Distribution amplitudes and electroproduction of the delta and other low-lying resonances

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(Received 2 May 1988)

Calculations are presented of certain moments of the wave function of the Δ and of the lowest-lying negative-parity resonances. These are used to obtain model distribution amplitudes for these resonances and to calculate the size of the high-Q^2 nucleon-resonance transition amplitude for each case. The $I=\frac{3}{2}$ resonances are less asymmetric than the $I=\frac{1}{2}$ ones, and indeed are compatible with total symmetry and even with being asymptotic; the agreement between calculation and measurement is good for the $S_{11}(1535)$ transition amplitude although inconclusive for the Δ. (For the isoquartet negative-parity ground state, the data presently hardly exist.)

I. INTRODUCTION

Perturbative QCD applied to baryon resonance transition form factors gives helicity[1] and scaling[2,3] results that are easy to obtain. One expects that the dominant transition amplitude at high-momentum transfer Q^2 is the one that preserves the hadronic helicity. In differing notation, this amplitude is called f_+, G_{MNR}, or $A_{1/2}$. Further, the Q^2 falloff (modulo powers of $\ln Q^2$) at high Q^2 is determined. One expects the dominant amplitude to fall like $1/Q^3$ or $1/Q^4$, depending on kinematic factors in its definition. The helicity and scaling results are not in poor agreement with existing data.[4]

In this paper, we want to report on the next problem, which is calculating the absolute normalization of the dominant helicity amplitudes. To begin, one needs a well-founded model for the wave function of each baryon resonance involved. Only the three-quark wave functions for very small transverse separation are needed for high-Q^2 processes, and for this the QCD sum-rule method gives relevant information about some low moments of the momentum-space wave function (or more precisely, moments of the distribution amplitude, which is the momentum-space wave function integrated over transverse momentum). This has been worked out for the nucleon by Chernyak and Zhitnitsky,[5,6] with significant augmentation by King and Sachrajda,[7] and we extend this work to three additional nonstrange-baryon states: namely, the Δ(1232) and the lightest negative-parity isospin-$\frac{1}{2}$ and -$\frac{3}{2}$ states.

The QCD sum-rule moments constrain the distribution amplitude. The distribution amplitude can be expanded in orthogonal functions[8] (Appell polynomials) whose coefficients decrease logarithmically with Q^2. The moments determine the coefficients of the few lowest polynomials, and one neglects the higher polynomials hoping that their coefficients are small enough at the Q^2 scales were are interested in.

Once the distribution amplitudes are known, then high-Q^2 baryon resonance electroproduction from nucleon targets,[3] among many other processes,[8] can be calculated. The procedure involves convoluting the distribution amplitude with a perturbatively calculated hard-scattering amplitude which, for the case of electroproduction, describes the absorption of a high-Q^2 photon by one quark and the subsequent sharing of momentum with the other two quarks by gluon exchange.

We have chosen to present our calculations of the moments, distribution amplitudes, and electroproduction amplitudes resonance by resonance. The Δ(1232) is presented in Sec. II, with individual sections devoted to kinematics, resonance and quark evaluations of a certain correlator, moments, the distribution amplitude, and the transition amplitude, the latter presented along with some data. Sections III and IV are devoted to the $S_{11}(1535)$ and the negative-parity ground state, respectively. They are short compared to Sec. II and rely on Sec. II for details. Readers chiefly interested in the "answers" and comparison with data will find them mainly in Secs. II F and III C. Conclusions are given in Sec. V.

II. THE DELTA

A. Kinematics and definitions

The three-quark Fock-space components of the proton and Δ are

$$|p(\lambda=\tfrac{1}{2})\rangle=\int[dx][d^2k_T]\left[\frac{\psi_S(x_i,k_{iT})}{\sqrt{x_1x_2x_3}}\frac{1}{\sqrt{6}}\,|\,2uud-udu-duu\,\rangle_{\uparrow\downarrow\uparrow}+\frac{\psi_A(x_i,k_{iT})}{\sqrt{x_1x_2x_3}}\frac{1}{\sqrt{2}}\,|\,uud-duu\,\rangle_{\uparrow\downarrow\uparrow}\right] \quad (1)$$

and

$$|\Delta(\lambda=\tfrac{1}{2})\rangle=\int[dx][d^2k_T]\frac{\psi_\Delta(x_i,k_{iT})}{\sqrt{x_1x_2x_3}}\frac{1}{\sqrt{3}}|uud+udu+duu\rangle_{\uparrow\downarrow\uparrow} \tag{2}$$

from which we may define the distribution amplitudes ϕ_S, ϕ_A, and ϕ_Δ by, for example,

$$\phi_\Delta^{\rm BL}(x_i)=\int[d^2k_T]\psi_\Delta(x_i,k_{iT}) \tag{3}$$

in the manner favored by Brodsky and Lepage where the normalization follows from

$$\int[dx][d^2k_T]|\psi_\Delta(x_i,k_{iT})|^2=P_{3q} \tag{4}$$

and where P_{3q} is the probability of finding the three-quark Fock component of the Δ. Note that ϕ_Δ, like ϕ_S, is not forced to be totally symmetric although it must be symmetric under the interchange of x_1 and x_3.

