Tests of compositeness at e^+e^- colliders

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(Received 13 May 1988)

In this paper we carry out a systematic study of the general phenomenological $ZZ\gamma$ coupling and derive the helicity amplitudes for γZ production. We examine the reaction $e^+e^- \to \gamma\mu^+\mu^-$ with polarized beams and search for signals of effects beyond the standard model. The results obtained may be regarded as further evidence in support of the usefulness of polarized beams.

I. INTRODUCTION

So far the $SU(2)\times U(1)$ electroweak theory has survived all tests and it shows a remarkable agreement with the experimental data (see Ref. 1 for a review). However, there are well-known reasons to look beyond the standard model and, over the last few years, we have seen the development of two main concepts attempting to do it. The first one—supersymmetry, supergravity, etc.—is based, essentially, on the extension of the gauge principle to the boson-fermion symmetry. The second attempt—compositeness—proposes a further and deeper layer of constituents in the structure of matter.

The phenomenological consequences of the first extrapolating route have been analyzed by several authors (see Ref. 2, and references therein). On the contrary, tests of compositeness (for a review see Ref. 3) did not attract a comparable amount of work yet. This reflects the current fashion in theoretical physics but this fashion could be a prejudice against the analysis of the forthcoming experimental results of the new colliders.

Clearly, if the scale associated with the substructure is of the order of the grand unification scale or higher, the theory, even at energies of the CERN e^+e^- collider LEP II, will be completely indistinguishable from a renormalized gauge model. Such a conservative version of compositeness tries to explain the spectrum of quarks and leptons in terms of constituents (haplons, for instance) but maintains the W and Z particles as elementary gauge bosons. On the other hand, there are more radical models[4,5] where even the W and Z are composite particles. Then, the compositeness scale is of the order of a few hundreds of GeV and, in this case, deviations from the standard model should be observed in the next run of collider experiments. In fact, these models imply (i) the existence of other spin-1 bosons and, in particular, of a weak-isospin singlet, and (ii) different $WW\gamma$, $ZZ\gamma$, and $Z\gamma\gamma$ couplings.

The influence of extra spin-1 particles in the M_W/M_Z mass shift, in the partial width $\Gamma(Z\to e^+e^-)$, and in the forward-backward and left-right asymmetries have been studied by several authors.[6] Recently, three different groups[7,8] have also reexamined the W pair production in e^+e^- collisions. However, such systematic study of the $ZZ\gamma$ and $Z\gamma\gamma$ vertices has not yet been done. This is our aim.

The pioneer work in this field is a paper due to Renard[9] where it is explicitly shown that an anomalous Z coupling will be reflected in the photon spectrum of the $e^+e^-\to\gamma\mu^+\mu^-$ reaction. After that, the $ZZ\gamma$ and $Z\gamma\gamma$ three-point functions were correctly calculated[10] in the standard model and, in 1987, the $e^+e^-\to\gamma Z$ reaction was used[5,11] as a testing ground of compositeness. With a similar purpose we[12] have pointed out the importance of the neutrino-counting reaction. In this paper we follow this work and extend it. In Sec. II we discuss the question of extracting information about the Z polarizations from a measurement of the differential cross section $e^+e^-\to\gamma f\bar{f}$, where the invariant mass of the final fermion pair is in the vicinity of M_Z. In particular, we show that it is possible to separate the contribution due to longitudinal Z's. This could be important because a deviation from the standard model is more likely to be shown there. Notice that, in the standard model, it requires the cancellation mechanism of a gauge structure to prevent the cross section for the production of a longitudinal Z to rise with the c.m. energy. In Sec. III we study the general phenomenological $ZZ\gamma$ coupling and derive the helicity amplitudes for γZ production. Then, in Sec. IV we consider polarized beams. Our aim is to search for signals of effects beyond the standard model and the results obtained can be regarded as a further evidence in support of beam polarization. Finally, Sec. V discusses our results which are summarized in Sec. VI.

II. $e^+e^-\to\gamma\mu^+\mu^-$ vs $e^+e^-\to\gamma Z$

It is clear that the Z particle produced in a e^+e^- collision is not directly observed since it decays promptly into a fermion-antifermion pair. Then, any information about the γZ production has to be obtained from a study of the reactions $e^+e^-\to\gamma f\bar{f}$.

Let us consider, in particular, the $\gamma\,\mu^+\mu^-$ channel. At tree level and for center-of-mass (c.m.) energies above the Z threshold one can neglect the diagrams where the final-state fermions do not couple to the Z resonance. Hence, the amplitude can be written in the form

$$T(\gamma\mu^+\mu^-)=J_\mu(p)(g^{\mu\nu}-p^\mu p^\nu/M_Z^2)J'_\nu(p)(m^2-M_Z'^2)^{-1}\,, \tag{1}$$

where J and J' are the currents coupled to the Z in the processes γZ^* production and $Z^* \to \mu^+\mu^-$, respectively, p is the momentum of the muon pair, $m^2=p^2$ its invariant mass and

$$M_Z' = M_Z - i\Gamma_Z/2 \tag{2}$$

with Γ_Z the Z-boson width. In the c.m. of the colliding beams the differential cross section $d\sigma(\gamma\mu^+\mu^-)$ is

$$d\sigma(\gamma\mu^+\mu^-) = \frac{|J\cdot J'|^2}{4\pi s}\frac{dQ_{\gamma Z}dQ_{\mu\mu}dm^2}{|m^2-M_Z'^2|^2} \tag{3}$$

