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Brief Reports

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Testing the principle of equivalence with neutrino oscillations

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If the equivalence principle is violated, and gravity is not universally coupled to all leptonic flavors, a gravitational field may contribute to neutrino oscillations. The laboratory limits on the oscillation process can thus be interpreted as tests of the equivalence principle in the quantum-relativistic regime, and put severe constraints on a maximal violation of this principle in the case of massless neutrinos coupled to the Earth's gravitational field.

I. INTRODUCTION

It is well known that neutrino oscillations[1] can occur, in a vacuum, if the eigenvalues of the mass matrix are not all degenerate, and the corresponding mass eigenstates are different from the weak-interaction eigenstates $\nu_e, \nu_\mu, \ldots$. Inside matter oscillations can be induced also by weak interactions, if the leptonic current has an off-diagonal part connecting different neutrino types;[2] but even if the current is diagonal the oscillations in matter are affected by weak interactions, because the different neutrinos are differently scattered by the electrons,[2] and, in particular, the oscillations can be resonantly enhanced at a given critical density.[3]

The only other interaction a neutral lepton can feel, besides the weak, is the gravitational interaction. A gravitational field, however, cannot induce nor affect neutrino oscillations if gravity couples universally to all types of matter, according to the principle of equivalence. A gravitational field could contribute to neutrino oscillations only if the different neutrino types would be differently affected by gravity, that is only if the equivalence principle would be violated in neutrino interactions.

The validity of the principle of equivalence is very well tested for macroscopic bodies, but this does not necessarily imply that such a principle continues to hold at a microscopic scale and in the quantum regime. It has been shown in fact that the equality of inertial mass and gravitational mass is no longer valid in the context of quantum field theory at finite temperature,[4] and the possibility that this equality is violated also in the case of the gravitational interactions of antimatter[5] has been recently suggested, and shown not to be in contrast with *CPT* invariance.[6]

The propagation of a neutrino beam through a gravitational field probes the validity of the Einstein principle of equivalence for quantum and relativistic test particles. As pointed out recently, the observations of neutrinos from the supernova 1987A has provided a test of the universality of the gravitational time delay for photons and neutrinos,[7,8] and for neutrinos with different energies.[7] The aim of this paper is to point out that experiments on neutrino oscillations test the universality of the gravitational red-shift for different neutrino flavors.

The interesting result is that, from the present laboratory limits on neutrino oscillations, in the hypothesis of massless neutrinos one can exclude a maximal violation (i.e., with maximum mixing angle) of the equivalence principle up to one part in 10^{11}, that is with a precision comparable to that of the Dicke-Braginskii experiments[9,10] which test the universality of the accelerations of different macroscopic bodies in the solar gravitational field. For comparison, in the time delays of the supernova neutrinos and photons the principle of equivalence is tested[7,8] only up to 1 part in 10^3, and 1 part in 10^6 for neutrinos of different energies in the hypothesis they are massless.[7]

II. GRAVITY-INDUCED OSCILLATIONS OF MASSLESS NEUTRINOS

If neutrinos are massless, in vacuum the eigenstates of the energy E coincide with the weak-interaction eigenstates $|\nu_W\rangle$: considering for simplicity two flavors only, we can set

$$|\nu_W\rangle = \begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix}, \quad E = \begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix}, \tag{1}$$

where p is the neutrino momentum, and the time evolution is then

$$|\nu_W(t)\rangle = e^{-iEt}|\nu_W(0)\rangle . \tag{2}$$

There is no mixing, and no oscillation occurs.

Suppose now that neutrinos propagate through a given gravitational field. The neutrino energies are red-shifted with respect to the vacuum, $E \rightarrow E' = \sqrt{g_{44}} E$, but according to the equivalence principle the energy shift should be the same for all the neutrino flavors, because of the universality of the gravitational coupling: therefore the energy should be still diagonal in the $|\nu_W\rangle$ basis, the only effect being an overall shift of the momenta, given by $\Delta p/p = -GM/r$ (if we are working in the weak field of a static source), which does not contribute to the oscillations.

If the principle of equivalence is violated, however, and gravity is not minimally coupled to all kinds of energy, we can have different red-shifts for different neutrino types: in this case there is no reason to believe that the weak-interaction eigenstates and the gravitational eigenstates are identical or, in other words, that the shifted kinetic energy E' is diagonal in the $|\nu_W\rangle$ basis.

