Astrophysical evidence for a weak new force?

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(Received 21 March 1988)

Discrepancies between measurements of and theoretical predictions for the orbital precession in binary-star systems are reexamined assuming the existence of a hitherto undiscovered, very weak long-range force. The binary-star data are consistent with the existence of such a force only if the internal density parameter k_2, computed using stellar models, is uncertain to 80% for some of the stars involved. If so, the observations are compatible with a repulsive force that couples to electrically neutral bulk matter through a linear combination of neutron and proton number with 10^{-5}–10^{-4} the strength of gravity and a range of $(3-6)\times10^6$ km. Surprisingly, such a force is consistent with the binary pulsar and extraterrestrial solar-system tests of general relativity. It is ruled out only by very recent tests of the principle of equivalence on Earth. Such binary-star systems are extremely sensitive to, and so furnish strong constraints on, new forces.

I. INTRODUCTION

The results of measurements of the relativistic periastron shift in eclipsing binary-star systems have become available over the last few years,[1–3] opening a new experimental window for gravitational physics.[4] Perhaps the most interesting feature of these measurements is that, of the handful of systems measured to date, two (DI Her and AS Cam) appear to be in significant disagreement with theoretical expectations,[1,2] with experiment falling *short* of the predicted value. Ironically, the discrepancy is clearest for DI Her, a system that was identified thirty years ago as potentially being the cleanest for the purposes of testing general relativity[5] (GR).

In the present paper we wish to explore the possibility that what is being seen is a signal of new fundamental physics. In particular we adopt a phenomenological approach and ask whether the observed discrepancies can be accommodated by a new, long-ranged interaction that couples coherently to matter in bulk. This involves no loss of generality since the systems involved are of astrophysical dimensions, and so can be understood in terms of a low-energy effective theory in which all but the very long-wavelength modes are "integrated out." *Any* microscopic theory of Planck-scale physics which aspires to account for these measurements must then do so in terms of a very light Bose degree of freedom whose exchange can produce such a force.

We wish to learn two things in this analysis: (i) whether the binary-star measurements presently available are consistent with the systematics of a new force, and, if so, (ii) whether the properties required of such a force in order to account for the discrepancies are consistent with the other tests available concerning the accuracy of general relativity. In a nutshell our conclusions are (i) the measurements are only consistent with what is expected with a new force provided that the errors in computing the Newtonian contribution are about 80% and (ii) the properties of the force necessary to account for the binaries are consistent with the other bounds on new forces coming from the binary pulsar and extraterrestrial solar-system measurements. The force can, however, be ruled out by two recent searches for a dependence on chemical composition in the gravitational acceleration of two bodies.

The fact that only very recent terrestrial experiments conflict with the forces necessary to account for the binary-star anomalies illustrates how surprisingly sensitive these binary systems are to forces much weaker than gravity. [A simple reason for why this is so is given below—see the discussion following Eq. (9).] Once the discrepancy is properly understood, these binary systems potentially furnish limits on new forces whose sensitivity is comparable to more conventional Earth-bound tests. This is of some interest in itself given the scarcity of precision tests outside of the solar system.

We organize our results as follows: Sec. II describes the binary systems and their properties, Sec. III gives their interpretation in terms of a hypothetical new force, and Sec. IV summarizes the confrontation of the resulting force with the other tests of general relativity. Our conclusions are presented in Sec. V.

II. BINARY-STAR MEASUREMENTS AND CLASSICAL THEORY

For convenience the binary-star data[1–3] are listed in Tables I and II. The final columns of these tables contain the corresponding results, where appropriate, for Mercury[6] and the binary pulsar[4,7] 1913 + 16. The following remarks should be kept in mind.

(i) The special feature of binary-star systems is that their stellar and orbital parameters can be directly inferred from plots of light intensity and radial velocity (Doppler shift) against time. They are found by fitting a model of the eclipses to the observed light curves.[8] The errors quoted in the tables represent a standard deviation in these fits. This generally underestimates the real un-

TABLE I. Orbital and stellar properties. The properties of the binary stars and their orbits are listed here, with the corresponding symbols defined in column two. Subscripts 1, 2, and ⊙, refer to the primary and secondary stars and to the Sun, respectively. ω_0 denotes the average orbital angular frequency $2\pi/P$. The data for Mercury and the binary pulsar 1913+16 are also included where appropriate.