with

$$dQ_{\gamma Z} = d\Omega_\gamma \omega/16\pi^2\sqrt{s} \ , \tag{4a}$$

$$dQ_{\mu\mu} = d\Omega_\mu q/16\pi^2 m \ , \tag{4b}$$

and

$$m^2 = s - 2\sqrt{s}\,\omega \ , \tag{4c}$$

where Ω_γ and ω are the photon solid angle and energy, respectively, and $\Omega_\mu(\theta,\phi)$ and q are the solid angle and the momentum of the μ^- in the $\mu^+\mu^-$ c.m. frame. Because of our interest in the study of the Z boson we calculate the cross section for an on-shell Z; i.e., we integrate over the photon spectrum and everywhere else we set $m^2=M_Z^2$. Therefore, we obtain

$$d\sigma(\gamma\mu^+\mu^-) = \beta \sum d\sigma_{\lambda\lambda'} d\Gamma_{\lambda\lambda'} \tag{5}$$

with $(m = M_Z)$

$$d\sigma_{\lambda\lambda'} = T_\lambda T^*_{\lambda'} dQ_{\gamma Z}/2s \ , \tag{6}$$

$$d\Gamma_{\lambda\lambda'} = T'_\lambda T'^*_{\lambda'} dQ_{\mu\mu}/2M_Z\Gamma_Z \ , \tag{7}$$

and

$$\beta = (M_Z\Gamma_Z/\pi)\int dm^2/|m^2-M_Z'^2|^2 \ . \tag{8a}$$

T and T' are the amplitudes for the γZ^* production and Z^* decay, respectively, and $\lambda=0,\pm1$ denote the helicity of the Z^*. Obviously, when $\lambda=\lambda'$ $d\sigma_{\lambda\lambda'}$ is the production cross section for a Z with helicity λ and $\Gamma_{\lambda\lambda}$ is the branching fraction for its decay into $\mu^+\mu^-$. In the zero-width limit $\beta=1$ and a better approximation valid up to order $(\Gamma_Z/M_Z)^3 \approx 10^{-6}$ is

$$\beta = 1 - s\Gamma_Z/[\pi M_Z(s-M_Z^2)] + O((\Gamma_Z/M_Z)^3) \ . \tag{8b}$$

From a measurement of $d\sigma(\gamma\mu^+\mu^-)$ we would like to obtain the values of the tensor $\sigma_{\lambda\lambda'}$. This is quite simple because one knows the amplitude for the Z decay. Neglecting the muon mass both particles are emitted with opposite helicities. Then, denoting the μ^- helicity by $h/2=\pm\frac{1}{2}$, we obtain

$$T'(h,0) = -(g/\cos\theta_W)g_h M_Z \sin\theta \ , \tag{9a}$$

$$T'(h,\lambda=\pm1) = (g/\cos\theta_W)g_h M_Z \lambda h \times \exp(i\lambda\phi)(1+\lambda h\cos\theta)/\sqrt{2} \ , \tag{9b}$$

where θ is measured using the Z direction in the laboratory as the z axis, ϕ is defined in such a way that the electron momentum has $\phi=0$ and

$$g_+ = g_V - g_A = \sin^2\theta_W \ , \tag{10a}$$

$$g_- = g_V + g_A = -\tfrac{1}{2} + \sin^2\theta_W \ . \tag{10b}$$

The Z polarization vectors $\epsilon_{\pm1}=(\epsilon_x \pm i\epsilon_y)/\sqrt{2}$ are defined in the previous frame. Inserting Eqs. (9) into Eq. (7) and summing over the polarizations of the muons, the result is

$$d\Gamma_{\lambda\lambda'} = \alpha\frac{g_V^2+g_A^2}{(\sin 2\theta_W)^2}\frac{M_Z}{\Gamma_Z}e^{i(\lambda-\lambda')\phi}G_{\lambda\lambda'}d\Omega_\mu \ , \tag{11}$$

with

$$G_{00} = 2\sin^2\theta \ , \tag{12a}$$

$$G_{0\pm} = G_{\pm0} = -\sqrt{2}\sin\theta(\cos\theta \pm \Delta) \ , \tag{12b}$$

$$G_{\lambda\lambda'} = \lambda\lambda' + \cos^2\theta + \Delta(\lambda+\lambda')\cos\theta \ , \tag{12c}$$

and

$$\Delta = -2g_V g_A/(g_V^2+g_A^2) \ . \tag{13}$$

Finally, using these relations in Eq. (5), it is possible to project out the components of $\sigma_{\lambda\lambda'}$ integrating $d\sigma(\gamma\mu^+\mu^-)$ with appropriate weight functions $f(\Omega_\mu)$. In particular, we obtain

$$d\sigma_{00} = 2\,d\sigma - 5\int_{\Omega_\mu} d\sigma(\gamma\mu\mu)\cos^2\theta/B \ , \tag{14a}$$

$$d\sigma_{-+} = -\tfrac{5}{2}\int_{\Omega_\mu} d\sigma(\gamma\mu\mu)\sin^2\theta\exp(2i\phi)/B \ , \tag{14b}$$

$$d\sigma_{\pm\pm} = -\tfrac{1}{2}d\sigma + \int_{\Omega_\mu} d\sigma(\gamma\mu\mu)(\tfrac{5}{2}\cos^2\theta \pm \cos\theta/\Delta)/B \ , \tag{14c}$$

$$d\sigma_{0\pm} = -\int_{\Omega_\mu} d\sigma(\gamma\mu\mu)(\tfrac{5}{2}\cos\theta \pm 1/\Delta)\sin\theta\exp(\pm i\phi)/B \ , \tag{14d}$$

where $d\sigma = d\sigma_{++} + d\sigma_{--} + d\sigma_{00}$ is the differential γZ cross section and B is the branching fraction of the $Z\to\mu^+\mu^-$ channel. Results for other $\lambda\lambda'$ combinations can be obtained from Eqs. (14) noting that $\sigma_{\lambda\lambda'}$ is Hermitian. Let us point out that $d\sigma_{\lambda\lambda'}/d\sigma = \rho_{\lambda\lambda'}$ is the spin-density matrix for the Z production.

