Phenomenological aspects of new gravitational forces. IV. New terrestrial experiments

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We use the formalism of spin-1 (graviphoton) and spin-0 (graviscalar) partners of the graviton developed in this series to analyze four terrestrial experiments. This analysis is aided by the results of Stacey, Tuck, and Moore of Australian mine data, which allow interaction ranges up to 450 km. We find that an anomalous result of order 10% could occur for the gravitational acceleration of antimatter. Such ranges could also explain the apparently contradictory results of the Brookhaven and Washington Eötvös experiments in terms of the local geology. In the appendixes we rule out two other possibilities. The first would perform a gravity measurement inside a spherical shell: a "Gauss's law" test. The second would be a new rotating-disk Cavendish experiment.

I. INTRODUCTION

Modern theories of quantum gravity generally predict that there will be spin-1 (graviphoton) and spin-0 (graviscalar) partners of the graviton.[1,2] These new partners are expected to couple with gravitational strength to some conserved charge (such as baryon-quark and/or lepton number), and to be massive (produce Yukawa potentials). Phenomenologically, this means that the gravitational potential between two particles, in linear approximation, can be parametrized as[2–5]

$$I(r)=\frac{-Gm_1m_2}{r\gamma_1\gamma_2}\{[2(u_1\cdot u_2)^2-1]\mp a(u_1\cdot u_2)e^{-r/v}+be^{-r/s}\} . \tag{1.1}$$

In (1.1) u_i are four-velocities and γ_i are relativistic factors. a (b) and v (s) are the coupling constant and range of the graviphoton (graviscalar), respectively. The coupling constants represent the products of the charges normalized to ordinary gravity. The signs in front of a represent matter-matter ($-$) and antimatter-matter ($+$) interactions. Note that we have included only *one* partner of each spin, although, in general, there could be many, and, in principle, even a tensor partner. In the static limit Eq. (1.1) becomes

$$I(r,\beta=0)=\frac{-Gm_1m_2}{r}(1\mp ae^{-r/v}+be^{-r/s}) . \tag{1.2}$$

In this series we have investigated the physical implications of this phenomenology. In paper I,[3] we studied rapidly rotating compact objects. From the millisecond pulsar we found that if the coupling constants a and b were $\gtrsim 100$, then a completely new analysis of such objects would be necessary, as the macroscopic stability solutions would need to be recalculated.[3]

In paper II we did both a multipole and also a spherically symmetric analysis of static Yukawa potentials.[4] In paper III we studied the potential due to slowly rotating astronomical objects.[5] In particular, we found that Earth's slow rotation was not large enough to produce velocity-dependent effects which could reasonably account for the correlation observed in the Eötvös data.[6] Others, too, have been obtaining interesting results.[7–9]

In this concluding paper, we will analyze four types of terrestrial experiments. The first (Sec. II) concerns experiments to measure the gravitational acceleration of antimatter. From the reanalysis[10] of the Australian mine data in terms of Eq. (1.2), Stacey, Tuck, and Moore found one could have ranges $v\simeq s$ up to 450 km, for $a\simeq b$. Using this, we find that a significant, measurable matter-antimatter acceleration difference may be seen of order

$$\frac{\Delta g}{g}=2a\left[\frac{\lambda}{450\ \text{km}}\right](0.14) , \tag{1.3}$$

where $\lambda\equiv v\simeq s$.

The recent Eötvös experiments by the Adelberger group[11] have constrained a *single* new vector potential to have a coupling of $|\alpha|\lesssim 10^{-3}$ for ranges $\sim 100-1000$ m. Contrariwise, the Thieberger[12] experiment at the edge of the Palisades Cliff in New Jersey found a positive result ($|\alpha|\sim 10^{-2}$, for such ranges). In Sec. III we elucidate the observation[13] that these experiments could be consistent if Thieberger is observing the effects of a much larger-sized geologic object than Adelberger's group.

Note that the recently announced Air Force Geophysics Laboratory tower experiment[14] bears on this matter. They find evidence for a new, finite-range, attractive force. Thus, evidence now exists for both graviphoton (repulsive) and graviscalar (attractive) forces, independent of whether or not the ranges of these partners are long.

In the appendixes we discuss two experiments which appear not to be feasible. The first is a static experiment discussed by McCullen.[15] This consists of a gravimeter (in the form of a suspended torsion-balance pendulum) in the middle of a hollow spherical shell of a few meters radius. Because of Gauss's law there will be no force inside the shell from Newtonian gravity. This statement does not hold for the Yukawa potentials.