System		DI Her	As Cam	EK Cep	V1143 Cyg	V889 Aql	Mercury	Pulsar
Period	P (days)	10.6	3.4	4.4	7.6	11.1	88.0	0.323
Eccentricity	e	0.489±0.002	0.1695±0.0014	0.109±0.003	0.540±0.005	0.37±0.01	0.2056	0.617 14
Semimajor axis	$a/R_\odot$	43.2	17.1	16.6±0.1	22.4	35.8±0.3	83.2	2.8
Stellar radius	R_1/a	0.0621±0.001	0.1499±0.0004	0.095±0.003	0.062±0.003	0.05±0.005		
	R_2/a	0.0574±0.001	0.1111±0.0004	0.079±0.003	0.054±0.003	0.05±0.005		
Stellar mass	$M_1/M_\odot$	5.15±0.10	3.3±0.1	2.03±0.02	1.33±0.03	2.5±0.5		1.42
	$M_2/M_\odot$	4.52±0.06	2.5±0.1	1.12±0.01	1.29±0.03	2.5±0.5		1.41
Rotational frequency	ω_1/ω_0	3.5±1.1	~1.0	1.4±0.5	~1.0	~1.6		
	ω_2/ω_0	3.8±1.3	~1.0	0.8±0.3	~1.0	~1.6		

certainty due, for example, to systematic errors. Where measurements for the same systems are available from several sources, the spread in the inferred parameters is used as a more realistic estimate. If a deviation is not quoted, then the corresponding error is negligible.

(ii) The uncertainties in the stellar properties of the system V889 Aql are the largest. This is because this system is only visible in the northern hemisphere during the summer. Because of the short nights in this season no measurements containing an entire eclipse are available, precluding the direct determination of quantities such as the stellar mass. These are instead inferred from stellar theory given the measured spectral type. The quoted error in this case reflects the possible spectral misidentification by ± one subtype.

(iii) The stellar rotational frequencies can be inferred from the broadening of spectral lines, although this measurement has not been done for all of the systems listed. When data are not available it is assumed that the rotational angular velocities are synchronized with the orbital angular velocity at the point of closest approach (periastron). This assumption is motivated by both theoretical and observational evidence. Theoretically, tidal interactions quickly synchronize the rotational frequency to the orbital frequency. For eccentric orbits, where the orbital frequency varies significantly at different points in the orbit, ω_{rot} approaches the orbital frequency at periastron since this is the point at which the tidal forces are largest. Observationally it is found that binary systems indeed tend to be synchronous.[8] We return to the uncertainties introduced by this assumption below.

(iv) DI Her observations are available[1] dating back to the turn of the century. These earlier observations are taken from photographic plates and furnish eclipse timings that are less accurate than the more recent photoelectric measurements. In Ref. 1 the apsidal precession is first computed using just the photoelectric measurements, and then again using both the photographic *and* the photoelectric data. Both of these results are listed in Table II. Inspection of the tables shows that the discrepancy with theory is worse when both of the measurements are used rather than just the photoelectric ones. Only the more accurate photoelectric measurements are used in the theoretical analysis reported here.

We turn now to a discussion of the theoretical periastron shift. For the systems under consideration the static, post-Newtonian approximation is very good, since the typical orbital velocities are[9] $v^2 \sim GM_\odot/R_\odot \sim 10^{-6}$. Furthermore, the relaxation time for the stellar interior is short compared to the orbital period and so the stars can be considered to be instantaneously at equilibrium with the perturbing forces.[10] Let Ω denote the angle in radians through which the direction to the point of closest approach, the periastron, precesses over one (unperturbed) orbital period. The dominant Newtonian source for this precession arises from the distortion of each star

TABLE II. Apsidal motion. The internal density parameter, as quoted by Refs. 1–3, are listed here together with the resulting Newtonian, relativistic, and total precession of the direction of closest approach. The final row gives the observed precession. For DI Her the precession inferred with and without older photographic measurements is given.