Therefore, let $|\nu_G\rangle$ be the eigenstates of the total energy in the presence of gravity, related to the $|\nu_W\rangle$ basis by a rotation angle θ_G, that is $|\nu_W\rangle = R(\theta_G)|\nu_G\rangle$, where

$$|\nu_G\rangle = \begin{bmatrix} \nu_{1G} \\ \nu_{2G} \end{bmatrix}, \quad R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{3}$$

and let $p'_{1,2}$ be the corresponding eigenvalues,

$$p'_{1,2} = p\left[1 - \frac{GM}{r}\epsilon_{1,2}\right] \tag{4}$$

where the dimensionless coefficients ϵ_1 and ϵ_2 parametrize the strength of a possible violation of the equivalence principle. If this principle is violated, and $\Delta\epsilon = \epsilon_1 - \epsilon_2 \neq 0$, flavor oscillations can occur, induced by gravity, even if neutrinos are massless. Suppose in fact we have a state that, at $t = 0$, is a pure $|\nu_e\rangle$, and propagates through a gravitational potential GM/r; at a time t later we have the mixture (modulo an overall phase factor)

$$|\nu(t)\rangle = \cos\theta_G |\nu_{1G}\rangle + e^{-i\Delta E_G t}\sin\theta_G |\nu_{2G}\rangle , \tag{5}$$

where

$$\Delta E_G = p'_2 - p'_1 = \frac{GM}{r} p \Delta\epsilon \tag{6}$$

corresponding to oscillations, with characteristic length $L_G = 2\pi/\Delta E_G$, and oscillation probability

$$\begin{aligned} P_{\nu_e \rightarrow \nu_\mu}(t) &= |\langle \nu_\mu | \nu(t)\rangle|^2 \\ &= \sin^2(2\theta_G)\sin^2(\tfrac{1}{2}\Delta E_G t) . \end{aligned} \tag{7}$$

Assuming that neutrinos are massless, the laboratory limits on the oscillations can be directly interpreted therefore as limits on a possible violation of the equivalence principle in the Earth's gravitational field. Among the experimental results presently available (for a recent review see Ref. 11), the best limits on $\Delta\epsilon$ can be obtained by considering the limits on $\nu_\mu \leftrightarrow \nu_e$ oscillations recently obtained[12] with neutrinos of average energy $p = 1.5$ GeV. Using Eq. (7) to interpret the data of Ref. 12, and setting $GM/R = 0.69 \times 10^{-9}$ for the Earth average potential, one obtains, in the case of maximal mixing $(\theta_G = \pi/4)$, the upper limit

$$\Delta\epsilon \leq 3 \times 10^{-11} . \tag{8}$$

A maximal violation of the equivalence principle for massless neutrinos is thus excluded, at the GeV scale, with the same accuracy achieved in the case of macroscopical test bodies.[9,10] It should be pointed out, however, that the neutrino oscillations probe the universality of the gravitational coupling to different neutrino flavors, but give no informations on the possibility that neutrinos have an anomalous gravitational red-shift with respect to other fields, that is $\epsilon_1 = \epsilon_2 \neq 1$.

III. RESONANT OSCILLATIONS OF MASSLESS NEUTRINOS IN MATTER

The result (8) is valid if there is no mass contribution to the oscillations. If this is the case, one may wonder what happens in the case of the solar-neutrino problem, which can find explanations based just on the oscillations of massive neutrinos, in a vacuum[1] or inside the solar matter[13,14] according to the so-called Mikheyev-Smirnov-Wolfenstein (MSW) resonance mechanism.[2,3]

Of course the solution of the problem might be due to different mechanisms (for example, as recently suggested,[15] to the existence of exotic, weakly interacting, massive particles called in general cosmions[16]); but if one insists in looking for a solution based on neutrino oscillations, it must be noted first of all that the condition (8), in the case of solar neutrinos ($p \sim 10$ MeV) traveling to the Earth in the solar gravitational potential, is still compatible with an oscillation length smaller than the Sun-Earth distance; moreover, even without mass contributions, neutrinos can have resonant oscillations inside the Sun if the principle of equivalence is violated.

