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BRS and anti-BRS symmetries in the planar gauge

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The planar gauge is reexamined from various points of view. First, we find an annoying ambiguity in the definition of the product of two propagators. Second, Becchi-Rouet-Stora (BRS) invariance can be implemented only at the price of unavoidable second-order derivatives in the Lagrangian. BRS and anti-BRS symmetries cannot be realized simultaneously. If, instead of BRS, anti-BRS symmetry is implemented, the ambiguity does not give rise to different results. There is some simplification in the calculation but the gluon self-energy is neither conserved nor orthogonal to n. Again, second-order derivatives are unavoidable in the invariant Lagrangian. For all these reasons, the planar gauge with its usual propagator either with BRS or with anti-BRS symmetry does not seem to be a true gauge for Yang-Mills theory.

I. INTRODUCTION

The planar gauge, whose propagator is

$$D_{\mu\nu}(k)=-\frac{i}{k^2+i\epsilon}\left[g_{\mu\nu}-\frac{n_\mu k_\nu+n_\nu k_\mu}{n\cdot k}\right] \quad (1)$$

was introduced[1] as a Faddeev-Popov ghost-free gauge for which the infrared unphysical pole is only simple, independent of the value of n^2. However, it soon appeared[2] that there was an internal difficulty within the formalism. Indeed, dimensionally regularized ghost loops formally vanish while ghosts contribute to the Slavnov-Taylor identities.

On another hand, more recent calculations[3,4] show that, in the *temporal* gauge, ghosts cannot be neglected as easily as usually assumed. If we follow the Cheng and Tsai reasoning,[5] the ghost contribution is the same in the planar and temporal gauges if, in both gauges, the infrared poles at $n\cdot k=0$ are regularized by the principal-value prescription. To what extent then can ghost loops be neglected in the planar gauge?

This intriguing situation led us to reexamine the problem of the planar gauge. In the course of this reexamination, we found two interesting points. The first one concerns Becchi-Rouet-Stora[6] (BRS) and anti-BRS (Ref. 7) symmetries. In contrast with usual gauges, [8] both cannot be realized simultaneously in the planar gauge. The reason is that the Nakanishi-Lautrup[9] field occurs through derivatives. This is not in itself a problem since, in relativistic gauges or their extensions, each of these symmetries leads to the same set of Ward identities.[8] One of them can therefore be considered as redundant. The realization either of BRS or of anti-BRS symmetry leads to two different gauges with the same propagator but very different properties.

The second point deals with a so-far-neglected subtlety in nonrelativistic gauge calculations. When infrared poles occur in the product of two propagators, one is led to make the decomposition[10]

$$\frac{1}{n\cdot p}\frac{1}{n\cdot(p+q)}=\frac{1}{n\cdot q}\left[\frac{1}{n\cdot p}-\frac{1}{n\cdot(p+q)}\right], \quad (2)$$

which holds even in the limit $n\cdot q\to 0$. However, in the real situation, such poles must be regularized. There is a problem of timing for this regularization. Should it be made before or after the decomposition (2)?

This is a crucial problem because, if we use the principal-value prescription, we have

$$\begin{aligned}\lim_{\alpha\to 0}\frac{n\cdot p}{(n\cdot p)^2+\alpha^2}\frac{n\cdot(p+q)}{[n\cdot(p+q)]^2+\alpha^2} &= \lim_{\alpha\to 0}\frac{1}{n\cdot q}\left[\frac{n\cdot p}{(n\cdot p)^2+\alpha^2}-\frac{n\cdot(p+q)}{[n\cdot(p+q)]^2+\alpha^2}\right]\\ &\quad+\lim_{\alpha\to 0}\frac{\alpha^2}{[(n\cdot p)^2+\alpha^2]\{[n\cdot(p+q)]^2+\alpha^2\}}\ . \quad (3)\end{aligned}$$

There is therefore a difference between the two methods since the last term of Eq. (3) does not necessarily vanish. When the limit $\alpha\to 0$ can be taken, it gives $\pi^2\delta(n\cdot p)\delta(n\cdot q)$ and the integration over $n\cdot p$ gives rise to $\delta(n\cdot q)$ terms which do not necessarily cancel.

If, for $\alpha\neq 0$, the theory is also a gauge, the regularization must of course be taken before the decomposition. If the theory is only a gauge in the limit $\alpha\to 0$, there is no way to solve this ambiguity. Therefore, if all the contributions coming from the last term of Eq. (3) do not cancel, there is an unacceptable arbitrariness which makes the results very suspicious.