System		DI Her	AS Cam	EK Cep	V1143 Cyg	V889 Aql	Mercury
Density Parameter	$k_{2,1}$	0.0083±0.0010	0.0056±0.0010	0.005±0.001	0.0060±0.0005	0.0054±0.002	
	$k_{2,2}$	0.0078±0.0010	0.0056±0.0010	0.010±0.005	0.009±0.001	0.0054±0.002	
Newtonian precession	deg/cent.	1.93±0.26	35.7±3.1	4.3±3.0	2.4±1.4	0.33±0.07	1.543 78±0.000 06
Relativistic precession	deg/cent.	2.34±0.15	7.9±1.6	3.6±0.06	1.8±0.05	1.2±0.2	0.011 95
Total theory	deg/cent.	4.27±0.41	43.6±4.7	7.9±3.1	4.2±1.5	1.53±0.27	1.555 74±0.000 06
Observed precession	deg/cent.						
all data		0.65±0.18	16.0±1.3	8.8±2.6	3.4±0.2	1.5±0.5	1.555 76±0.000 11
photoelectric data		1.75±0.11					

due to its rotation and due to the tidal forces of its companion. These contribute[8]

$$\Omega_{\text{tidal}}=30\pi\left[\frac{1+\frac{3}{2}e^2+\frac{1}{8}e^4}{(1-e^2)^5}\right]\times\left[k_{2,1}\left[\frac{M_2}{M_1}\right]\left[\frac{R_1^5}{a^5}\right]+k_{2,2}(1\leftrightarrow 2)\right], \quad (1)$$

$$\Omega_{\text{rot}}=\frac{2\pi}{(1-e^2)^2}\left[k_{2,1}\left[\frac{\omega_1^2}{\omega_0^2}\right]\left[1+\frac{M_2}{M_1}\right]\left[\frac{R_1^5}{a^5}\right]+k_{2,2}(1\leftrightarrow 2)\right]. \quad (2)$$

The symbols are defined in the tables. The first-order relativistic shift is

$$\Omega_{\text{GR}}=6\pi\frac{G(M_1+M_2)}{a(1-e^2)} . \quad (3)$$

These expressions combined with the values in Table I give the results of Table II. The dimensionless constants, $k_{2,i}$, $i=1,2$, appearing in Eqs. (1) and (2) are the so-called internal-density parameters for each star. Given the total mass, age, and chemical composition of a star, $k_{2,i}$ is computable from a theoretical model of the stellar interior.[10,11] Table II lists these values. The uncertainty here is due to both imperfect knowledge of the stellar parameters and to differences between various stellar models. For the stars quoted this estimated error ranges between about 10% and 70%.

The principal uncertainties in the theoretical periastron shift lie in the knowledge of $k_{2,i}$ and ω_i. The best way to minimize their effect is to choose systems for which Ω_{GR} is as big as possible compared to $\Omega_{\text{Newt}}=\Omega_{\text{tidal}}+\Omega_{\text{rot}}$. Reference to Table II shows that this is least well satisfied by AS Cam. It should therefore be borne in mind that a relatively minor uncertainty in the Newtonian term can overwhelm the relativistic effect for this system, thereby bringing it into agreement with theory. DI Her and V889 Aql, on the other hand, are the opposite extreme. For these systems the relativistic shift is the dominant part of the apsidal motion, and so their interpretation is the least dependent on the details of stellar models. In particular, the discrepancy for DI Her still persists even in the extreme case that Ω_{Newt} vanishes.

Notice that when the masses, radii, and angular velocities are roughly equal, the rotational precession is less than the tidal precession provided that $\omega_i^2/\omega_0^2\leq 8$. A relatively large uncertainty in ω_i in this case does not cause a correspondingly large error in Ω_{Newt}. If we mistakenly take $\omega_i/\omega_0\approx 1$, for a system for which $\omega^2/\omega_0^2\geq 8$, we *underestimate* the theoretical prediction and so *underestimate* the disagreement with experiment. Since observational and dynamical evidence suggest that the rotational frequencies synchronize with the maximum orbital frequency, and since the assumption of synchronism will at worst give a conservative estimate of the discrepancy with theory, we follow astronomical practice and assume the synchronous value[8] for ω_i/ω_0, for those systems in which the rotation is poorly measured. Clearly better measurements of this quantity are required.