It is sometimes useful, particularly when discussing moments, to use a differently normalized distribution amplitude following Chernyak and Zhitnitsky wherein

$$\int[dx]\phi_\Delta^{\rm CZ}(x_i)=1 \ . \tag{5}$$

The two notations are related by a constant f_Δ which can be defined by

$$\phi_\Delta^{\rm BL}=(f_\Delta/\sqrt{8})\phi_\Delta^{\rm CZ} \ . \tag{6}$$

Interesting questions about ϕ_Δ include what is its width compared to the asymptotic distribution amplitude $x_1x_2x_3$, what is the actual overall symmetry of ϕ_Δ, and what is its normalization or the value of f_Δ?

We will adopt the notation of Ref. 7 as much as possible. The starting point is the matrix elements of three-quark fields between the vacuum and Δ^+ in light-cone gauge:

$$\langle 0|u_\alpha^i(z_1)u_\beta^j(z_2)d_\gamma^k(z_3)\epsilon_{ijk}|\Delta(p,\lambda)\rangle=\tfrac{1}{4}f_\Delta\{(\gamma_\mu C)_{\alpha\beta}N_\gamma^\mu(p,\lambda)m_\Delta V(z_i\cdot p)+(\gamma_\mu\gamma_5C)_{\alpha\beta}[\gamma_5N^\mu(p,\lambda)]_\gamma m_\Delta A(z_i\cdot p)$$
$$+(i\sigma_{\mu\nu}p^\nu C)_{\alpha\beta}N_\gamma^\mu(p,\lambda)T(z_i\cdot p)\} \ , \tag{7}$$

where z_1, z_2, and z_3 are on the same light line and N_γ^μ is the Rarita-Schwinger wave function of the Δ. (For future use, we can note that if we were interested in a negative-parity spin-$\frac{3}{2}$ state we could use the same expression save for multiplying each Rarita-Schwinger wave function by γ_5.) Functions V and T are symmetric and A antisymmetric in their first two arguments and each can be Fourier transformed as

$$V(x_1,x_2,x_3)\delta\left[1-\sum x_j\right]=\int\prod_j\left[\frac{d(z_j\cdot p)}{2\pi}\right]\exp\left[i\sum x_i(z_i\cdot p)\right]V(z_i\cdot p) \ . \tag{8}$$

Also, since the Δ is isospin $\frac{3}{2}$, only one of the three functions is independent and

$$T(x_1,x_2,x_3)\equiv T(123)$$
$$=-V(123)+V(231)+V(312) \ , \tag{9a}$$

$$A(123)=V(312)-V(231) \ . \tag{9b}$$

The reader will doubtless be able to show that

$$\phi_\Delta^{\rm CZ}(123)=V(123)-A(123) \ . \tag{10}$$

We cannot expect to calculate the full wave function using perturbative QCD, but we might expect that the wave function and some of its derivatives at short range are amenable to calculation. In momentum space this means that some of the lower moments might be amenable to calculation, and we note that

$$V(z_i\cdot p=0)=\int[dx]V(x_1,x_2,x_3)\equiv V^{(0,0,0)} \tag{11}$$

or generally

$$V^{(n_1,n_2,n_3)}\equiv\int[dx]x_1^{n_1}x_2^{n_2}x_3^{n_3}V(x_1,x_2,x_3)$$
$$=(z\cdot p)^{-N}\prod(iz\cdot\partial_j)^{n_j}V(z_i\cdot p)|_0 \ , \tag{12}$$

where $N=n_1+n_2+n_3$ and z is a lightlike vector with $z\cdot q=q^+=q^0+q^3$ for any vector q.

We use the QCD sum-rule method to get the moments. An operator $\hat{V}$ that is useful in projecting out the moments is

$$\hat{V}_\tau^{(n_1,n_2,n_3)}=[(iz\cdot D)^{n_1}u]^iC\gamma\cdot z[(iz\cdot D)^{n_2}u]^j$$
$$\times[(iz\cdot D)^{n_3}d]_\tau^k\epsilon_{ijk} \ , \tag{13}$$

where D is the covariant derivative, $D=\partial-igt\cdot A$. (Again, if we were interested in a different parity state, we could insert a γ_5 just before the d field.) Matrix elements are

$$\langle 0|\hat{V}_t^{(\bar n)}|\Delta(p,\lambda)\rangle=-[m_\Delta f_\Delta z\cdot N_\tau(p,\lambda)(z\cdot p)^N]V^{(\bar n)} \tag{14}$$