We have computed the contribution of the last term of Eq. (3) to the gluon self-energy for a planar gauge either with BRS or with anti-BRS symmetry. For the usual planar gauge, its contribution does not vanish while it does for the planar gauge with anti-BRS symmetry. The conclusion will be clearer after the discussion. Therefore, we report it in the last section of this paper.

We organize our work as follows. In Sec. II we study the Abelian version of the planar gauge. In Sec. III we give all the details concerning the non-Abelian extension which realizes the BRS symmetry, what we call the BRS planar gauge. The anti-BRS planar gauge is described in Sec. IV while Sec. V summarizes our results and concludes.

II. THE PLANAR GAUGE IN THE ABELIAN CASE

Let us first consider the planar gauge in the free Abelian case, a restriction which is sufficient in view of discussing the bare propagator properties. The Lagrangian is

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\partial_\mu n\cdot A\,\partial^\mu S-\tfrac{1}{2}\partial_\mu S\partial^\mu S \ , \tag{4}$$

where S is the Nakanishi-Lautrup field. It occurs here with derivatives. The resulting field equations are

$$\partial^\mu F_{\mu\nu}-n_\nu\Box S=0 \ , \tag{5}$$

$$\Box n\cdot A=\Box S \ , \tag{6}$$

or, if we introduce Eq. (6) inside Eq. (5),

$$\partial^\mu F_{\mu\nu}-n_\nu\Box n\cdot A=0 \ . \tag{7}$$

This last equation can also be derived from the Lagrangian

$$\mathcal{L}'=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}\partial_\mu n\cdot A\,\partial^\mu n\cdot A \tag{8}$$

from which the S field is removed.

With the Lagrangian (4) where n is (1,0,0,0), the canonically conjugate variables are, in a self-explanatory notation,

$$\pi^k=F^{k0}, \quad \pi^0=\partial_0 S, \quad \pi_S=\partial_0 A_0-\partial_0 S \ . \tag{9}$$

These equations lead to

$$\partial_0 A_0=\pi_S+\pi^0 \ . \tag{10}$$

The nonvanishing canonical commutators are

$$[A_\mu(x),\pi^\nu(y)]_{x_0=y_0}=i\delta_\mu^\nu\delta^{(3)}(\mathbf{x}-\mathbf{y}) \ , \tag{11}$$

$$[S(x),\pi_S(y)]_{x_0=y_0}=i\delta^{(3)}(\mathbf{x}-\mathbf{y}) \ . \tag{12}$$

With the Lagrangian (8), we can also derive Eq. (11) while Eq. (10) is replaced by

$$\partial_0 A_0=\pi^0 \ . \tag{13}$$

It is clear, from the canonical commutation relations, that, in both cases, we will have the same equal-time commutation relations between A_μ and $\partial_0 A_\nu$. Since the equation of motion (7) holds also in both cases, the quantum theories described by (4) and (8) are equivalent. With path-integral methods, it is very easy to prove the equivalence. $\mathcal{L}'$ is obtained from $\mathcal{L}$ after a path integral over S.

In the case of timelike n, four degrees of freedom are involved with the Lagrangian (8), as in relativistic gauges where the Gupta-Bleuler formalism[11] is used to get a description of the photon. The case of spacelike n is different and will not be considered here although, formally, the rules for perturbation theory are very similar. However, a consistent canonical formalism is, as for the spacelike axial-vector gauge, more difficult to introduce.[12]

Let us now proceed, without details, to the determination of the propagator. In the path-integral formalism, it is defined as the causal Green's function of the operator involved in the free field equations. This gives Eq. (1) where the regularization of the singularity at $n\cdot k=0$ is not fixed. The canonical formalism, which defines the propagator as the Fourier transform of $\langle 0\,|\,TA_\mu(x)A_\nu(0)\,|\,0\rangle$ allows us to fix the regularization of this pole.[13] The singularity $1/(n\cdot k)$ must be interpreted as the principal value

$$P\left[\frac{1}{n\cdot k}\right]=\lim_{\alpha\to 0}\frac{n\cdot k}{(n\cdot k)^2+\alpha^2} \ . \tag{14}$$

This again holds only for timelike n.

