# Neutrino tridents, conserved vector current, and partially conserved axial-vector current

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The trident reaction $\nu+Z\to\nu+l^++l^-+Z$ is analyzed from the viewpoint of the conservation properties of the weak lepton current. The conserved part of the current yields a cross section that is related to the Bethe-Heitler process $\gamma+Z\to l^++l^-+Z$. The nonconserved part, with the divergence $\partial_\mu A_\mu=2m_l i\bar{l}\gamma_5 l$, gives an additional contribution which is expressible in terms of an axion-like reaction $a^0+Z\to l^++l^-+Z$. We calculate and compare the two pieces using an equivalent-photon approximation.

The dissociation of a neutrino into a pair of charged leptons in the Coulomb field of a nucleus (Fig. 1) has been the subject of numerous theoretical investigations.[1–5] The process was envisaged as a probe of the purely leptonic interaction $\nu+l\to\nu+l$, which we may characterize by a Lagrangian[6]

$$\mathcal{L}=-\frac{G}{\sqrt{2}}[\bar{\nu}\gamma_\alpha(1+\gamma_5)\nu][\bar{l}\gamma_\alpha(C_V+C_A\gamma_5)l] \ . \qquad (1)$$

The cross section of the trident reaction $\nu+Z\to\nu+l^++l^-+Z$ has the general form

$$\sigma=C_V^2\sigma_V+C_A^2\sigma_A+C_VC_A\sigma_I \ . \qquad (2)$$

The interference term $\sigma_I$ vanishes if one measures the total cross section or a distribution that is symmetric in $l^+$ and $l^-$ (e.g., the invariant mass). This is because the vector and axial-vector currents, combining with the Coulomb photon, produce $l^+l^-$ states with $C=+1$ and $-1$, respectively. Our concern in this paper is with the relative magnitude of $\sigma_V$ and $\sigma_A$.

The difference between $\sigma_V$ and $\sigma_A$ arises from the fact that the leptonic vector current is conserved while the axial-vector current is not:

$$\partial_\mu(\bar{l}\gamma_\mu l)=0 \ , \qquad (3a)$$

$$\partial_\mu(\bar{l}\gamma_\mu\gamma_5 l)=2m_l i\bar{l}\gamma_5 l \ . \qquad (3b)$$

[We will not be concerned here with the additional term $\alpha/2\pi F_{\mu\nu}\tilde{F}_{\mu\nu}$ in Eq. (3b) arising from the axial-vector-current anomaly.] It is natural to suppose that if the lepton mass is in some sense negligible, one will obtain $\sigma_V=\sigma_A$ and a total cross section proportional to $C_V^2+C_A^2$. This, indeed, is assumed in most approximate calculations of the process.[7] On the other hand, numerical work based on the exact matrix element reveals[5] that, while there is little discernible difference between $\sigma_V$ and $\sigma_A$ for $e^+e^-$ pairs, there is a significant difference for $\mu^+\mu^-$ production, the ratio $\sigma_A/\sigma_V$ being typically 1.2 at a neutrino energy of 50 GeV.

To quantify the difference between $\sigma_V$ and $\sigma_A$ it is expedient to write the cross section as

$$\sigma=(C_V^2+C_A^2)\sigma_{\rm CVC}+C_A^2\sigma_{\rm PCAC} \ . \qquad (4)$$

The first part treats the vector and axial-vector currents on the same footing, and may therefore be related, through current conservation, to the electromagnetic analog reaction. Denoting the initial and final neutrino momenta by $k$ and $k'$, and defining $Q^2=-(k-k')^2$, $\nu=E-E'=Ey$ ($E$ and $E'$ being the initial and final neutrino energies in the laboratory frame), the conserved-vector-current cross section $\sigma_{\rm CVC}$ is

$$\sigma_{\rm CVC}=\int dQ^2d\nu\frac{G^2Q^2}{16\pi^3\alpha}\frac{1}{(Q^2+\nu^2)^{1/2}}\times\left[1+(1-y)^2+\frac{Q^2}{2E^2}\right]\times\sigma(\gamma^*+Z\to l^++l^-+Z) \ , \qquad (5)$$

where the last factor is the Bethe-Heitler cross section for a virtual photon of mass $-Q^2$ and energy $\nu$.

Turning to the term denoted by $\sigma_{\rm PCAC}$, we observe that

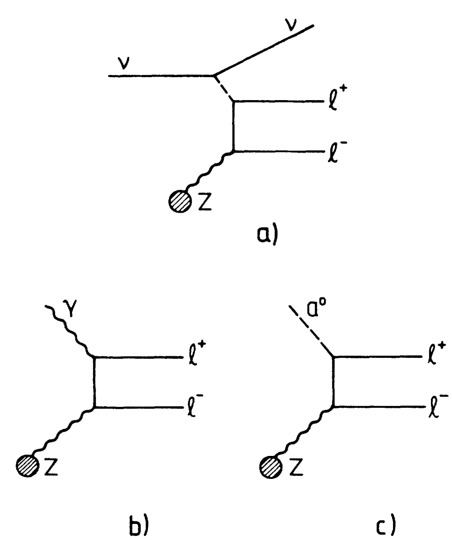


FIG. 1. Diagrams showing (a) neutrino-induced trident process, (b) pair production by photon, (c) pair production by axion.