with $(\bar n)=(n_1,n_2,n_3)$. Next we need to define a vacuum expectation value or "correlator" that we shall evaluate in two ways, once by saturating it with baryons and once by saturating it with quarks. We choose

$$J^{(\bar n,n')}=i\int d^4y\, e^{iqy}\langle 0\,|\,T\hat V_\tau^{(\bar n)}(y)\bar J_{\tau'}^{(n')}(0)\,|\,0\rangle \not z_{\tau'\tau}\ , \tag{15}$$

where the auxiliary operator is

$$J_\tau^{(n)}=\{a[(iz\cdot D)^n u]^i C\not z u^j d_\tau^k + b[(iz\cdot D)^n u]^i C\not z d^j u_\tau^k + c[(iz\cdot D)^n d]^i C\not z u^j u_\tau^k\}\epsilon_{ijk}\ . \tag{16}$$

We choose the constants $a=b=c=\frac{1}{4}$ to make this operator isospin $\frac{3}{2}$ for the Δ calculation. Choosing $a=-b=1$ and $c=0$ makes the operator isospin $\frac{1}{2}$ and is useful for checking against the nucleon calculation of King and Sachrajda.

B. Resonance saturation

If we saturate the correlator with ordinary hadrons, we get $(N'=N+n')$

$$(1/\pi)\mathrm{Im}J^{(\bar n,n')}=r^{(\bar n,n')}(z\cdot q)^{N'+3}\delta_+(q^2-m_\Delta^2) + \text{other resonances} + \text{continuum}\ , \tag{17}$$

where the residue is

$$r^{(\bar n,n')}=2\,|\,f_\Delta\,|^2V^{(\bar n)}V^{(n',0,0)}\ . \tag{18}$$

[Equation (5) implies $V^{(0,0,0)}=1$.]

C. Quark saturation

The Feynman diagrams corresponding to a quark saturation of the correlator are shown in Figs. 1–3 and we shall evaluate them in turn. Figures 2 and 3 involve situations where the nonperturbatively calculated structure of the vacuum is parametrized in terms of gluon density and quark density matrix elements which are taken from other studies.

The purely perturbative evaluation (Fig. 1) can be done in either momentum or coordinate space and the terms that will contribute to the imaginary part are

$$J_1^{(\bar n,n')}=(1/\pi^4)c_1^{(\bar n,n')}(z\cdot q)^{N'+3}q^2\ln(-q^2/\Lambda^2)\ . \tag{19}$$

We have calculated c_1 for the auxiliary operator having zero or one derivative and tabulated the results in Table I. For zero derivatives the results are the same for nucleon and Δ and we use the notation

$$a_i^{(\bar n)}=c_i^{(\bar n,0)}\ ; \tag{20}$$

for one derivative, the nucleon case is given for completeness and checks against King and Sachrajda and we are using the notation

$$b_i^{(\bar n)}=c_i^{(\bar n,1)}\ . \tag{21}$$

The diagrams of Fig. 2 are calculated in fixed-point gauge $y_\nu A^\nu(y)=0$ so that gluons attach at only one of the vertices. The calculation except for the parametrization of the gluon density in the vacuum is perturbative and yields

$$J_2^{(\bar n,n')}=(1/\pi^2)c_2^{(\bar n,n')}(z\cdot q)^{N'+3}q^{-2}\langle(\alpha_s/\pi)G^2\rangle\ , \tag{22}$$

where the last piece of notation means $\langle 0\,|\,(\alpha_s/\pi)G_{\mu\nu}(0)G^{\mu\nu}(0)\,|\,0\rangle$. The results for c_2 are also given in Table I.

Finally we evaluate the diagrams in Fig. 3 which contain four quark lines or else two quark lines and a gluon line disappearing into or emerging from the vacuum. The gluon equation of motion may be used to express both cases in terms of $\langle\bar qq\rangle^2$. The result is

$$J_3^{(\bar n,n')}=(1/\pi)c_3^{(\bar n,n')}(z\cdot q)^{N'+3}q^{-4}\langle\sqrt{\alpha_s}\bar qq\rangle^2 \tag{23}$$

and the coefficients c_3 are also in Table I.

D. Matching and moments

The two ways of calculating the same correlator should give the same results. Thus (displaying the imaginary parts for both evaluations of the correlator)

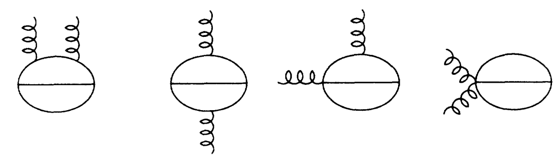

FIG. 2. Diagrams for the quark evaluation of the correlator.

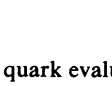

FIG. 1. Diagram for the quark evaluation of the correlator.