III. PLANAR GAUGE WITH BRS SYMMETRY

A. Lagrangian and usual Feynman rules

Let us now go to the non-Abelian case. It is clear that the gauge-fixing terms in (4) can be extended in two distinct ways, either by keeping the ordinary derivatives or by replacing them by covariant derivatives. In this section, we consider only the first possibility. The Lagrangian is

$$\mathcal{L}=-\tfrac{1}{4}F^\alpha_{\mu\nu}F^{\mu\nu}_\alpha+\partial_\mu n\cdot A^\alpha\partial^\mu S_\alpha-\tfrac{1}{2}\partial_\mu S^\alpha\partial^\mu S_\alpha+\mathcal{L}_{\text{ghost}} \ , \tag{15}$$

where, as usual,

$$F^\alpha_{\mu\nu}=\partial_\mu A^\alpha_\nu-\partial_\nu A^\alpha_\mu+gf^\alpha{}_{\beta\gamma}A^\beta_\mu A^\gamma_\nu \ , \tag{16}$$

$$D^\alpha_{\mu\beta}=\partial_\mu\delta^\alpha_\beta+gf^\alpha{}_{\beta\gamma}A^\gamma_\mu \ . \tag{17}$$

The ghost part $\mathcal{L}_{\text{ghost}}$ is obtained by requiring invariance under the BRS transformations:

$$\delta A^\alpha_\mu=(D_\mu\eta)^\alpha\delta\lambda, \quad \delta S_\alpha=0 \ ,$$
$$\delta\xi_\alpha=-S_\alpha\delta\lambda, \quad \delta\eta^\alpha=\frac{g}{2}f^\alpha{}_{\beta\gamma}\eta^\beta\eta^\gamma\delta\lambda \ . \tag{18}$$

Here, η is the ghost and ξ the antighost. As $\delta\lambda$, they are, in the classical case, anticommuting Grassmann variables. Invariance implies

$$\mathcal{L}_{\text{ghost}}=-\partial^\mu\xi_\alpha\partial_\mu(n\cdot D\eta)^\alpha \ . \tag{19}$$

It involves second-order derivatives of the fields, a fact which is in general unwanted in usual field theories. Therefore, one tries to remove them by rewriting (19) up to a four-divergence as

$$\mathcal{L}_{\text{ghost}}=\Box\xi_\alpha(n\cdot D\eta)^\alpha=\xi'_\alpha(n\cdot D\eta)^\alpha \ , \tag{20}$$

where the second-order derivative is removed by introducing the new field

$$\xi'_\alpha=\Box\xi_\alpha \ .$$

The removal of second-order derivatives is not necessary when one makes formal manipulations in the path-integral formalism. In that formalism, a BRS-invariant Lagrangian is

$$\mathcal{L}=-\tfrac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu}_{\alpha}-n\cdot A^{\alpha}\Box S_{\alpha}+\tfrac{1}{2}S_{\alpha}\Box S^{\alpha}+\xi'_{\alpha}(n\cdot D_{\eta})^{\alpha} \quad (21)$$

or, by removing the S field by integration,

$$\mathcal{L}'=-\tfrac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu}_{\alpha}-\tfrac{1}{2}n\cdot A_{\alpha}\Box n\cdot A^{\alpha}+\xi'_{\alpha}(n\cdot D_{\eta})^{\alpha} . \quad (22)$$

This is the Lagrangian used by Andrasi and Taylor.[2] It formally gives rise to the following Feynman rules: (1) the gluon propagator is given by Eq. (1); (2) the ghost propagator is

$$D^{\alpha}_{\beta}(k)=-i\frac{\delta^{\alpha}_{\beta}}{n\cdot k} ; \quad (23)$$

(3) the gluon vertices are the usual ones; (4) the ghost-ghost-gluon vertex is $-gf^{\alpha}{}_{\beta\gamma}n_{\mu}$. The infrared poles at $n\cdot k=0$ are regularized by the principal-value prescription (14).

B. Ambiguity in the gluon self-energy

Let us proceed to the computation of the gluon self-energy on the basis of these rules by taking also into account the remarks made in the Introduction. Of course, details will be omitted and we concentrate our attention only on the crucial points.

1. Ghost contribution

The ghost contribution involves the integral

$$I=\int d^4p\frac{n\cdot p}{(n\cdot p)^2+\alpha^2}\frac{n\cdot(p+q)}{[n\cdot(p+q)]^2+\alpha^2} . \quad (24)$$

Using Eq. (3) and making the substitution $n\cdot p'=n\cdot(p+q)$ in the convergent integral over $n\cdot p$ involved by the second term of the right-hand side of Eq. (3), we see that the first two terms mutually cancel. If we put $n=(1,0,0,0)$, the third term gives

$$I=\pi^2\int d^3p\, \delta(q_0) . \quad (25)$$

If the divergent integral over d^3p is dimensionally regularized (and only in this case), we get $I=0$, so that there is no ambiguity in the ghost contribution, at least in the frame of dimensional regularization.