III. NEW FORCES

The exchange of a very light particle of mass μ would manifest itself as a Yukawa potential energy of interaction between particles of range $\rho=1/\mu$. Consider two spherical bodies of radii R_i $(i=1,2)$ containing N_i identical particles distributed with densities $n_i(r)$ and with centers separated by a distance l. The resulting potential energy due to a Yukawa force is

$$V(l)=\alpha Y_1 Y_2 B_1 B_2\frac{e^{-l/\rho}}{l}$$

with (4)

$$B_i\equiv B\left[\frac{R_i}{\rho}\right]=\frac{4\pi\rho}{N_i}\int_0^{R_i}n_i(r)r\sinh\left[\frac{r}{\rho}\right]dr .$$

Here $\alpha=\pm g^2/4\pi$ where g is the coupling strength per particle, and the sign is positive for a repulsive force (spin-one exchange) and negative for an attractive force (spin-zero exchange). Y denotes the charge to which the new particle couples. Its precise form is given below. The function $B(x)$ expresses the fact that, for Yukawa potentials, a spherical distribution of matter is not equivalent to a point particle, positioned at its center, with the same total charge. For illustrative purposes consider the two extreme cases in which all of the particles are either concentrated at the center or are uniformly distributed throughout the star. For these cases we find $B_{\text{center}}(x)=1$ and $B_{\text{const}}(x)=3(x\cosh x-\sinh x)/x^3$, respectively. For small x, $B_{\text{const}}(x)\approx 1+x^2/10+O(x^4)$, so for all cases we may ignore B if R/ρ is sufficiently small.

The potential energy of Eq. (4) produces a periastron shift given by

$$\Omega_{\text{Yuk}}=-2\pi B_1 B_2\eta\left[\frac{\sqrt{1-e^2}}{e}\right]\times\left[\frac{a}{\rho}I_1\left[\frac{ae}{\rho}\right]\exp(-a/\rho)\right] \quad (5)$$

with

$$\eta=\frac{\alpha Y_1 Y_2}{GM_1M_2} . \quad (6)$$

$I_1(x)$ is the modified Bessel function of the first kind, normalized in the standard way.[12]

We now ask whether the data are consistent with a precession given by the classical results, Eqs. (1)–(3), plus a Yukawa precession, Eqs. (5) and (6). Since the data are slim enough to hardly merit a full-blown fit of parameters, our purpose here is to simply make a preliminary estimate of whether a new force could be responsible for the observed discrepancies. There are several immediate observations.

(i) Since the measured shift in the anomalous systems is smaller than predicted by standard theory, a retrograde precession is necessary, implying a repulsive force. For two bodies of similar particle content, such as two main-sequence stars, this means that the hypothetical exchanged particle has spin one. Theoretical prejudice then suggests that this particle couples to a conserved charge[13] which, for electrically neutral bulk matter, amounts to some function of proton and neutron numbers, $Y=Y(N,Z)$. For simplicity we assume this function to be linear, and so parametrize it by[14] $Y\equiv N\cos\theta+Z\sin\theta$.

(ii) The factor η defined in Eq. (6) involving the coupling strength α of the new force simplifies under the assumption that the stellar mass is dominated by the rest mass of its constituents, $M\approx m(N+Z)$, in which m denotes the nucleon mass. This factor becomes

$$\eta\approx(\alpha/Gm^2)[Y_1/(N+Z)_1][Y_2/(N+Z)_2] . \quad (7)$$

The first term is the ratio of the coupling strength per nucleon of the Yukawa and gravitational forces, and the remaining factors just depend on the chemical makeup of the star. In terms of the neutron excess, defined by

$$\xi\equiv\frac{N-Z}{N+Z} , \quad (8)$$

we have

$$\frac{Y}{N+Z}=\frac{1}{\sqrt{2}}\left[\cos\left[\theta-\frac{\pi}{4}\right]-\xi\sin\left[\theta-\frac{\pi}{4}\right]\right] . \quad (9)$$

The eclipsing binaries discussed (and the Sun) are main-sequence stars with about 75% hydrogen and 25% helium (by mass) together with trace amounts of heavier elements, and so $[Y/(N+Z)]_\odot\approx\frac{1}{8}\cos\theta+\frac{7}{8}\sin\theta$.