The second is an experimental test of the velocity dependence of the potentials, by studying the gravitational effect which results from placing two rotating disks next to the masses of a Cavendish balance.

II. THE GRAVITATIONAL ACCELERATION OF ANTIMATTER

Recently, Stacey, Tuck, and Moore have reanalyzed[10] their mine data in terms of the phenomenology we have been discussing: a new repulsive force from graviphoton exchange and a new attractive force from graviscalar exchange, both with relatively long ranges. Using their mine data, Lunar ranging data, Laser Geodynamics Satellite (LAGEOS) data, Earth measurements of g, and their lake experiment, they found that, for $a \simeq b$ and $v \simeq s$, ranges of v up to 450 km are allowed, with the restrictions that[10]

$$a-b=d, \quad (s/v-1)=\delta/a, \quad (d,\delta)\simeq 0.01 \ . \tag{2.1}$$

For the gravitational acceleration of antimatter, the sign of the a term changes, although the magnitude is the same. Therefore, the new gravitational effects could be significant for the acceleration of antimatter even though small for matter. To calculate what size effect might be seen, we used the preliminary reference Earth model (PREM). This model[16] is the standard spherically symmetric geophysical model of the Earth, obtained from seismic, gravity, satellite and other data. It consists of a series of 10 shells, and within the ith shell the density is given as a polynomial up to order 3 in radius:[16]

$$\rho(r)=\sum_{n=0}^{3}\rho_{ni}(r/R)^n, \quad B_i \le r \le A_i \ , \tag{2.2}$$

where the ρ_{ni} are determined constants and the B_i and A_i are the inner and outer radii of the ith shell. The values of the ρ_{ni} and graphs of $\rho(r)$ are given in Ref. 16. The radius of Earth is taken as 6371 km and the mass as 5.974×10^{24} kg. [We have checked that the integration over the volume of the Earth of the density of Eq. (2.2) gives the correct total mass.]

Since for antimatter there are no significant cancellations, we can take $a=b$ and $v=s\equiv\lambda$. Then the difference between the local gravitational acceleration of antimatter (say antiprotons) $\bar{g}$ and of matter g, normalized to that for matter, can be calculated from the formulas[4] in Sec. IV of paper II. In particular,

$$\frac{\Delta g}{g}\equiv\frac{\bar{g}-g}{g}=\left[\frac{2aR^2}{M}\right]\left[\frac{1}{R^2}+\frac{1}{R\lambda}\right]e^{-R/\lambda}(4\pi\lambda^3) \times\sum_{i=1}^{10}\sum_{n=0}^{3}\rho_{ni}\left[\frac{\lambda}{R}\right]^n E_{ni} \ , \tag{2.3}$$

where for $n\ge -1$, E_n is defined by

$$E_n=\sum_{k=0}^{[(n+1)/2]}t^{n+1-2k}\frac{\Gamma(n+2)}{\Gamma(n+2-2k)}\cosh t\Bigg|_{B/\lambda}^{A/\lambda} -\sum_{k=0}^{[n/2]}t^{n-2k}\frac{\Gamma(n+2)}{\Gamma(n+1-2k)}\sinh t\Bigg|_{B/\lambda}^{A/\lambda} \ . \tag{2.4}$$

So, for each of the 10 shells,

$$E_{0i}=(t\cosh t-\sinh t)\,\Big|_{B_i/\lambda}^{A_i/\lambda} \ , \tag{2.5}$$

$$E_{1i}=[(t^2+2)\cosh t-2t\sinh t]\,\Big|_{B_i/\lambda}^{A_i/\lambda} \ , \tag{2.6}$$

$$E_{2i}=[(t^3+6t)\cosh t-(3t^2+6)\sinh t]\,\Big|_{B_i/\lambda}^{A_i/\lambda} \ , \tag{2.7}$$

$$E_{3i}=[(t^4+12t^2+24)\cosh t-(4t^3+24t)\sinh t]\,\Big|_{B_i/\lambda}^{A_i/\lambda} \ . \tag{2.8}$$

Combining all these equations and evaluating (2.3) with vector coupling $a=1$, we obtain the solid curve in Fig. 1. For comparison, the dashed curve shows the effect that would result from a uniform Earth of the same total mass.

For small ranges, the idealized uniform spherical Earth differs by a factor of 2, essentially because the density near the surface of the Earth is a factor of 2 smaller than the average density of 5.5 g/cm^3. The real Earth's curve is wavy, corresponding to the contribution of different shells becoming significant as the range is changed. The important point is that for a range of 40 km there would a 1% effect in the antiproton experiment, which should be measureable. This is for $a=b=1$, and the effect scales with $a=b$.