2. Gluon contribution

For the gluon contribution, we will assume that the standard calculation[14] is correct, in order to avoid an unnecessary recalculation of the contribution of the first two terms of Eq. (3). We only compute the possible additional singular contribution coming from the last term:

$$\pi^{bb'}_{\mu\mu'\mathrm{sing}}(q)=\frac{\pi}{2}\delta(q_0)g^2f_a{}^{bc}f^a{}_{b'c}\int\frac{d^3p}{(2\pi)^3}\frac{1}{|\mathbf{p}|^2|\mathbf{p}+\mathbf{q}|^2}\int dp_0\delta(p_0)d_{\mu\mu'}(p,q) , \quad (26)$$

where

$$\begin{aligned}d_{\mu\mu'}(p,q)=&(p_\mu n_{\mu'}+n_\mu p_{\mu'})[n\cdot p(q^2+2p\cdot q+2p^2)+p^2n\cdot q]\\&-(q_\mu n_{\mu'}+n_\mu q_{\mu'})[n\cdot p(q^2+2p\cdot q-p^2)+n\cdot q(3p\cdot q+p^2)]\\&-(q_\mu p_{\mu'}+q_{\mu'}p_\mu)[n^2(q^2-2p\cdot q)+(n\cdot p)^2-(n\cdot q)^2-n\cdot pn\cdot q]\\&+2n_\mu n_{\mu'}(q^2-p^2)p\cdot(2q+p)-p_\mu p_{\mu'}[4n^2q^2+2(n\cdot p)^2+2n\cdot pn\cdot q]+2q_\mu q_{\mu'}(n^2p\cdot q+n\cdot pn\cdot q) . \end{aligned} \quad (27)$$

If the integration over d^3p is dimensionally regularized, we get

$$\pi^{bb'}_{00\mathrm{sing}}(q)=\pi^2g^2f^a{}_{bc}f_a{}^{b'c}\tfrac{297}{3072}|\mathbf{q}|^3\delta(q_0) , \quad (28a)$$

$$\pi^{bb'}_{0k\,\mathrm{sing}}(q)=\pi^2g^2f^a{}_{bc}f_a{}^{b'c}\tfrac{21}{256}q_0q_k|\mathbf{q}|\delta(q_0) , \quad (28b)$$

$$\pi^{bb'}_{kl\,\mathrm{sing}}(q)=\pi^2g^2f^a{}_{bc}f_a{}^{b'c}\left[\tfrac{1}{32}(q^2g_{kl}-q_kq_l)|\mathbf{q}|+\frac{3}{32}\frac{q_kq_l}{|\mathbf{q}|}q_0^2\right]\delta(q_0) . \quad (28c)$$

This singular contribution does not vanish. Since it is undesirable, one can argue that it should not be taken into account and that the correct way to compute axial-vector gauge integrals consists of regularizing the poles only after the description (2). No other compelling argument can be advanced in favor of this procedure. On the contrary, if we remember a calculation one of us[11] made in the temporal gauge, the regularization is necessarily made before the decomposition and the singular term cancels with the ghost contribution. The difference between the two cases is that the temporal gauge is, in this approach, the limit of gauges also defined for $\alpha\neq 0$ while the planar gauge cannot be considered as the limit for $\alpha\to 0$ of theories which are gauges of Yang-Mills theory.

Actually, we did not succeed in finding a Lagrangian approach where the planar gauge is the limit for $\alpha \to 0$ of consistent gauges.

C. The Cheng and Tsai approach

There is however a non-Lagrangian approach which gives a meaning as a gauge for $\alpha \neq 0$ to the principal-value regularized propagator (1). It uses a theorem due to Cheng and Tsai:[5] if the gluon propagator is

$$D^{a}_{\mu\nu b}(k) = \frac{-i\delta^{a}{}_{b}}{k^2+i\epsilon}[g_{\mu\nu} - a_\mu(k)k_\nu - b_\nu(k)k_\mu + c(k)k_\mu k_\nu] \tag{29}$$

with

$$a_\mu(k) = -b_\mu(-k), \quad c(k) = c(-k) , \tag{30}$$

the ghost propagator is

$$\frac{-i\delta^{a}{}_{b}}{k^2+i\epsilon}$$

and the ghost-ghost-gluon vertex reads

$$g f^{a}{}_{bc}[(a\cdot k - 1)k^\mu - k^2 a^\mu] .$$

In our particular case,

$$a_\mu(k) = b_\mu(k) = \frac{n_\mu n\cdot k}{(n\cdot k)^2+\alpha^2}, \quad c(k) = 0 . \tag{31}$$