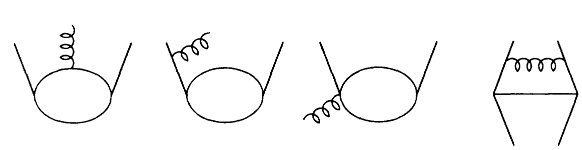

FIG. 3. Diagrams for the quark evaluation of the correlator.

$$\frac{1}{\pi}\text{Im}\left[\frac{c_1}{\pi^4}q^2\ln(-q^2/\Lambda^2)+\frac{c_2}{\pi^2}\frac{1}{q^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle+\frac{c_3}{\pi}\frac{1}{q^4}\langle\sqrt{\alpha_s}\bar{q}q\rangle^2\right]\approx r\delta_+(q^2-m_\Delta^2)-\frac{c_1}{\pi^4}q^2\theta(q^2-s_0)\ , \tag{24}$$

where the contribution of higher resonances and the continuum is approximated by the perturbative contribution above some threshold s_0. Superscripts on the c_i, r, and s_0 are implied. The two sides of the above equation cannot be matched as they stand, but a smoothing will be done with some extra weighting in the smoothing given to lower q^2 contributions. Each side will be Borel transformed, where the Borel transform of a function $f(q^2)$ is given by

$$\mathcal{B}f(M^2)\equiv\frac{1}{\pi}\int_0^\infty dq^2 e^{-q^2/M^2}\text{Im}f(q^2)\ . \tag{25}$$

Then, transferring one term to the opposite side of the equation, we have

$$-\frac{c_1}{\pi^4}M^4[1-(1+s_0/M^2)e^{-s_0/M^2}]-\frac{c_2}{\pi^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle+\frac{c_3}{\pi}\frac{1}{M^2}\langle\sqrt{\alpha_s}\bar{q}q\rangle^2=re^{-m_\Delta^2/M^2}\ . \tag{26}$$

Matching the left-hand side and the right-hand side is now possible. However, and unlike the nucleon case, a stable match (i.e., one that works over some reasonable range of M^2) cannot be obtained for all the moments studied. A stable match is always obtained when c_2 and c_3 have opposite sign. The results in terms of f_Δ and moments of V are given in Table II. Also included in Table II are the corresponding quantities from the nucleon case taken (and confirmed) from King and Sachrajda, and the moments that follow from the asymptotic distribution[5,6,8] amplitude $\phi_a=120x_1x_2x_3$.

We can summarize here the nontrivial stable moments for the Δ:

$$V^{(001)}=0.35\pm0.02\ ,$$
$$V^{(002)}=0.16\pm0.02\ , \tag{27}$$
$$V^{(101)}=0.092\pm0.02\ .$$

These three moments are dependent since

TABLE I. Coefficients for the quark evaluation of the correlators for the nucleon and delta. For the case of no derivatives in the correlator, the nucleon and Δ coefficients are the same.

	a_1	$b_1(N)$	$b_1(\Delta)$	a_2	$b_2(N)$	$b_2(\Delta)$	a_3	$b_3(N)$	$b_3(\Delta)$
$V^{(0,0,0)}$	$-\frac{1}{160}$			$-\frac{1}{288}$			$\frac{2}{9}$		
		$-\frac{1}{480}$			$-\frac{1}{576}$			$\frac{44}{243}$	
			$-\frac{1}{480}$			$-\frac{1}{1152}$			$\frac{8}{243}$
$V^{(1,0,0)}$	$-\frac{1}{480}$			$-\frac{1}{1152}$			$\frac{2}{81}$		
		$-\frac{1}{1344}$			$-\frac{1}{1920}$			$\frac{77}{972}$	
			$-\frac{19}{26\,880}$			$-\frac{1}{5760}$			$-\frac{29}{3888}$
$V^{(0,0,1)}$	$-\frac{1}{480}$			$-\frac{1}{576}$			$\frac{14}{81}$		
		$-\frac{1}{1680}$			$-\frac{1}{1440}$			$\frac{11}{486}$	
			$-\frac{3}{4480}$			$-\frac{1}{1920}$			$\frac{31}{648}$
$V^{(2,0,0)}$	$-\frac{1}{1120}$			$-\frac{1}{1440}$			$\frac{2}{81}$		
		$-\frac{3}{8960}$			$-\frac{7}{17\,280}$			$\frac{26}{405}$	
			$-\frac{11}{35\,840}$			$-\frac{11}{69\,120}$			$-\frac{13}{3240}$
$V^{(0,0,2)}$	$-\frac{1}{1120}$			$-\frac{1}{720}$			$\frac{4}{27}$		
		$-\frac{1}{4480}$			$-\frac{1}{2160}$			$\frac{2}{135}$	
			$-\frac{1}{3584}$			$-\frac{1}{2304}$			$\frac{23}{540}$
$V^{(1,1,0)}$	$-\frac{1}{1680}$			0			$-\frac{1}{81}$		
		$-\frac{1}{4480}$			0			$\frac{1}{90}$	
			$-\frac{11}{53\,760}$			$\frac{1}{34\,560}$			$-\frac{59}{9720}$
$V^{(1,0,1)}$	$-\frac{1}{1680}$			$-\frac{1}{5760}$			$\frac{1}{81}$		
		$-\frac{1}{5376}$			$-\frac{1}{8640}$			$\frac{19}{4860}$	
			$-\frac{1}{5120}$			$-\frac{1}{23\,040}$			$\frac{17}{6480}$