These equations enable us to obtain information about $d\sigma_{\lambda\lambda'}$ from measurements of the differential cross section for the reaction $e^+e^-\to\gamma\mu^+\mu^-$ at c.m. energies larger than M_Z. Clearly, our expressions above involve an approximation since, even in the lowest order, there are other diagrams where the $\mu^+\mu^-$ pair does not form a Z. The complete analysis was done by Renard[9] and there is no need to repeat it here. It is enough to recall that, at $\sqrt{s}=140$ GeV and at the Z peak ($\omega\approx 42$ GeV) the background contribution to $d\sigma(\gamma\mu^+\mu^-)/d\omega$, integrated with a cut $30° \le \theta_\gamma \le 150°$, is roughly 100 times smaller than the signal.

III. ANOMALOUS $ZZ\gamma$ COUPLINGS

In the standard model the $ZZ\gamma$ vertex does not exist at the tree level and at one loop it is exclusively induced by the fermionic triangle diagram. Examining this $ZZ\gamma$ three-point function[10] one can see that there is an electric dipole transition (EDT) which vanishes if both Z bosons are on shell. Furthermore, one can show[13] that, for a real photon, the lowest dimension $ZZ\gamma$ couplings, which satisfy electromagnetic gauge invariance and Bose symmetry, are

$$e(p^2-p'^2)/M_Z^2[f_+\epsilon^{\alpha\beta\mu\nu}e_\alpha e'_\beta k_\mu \epsilon_\nu + f_-(\epsilon\cdot e\, k\cdot e' - \epsilon\cdot e' k\cdot e)] , \quad (15)$$

where $e\ (p)$, $e'\ (p')$, and $\epsilon\ (k)$ are the polarization vectors (momenta) of the initial Z, final Z, and photon, respectively. Furthermore, we assume orthogonality between the polarization and the momentum of the Z's despite the fact that they are not on shell. This is appropriate because we are neglecting the fermion masses. The first form factor f_+ which is parity violating but CP conserving, is the EDT and f_- is CP violating and parity conserving. Of course, in the standard model f_- is zero and f_+ is of the order 10^{-5}. However, as Boudjema and Dombey[5] have pointed out, f_+ could be enhanced by a large factor if the Z is a composite particle. To see what are the effects of such a large EDT is our main goal but, before we do that it is interesting to mention that, for an outgoing photon with helicity $\gamma=\pm 1$, Eq. (15) becomes

$$e(p^2-p'^2)/M_Z^2(i\gamma f_+ - f_-)(k\cdot e\, \epsilon_\gamma^*\cdot e' - k\cdot e' \epsilon_\gamma^*\cdot e) . \quad (16)$$

In this form, it is easy to convince ourselves that, with the previous conditions, this is the most general $ZZ\gamma$ interaction.[13]

Let us now consider the helicity amplitudes for γZ production, $T(h,\lambda,\gamma)$, where $h/2$ is the e^- helicity. Since the e^+e^- pair is coupled by a vectorial interaction and we have neglected the masses, the e^+ helicity must be $-h/2$. It is convenient to separate out the dependence on the photon azimuthal angle ϕ_γ. So, writing

$$T(h,\lambda,\gamma) = \exp(ih\phi_\gamma)\tau(h,\lambda,\gamma) \quad (17)$$

and using[14] the spinor formalism,[15] we obtain

$$\tau(h,0,\gamma) = \frac{eg}{\cos\theta_W} g_h 2\sqrt{2s_0} \times \left[\frac{1}{s_0 x} + h(f_+ + i\gamma f_-)(1+\gamma h z)\frac{s_0 x}{8} \right] \quad (18a)$$

and

$$\tau(h,\pm 1,\gamma) = -\frac{eg}{\cos\theta_W} g_h \lambda y \times \left[\frac{2h(s_0\delta_{-\gamma\lambda}+\delta_{\gamma\lambda})}{s_0 x(1+\lambda h z)} + (f_+ + i\gamma f_-)\delta_{\gamma\lambda}\frac{s_0 x}{2} \right] , \quad (18b)$$

with $s_0 = s/M_Z^2$, $x = 1-1/s_0$ is the photon energy in units of beam energy, $z=\cos\theta_\gamma$, and $y=\sin\theta_\gamma$. The amplitudes corresponding to the standard model, i.e., with $f_+ = f_- = 0$, agree, except for the exponential factor in Eq. (17), with the ones derived by Hagiwara *et al.*[7] This exponential simply reflects the fact that the initial state is an eigenstate of the angular momentum J_Z and it is irrelevant except when one has transversely polarized beams. The same factor is missing in Ref. 5, where the EDT amplitude is also given. However, in Ref. 5 the relative sign of the z coefficients in the standard and the EDT contributions is not correct.

With the helicity amplitudes one can easily write the γZ-production cross section and look for possible effects of the abnormal couplings. So far, this has been done only with the EDT coupling. Moreover, the calculations were done either for unpolarized e^+e^- beams[5,11] or summing over the final Z spin states.[9,11] In both cases, terms linear in f_+ are suppressed because they are proportional to Δ and so, they were neglected. Here we lift both restrictions.

IV. TRANSVERSE AND LONGITUDINAL POLARIZATION

In circular colliders the electron and positron beams have a natural transverse polarization in opposite directions. On the other hand, longitudinal polarization is particularly interesting to study parity-violating effects and its implementation is presently being considered. We will see that the use of polarized beams is of great help in the search for deviations from the standard model.