If one combines this with the analysis of Stacey, Tuck, and Moore,[10] then one can say that the expected acceleration difference Δg for the antiproton can be parametrized as

$$\frac{\Delta g}{g}=a\left[\frac{v}{450\text{ km}}\right](0.14) \ . \tag{2.9}$$

From our analysis of rotating pulsars,[3] Eq. (2.9) is valid for $a \lesssim 100$. Before going on, we make a brief comment

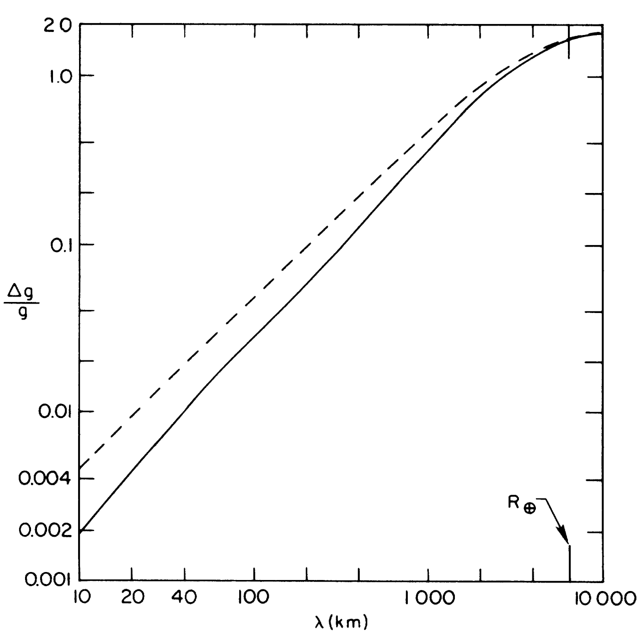

FIG. 1. The size of the new effect due to the graviphoton and graviscalar interactions for antimatter as a function of the length scale $v=s=\lambda$. This result is for new coupling constants $a=b=1$, and scales with their values. The lower solid line is for Earth's real mass distribution, whereas the dashed line is for a uniform mass distribution of the same total mass.

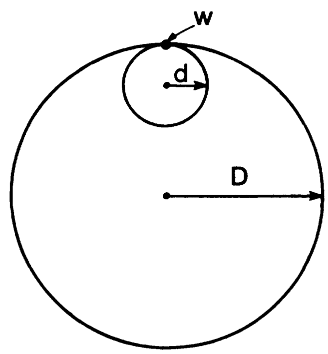

FIG. 2. Two spheres of the same uniform density with radii d and D, tangent to the point w. In the text we calculate the relative force exerted on a test mass at point w.

on whether a $\sim 1\%$ gravitational effect for a range λ on the order of 1% of the Earth's radius makes sense. This effect is similar to the Newtonian force from an adjacent radius-λ half-sphere of matter. For an order-of-magnitude estimate, suppose one asks what is the relative Newtonian force from two spheres with the same density, of radii d and D, on a tangent point w (see Fig. 2):

$$\frac{F_d}{F_D}=\frac{M_d}{d^2}\frac{D^2}{M_D}=\frac{d^3}{d^2}\frac{D^2}{D^3}=\frac{d}{D} \ . \tag{2.10}$$

The forces scale with the radii of the effective objects. Therefore, it is not surprising that $\lambda \sim 10^{-2}R$ gives an approximately 1%-sized effect in the change of gravity.

III. THE BROOKHAVEN AND WASHINGTON EÖTVÖS EXPERIMENTS

Two new Eötvös experiments have been reported by Thieberger[12] and by Adelberger's group[11] at the University of Washington. They obtained seemingly contradictory results. Thieberger has found that a copper sphere neutrally buoyant in water on top of a cliff is repelled in the outward direction normal to the cliff.[12] The strength of repulsion is found to be consistent with the proposed violation of the inverse-square law when parametrized with a single new Yukawa potential:[17]

$$V=\frac{-GmM}{r}(1+\alpha e^{-r/\lambda}) \ , \tag{3.1}$$

$$\lambda \simeq 200 \text{ m}, \ \alpha=-0.01 \ . \tag{3.2}$$

Adelberger's group compared the differential gravitational effect of a hill on two materials.[11] However, they saw no horizontal non-Newtonian differential acceleration. We proposed with Ander[13] that there may be no unrecognized systematic errors in either experiment and that both experiments may be correct. Of course, for us this possibility was motivated by our proposal that the most likely origin for new gravitational-strength forces may be found in quantum theories of gravity,[2] which could be longer ranged. The difference between the Thieberger and Adelberger *et al.* results would then be due to differences in the local topography and geology over distance scales larger than 1 km (Ref. 18).