TABLE II. Moments of the distribution amplitude with normalization $V^{(0,0,0)}=1$. Typical errors are ±0.02 or ±10%, whichever is larger, for each moment. The constants f_i are given in units of 10^{-3} GeV^2.

Moment	Nucleon	Δ(1232)	$S_{11}(1535)$	$\{\mathcal{P}=-,I=\frac{3}{2}\}$	Asymptotic
$V^{(0,0,0)}$	1.00	1.00	1.00	1.00	1.00
$V^{(1,0,0)}$	0.38	Unstable	0.36	0.33	$\frac{1}{3}$
$V^{(0,0,1)}$	0.24	0.35	0.27	0.34	$\frac{1}{3}$
$V^{(2,0,0)}$	0.22	Unstable	0.17	0.14	$\frac{1}{7}$
$V^{(0,0,2)}$	0.12	0.16	0.11	0.15	$\frac{1}{7}$
$V^{(1,1,0)}$	0.10	Unstable	0.10	0.09	$\frac{2}{21}$
$V^{(1,0,1)}$	0.06	0.09	0.08	0.09	$\frac{2}{21}$
$A^{(1,0,0)}$	−0.17	Unstable	−0.08	Unstable	0
$A^{(2,0,0)}$	−0.13	Unstable	−0.06	Unstable	0
$A^{(1,0,1)}$	−0.04	Unstable	−0.02	Unstable	0
f_i	5.1	11.5	10.2	17.0	

$V^{(001)}=V^{(002)}+2\times V^{(101)}$, which within errors is compatible with the above results.

The remarkable feature of the nucleon was the asymmetry discovered in the distribution amplitude. No asymmetry, given our uncertainties and missing moments, is forced by the Δ results. Recall that symmetry in the first and third arguments of the Δ distribution amplitude is required but that in these light-cone frame quantities total symmetry is not. However, we seem to have it.

E. Distribution amplitude

The moments may be used to give a model distribution amplitude for the Δ. The standard technique is to expand in Appell polynomials as

$$\phi_\Delta^{CZ}=120x_1x_2x_3\sum B_i\tilde{\phi}_i(x_1,x_2,x_3)\ . \tag{28}$$

The sum is over only the symmetric Appell polymonials, so that B_1 and B_4 are zero, and the factor 120 ensures that $B_0=1$. In general, the linear moments determine the coefficients of the linear Appell polynomials B_1 and B_2, and then the quadratic moments determine B_3, B_4, and B_5, the coefficients for the quadratic Appell polynomials, etc. We have[9]

$$\phi_\Delta^{CZ(100)}=V^{(001)}=\tfrac{1}{3}-\tfrac{1}{21}B_2=0.35\pm0.02\ . \tag{29}$$

Thus,

$$B_2=21(\tfrac{1}{3}-V^{(001)})=-0.35\pm0.42\ . \tag{30}$$

For the quadratic polynomials we have one remaining independent moment and two coefficients to determine. If we suppose that the quadratic terms of the distribution amplitude are totally symmetric (at some relevant scale), then[10] $B_3=\frac{7}{3}B_5$ and[9]

$$B_3=\tfrac{189}{20}(1-7\phi_\Delta^{CZ(100)}+\tfrac{28}{3}\phi_\Delta^{CZ(200)}) \tag{31}$$

leads to

$$B_5=0.18\pm1.32 \tag{32a}$$

and

$$B_3=0.41\pm3.09\ . \tag{32b}$$

Taking central values, this result in Brodsky-Lepage notation is

$$\phi_\Delta^{BL}=x_1x_2x_3(0.34\tilde{\phi}_0-0.12\tilde{\phi}_2+0.14\tilde{\phi}_3+0.06\tilde{\phi}_5)\ . \tag{33}$$

For comparison, the symmetric part of the King-Sachrajda amplitude is[7]

$$\phi_s=x_1x_2x_3(0.11\tilde{\phi}_0-0.14\tilde{\phi}_2+0.44\tilde{\phi}_3+0.11\tilde{\phi}_5)\ . \tag{34}$$