(iii) If the range is not too much smaller than the orbital size, then $\Omega_{\rm Yuk}/2\pi\sim-(\alpha/Gm^2)$. For comparison, the relativistic precession is $\Omega_{\rm GR}/2\pi\sim GM_\odot/R_\odot\sim10^{-6}$. Since the data suggest that $\Omega_{\rm Yuk}\sim-\Omega_{\rm GR}$, we can estimate that $\alpha/Gm^2\sim10^{-6}$. This implies that the new force can be much *weaker* than gravity and still produce a comparable apsidal motion. This somewhat surprising observation explains why the resulting potential turns out to be compatible with all but the most recent terrestrial bounds on new forces. The reason for this sensitivity to new forces is that the interaction energy in general relativity only deviates from Newton's law in velocity-dependent terms, while the Yukawa force perturbs the orbit in the static limit. This implies that gravitational contributions to orbital precession are suppressed by $v^2\sim GM_\odot/R_\odot\ll1$, when compared to the effect due to a new force of comparable strength.

(iv) Finally, Eq. (5) decreases exponentially with orbital size. If, for example, $R\ll\rho\ll ae$, then $\Omega_{\rm Yuk}$ falls off like $\sqrt{(a/\rho)}\exp(-r_m/\rho)$, where $r_m=a(1-e)$ is the distance of closest approach. We would expect, then, that the anomalous effect is largest for the *smallest* orbits.[15] Although the data are sparse, they do *not* appear to reflect this trend since the two anomalous systems, DI Her and AS Cam, have close to the largest and smallest orbits, respectively. As we shall see, this is the basic difficulty in fitting these systems.

We turn now to a determination of the parameters ρ and α from the binary-star observations. For these purposes it is convenient to consider the ratio of $\Omega_{\rm Yuk}$'s for different binary systems. This is because α and θ both cancel in the ratios, assuming that all of the stars involved have roughly the same abundance of hydrogen and helium. We find that, using the quoted errors, no choice for ρ can account for all of these ratios. A simple calculation shows that Ω(DI Her)/Ω(AS Cam) can only lie within its experimental range provided that $5\le\rho/R_\odot\le7.5$. On the other hand, consistency of DI Her with the null result for EK Cep and V1143 Cyg requires $\rho/R_\odot\ge16.4$, giving an inconsistency.

As was noted earlier, however, the significance of the observed discrepancy depends very sensitively on the uncertainty with which the Newtonian contribution is known. This is dominated by the uncertainty in $k_{2,i}$, an unmeasured parameter. In particular, the upper limit on ρ found above comes from the requirement that the discrepancy in AS Cam be as large as was measured. This is somewhat suspicious since AS Cam has $\Omega_{\rm GR}/\Omega_{\rm Newt}\approx22\%$, and so is the system most sensitive to small errors in the Newtonian term. To illustrate this point, suppose the error in $\Omega_{\rm Newt}$ was $\pm80\%$, rather than the quoted values. In this case AS Cam agrees with general relativity within the errors, and the binary-star data are *consistent* with a Yukawa force with a range $\rho/R_\odot\ge7.5$. (DI Her, on the other hand, is inconsistent for any positive $\Omega_{\rm Newt}$.) The size of the corresponding coupling strength depends on θ and is smaller the larger ρ is. We consider the two examples $\theta=\pm\pi/4$ for which $Y=Y_\pm=(N\pm Z)/\sqrt{2}$ in order to outline the θ dependence of the predictions. For Y_- and $\rho\ge7.5R_\odot$: $(\alpha/Gm^2)_-\le(5.6\pm2.2)\times10^{-5}$. For Y_+: $(\alpha/Gm^2)_+\le(3.2\pm1.3)\times10^{-5}$. Similarly, if the error in $\Omega_{\rm Newt}$ were 100% (i.e., a factor of 2) then the corresponding limit would be $\rho\ge4.7R_\odot$ for which $(\alpha/Gm^2)_-\le(2.6\pm1.0)\times10^{-4}$ and $(\alpha/Gm^2)_+\le(1.4\pm0.6)\times10^{-4}$.