 

this cross section, considered differentially, is purely longitudinal in character and, unlike $\sigma_{\rm CVC}$, does not vanish at $Q^2=0$. This is a reminder of the fact that in the configuration in which the neutrino is scattered forward, the cross section of any inelastic neutrino reaction is determined entirely by the divergence of the axial-vector current. According to the Adler theorem,[8] the cross section for $\nu+\alpha\to\nu+\beta$ in the forward configuration is

$$\lim_{Q^2\to 0} d\sigma(\nu+\alpha\to\nu+\beta)\propto |\langle\beta|\partial_\mu A_\mu|\alpha\rangle|^2 . \tag{6}$$

In cases where the states $\alpha$ and $\beta$ are hadronic, one is concerned with the hadronic part of the axial-vector current for which one writes

$$\partial_\mu A_\mu = f_{\pi^0} m_\pi^2 \phi_{\pi^0} , \tag{7}$$

$\phi_{\pi^0}$ denoting the pion field. The PCAC (partial conservation of axial-vector current) hypothesis allows one to rewrite $\langle\beta|\partial_\mu A_\mu|\alpha\rangle$ as $\lim_{Q^2\to 0}\langle\beta|j_\pi|\alpha\rangle$, $j_\pi$ being the source of the pion. In this manner the longitudinal axial-vector cross section is proportional to the cross section for the process $\pi^0+\alpha\to\beta$, i.e.,

$$\lim_{Q^2\to 0} d\sigma(\nu+\alpha\to\nu+\beta)\propto f_{\pi^0}^2\sigma(\pi^0+\alpha\to\beta) . \tag{8}$$

In the case of the trident reaction, we are dealing with a leptonic axial-vector current, the divergence of which is given by Eq. (3b). Imagine a massless axionlike particle which couples to leptons in accordance with the Lagrangian

$$\mathcal{L}_{\rm axion} = i\frac{m_l}{v} a^0 \bar{l}\gamma_5 l , \tag{9}$$

where $v$ is a mass scale that we need not specify further. We then notice that $\partial_\mu A_\mu = 2m_l i\bar{l}\gamma_5 l$ is just $2v$ times the source ($=j_{\rm axion}$) of the axion field appearing in Eq. (9). Accordingly, we can replace $\langle\beta|\partial_\mu A_\mu|\alpha\rangle$ in the present problem by $2v\langle\beta|j_{\rm axion}|\alpha\rangle$. It follows that the standard PCAC formula for the longitudinal axial-vector contribution to hadronic neutrino interactions can be adapted to neutrino tridents, with the substitution

$$\begin{aligned} &f_{\pi^0}\to 2v , \\ &\sigma(\pi^0+\alpha\to\beta)\to\sigma(a^0+Z\to l^+ + l^- + Z) . \end{aligned} \tag{10}$$

With this correspondence we obtain for $\sigma_{\rm PCAC}$ the result

$$\begin{aligned}\sigma_{\rm PCAC} = \int dQ^2 d\nu \frac{G^2}{\pi}\frac{1}{\nu}\left[1-\frac{\nu}{E}\right]\frac{4v^2}{\pi} \\ \times\sigma(a^{0*}+Z\to l^+ + l^- + Z) ,\end{aligned} \tag{11}$$

the axion cross section being at energy $\nu$ and virtual mass $-Q^2$.

It should be stressed that the relation (11) between $\sigma_{\rm PCAC}$ and the cross section for axion-induced lepton pair production holds regardless of whether an axionlike object exists or not.

In order to evaluate and compare Eqs. (5) and (11), we use a simplified version of these formulas based on the equivalent-photon approximation. The process is treated as neutrino scattering on a photon,[9] the spectrum of photons being given by the Weizsäcker-Williams formula. We then obtain

$$\begin{aligned}\sigma_{\rm CVC} = \int \frac{Z^2\alpha}{\pi}\frac{ds}{s}\ln\left[\frac{t_{\max}}{t_{\min}}\right] \\ \times\int dx\, dy \frac{G^2 s^2}{16\pi^3\alpha} xy[1+(1-y^2)]\sigma_{\gamma\gamma}(W^2) ,\end{aligned} \tag{12a}$$

$$\begin{aligned}\sigma_{\rm PCAC} = \int \frac{Z^2\alpha}{\pi}\frac{ds}{s}\ln\left[\frac{t_{\max}}{t_{\min}}\right] \\ \times\int dx\, dy \frac{G^2 s}{2\pi}(1-y)\frac{4v^2}{\pi}\sigma_{a^0\gamma}(W^2) .\end{aligned} \tag{12b}$$