F. N-Δ transition with data

The consequences of the Δ distribution amplitude for the leading $N\rightarrow\Delta$ electromagnetic transition amplitude can be determined using formulas in Ref. 3. That reference gave results in terms of a certain $G_{Mp\Delta^+}$ chosen to be analogous to the proton elastic G_{Mp}. The amplitude $A_{1/2}$ is equivalent and probably more common. We have

$$\begin{aligned}Q^3A_{1/2}&=e[2m_N(m_\Delta^2-m_N^2)]^{-1/2}Q^4G_{Mp\Delta^+}\\&=0.277\ \text{GeV}^{-3/2}Q^4G_{Mp\Delta^+}\end{aligned} \tag{35}$$

and for several choices of the nucleon distribution amplitude and the present Δ distribution amplitude, we get

$$Q^3\,|\,A_{1/2}\,|=\begin{cases}0.02\ \text{GeV}^{5/2}\ \ (\text{Chernyak-Zhitnitsky}^{5,6})\ ,\\0.03\ \text{GeV}^{5/2}\ \ (\text{King-Sachrajda}^{7})\ ,\\0.17\ \text{GeV}^{5/2}\ \ (\text{Gari-Stefanis}^{11})\ .\end{cases} \tag{36}$$

We used $\alpha_s=0.3$. The result is small if we use the Chernyak-Zhitnitsky or King-Sachrajda nucleon distribution amplitudes, which are distribution amplitudes based on the QCD sum-rule moment results for the nucleon. The smallness has to do with cancellations between the symmetric and antisymmetric parts of the nucleon amplitude which would occur for almost any Δ wave function. For the Gari-Stefanis[11] distribution amplitude, which was partly inspired by a particular analysis[12] of the data[13] hinting that the neutron magnetic form factor is small, the result is typical of what one sees experimentally for other nucleon-resonance transitions.[4] (In all cases the result is roughly what was estimated earlier when the Δ amplitude was guessed to be the same as the symmetric part of the nucleon one, and the "anticorrelation" between G_{Mn} and $Q^3 A_{1/2}$ for the $N \rightarrow \Delta$ still holds.[14] Also, if the Δ distribution amplitude is purely asymptotic with our f_Δ the Gari-Stefanis result becomes $\frac{1}{2}$ of what is given above.)

Data for the Δ are shown in Fig. 4. It is based[15] on cross-section measurements for $e+N \rightarrow e'+X$, with the background subtracted incoherently. If $A_{3/2}$ is small compared to $A_{1/2}$, which one should understand is predicted by perturbative QCD (PQCD) but not yet established experimentally, then the central values of $Q^3 A_{1/2}$ fall between Q^2 of 3 and 6 GeV^2. The falloff is by a factor of 2 if squared and is clearly visible in the falling peak-to-background ratio in the data. In contrast, for the other resonance bumps $Q^3 A_{1/2}$ is roughly constant at higher Q^2 and the peak-to-background ratio is also roughly constant, both in accord with PQCD. However, a constant $Q^3 A_{1/2}$ for the $\Delta(1232)$ is still compatible with the edges of the error bars, so that while awaiting more precise data, higher Q^2 data, or separated data for the individual amplitudes, we should not conclude if $Q^3 A_{1/2}$ for the $N \rightarrow \Delta$ transition has already stabilized at a normal sized value or has yet to stabilize at a lower value.

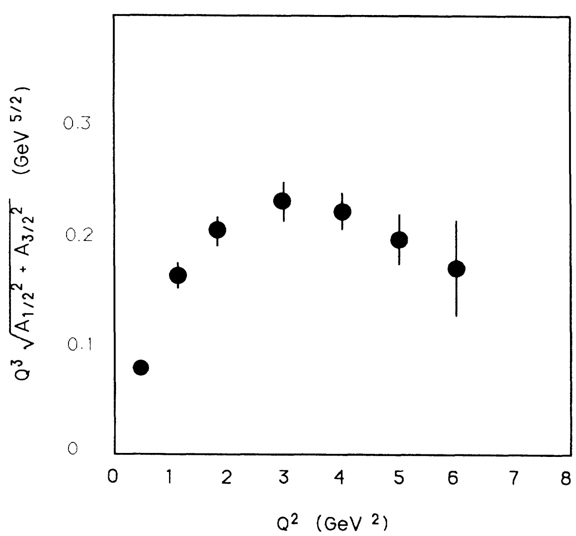

FIG. 4. Plot of $Q^3(A_{1/2}^2 + A_{3/2}^2)^{1/2}$ for the $\Delta(1232)$. Perturbative QCD predicts that $A_{1/2}$ falls like $1/Q^3$ and $A_{3/2}$ falls like $1/Q^5$ (both modulo ln Q^2 to a power) if we are in the asymptotic region. The data for Figs. 4–6 is from Ref. 15.

III. THE $S_{11}(1535)$

This section is devoted to the ground-state negative-parity $I=\frac{1}{2}$ baryon. There are two candidates for this title: the $D_{13}(1520)$ and the $S_{11}(1535)$. The light-cone formalism distinguishes helicities but not total angular momentum, so that the state we will consider is really a linear combination of these two. However, experiments at lower Q^2 where a separation of amplitudes is possible show that the cross section at increasing Q^2 is dominantly the S_{11} and so we shall call our state by that name.