IV. OTHER TESTS

For the remainder of this paper we adopt the point of view that the uncertainties in $\Omega_{\rm Newt}$ are large enough to allow consistency with the binary-star data. We wish to explore whether the resulting force is consistent with the other known tests of general relativity.[4] These tests fall into three broad classes: the motion of bodies within the solar system, the rate of period decrease in the binary pulsar, and direct terrestrial searches for composition-dependent forces. We deal with each of these in turn.

(i) Not surprisingly, one of the best constraints comes from perihelion-shift measurements. These are available for Mercury, Venus, Earth, Mars, and Icarus.[4,6,16] Since these all have orbits that are larger than those of the binary-star systems, absence of any anomalous precession implies an upper limit on the range ρ. Icarus is particularly interesting in this regard because its distance of closest approach to the Sun is smaller than that for Mercury. This makes it a potentially more sensitive probe of forces in the range of $(5-10)R_\odot$. At present, however, the lowest limit for ρ comes from Mercury due to the greater precision of the measurements available.

The upper limit on ρ so determined depends on the parameter θ. When the error in $\Omega_{\rm Newt}$ is 80%, θ must lie in some interval around $-\pi/4$. This is because the upper and lower limits on the range are inconsistent for Y_+, but not for Y_-. In the allowed range of θ, the upper and lower limits for ρ and α coincide. For slightly larger uncertainty, $\Omega_{\rm Newt}$ uncertain up to a factor of 2, all values of θ are allowed. We find that Y_- implies $\rho \le 8.7R_\odot$, $\alpha \ge (4.0\pm1.6)\times10^{-5}$; for Y_+ the corresponding numbers are $5.4R_\odot$ and $(8.4\pm3.4)\times10^{-5}$. We use $\xi \approx -0.75$ for the Sun and 0.05 for Mercury.[17]

In passing we note that the sign of the anomalous perihelion shift for Mercury is $\mp$ for $Y_\pm$, giving retrograde precession for Y_+. In fact the upper bounds on ρ can be relaxed somewhat if the solar quadrupole moment turns out to be large, as has been suggested by Dicke and others,[18] since in this case a larger retrograde perihelion shift could partially cancel a positive solar contribution.

(ii) Precision position measurements of bodies within the solar system now allow accurate tests of their equations of motion.[4,16] For a new force with the range of interest here, the best limit on η comes from Earth-moon measurements and from the Laser Geodynamics Satellite (LAGEOS). The bound is $\eta_{\rm expt} \le 6\times10^{-5}$. Since $\xi \sim 10^{-2}$ for planets and satellites, we expect that $\eta_- \approx 10^{-9}$, for Y_- and $\eta_+ \approx 4\times10^{-5}$ for Y_+. These predictions are just compatible with experiment for Y_+ and are well below the experimental limit for any other value of θ. Since the force described here does not couple directly to photons, it is not detectable in red-shift or radar-echo delay measurements.

(iii) The agreement of the rate of change of the orbital period of the binary pulsar 1913+16 with that predicted by gravitational radiation[4,7] puts another constraint on any new force. This is a somewhat indirect test because the orbital parameters of this binary system are not directly measurable. Indeed the character of the pulsar's partner is itself not known. Although it is most likely another neutron star or black hole, a helium star or white dwarf could be consistent with the data. The logic of the analysis used to test general relativity is as follows:[19] If the pulsar's partner is a compact object, as seems likely, then the observed periastron shift should be completely due to general relativity. If this is *assumed* to be the case, then the orbital parameters can be inferred from Eq. (3) together with the Doppler measurements of the pulsar period. Using these parameters the gravitational radiation reaction is calculated, giving agreement with the observed decrease in orbital period. Two things must be checked in the presence of a new force. $\Omega_{\rm Yuk}$ must be small enough to be dominated by $\Omega_{\rm GR}$, and the energy loss into radiation of the new particles must be small compared to the gravitational rate. These are both potentially quite restrictive because the small size of the binary pulsar orbit enhances its anomalous apsidal motion and quadrupole spin-two particle emission is suppressed relative to dipole spin-one radiation for slowly moving sources.