In particular, the cliff used by Thieberger is very special. It is part of the Palisades, on the western shore of the Hudson River near the New Jersey–New York border (see Fig. 3). The critical point about the geology is that this cliff is the eastern terminus of a diabase sill, with density ~ 2.9 g/cm^3, that has been extruded to form the Palisades outcrop.[13] In comparison, the rocks to the East consist of Precambrian granites and gneisses having a density of about 2.7 g/cm^3. The Palisades site may be sensitive to possible new long-range forces because it sits at the eastern terminus of the diabase sill which extends to the west into Pennsylvania. The excess of this sill over the "average" surrounding material can be represented as a 275-m-thick semi-infinite horizontal slab, extending to the west, of density $\rho_+ = 0.2$ g/cm^3.

As emphasized by Milgrom[19] an Eötvös experiment testing for finite-range forces is sensitive to the lack of horizontal symmetry in the local hemisphere of mass whose radius is of order the range of the force. Therefore, if there exists a distinct and obvious long-range density contrast, this would produce a known effect. The

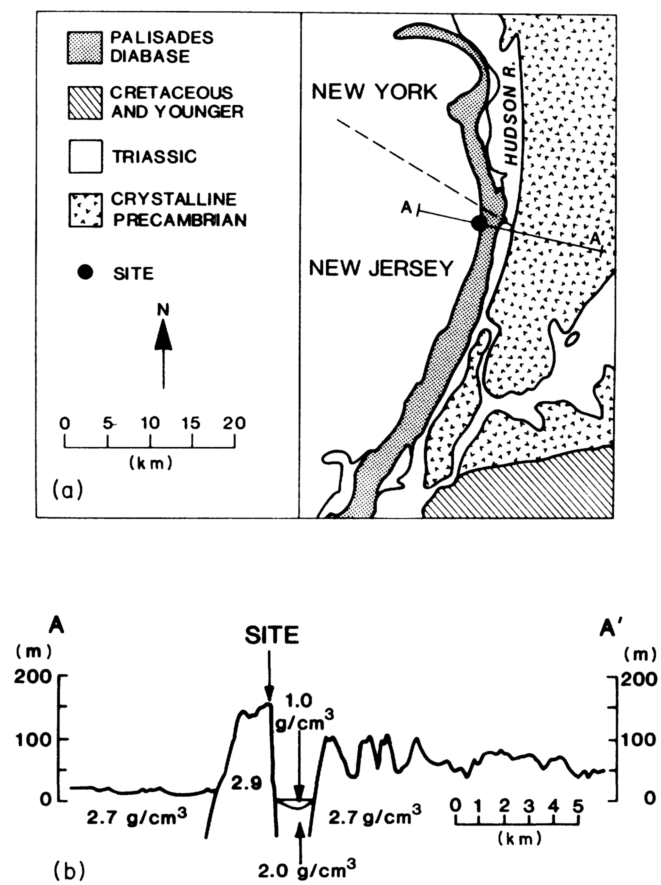

FIG. 3. Geologic and cross-sectional maps of the Palisades site [(a) and (b)]. The line (A-A') labels the trace of the cross section in the geologic maps for the Palisades site. Note the exaggerated vertical scale of the cross section relative to the horizontal scale. Taken from Ref. 13; see footnote 13 therein.

Palisades diabase sill is just such an obvious feature. At the Washington sites,[11,18] as of now, one does not have sufficient knowledge of the geology to long-range depths to discuss such an effect. What we are saying is only that there exists a known density contrast at the Palisades site which could produce the size of effect seen. Not enough is known about the Washington sites to make a similar comment. (See Ref. 13 for further geophysical details.)

In this section we will give more details on the calculation of the force from the Palisades diabase sill. Consider the Thieberger experiment to be at the midpoint of the flat edge of a half-disk with radius R and thickness t. This half-disk represents the diabase sill. For convenience divide the force F by $(aGm\rho_+)$ to yield the "force function"

$$\mathcal{F}=F/(aGm\rho_+) \ . \tag{3.3}$$

From graviphoton exchange this is

$$\mathcal{F}=-\left[\frac{d}{dx}\int_{\pi/2}^{3\pi/2}d\phi\int_0^R\rho' d\rho'\int_{-t/2}^{t/2}dz'\frac{\exp(-r/v)}{r}\right]_{x=0} , \tag{3.4}$$

