order one is of order $\bar{\tau}$. But the restoring forces have a negligible effect in a time that short. Thus long before the restoring forces have had a chance to act significantly the linear analysis already has broken down. The correct thing to do for perturbations where $\bar{\tau}\ll t$ is to drop higher powers in $\bar{\tau}/t$ but *not* to linearize with respect to a and b. That is what we have done.

What we have found is that the nonlinearities in fact accelerate the growth of the perturbations so that $a(t)$ blows up after a time $c \sim \bar{\tau}$. The restoring forces revealed by the linear analysis are irrelevant for $\bar{\tau} \ll t$; it is the nonlinearities that play the dominant role.

Why does k not appear in Eq. (C3)? Simply because those terms are higher order in $\bar{\tau}/t_0$.

The upshot is that there is stability with respect to slowly growing perturbations ($\dot{a} \sim \dot{b} \sim \bar{\tau}^{-1} < t^{-1}$), as shown in Ref. 15. But rapid enough perturbations are unstable. It is a valid question whether such modes would in fact get excited.

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