The condition that spin-one radiation not dominate the graviton emission rate turns out to be easily satisfied for the binary pulsar with a spin-one particle mass in the range considered here. This is because the range of the new force is of the order of the orbital size in this system: $a/\rho = a\mu \sim 1$. The orbital angular frequency, however, satisfies $(a\omega_0)^2 = G(M_1+M_2)/a \approx v^2 \sim 10^{-6} \ll 1$. This implies that $\omega_0 \ll \mu$, whereas radiation is suppressed unless $\omega_0 > \mu$. This suppression arises because dipole radiation would like to be emitted with the frequency of the driving source ω_0, but this frequency is not available since all propagating modes have frequencies greater than μ.

Since general relativity correctly predicts the observed period change to within the 10% error, we infer that assuming the periastron shift to be of relativistic origin is correct to the 10% level. This requires that $\eta \le (2-3)\times10^{-5}$ [for $\rho = (5-8)R_\odot$]. Taking $Z \approx 0$, and parametrizing the neutron-star binding energy by $M = fNm$, with $0 \le f \le 1$, gives $Y/M \approx \cos\theta/mf$ so $\eta \approx (\alpha/Gm^2)(\cos^2\theta/f_1 f_2)$. This is small enough to be acceptable, although $\theta \approx 0$ is getting uncomfortable.

(iv) The terrestrial tests of most interest for the present purposes are searches for a dependence on chemical composition in the gravitational attraction between two objects. These tests are potentially quite sensitive to a force that couples to $N-Z$, since this quantity varies strongly from element to element. The most sensitive such tests measure the relative rate with which bodies fall toward the Sun.[20] These do not constrain a force with a range as short as that considered here.

The best limits, for our purposes, come from Eötvös-type experiments[21] that measure the differential acceleration of two bodies towards the Earth. These limit the difference, $\epsilon \equiv |a_1 - a_2|/g \approx |\eta_1 - \eta_2|$. Here a_i/g is the measured acceleration of body i in units of g, the gravitational acceleration at the Earth's surface, and

$$\eta_i = \frac{\alpha Y_\oplus Y_i}{GM_\oplus M_i} \approx (\alpha/Gm^2)\left[\frac{Y}{N+Z}\right]_\oplus \left[\frac{Y}{N+Z}\right]_i \tag{10}$$

is the coupling strength between the Earth and body i. The order of magnitude of the predicted effect is easily computed. Since $(\alpha/Gm^2) \sim 10^{-5}$ and $Y_-/(N+Z) \approx 10^{-2}$ for the Earth, and ranges from 0 to 10^{-1} for the materials compared in the experiment, we expect $\epsilon_- \sim (1-10)\times10^{-9}$. For Y_+, ϵ_+ is zero in the approximation that $M \approx m(N+Z)$ so a more detailed calculation is required. Generically M differs from $m(N+Z)$ by about one part in a thousand for different elements on Earth so $[Y_+/(N+Z)]_\oplus \approx 1$ and the difference between $Y_+/(N+Z)$ for body one and body two is $\sim 10^{-3}$. Therefore we typically find $\epsilon_+ \sim \epsilon_-$.

For torsion-balance experiments these predictions must be multiplied by an additional factor of 10^{-3}. This further suppression arises because the experiment can only detect the component of the composition-dependent force perpendicular to the torsion wire: $\epsilon_\perp \equiv |(a_1 - a_2)_\perp|/g$. For forces whose range is bigger than $R_\oplus \approx 6\times10^3$ km, as is the case here, the anomalous force is directed towards the center of the Earth, so the angle between the new force and the torsion wire is given by the small angle: $\sin\delta \approx a_{\rm centripetal}/g \sim 10^{-3}$. The prediction in this case is therefore $\epsilon_\perp \sim (1-10)\times10^{-12}$.

Experimentally, the best limits come from recent searches for a (relatively) short-ranged "fifth force."[22] Although the experimental situation is muddied by contradictory results,[22,23] we take the conservative route and consider only those experiments that see a null result. Of these the best limits are $(\epsilon_1)_{\rm expt} \leq 1\times 10^{-12}$ (torsion-balance)[23] and $\epsilon_{\rm expt} \leq 5\times 10^{-10}$ (free-fall).[24] A calculation of the predicted effect for these two experiments gives

$$\text{torsion-balance: } \epsilon_1 \approx \begin{cases} 1\times 10^{-10} & \text{for } Y_+, \\ 5\times 10^{-12} & \text{for } Y_-; \end{cases}$$
$$\text{free-fall: } \epsilon \approx \begin{cases} 2\times 10^{-8} & \text{for } Y_+, \\ 3\times 10^{-8} & \text{for } Y_-. \end{cases} \quad (11)$$

We see that the predictions are ruled out, most convincingly by the free-fall result. It is nevertheless noteworthy that the binary-star measurements can see forces which only the most recent generation of terrestrial experiments could detect.