Consider in fact the most general situation in which neutrinos are massive, and the equivalence principle is violated in such a way that $|\nu_W\rangle \neq |\nu_G\rangle \neq |\nu_M\rangle$, where $|\nu_M\rangle$ are the mass eigenstates. Given a medium characterized by a gravitational potential ϕ, and electron density N_e, the propagation eigenstates are obtained by diagonalizing the matrix which contains the contributions of mass, weak and gravitational interactions to the total energy. In the $|\nu_W\rangle$ basis, and for mass eigenvalues $m_{1,2} \ll p$, this matrix can be written (modulo a multiple of the identity, which only contributes to an overall phase and does not affect oscillations)

$$\begin{bmatrix} \sqrt{2}G_F N_e - \dfrac{\Delta m^2}{2p}\cos^2\theta_M - \Delta E_G \cos^2\theta_G & \dfrac{\Delta m^2}{4p}\sin 2\theta_M + \dfrac{\Delta E_G}{2}\sin 2\theta_G \\ \dfrac{\Delta m^2}{4p}\sin 2\theta_M + \dfrac{\Delta E_G}{2}\sin 2\theta_G & \dfrac{-\Delta m^2}{2p}\sin^2\theta_M - \Delta E_G \sin^2\theta_G \end{bmatrix} , \tag{9}$$

where G_F is the Fermi coupling constant, and

$$\begin{aligned} &\Delta m^2 = m_2^2 - m_1^2 \,, \\ &\Delta E_G = \phi p \Delta\epsilon = \phi p(\epsilon_1 - \epsilon_2) \,. \end{aligned} \tag{10}$$

Here $\sqrt{2}G_F N_e$ represents the charged current contribution of the weak interactions, which affects ν_e but not ν_μ, and θ_M, θ_G, are the rotation angles which diagonalize, respectively, the mass and gravitational part of the energy, such that

$$|\nu_W\rangle = R(\theta_M)|\nu_M\rangle, \quad |\nu_W\rangle = R(\theta_G)|\nu_G\rangle \,. \tag{11}$$

Taking into account also a possible violation of the equivalence principle (i.e., $\Delta E_G \neq 0$), the MSW condition of maximal mixing,[2,3] for neutrino propagation in the presence of a background gravitational field, is then generalized as follows:

$$\sqrt{2}G_F N_e = \frac{\Delta m^2}{2p}\cos2\theta_M + \phi p \Delta\epsilon \cos2\theta_G \,. \tag{12}$$

Starting from this equation with $\Delta m^2 = 0$ (corresponding to massless neutrinos, or massive but degenerate, $m_1 = m_2$), and applying the same arguments as in Ref. 13, one can find that

$$\Delta\epsilon \cos2\theta_G \le 2\times10^{-12}\left[\frac{\text{MeV}}{p}\right] \tag{13}$$

is a sufficient condition in order that the gravity-induced oscillations of neutrinos, created with momentum p in the solar interior, be resonantly enhanced by the solar matter, and that the passage through the resonance region is adiabatic[3,13] provided that

$$\Delta\epsilon \frac{\sin^2 2\theta_G}{\cos2\theta_G} \gg 2\times10^{-14}\left[\frac{\text{MeV}}{p}\right] \,. \tag{14}$$

The two conditions are compatible for $\tan(2\theta_G) \gg 10^{-1}$.

A suitable deviation from universality in the gravitational coupling of massless neutrinos may therefore induce resonant oscillations in the Sun interior, and explain the neutrino puzzle. Note that the violation of the equivalence principle is to be fine-tuned according to Eqs. (13) and (14), which must be satisfied in order to apply the MSW mechanism in its original version,[3,13] but a resonant solution to the puzzle may exist even without imposing the adiabatic condition.[14]

IV. CONCLUSION

The laboratory experiments on neutrino oscillations provide interesting tests of the equivalence principle, and put severe restrictions on a maximal violation of this principle for massless neutrinos in the terrestrial gravitational field, $\Delta\epsilon \lesssim 10^{-11}$.

This limit does not preclude however the possibility that the solar-neutrino problem may find a solution based on gravity-induced oscillations of massless neutrinos.

The experimental constraints on the oscillations could be reconciled with a higher value of $\Delta\epsilon$ in two ways: either assuming that neutrinos are massive, so that the propagation eigenstates in a vacuum are not $|\nu_G\rangle$ but correspond instead to the eigenstates of the matrix (9) (with $N_e = 0$), or supposing that neutrinos are massless but the gravitational mixing is not maximal, i.e., $\sin2\theta_G < 1$.

In the last case, however, a sufficiently high value of $\Delta\epsilon$ would imply $\cos2\theta_G \simeq 1$, so that an oscillatory solution of the solar-ν puzzle might be no longer possible, neither in vacuum nor through a resonance [see Eq. (13)].

Therefore, in the hypothesis that neutrino oscillations are the true explanation of the solar puzzle, an experimental evidence that gravity is not universally coupled to all lepton numbers, with $\Delta\epsilon \gg 10^{-11}$ (obtained independently from the oscillations experiments), would also indirectly suggest that neutrinos have a nonzero rest mass.

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