$$r=(\rho'^2+x^2-2x\rho'\cos\phi+z'^2)^{1/2} \ . \tag{3.5}$$

Evaluating the derivative with respect to x at $x=0$, doing the ϕ integration, and noting the symmetry about zero for z' yields

$$\mathcal{F}=4\int_0^{t/2}dz'\int_0^R\rho'^2 d\rho'\left[\frac{1}{r^3}+\frac{1}{vr^2}\right]e^{-r/v} , \tag{3.6}$$

$$r=(\rho'^2+z'^2)^{1/2} \ . \tag{3.7}$$

Now we perform an integration by parts with the ρ' variable, and take $\bar{u}=-[\exp(-r/v)]/r$ and $\bar{v}=\rho'$. Then, letting $z'/v=k$, $\rho'/v=p$, and $R/v=P$, we obtain

$$\mathcal{F}=4v\int_0^{t/2v}dk\left[\int_0^P dp\frac{\exp[-(k^2+p^2)^{1/2}]}{(k^2+p^2)^{1/2}}-\frac{P\exp[-(k^2+P^2)^{1/2}]}{(k^2+P^2)^{1/2}}\right] . \tag{3.8}$$

As we let R become large with respect to v, the quantity in the parentheses becomes $K_0(k)$, where K_0 is the modified Bessel function. Thus, the horizontal force per unit mass due to graviphoton exchange is

$$f_v=aG\rho_+4v\int_0^{t/(2v)}dk\,K_0(k)\equiv aG\rho_+4vI(t/2v) \ . \tag{3.9}$$

As t goes to infinity, the integral $I(t/2v)$ becomes $\pi/2$, as it should. The integral can be evaluated by using Eq. (9.6.13) of Abramowitz and Stegun[20] for K_0 and integrating. One finds

$$I(\epsilon)=\sum_{n=0}^{\infty}\frac{(\epsilon/2)^{2n+1}}{[\Gamma(n+1)]^2(n+\frac{1}{2})}\left[\frac{1}{2n+1}+\ln2-\gamma-\ln\epsilon+(1-\delta_{n,0})\sum_{j=1}^{n}\frac{1}{j}\right] , \tag{3.10}$$

where γ is Euler's constant. Equation (3.10) gives us an analytic result for our model. The numerically dominant terms for ϵ small are given by

$$I(\epsilon)\simeq\epsilon(1+\ln2-\gamma-\ln\epsilon) \ . \tag{3.11}$$

Now adding graviscalar exchange, one has

$$f_{v,s}=4G\rho_+[vaI(t/2v)-sbI(t/2s)] \ . \tag{3.12}$$

Taking the Stacey, Tuck, and Moore analysis[10] of Eq. (2.1), this yields the differential force per unit mass

$$\Delta f_{v,s}\simeq\Delta2\rho_+Gt\{d[1+\ln2-\gamma-\ln(t/2v)]-\delta\} , \tag{3.13}$$

where we have used Eq. (3.11) for small $(t/2v)$. In Eq. (3.13), Δ is the difference of baryon number per unit atomic mass, which is

$$\Delta=(B/\mu)_{\rm Cu}-(B/\mu)_{\rm water}=0.001\,71 \tag{3.14}$$

between copper and water.

As an example, take $t=250$ m and the conservative value $v=50$ km. (Indeed, Ref. 10 prefers a larger value of v in this situation.) Then, Eq. (3.13) yields 7×10^{-8} cm/sec^2 for $\Delta f_{v,s}$, compared to Thieberger's experimental value,[10] $(8.5\pm1.3)\times10^{-8}$ cm/sec^2. For $v\simeq200$ km, one obtains the best agreement with the Thieberger result.

The validity of this resolution depends on the detailed large-scale geology of the two sites. Ultimately, of course, it depends on the experiments. It is to be hoped that in the future both these and other experiments can be performed at more than one identical site. See Ref. 13 for further comments on this proposal.

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APPENDIX A: TEST OF GAUSS'S LAW

One of the features of an inverse-square force law is that it obeys Gauss's law. As a consequence of Gauss's law there is no force inside a spherically symmetric charge distribution due to that distribution. However, if there were a non-Newtonian Yukawa component to gravity then there would be a nonzero force inside and away from the center of the distribution. McCullen[15] has raised the question if such a force would be measurable.