Let n^- denote the electron degree of polarization in the direction specified by the solid angle (θ^-,ϕ^-). The positron degree of polarization, n^+, is in the direction $(\pi-\theta^+,\pi+\phi^+)$. The longitudinal $n_L^\pm$ and transverse $n_T^\pm$ degrees of polarization are

$$n_L^\pm = n^\pm\cos\theta^\pm \quad \text{and} \quad n_T^\pm = n^\pm \sin\theta^\pm . \quad (19)$$

In Eq. (6), $d\sigma_{\lambda\lambda'}$ depends also on the polarizations of the initial state. Hence, Eq. (5) gives the cross section for a particular spin-initial state. Introducing the e^- and e^+ density of states,[16] $\rho_{hh'}$, the relevant quantity is the product $T_\lambda T^*_{\lambda'}$ averaged over the initial spins and summed over the photon polarizations, given by

$$4\sum_{\gamma h h'} T(h,\lambda,\gamma)T^*(h',\lambda',\gamma)\rho_{hh'} = (1+n_L^-)(1-n_L^+)\Sigma(++;\lambda\lambda') + (1-n_L^-)(1+n_L^+)\Sigma(--;\lambda\lambda') - n_T^- n_T^+[\exp(2i\phi_\gamma)\Sigma(+-;\lambda\lambda') + \exp(-2i\phi_\gamma)\Sigma(-+;\lambda\lambda')] , \quad (20)$$

with

$$\Sigma(hh';\lambda\lambda')=\sum_{\gamma}\tau(h,\lambda,\gamma)\tau^*(h',\lambda',\gamma)\ . \quad (21)$$

Without loss of generality we have set $\phi^-+\phi^+=0$. The use of transverse and longitudinal beams enables us to determine all squared amplitudes $\Sigma(hh';\lambda\lambda')$. In fact, from Eq. (20) one clearly sees that integrating the cross section multiplied by the weighting factor $\exp(2ih\phi_\gamma)$ projects out the quantity $\Sigma(-hh;\lambda\lambda')$. On the other hand, the unpolarized cross section is proportional to the sum, $\Sigma(++;\lambda\lambda')+\Sigma(--;\lambda\lambda')$, whereas the longitudinal asymmetry $A_{\lambda\lambda'}$ is given by

$$A_{\lambda\lambda'}=\frac{\Sigma(++;\lambda\lambda')-\Sigma(--;\lambda\lambda')}{\Sigma(++;\lambda\lambda')+\Sigma(--;\lambda\lambda')}\ . \quad (22)$$

The Σ can be trivially calculated from Eqs. (18) and for completeness we give the results in the Appendix.

Possible deviations from known physics are better shown by suitable weighted integrals of the differential cross section $d\sigma_{\lambda\lambda'}$ which vanish in the standard model. If one is interested in the odd part of $d\sigma_{\lambda\lambda'}$ with respect to z, the signature is a forward-backward asymmetry which is obtained using the sign of z as a weighting factor. On the other hand, to get information about the transverse part of $d\sigma_{\lambda\lambda'}$ one uses the weighting factors $\cos(2\phi_\gamma)$ and $\sin(2\phi_\gamma)$. However, it is worth pointing out that the signs of these functions project out the same quantities. This alternative could be advantageous in view of the experimental errors in the angular measurements. We define the weighted cross sections

$$\sigma_s[\lambda\lambda']=\int d\sigma_{\lambda\lambda'}\mathrm{sgn}(\sin2\phi_\gamma) \quad (23a)$$

and

$$a_s[\lambda\lambda']=\int d\sigma_{\lambda\lambda'}\mathrm{sgn}(z\sin2\phi_\gamma)\ , \quad (23b)$$

where the integration is over the photon solid angle with a cut $-z_0\le z\le z_0$ and similar ones, σ_c and a_c, with $\sin2\phi_\gamma$ replaced by $\cos2\phi_\gamma$. To simplify the notation, the values of the Z polarization previously given as subscripts are now written in the square brackets. Then, with the help of the expressions in the Appendix we show that the weighted cross sections listed in Table I are identically zero in the standard model. Notice that the a's are forward-backward asymmetries. Some of these quantities signal the $ZZ\gamma$ coupling. In fact, it is quite easy to obtain contributions proportional to the constants $f_\pm$. These are

$$a_c[++]-a_c[--]=Cf_+z_0^2\ , \quad (24a)$$

$$\sigma_s[++]-\sigma_s[--]=-2Cf_-z_0\ , \quad (24b)$$

$$\mathrm{Re}(\sigma_c[0+]-\sigma_c[0-]) = -C\sqrt{s_0/2}f_+\int_0^{z_0}dz[2/y+(s_0-3)y]\ , \quad (24c)$$

$$\mathrm{Im}(\sigma_c[0+]+\sigma_c[0-]) = -C\sqrt{s_0/2}f_-\int_0^{z_0}dz(2/y-s_0xy)\ , \quad (24d)$$

$$\mathrm{Im}(a_s[0+]+a_s[0-])=-C\sqrt{2s_0}f_+(1-y_0)\ , \quad (24e)$$

$$\mathrm{Re}(a_s[0+]-a_s[0-])=C\sqrt{2s_0}f_-(1-y_0)\ , \quad (24f)$$

where Re and Im mean real and imaginary part, respectively, y_0 is the minimum value of $\sin\theta_\gamma$,

$$C=8(\alpha/\sin2\theta_W)^2g_+g_-x/s\ , \quad (25)$$

and, for simplicity, we assumed $n_T^\pm=1$.

Let us look at longitudinal polarization. To be precise, we consider a right- (left-) helicity electron beam scattering off unpolarized positrons. Let $\sigma_{R,L}[\lambda\lambda']$ denote the total cross sections corresponding to these initial densities of states ($n_L^-=+1,-1$). In order to derive quantities which are forbidden in the standard model we define the forward-backward asymmetries

$$a_{R,L}[\lambda\lambda']=\int d\sigma_{R,L}[\lambda\lambda']\mathrm{sgn}(z)\ . \quad (26)$$

In the standard model the left and right cross sections are proportional to different coupling constants. Hence, it is convenient to factorize out these couplings and define "normalized" cross sections given by

$$\hat{\sigma}_L[\lambda\lambda']=(1+\Delta)\sigma_L[\lambda\lambda']\ , \quad (27)$$

and similarly for $\hat{\sigma}_R[\lambda\lambda']$ with $(1+\Delta)$ replaced by

TABLE I. γZ weighted total cross sections forbidden in the standard model.