A. Kinematics, etc.

The S_{11} has the same spin as the nucleon but the opposite parity. Hence we can use operators to define and project V, A, and T that are the same as the nucleon ones except for removing some factors of γ_5 where they exist and inserting them in some places where they do not. The γ_5's cancel in actual evaluations, so that the end result for the coefficients is the same as for the nucleon case. The matching is changed because the mass put in for the baryon is different, and we need to use a larger s_0 to get stable solutions with both the nonperturbative contributions and continuum contributions below 25% simultaneously. The best results for the moments are shown in Table II. The moments still show the asymmetry that made the nucleon case so remarkable, but the effect is less great at the higher mass.

B. Distribution amplitude

Fitting the central values of the moments to the first six Appell polynomials gives a distribution amplitude

$$\phi(S_{11})^{\mathrm{CZ}} = 120x_1x_2x_3(\tilde{\phi}_0 + 1.68\tilde{\phi}_1 - 0.56\tilde{\phi}_2 + 0.63\tilde{\phi}_3 + 0.27\tilde{\phi}_5) \quad (37)$$

(coefficient B_4 came out zero). Compared to the nucleon case, the higher-order Appell polynomials are less important. Coefficients for both quadratic compared to linear polynomials and linear compared to zeroth-order polynomials have decreased.

C. Transition amplitude and data

The expression for $G_{Mp \rightarrow S_{11}}$ is similar to the expression for the magnetic form factor of the proton and is given in terms of the Brodsky-Lepage style B_i in Ref. 16. They are there called N_i and one needs to symmetrize the expression

$$N_i N_j \rightarrow [N_i(p)N_j(S_{11}) + N_j(p)N_i(S_{11})]/2 \ . \quad (38)$$

With

$$Q^3 A_{1/2} = 0.182 \ \mathrm{GeV}^{-3/2} Q^4 G_{Mp \rightarrow S_{11}} \quad (39)$$

one gets

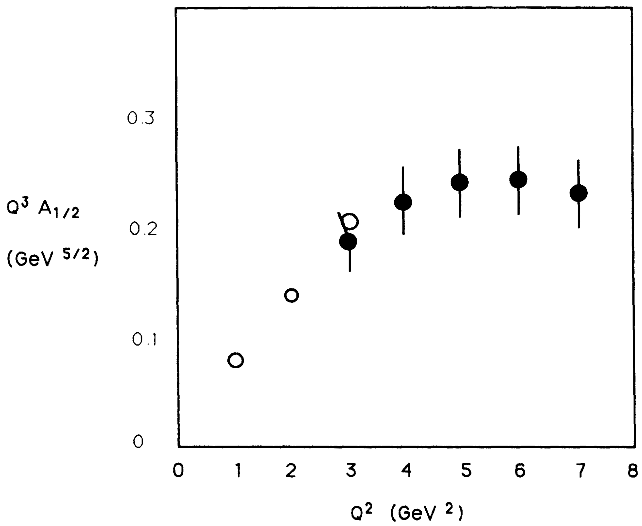

FIG. 5. Plot of $Q^3 A_{1/2}$ for the $S_{11}(1535)$. The open circles are separated data that are just the S_{11} and the high Q^2 points are from total-cross-section data using an assumption justified from the low Q^2 data that the S_{11} dominates the total cross section in this bump.

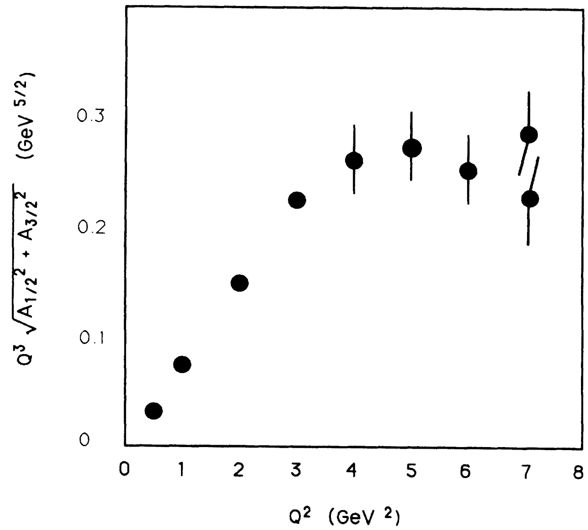

FIG. 6. Plot of $Q^3(A_{1/2}^2+A_{3/2}^2)^{1/2}$ for the bump around 1688 MeV. A number of resonances lie under this bump, and the total should scale the same way with Q^2 as the individual contributions.

$$Q^3\,|\,A_{1/2}(p\to S_{11})\,| = \begin{cases} 0.11\ \mathrm{GeV}^{5/2} & (\text{Chernyak-Zhitnitsky})\,, \\ 0.14\ \mathrm{GeV}^{5/2} & (\text{King-Sachrajda})\,, \\ 0.12\ \mathrm{GeV}^{5/2} & (\text{Gari-Stefanis}) \end{cases} \tag{40}$$

at high Q^2 and for the same selection of nucleon distribution amplitudes as before. The error range on the moments allows large variations in the B_i and the $A_{1/2}$'s quoted above may be as much as a factor of 3 larger (or they may be smaller). The available data are shown in Fig. 5.