To answer this question, let us use the single-Yukawa form. For a single new Yukawa, the standard notation is[17]

$$V(r)=\frac{-Gmm'}{r}(1+\alpha e^{-r/\lambda}) . \tag{A1}$$

Then, the potential as a function r from the center of a spherical shell of density ρ_0 and with outer and inner radii A and B is

$$V=-Gm\rho_0\alpha\int_B^A r'^2dr'\int_0^\pi \sin\theta\, d\theta\int_0^{2\pi}d\phi\frac{e^{-D/\lambda}}{D} , \tag{A2}$$

$$D=|\mathbf{r}-\mathbf{r}'|=(r'^2+r^2-2rr'\cos\theta)^{1/2} \tag{A3}$$

(see Fig. 4).

To evaluate (A3), we first integrate over ϕ to obtain 2π, and then perform two changes of variable. First let $\cos\theta=\mu$, or $d\mu=-\sin\theta\, d\theta$. Then let $y=D$, so that $dy=-d\mu(rr')/D$. One then has

$$V=-2\pi m\rho_0 G\alpha\int_B^A\frac{r'dr'}{r}\int_{r'+r}^{r'-r}dy\, e^{-y/\lambda} \tag{A4}$$

$$=-4\pi m\rho_0 G\alpha\left[\frac{\sinh(r/\lambda)}{r}\right]U , \tag{A5}$$

where

$$U=\lambda^3\left[e^{-B/\lambda}\left[\frac{B}{\lambda}+1\right]-e^{-A/\lambda}\left[\frac{A}{\lambda}+1\right]\right] , \tag{A6}$$

and we have used Eq. (5.9) of paper II.[4]

To obtain the radial force one takes

$$F_r=-\hat{\mathbf{r}}\cdot\nabla V$$

$$=-4\pi Gm\rho_0\alpha\left[\frac{\sinh(r/\lambda)}{r^2}-\frac{\cosh(r/\lambda)}{\lambda r}\right]U . \tag{A7}$$

Now performing a power-series expansion in A/λ and B/λ, one obtains

$$F=-4\pi Gm\rho_0\alpha\left[-\frac{(A^2-B^2)r}{6\lambda^2}+\frac{(A^3-B^3)r}{9\lambda^3}-\cdots\right] . \tag{A8}$$

This result agrees with a theorem obtained for Yukawa potentials while studying massive-photon electrodynamics.[21] The force due to a photon of mass $(\hbar/c\lambda)$ scales as $(D/\lambda)^2$, where D is the dimension of the apparatus or the size over which measurements are made. In that case, the result made clear that it is very difficult to perform a precise measurement of the photon mass with a small apparatus. In this case, Eq. (4.8) similarly implies that it is difficult to measure a large-scale non-Newtonian gravitational effect with an apparatus on the size of meters. This holds even if the range is only hundreds of meters, as discussed by the single-Yukawa "fifth-force" advocates.[6]

To be precise, McCullen considered[15] measuring the deflection δr of a torsion pendulum with characteristic oscillation frequency Ω. Then the distance measured would be (taking $B=A/2$)

$$\frac{\delta r}{r}=\frac{F}{m\Omega^2}\simeq\frac{\pi G\rho_0\alpha}{2\Omega^2}\left[\frac{A}{\lambda}\right]^2 . \tag{A9}$$

For $\Omega=2\pi$ sec^{-1}, $A=1.5$ m, $\lambda=200$ m, $\alpha=0.01$, and $\rho_0=1$ g/cm^3, this number is 1.5×10^{-15}. Therefore, such an experiment appears to be too difficult to perform with present technology.

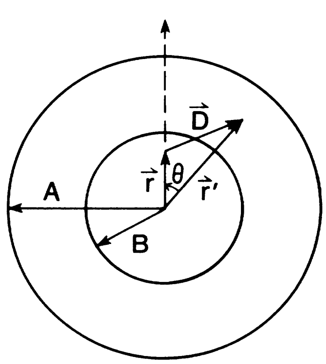

FIG. 4. The geometry of a spherical shell of uniform density with outer and inner radii A and B. The coordinate vectors are as in Eqs. (4.2) and (4.3).

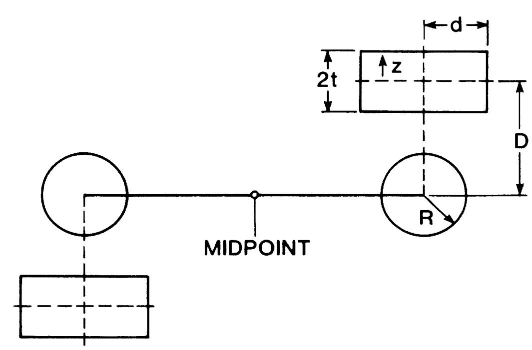

FIG. 5. The geometry of the rotating-disk Cavendish experiment. The spheres have radii R. Each disk has cylindrical radius d, thickness $2t$, and midpoint a distance D away from the center of the sphere. The axial coordinate of the disk is z.