Transverse polarization $\propto\sin2\phi_\gamma$	Transverse polarization $\propto\cos2\phi_\gamma$	Longitudinal polarization LR asymmetry	Longitudinal polarization h independent
$\sigma_s[\lambda\lambda]$	$\sigma_c[++]-\sigma_c[--]$	$\hat{\sigma}_R[\lambda\lambda]-\hat{\sigma}_L[\lambda\lambda]$	$\sigma_h[++]-\sigma_h[--]$
$a_s[\lambda\lambda]$	$a_c[\lambda\lambda]$	$\hat{a}_R[\lambda\lambda]+\hat{a}_L[\lambda\lambda]$	$a_h[++]+a_h[--]$
			$a_h[00]$
$\sigma_s[-+]$	$a_c[-+]$	$\hat{\sigma}_R[+-]-\hat{\sigma}_L[+-]$	$a_h[+-]$
$a_s[+-]+a_s[-+]$	$\sigma_c[+-]-\sigma_c[-+]$		$\sigma_h[+-]-\sigma_h[-+]$
$\sigma_s[0+]+\sigma_s[0-]$	$a_c[0+]-a_c[0-]$		$\sigma_h[0+]+\sigma_h[0-]$
$a_s[0\pm]$	$\sigma_c[0\pm]$	$\hat{\sigma}_R[0\pm]+\hat{\sigma}_L[0\pm]$	$a_h[0+]-a_h[0-]$
$\sigma_s[0\pm]+\sigma_s[\pm0]$	$a_c[0\pm]-a_c[\pm0]$	$\hat{a}_R[0\pm]-\hat{a}_L[0\pm]$	$a_h[0\pm]-a_h[\pm0]$
			$\sigma_h[0\pm]-\sigma_h[\pm0]$

$(1-\Delta)$. Therefore, both are proportional to the constant $(1-\Delta^2)(g_+^2+g_-^2)$. With the same convention, we define the "normalized" asymmetries, $\hat{a}_{L,R}$. Now, it is easy to see that, in the standard model, the quantities listed on the right-hand side of Table I also vanish. Furthermore, it turns out that the contributions sensitive to the abnormal $ZZ\gamma$ coupling are

$$\hat{\sigma}_R-\hat{\sigma}_L=8\hat{C}(s_0+1)f_+z_0 \ , \qquad (28a)$$

$$\hat{a}_R[++]-\hat{a}_R[--]+(R\to L)=-4\hat{C}f_+z_0^2 \ , \qquad (28b)$$

$$\hat{\sigma}_R[+-]-\hat{\sigma}_L[+-]=-4\hat{C}s_0(f_++if_-)z_0 \ , \qquad (28c)$$

$$\mathrm{Re}\{\hat{\sigma}_R[0+]-\hat{\sigma}_R[0-]+(R\to L)\}$$
$$=-2\hat{C}\sqrt{2s_0}f_+\int_0^{z_0}dz[3y+s_0(1+z^2)/y] \ , \qquad (28d)$$

$$\mathrm{Im}\{\hat{\sigma}_R[0+]+\hat{\sigma}_R[0-]+(R\to L)\}$$
$$=2\hat{C}\sqrt{2s_0}f_-\int_0^{z_0}dz[y+s_0(1+z^2)/y] \ , \qquad (28e)$$

$$\mathrm{Re}\{\hat{a}_R[0+]+\hat{a}_R[0-]-(R\to L)\}$$
$$=4\hat{C}\sqrt{2s_0}s_0f_+(1-y_0) \ , \qquad (28f)$$

$$\mathrm{Im}\{\hat{a}_R[0+]-\hat{a}_R[0-]-(R\to L)\}$$
$$=-4\hat{C}\sqrt{2s_0}s_0f_-(1-y_0) \ , \qquad (28g)$$

where

$$\hat{C}=\pi(\alpha/\sin2\theta_W)^2(1-\Delta^2)(g_+^2+g_-^2)x/s \ . \qquad (29)$$

A relevant and interesting question is to use these results to find out what are the minimum values of the coupling constants accessible in a collision experiment. For this, we need to consider the statistical errors. The experimental measured signals are in general, given by integrals of the number of events per phase-space element, $dn/d\Omega$, weighted by some function $f(\Omega)$ of the $\gamma\mu^+\mu^-$ phase space. For each direction, the number of events, with mean value equal to the differential cross section times the integrated luminosity, is assumed to be an independent statistical variable. Therefore, the resulting squared standard deviation is the integral of the squared standard deviations, given by the mean number of events times $f^2(\Omega)$. If, as a criterion of discovery, we require that the signal is 2 standard deviations larger than the background then, for an integrated luminosity L_i, the discovery condition is

$$\left|L_i\int[d\sigma(\gamma\mu\mu)-d\sigma_{\mathrm{SM}}(\gamma\mu\mu)]f(\Omega)\right|$$
$$>2\left[L_i\int d\sigma(\gamma\mu\mu)_{\mathrm{SM}}f^2(\Omega)\right]^{1/2} \ . \qquad (30)$$

For illustrative purposes, we will use this criterion to obtain some bounds on the abnormal $ZZ\gamma$ couplings.