In case data for the transition to the neutral S_{11} from a neutron target become available, we wish to quote the results

$$Q^3\,|\,A_{1/2}(n\to S_{11})\,| = \begin{cases} 0.08\ \mathrm{GeV}^{5/2} & (\text{Chernyak-Zhitnitsky})\,, \\ 0.06\ \mathrm{GeV}^{5/2} & (\text{King-Sachrajda})\,, \\ 0.05\ \mathrm{GeV}^{5/2} & (\text{Gari-Stefanis})\,. \end{cases} \tag{41}$$

IV. THE $I=\frac{3}{2}$ NEGATIVE-PARITY STATE

This state bears the same relation to the $\Delta(1232)$ that the $I=\frac{1}{2}$ negative-parity state bears to the nucleon and so the bulk of the calculation is already done. The candidates for this state are the $S_{31}(1650)$ and the $D_{33}(1670)$, and there is no evidence that either dominates the other or that they collectively dominate the cross section for the 1688 (nominal) bump. The matching procedure using a mass of 1660 MeV and an s_0 of 6 GeV^2 leads to moments (Table II) that are compatible with an asymptotic distribution amplitude, and the transition amplitude from the nucleon is small. The bump seen at 1688 is at high Q^2 very possibly dominated by $I=\frac{1}{2}$ resonances and unless the individual contributions can be separated is not relevant to this section. There is still some interest in seeing the data plotted to check the $1/Q^3$ scaling of the leading amplitude and so we present Fig. 6.

V. CONCLUSIONS

The Δ distribution amplitude has the same symmetry as the symmetric (under the interchange $x_1 \rightleftarrows x_3$ if 1 and 3 represent the parallel helicity quarks) part of the nucleon distribution amplitude. In the nucleon case all of the moment sum rules led to stable results, so that there were four pieces of information up to the quadratic moments—one normalization and three independent moments. In the Δ case some of the moment sum rules do not give stable results, so we lose some information on the moments and some of the cross-checks that helped reduce the uncertainties. However, we are still left with the normalization and two independent moments.

The moments are consistent with the distribution amplitude being totally symmetric and even with the distribution amplitude already having its ultimate asymptotic form. If we expand the distribution amplitude in Appell polynomials and fit to the moments, then while the asymptotic form is very possible, the best fit has a significant amount of quadratic polynomial mixed in.

For the $S_{11}(1535)$ the asymmetry seen in the nucleon case is still plainly there although the amount of higher-order polynomial in the distribution amplitude has decreased. For the lowest negative-parity $I=\frac{3}{2}$ state the distribution amplitude is compatible with being asymptotic.

For the leading nucleon resonance electroproduction transition amplitudes, the results from perturbative QCD and the distribution amplitudes based on the sum rules is in the ballpark of the data.

The high Q^2 $N\to\Delta$ transition of course depends on the nucleon as well as the Δ distribution amplitude. The Δ amplitude is important for setting the overall scale, and the relative sign of the symmetric and antisymmetric parts of the nucleon amplitude leads to self-cancellations and small asymptotic $N\to\Delta$ transitions for some distribution amplitudes such as those of Chernyak and Zhitnitsky and of King and Sachrajda. Other nucleon distribution amplitudes such as that of Gari and Stefanis give asymptotic $N\to\Delta$ transitions that are similar in size to other $N\to$resonance transitions.

If the Chernyak-Zhitnitsky or King-Sachrajda distribution amplitudes are correct for the nucleon, then there will be substantial self-cancellation in the calculation of the leading electromagnetic transition amplitude for any $I=\frac{3}{2}$ state, leading to a prediction that the $I=\frac{1}{2}$ resonances will always be more visible at high Q^2 than the $I=\frac{3}{2}$ ones.

For the nucleon to S_{11} transition amplitude there is little spread among the calculations using different nucleon distribution amplitudes and the S_{11} distribution amplitude presented here and all are in the ballpark of the data.

ACKNOWLEDGMENTS

We wish to thank V. Burkert for many useful conversations, questions, and help with the data. We have received reports of overlapping work by Farrar, Zhang, Ogloblin, and Zhitnitsky which also show the Δ distribution amplitude compatible with an asymptotic form and which calculate a number of physical processes dependent upon the distribution amplitudes for the Δ and other octet and decuplet baryons. We thank the authors for sending us these papers.

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