APPENDIX B: ROTATING DISK CAVENDISH EXPERIMENT

Finally, we wish to investigate whether a Cavendish experiment, using rotating disks instead of spheres for the test masses, could detect anomalous velocity-dependent forces. (The difficulty of measuring even normal velocity-dependent forces is well known.[22]) Consider the geometry of Fig. 5, with $\beta c = 2\pi v d$ the velocity of the edge of the rotating disk. We consider the force between one sphere and one disk.

For the interaction of Eq. (1.1), we first state the force between the two objects, and then will show how the results are obtained. With M_S and M_D being the masses of the sphere and disk, the force between the two objects to order β^2 is

$$F = \frac{-GM_S M_D}{D^2}\{F^N + \tfrac{3}{2}\beta^2 F_\beta^N - af(R/v)F^Y(v) + bf(R/s)[F^Y(s) - \tfrac{1}{2}\beta^2 F_\beta^Y(s)]\}\ , \tag{B1}$$

where

$$D_\pm = [(D \pm t)^2 + d^2]^{1/2}\ , \tag{B2}$$

$$F^N = \left[\frac{D^2}{d^2 t}\right](D_- - D_+ + 2t) \tag{B3}$$

$$\simeq 1 + \frac{t^2 - \frac{3}{4}d^2}{D^2} + \frac{t^4 - \frac{5}{2}d^2 t^2 + \frac{5}{8}d^4}{D^4} + O\left[\frac{1}{D^6}\right]\ , \tag{B4}$$

$$F_\beta^N = \left[\frac{D^2}{d^4 t}\right]\{\tfrac{2}{3}(D_+^3 - D_-^3) - d^2(D_+ - D_-) - \tfrac{2}{3}[(D+t)^3 - (D-t)^3]\} \tag{B5}$$

$$\simeq \frac{1}{2} + \frac{t^2 - d^2}{2D^2} + \frac{45d^4 - 160d^2 t^2 + 48d^4}{96D^4}\ , \tag{B6}$$

$$f(x) = \frac{3}{x^3}(x\cosh x - \sinh x) \tag{B7}$$

$$\to 1 \quad \text{as } x \to 0$$
$$\to 3e^x/(2x^2) \quad \text{as } x \to \infty\ , \tag{B8}$$

$$F^Y(\lambda) = \frac{\lambda D^2}{d^2 t}\left[2e^{-D/\lambda}\left[\sinh\frac{t}{\lambda}\right] + \exp(-D_+/\lambda) - \exp(-D_-/\lambda)\right] \tag{B9}$$

$$\to F^N \quad \text{as } \lambda \to \infty\ , \tag{B10}$$

$$F_\beta^Y(\lambda) = \frac{\lambda^3 D^2}{d^4 t}\{\exp(-D_+/\lambda)[(2D_+/\lambda) + 2 + (d/\lambda)^2] - \exp(-D_-/\lambda)[(2D_-/\lambda) + 2 + (d/\lambda)^2] - 2\exp[-(D+t)/\lambda][(D+t)/\lambda + 1] + 2\exp[-(D-t)/\lambda][(D-t)/\lambda + 1] \tag{B11}$$

$$\to F_\beta^N \quad \text{as } \lambda \to \infty\ . \tag{B12}$$

In (B1) the β^2 terms come from the expansion of the γ factors. Note that $u_1 \cdot u_2$ will only have one gamma factor because only one of the pair of objects is moving. Since the two F^Y terms go to the F^N terms as $\lambda \to \infty$, it suffices to calculate only the F^Y terms.

For $F^Y(\lambda)$, first recall that the radial force from the sphere to any point in the disk goes as $f(R/\lambda)$, and so the integrated force is

$$\mathbf{F}^Y = \int dV_{\text{disk}} f(R/\lambda)\left[e^{-r/\lambda}\left[\frac{1}{r^2} + \frac{1}{r\lambda}\right]\hat{\mathbf{r}}\right]\left[\frac{GM_S M_D}{td^2 2\pi}\right] \tag{B13}$$

or

$$\mathbf{F}^Y \cdot \hat{\mathbf{r}} \equiv \frac{GM_S M_D}{D^2} f(R/\lambda) F^Y(\lambda)\ , \tag{B14}$$

where

$$r = [(D+z)^2 + \rho^2]^{1/2}\ . \tag{B15}$$

However, in integrating over the disk, the symmetry of the problem eliminates all components not parallel to $\hat{\mathbf{z}}$. Therefore, $\hat{\mathbf{r}}$ can be replaced by $\hat{\mathbf{z}}(D+z)/r$, and the integration simplifies. After doing the ϕ integration, one has for the force in the z direction

$$F^Y(\lambda) = \frac{D^2}{td^2}\int_{-t}^{t} dz \int_0^d \rho\, d\rho\, e^{-r/\lambda}\left[\frac{1}{r^2} + \frac{1}{r\lambda}\right]\frac{D+z}{r}\ . \tag{B16}$$