V. RESULTS AND DISCUSSION

In Fig. 1 we plot the γZ-production cross sections as a function of the center-of-mass energy, with a cut $z_0=0.8$. The solid curve corresponds to the prediction of the standard model, with $M_Z=92.5$ GeV/c^2 and $\sin^2\theta_W=0.226$, and the dashed curve includes also the extra Z amplitude with $F^2=f_+^2+f_-^2=1$. As one would have expected, for high energies, the standard unpolarized cross section goes like $1/s$ while the contribution quadratic in the f increases with s^2. So, around 300 GeV the standard and nonstandard cross sections differ from each other by 1 order of magnitude.

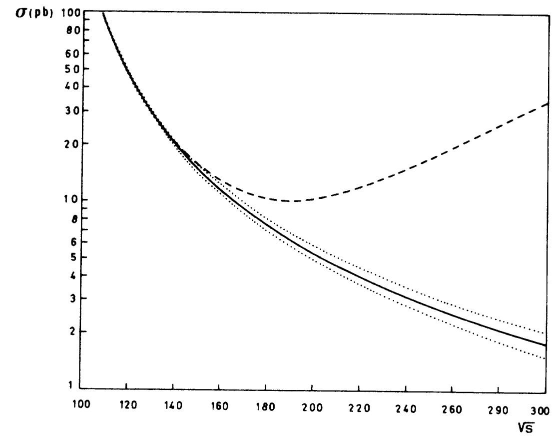

FIG. 1. $e^+e^-\to\gamma Z$ cross section as a function of $\sqrt{s}$, for $f_\pm=0$ (solid curve) and $F^2=f_+^2+f_-^2=1$ (dashed curve); the dotted curves show a statistical fluctuation of two standard deviations from the standard-model result.

In Fig. 1 the dotted curves give the two standard deviation limits of the standard-model (SM) result, calculated using an integrated luminosity of 100 pb^{-1}. According to our previously defined discovery criterion, a result outside these dotted lines should be attributed to some new physics. Obviously, we can turn the question around and ask for the minimum value of F such that the dashed curve lies just outside the two σ band. The resulting F minimum is plotted in Fig. 2. Notice that $F=1$ is reached at an energy of 140 GeV and, even at 200 GeV one has $F=0.3$. This value is comparable with the limit derived[12] using the neutrino counting reaction. Since the SM amplitude is a typical bremsstrahlung the photons tend to be rather soft. So, increasing the z_0 cut enhances the signal-to-background ratio, but, a smaller z_0 gives a reduction in phase space which implies poorer statistics. In such a situation a compromise is needed and we have checked that $z_0=0.8$ is always close to the best possible values.

It should be noted that the cross section also has a contribution linear in f_+ arising from the interference between the standard and nonstandard amplitudes. Clearly, such an interference term does not exist for the *CP*-odd coupling f_-. Including this linear term we obtain the bounds represented by the dotted curves in Fig. 2, where the lower one corresponds to $f_+<0$. Although the linear term could be important for $|f_+|<1$, it is suppressed by Δ and its contribution is further washed out by the s^2 dependence of the quadratic term.

In the search for signals that could give information about the signs of the constants f, discriminate their contributions, or eventually ascertain more stringent evi-

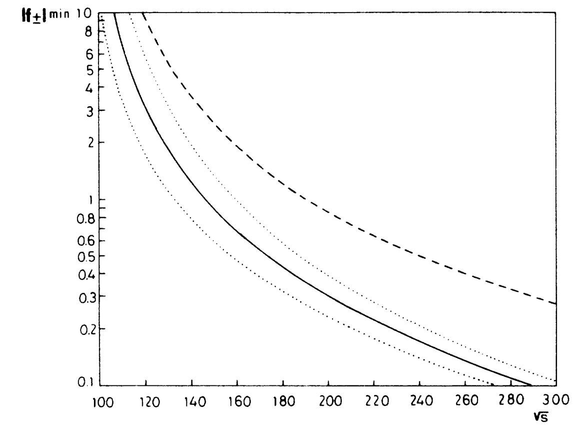

FIG. 2. Discovery limits of $|f_\pm|$. The solid and dotted curves are obtained from the unpolarized γZ cross section and the dashed curve is the minimum visible $|f_+|$ from the process $e^+e^- \to \gamma f\bar{f}$ with transverse polarization.

dence of them we were lead to consider polarized beams and Z spin effects. With complete transverse polarization ($n_T^\mp = 1$) the best limits are obtained with the signal corresponding to Eq. (24c): namely,

$$\int d\sigma(\gamma\mu\mu)\mathrm{sgn}(\cos\phi \cos 2\phi_\gamma) = \tfrac{3}{4} B\Delta C\sqrt{s_0/2} f_+ \int_0^{z_0} dz[2/y + (s_0-3)y] . \quad (31)$$

The reason why this gives the best limit is the energy dependence of Eq. (24c) which increases with $\sqrt{s}$, while all the others, except Eq. (24d), decrease with energy. Equation (24d) gives a poorer limit on f_- because of a cancellation between the two terms in the integral. In both cases the signature is attenuated by the factor $\Delta \approx 0.2$ and also by the branching fraction B of the Z decay into $\mu^+\mu^-$. Since the signal has to be compared with the statistical error proportional to $\sqrt{B}$, the relevant factor is $\sqrt{B}$ times Δ. Its value is extremely small, -0.032 for $\sin^2\theta_W = 0.226$. In order to overcome this handicap one can consider other Z decay modes into $f\bar{f}$ pairs, with a negligible fermion mass in comparison with the Z mass. Without this restriction the previously given amplitude for the Z decay does not hold. Hence, the most one can do is to sum the contributions of the first two generations of charged leptons and quarks. In this case, the relevant parameter

$$\sum_f \frac{B_f}{\sqrt{B}} \Delta_f = -0.54$$

is more than 16 times larger than the previous one. Taking all these four channels we obtained the limit on $|f_+|$ which is represented by the dashed curve of Fig. 2.