Here, $t_{\max}=R^{-2}$, $t_{\min}=s^2/(4E^2)$, $s_{\max}=2ER^{-1}$, and $s_{\min}=4m_l^2$, $R$ being the nuclear radius. $\sigma_{\gamma\gamma}(W^2)$ and $\sigma_{a^0\gamma}(W^2)$ are the cross sections of the reactions $\gamma+\gamma\to l^+ + l^-$ and $a^0+\gamma\to l^+ + l^-$, respectively, and are given by[10,11]

$$\begin{aligned}\sigma_{\gamma\gamma}(W^2) = \frac{4\pi\alpha^2}{W^2}\Bigg[\left[1+\frac{4m_l^2}{W^2}-\frac{8m_l^4}{W^4}\right]L \\ -\left[1+\frac{4m_l^2}{W^2}\right]\left[1-\frac{4m_l^2}{W^2}\right]^{1/2}\Bigg] , \\ L = 2\ln\left[\frac{W}{2m_l}+\left[\frac{W^2}{4m_l^2}-1\right]^{1/2}\right] ,\end{aligned} \tag{13a}$$

$$\sigma_{a^0\gamma}(W^2) = \frac{2\alpha m_l^2}{v^2}\frac{1}{W^2}\ln\left[\frac{W+(W^2-4m_l^2)^{1/2}}{W-(W^2-4m_l^2)^{1/2}}\right] . \tag{13b}$$

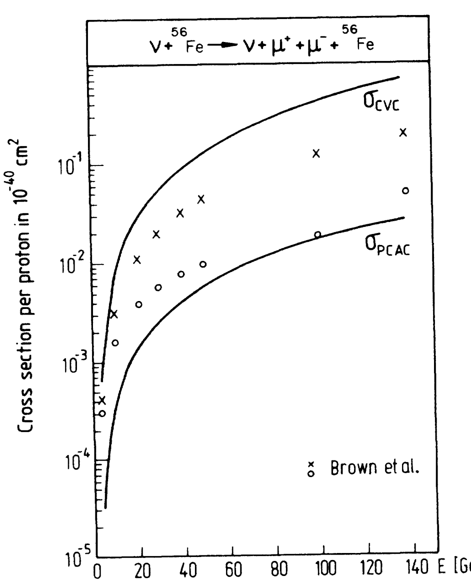


FIG. 2. Cross sections $\sigma_{\rm CVC}$ ($=\sigma_V$) and $\sigma_{\rm PCAC}$ ($=\sigma_A-\sigma_V$) as functions of neutrino energy. Also indicated are the results of Brown, Hobbs, Smith, and Stanko (Ref. 5).

The variable $W^2$ is related to $x$ and $y$ by $W^2=sy(1-x)$, the domain of integration being $4m_l^2 < W^2 < s$. Note that the scale parameter $v$ cancels out when Eq. (13b) is inserted in Eq. (12b).

Our results for $\sigma_{\rm CVC}$ and $\sigma_{\rm PCAC}$ for $\mu^+\mu^-$ production on a $^{56}$Fe target are shown in Fig. 2 as a function of the neutrino energy. It is clear that the nonconserved part of the axial-vector current makes a certain small contribution to the $\mu^+\mu^-$ cross section. We have compared our results with those of Brown, Hobbs, Smith, and Stanko[5] who computed $\sigma_V$ and $\sigma_A$ by exact treatment of the Feynman diagrams for the trident process. The agreement is reasonable, considering that we have made no allowance for the virtual nature ($Q^2\neq 0$) of the photon or axion. What is novel in our approach is the interpretation of the difference between $\sigma_V$ and $\sigma_A$ in terms of an axionlike cross section.

It is curious that although $\partial_\mu A_\mu$ is proportional to the lepton mass, the cross section $\sigma_{\rm PCAC}$ is not proportional to $m_l^2$ in all regions of phase space. It is easiest to see this by going to the forward neutrino configuration, in which the entire trident cross section is proportional to the cross section for $a^0+Z\to l^++l^-+Z$. At high energies ($E_a=\nu\to\infty$) the axion-induced cross section is[12]

$$\lim_{E_a\to\infty}\sigma(a^0+Z\to l^++l^-+Z)=\frac{1}{v^2}\frac{Z^2\alpha^2}{\pi}\left[\ln\frac{2E_a}{m_l}-\frac{5}{2}\right]. \tag{14}$$

This contains no factor of $m_l^2$ and so is just as large for $e^+e^-$ pairs as for $\mu^+\mu^-$. This extreme behavior of the trident cross section is, of course, limited to the domain $Q^2\lesssim m_l^2$. This region is a shrinking fraction of the total phase space as $m_l\to 0$. Correspondingly, the integrated cross section $\sigma_{\rm PCAC}$ is indeed small for $e^+e^-$ compared to $\mu^+\mu^-$ production.

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