Changing variables from ρ to r and then $y = r/\lambda$ allows the first integration to be done, and then the second is straightforward, if tedious. The result is Eq. (B9).

The $F_\beta^Y(\lambda)$ integration is of the same form as Eq. (B16), except that there is an added factor $(\rho/d)^2$ in the integrand:

$$F_\beta^Y(\lambda)=\frac{D^2}{td^4}\int_{-t}^{t}dz\int_0^d\rho^3 d\rho\, e^{-r/\lambda}\left[\frac{1}{r^2}+\frac{1}{r\lambda}\right]\frac{D+z}{r}\ . \tag{B17}$$

This is because we are integrating over $\beta^2(\rho)$. The integration is performed the same way as in Eq. (B16), except that this time it is more complicated. Use can be made of the relations obtained in paper II.[4] The end result is Eq. (B11).

Since experiment tells us that, if there are new gravitational forces, the ranges are much greater than the many tens of c.m. scale of this apparatus, Eq. (B1) can be approximated to

$$F=\frac{-GM_SM_D}{D^2}[F^N(1-a+b)+\tfrac{1}{2}\beta^2F_\beta^N(3-b)]\ . \tag{B18}$$

Therefore, this experiment will be sensitive to a graviscalar force, if b is not almost unity and β^2 can be made large enough.

It appears hard to obtain materials which can withstand centripetal forces to rotate fast enough. Further, in such an experiment it would be necessary to maintain (or know) the distances between the spinning disks and the test masses on the balance to a very high accuracy, even during spin up. Thus, although new materials may aid in this, and there are possibilities of doing interference detection[23] this experiment appears to be extremely unlikely to be done.

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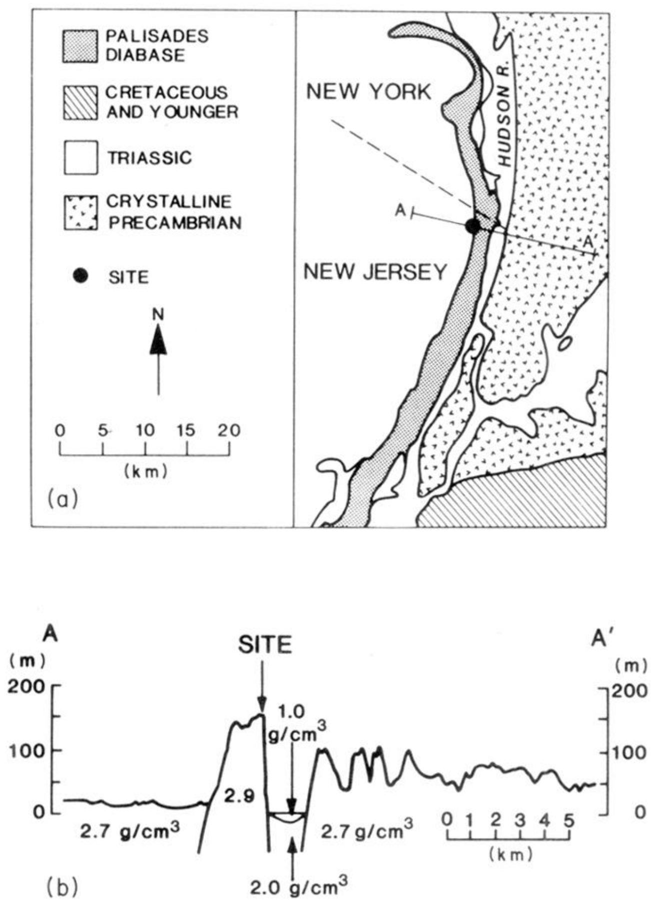

FIG. 3. Geologic and cross-sectional maps of the Palisades site [(a) and (b)]. The line (A-A') labels the trace of the cross section in the geologic maps for the Palisades site. Note the exaggerated vertical scale of the cross section relative to the horizontal scale. Taken from Ref. 13; see footnote 13 therein.