As far as the longitudinal polarization is concerned, the signals of the interference between the standard and the abnormal amplitudes are, as a general rule, weighted left-right asymmetries [see Eqs. (27) and (28)]. The weighting factor of the L (R) cross section is $1+(-)\Delta$, where Δ [see Eq. (13)] is the LR-asymmetry parameter with respect to the Z coupling. Curiously enough, not all of these interference signals arise from an asymmetry, i.e., a subtraction of the normalized cross sections, $d\hat{\sigma}_{R,L}$. Indeed, Eqs (28b), (28d), and (28e) show a summation over the electron helicity. Obviously, if the beams are unpolarized we cannot factorize the coupings. Nevertheless, it is still possible to obtain a signal for the presence of the extra Z couplings using similar Z helicity combinations but without the caret. In fact, these elements of the $d\sigma_{\lambda\lambda'}$ tensor have a nonstandard part linear in the f's and a standard-model contribution suppressed by Δ. Again, with an appropriated convolution, we obtain

$$\int d\sigma(\gamma f\bar{f})\mathrm{sgn}(z\cos\theta) = -\tfrac{3}{2} B_f\Delta_f C' f_+ z_0^2 + \text{SM contribution} , \quad (32a)$$

$$\int d\sigma(\gamma f\bar{f})\mathrm{sgn}(\cos\phi) = \tfrac{3}{2} B_f\Delta_f C'\sqrt{s_0/2} f_+ \int_0^{z_0} dz[3y + s_0(1+z^2)/y] + \text{SM contribution} , \quad (32b)$$

$$\int d\sigma(\gamma f\bar{f})\mathrm{sgn}(\sin\phi) = -\tfrac{3}{2} B_f\Delta_f C'\sqrt{s_0/2} f_- \int_0^{z_0} dz[y + s_0(1+z^2)/y] + \text{SM contribution} , \quad (32c)$$

with

$$C' = \pi(\alpha/\sin 2\theta_W)^2(g_+^2 + g_-^2)x/s . \quad (33)$$

The results are proportional to the product $B_f\Delta_f$. Thus, as we have already discussed, the only way of obtaining an interesting signal is to consider other $\gamma f\bar{f}$ channels. Furthermore, the last two expressions increase with energy, and so they give the best bounds. Their respective discovery limits for $f_\pm$ are plotted in Fig. 3 and it is interesting to point out that they are more stringent than the ones obtained with the γZ production cross section. The solid, dashed, and dashed-dotted curves correspond to the signals (32b), (32c), and γZ cross section, respectively.

The other quantities in Eqs. (28) are left-right asymmetries which require longitudinal polarization to be observed. Those with the best behavior as a function of energy are shown in Eqs. (28a), (28f), and (28g). The dotted curve in Fig. 4 displays the minimum values of $|f_\pm|$ arising from the signals corresponding to Eqs. (28f) and (28g); namely,

$$\int d\hat{\sigma}_R(\gamma f\bar{f})\mathrm{sgn}(z\cos\theta\cos\phi) - (R \to L) = -8/\pi B_f \hat{C} s_0 \sqrt{2s_0} f_+ (1-y_0) , \quad (34)$$

and a similar equation with $\cos\phi$ replaced by $\sin\phi$ and f_+ replaced by $-f_-$. Even for an ideal case with $B=1$, this puts a limit on $|f_\pm|$ similar to the one achieved with ex-

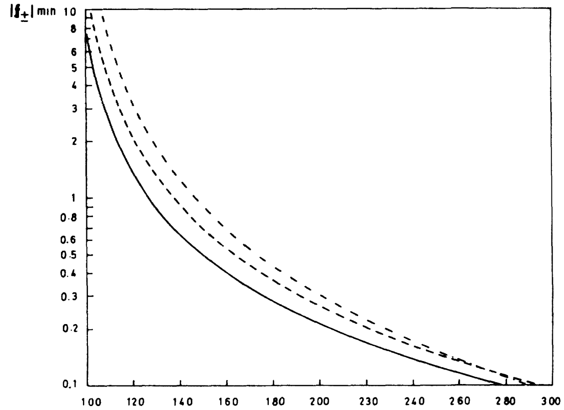

FIG. 3. Discovery limits of $|f_+|$ (solid curve) and $|f_-|$ (dashed curve) from the unpolarized angular asymmetries of the $\gamma f\bar{f}$ differential cross section. The dashed-dotted curve is the same as the solid curve in Fig. 2.

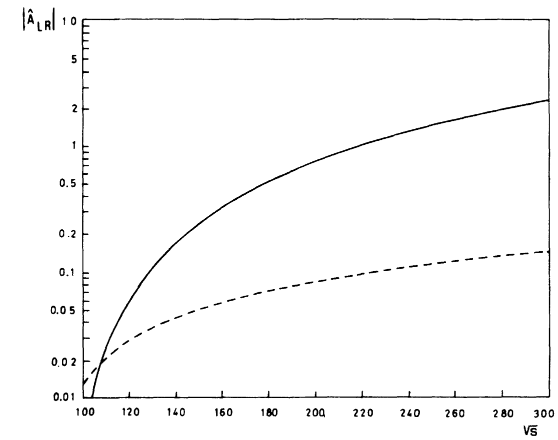

FIG. 5. The solid curve is the absolute value of the weighted left-right asymmetry with $f_+=\pm 1$ and the dashed one is its minimum value statistically discernible from the vanishing standard-model result.

pressions (32b) and (32c), and again it implies a weighted integration over the $f\bar{f}$ phase space. Indeed, at 200 GeV we obtain 0.28 whereas the previous limits were 0.21 and 0.26, respectively.