V. CONCLUSIONS

To summarize, we have analyzed whether the anomalous results recently found in apsidal motion measurements of eclipsing binary-star systems are consistent with the existence of a new weakly coupled force and, if so, whether such a force should have been seen elsewhere. We assume, for simplicity, a single new force with a linear coupling to a conserved charge which for electrically neutral bulk matter in the standard model amounts to a linear combination, $g(N\cos\theta + Z\sin\theta)$, of proton number and neutron number.

Our conclusion depends on the accuracy with which stellar models can calculate a parameter k_2 needed to compute the Newtonian contribution to the orbital precession. If the error in the computed value is less than about 80%, then the five binaries for which data are available are inconsistent with the expectation that systems with the smallest orbits should have the biggest anomalies. However, if the error is bigger than 80%, then the disagreement for AS Cam becomes within the errors and the five binaries are consistent with a new force. It should be stressed that for DI Her experiment and theory disagree for any positive k_2 and so the discrepancy cannot be eliminated in this way regardless of how poorly k_2 is known. The source of errors in this parameter comes both from differences between alternative stellar codes and from uncertainties in the measured properties of each star. The error estimated by astronomers due to both of these sources ranges from 10% to 60% for the stars discussed here, but larger errors cannot be ruled out with certainty.

Assuming, for the purposes of argument, that the uncertainty in k_2 is sufficiently large, the properties of the force necessary to produce the anomaly can be inferred. To account for the data the force must be repulsive for the stars involved, with coupling strength g satisfying $(g^2/4\pi) \approx (10^{-4}-10^{-5})Gm^2 \approx 10^{-42}-10^{-43}$ where G is Newton's constant and m is the nucleon mass. Its range must be $(5-8)R_\odot \approx (3-6)\times 10^6$ km.

Such a force is compatible with the extraterrestrial tests of GR but is close to the bounds set by the perihelion shift of Mercury and the binary pulsar. A coupling to $N+Z$ is also close to limits coming from the LAGEOS and the Earth-moon tracking data. The predictions conflict, however, with the most recent terrestrial Eötvös experiments, although the experimental situation is not entirely clear.

Regardless of the ultimate interpretation of the binary-star anomaly, this analysis shows that the periastron-shift measurements are surprisingly sensitive probes of new long-range forces. Their sensitivity is due to the fact that any new force must only compete with velocity-dependent gravitational effects to be observable. This implies that to produce a comparable effect the coupling strength per particle can be of order of $v^2 \sim GM_\odot/R_\odot \sim 10^{-6}$ smaller than that of gravity. As binary-star measurements improve, they may be expected to become more accurate extraterrestrial laboratories for new, long-range forces.

ACKNOWLEDGMENTS

The authors would like to thank P. Caldas, E. Guinan, A. Kshirsagar, C. S. Lam, J. Moffat, and E. Woolgar for useful discussions, and R. Mann for introducing us to the system DI Her. This research was supported in part by the Natural Sciences and Engineering Research Council.

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[13]This is by no means necessary since, although a light spin-one particle must be a spontaneously broken gauge boson, its coupling to ordinary matter could have the form $\mathcal{L}=(\lambda/M^2)J^\mu(\phi^* D_\mu \phi)$, in which J^μ is an arbitrary ordinary matter current that need not be conserved, and ϕ is some scalar field whose expectation value v gives the new spin-one field its mass. Such a coupling is indeed preferable to a direct gauge coupling since the resulting coupling strength is $g\sim\lambda(v^2/M^2)$ which can be naturally small. In order for a macroscopic body's charge not to change too quickly with time it is probably sufficient that J^μ be conserved by the strong and electromagnetic interactions.

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