Equation (28a) does not involve any angular asymmetries either in the $f\bar{f}$ phase space or in the photon scattering angles. This remarkable feature enables us to define a left-right weighted asymmetry

$$\hat{A}_{LR}=[(1+\Delta)\sigma_L-(1-\Delta)\sigma_R]/(\sigma_L+\sigma_R)_{SM}\ , \quad (35)$$

in terms of the total γZ cross section. Its value turns out to be

$$\hat{A}_{LR}=-4\hat{C}(s_0+1)f_+z_0/\sigma_{SM}\ , \quad (36)$$

where σ_{SM} is the γZ-production cross section with unpolarized beams. In Fig. 5 we plot the absolute value of this asymmetry with $f_+=1$. For the calculation of the statistical variance of the numerator in Eq. (35) we took an integrated luminosity of 50 pb^{-1} for each helicity configuration. The error variance is given by the product of $(1-\Delta^2)$ times the number of events with unpolarized beams. For comparison, the dashed curve, in Fig. 5, gives the upper limit of the standard-model asymmetry ($\hat{A}_{LR}=0$), with $B=1$, corresponding to a statistical fluctuation of two standard deviations. In other words, with this statistical significance, a new physical effect put in evidence by this asymmetry must give an absolute value above the dashed curve.

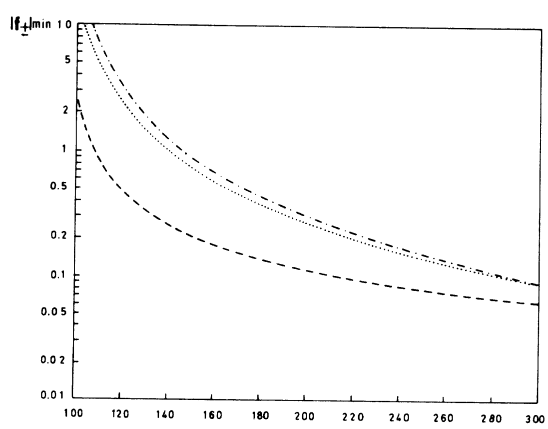

FIG. 4. The dashed curve is the minimum visible $|f_+|$ from a measurement of $\hat{A}_{LR}$, and the dotted one is the $|f_\pm|$ limit from the $\gamma f\bar{f}$ angular asymmetries with a longitudinally polarized e^- beam. The dashed-dotted curve is the same as the solid curve in Fig. 2.

VI. CONCLUSIONS

In this paper we have studied the reaction $e^+e^-\rightarrow\gamma f\bar{f}$ in order to learn about the abnormal $ZZ\gamma$ couplings, in particular the EDT. We have carried out a detailed analysis using both longitudinal and transverse polarized beams. The minimum visible value of the EDT coupling is obtained using longitudinal polarized beams. Its value as a function of c.m. energy is shown by the dashed curve in Fig. 4. At 200 GeV it gives 0.11 and this is the best limit for this coupling constant accessible with LEP II. Such a value of f_+ is 4 orders of magnitude larger than the SM result ($f_+\approx 4\times 10^{-5}$) but, unfortunately, it could very well be even larger than some current estimates[5] in composite models ($f_+\approx 0.05$).

We would like to stress that the best limits on the Z-boson EDT coupling, obtained at LEP II energy with a polarized e^- beam, is only a factor of 2 smaller than our previous limit[12] derived using the neutrino counting reaction at the Z peak. This enhances the importance of the $e^+e^-\rightarrow\gamma\nu\bar{\nu}$ reaction.

ACKNOWLEDGMENT

We thank F. Renard for a critical reading of the manuscript.

APPENDIX

The squared amplitudes defined in Eq. (21) are given in units of $(eg/\cos\theta_W)^2$ by

$$\Sigma(h\,-h\,;00)=16g_+g_-s_0[(s_0x)^{-2}-(Fs_0xy)^2/64]\ , \tag{A1}$$

$$\Sigma(h\,-h\,;\lambda\lambda)=4g_+g_-[-(s_0^2+1)(s_0x)^{-2}+(Fs_0xy)^2/16-(zf_++ihf_-)\lambda/2]\ , \tag{A2}$$

$$\Sigma(h\,-h\,;0\lambda)=4g_+g_-\sqrt{2s_0}/y\{(\lambda h+z)[(s_0+1)(s_0x)^{-2}-(Fs_0xy)^2/32]+[F_{-\lambda}(s_0-2)y^2+F_\lambda(1+\lambda hz)^2]\lambda/8\}\ , \tag{A3}$$

$$\Sigma(h\,-h\,;\lambda-\lambda)=8g_+g_-s_0(s_0x)^{-2}(1-\lambda hz)^2/y^2\ , \tag{A4}$$

$$\Sigma(hh\,;00)=16g_h^2s_0[(s_0x)^{-2}+(Fs_0x)^2(1+z^2)/64+hf_+/4]\ , \tag{A5}$$

$$\Sigma(hh\,;\lambda\lambda)=4g_h^2[(s_0^2+1)(s_0x)^{-2}(1-\lambda hz)^2/y^2+(Fs_0xy)^2/16+hf_+(1-\lambda hz)/2]\ , \tag{A6}$$

$$\Sigma(hh\,;0\lambda)=-4g_h^2\sqrt{2s_0}/y\{(s_0+1)(s_0x)^{-2}(\lambda h-z)+(Fs_0xy)^2(\lambda h+z)/32+[(2F_{-\lambda}+F_\lambda)y^2+F_{-\lambda}s_0(1-\lambda hz)^2]\lambda/8\}\ , \tag{A7}$$

$$\Sigma(hh\,;\lambda-\lambda)=-8g_h^2s_0[(s_0x)^{-2}+hF_\lambda/4]\ , \tag{A8}$$

where $F_\lambda=f_++i\lambda f_-$, and $F^2=f_+^2+f_-^2$. Notice that

$$\Sigma(hh';\lambda\lambda')=\Sigma^*(h'h;\lambda'\lambda)\ . \tag{A9}$$

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