In this approach, the limit $\alpha \to 0$ is taken at the very end of the calculations and the singular part (28) is present in the gluon contribution to the gluon self-energy. If the ghost contribution at the two-loop level does not vanish, the ghost contribution to the gluon self-energy at the one-loop level is however not affected and the singular term (28) is not canceled. The presence of such terms can be viewed as a serious obstruction against the use of Lagrangian (21) or (22) as describing a gauge of Yang-Mills theory. In addition, the nonconservation, even in the absence of the singular terms, of the gluon self-energy is already a difficulty.

D. BRS symmetry and the canonical Lagrangian

This kind of difficulty should always be considered as a manifestation of a hidden problem in the starting Lagrangian. If we go back to (22), we see that it is BRS symmetric but involves second-order derivatives of the fields. If only first-order derivatives are accepted as required by the usual canonical formalism, the Lagrangian giving rise to the same set of Feynman rules is

$$\mathcal{L} = -\tfrac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu}_{\alpha} + \partial_\mu n\cdot A_\alpha \partial^\mu S^\alpha - \tfrac{1}{2}\partial_\mu S_\alpha \partial^\mu S^\alpha + \xi'_\alpha (n\cdot D_\eta)^\alpha \tag{32}$$

or

$$\mathcal{L}' = -\tfrac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu}_{\alpha} + \tfrac{1}{2}\partial_\mu n\cdot A_\alpha \partial^\mu n\cdot A^\alpha + \xi'_\alpha (n\cdot D_\eta)^\alpha \tag{33}$$

if the S field is removed as explained in Sec. II. The Lagrangians (32) or (33) are not BRS invariant. We indeed have

$$\delta\mathcal{L} = \partial^\mu[\partial_\mu S_\alpha (n\cdot D_\eta)^\alpha]\delta\lambda , \tag{34}$$

i.e., the variation of the Lagrangian is a four-divergence. This does not matter for constructing a conserved current but, if field equations are invariant, canonical commutation relations are not. This last point can easily be checked at the level of commutation relations between ghosts. Since

$$\pi^{\alpha}_{\eta} = \xi'^{\alpha} , \tag{35}$$

we have

$$\{\xi_\alpha(x), \eta^{\alpha'}(y)\}_{x_0=y_0} = i\delta^{\alpha'}_{\alpha}\delta^{(3)}(\mathbf{x}-\mathbf{y}) \tag{36}$$

and Eq. (36) is obviously not invariant under the transformation (18). Consequently, the quantum theory based on (33) is not BRS invariant and cannot therefore be a gauge for Yang-Mills theory. One can also check this point by considering the consistency between Eqs. (18) and the brackets $[Q,\phi]$ when Q is the BRS charge

$$Q = \int d^3x\, j_0(x) \tag{37}$$

with

$$j_0(x) = \pi^i (D_i\eta)^\alpha + \frac{g}{2} f^{\alpha}{}_{\beta\gamma}\pi_{\eta\alpha}\eta^\beta \eta^\gamma \tag{38}$$

and ϕ an arbitrary field. We indeed get

$$[Q, A^{\alpha}_{0}] = 0 , \tag{39}$$

while

$$\delta A^{\alpha}_{0} = (D_0\eta)^\alpha \delta\lambda . \tag{40}$$

Equations (39) and (40) are consistent only if

$$D_0\eta = 0 , \tag{41}$$

i.e., if field equations are taken into account. Again, this means that field equations are invariant but canonical commutation relations are not.

Let us remark that it is possible to have the strict invariance of

$$\mathcal{L}'' = -\tfrac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu}_{\alpha} + \partial_\mu n\cdot A_\alpha \partial^\mu S^\alpha - \tfrac{1}{2}\partial_\mu S_\alpha \partial^\mu S_\alpha \tag{42}$$

under Eqs. (18) where η is restricted by (41). This is however not sufficient to get a consistent theory because η is an external field with respect to (41). If we introduce it with its field equation inside the Lagrangian with the help of a Lagrangian multiplier, we get Eq. (32) back and invariance is lost.

E. Conclusions

We conclude this section by noting the following two points.

(1) The planar gauge Lagrangian cannot be made BRS invariant without the use of second-order derivatives. Therefore, the usual canonical formalism is not of application.

(2) When the poles at $n \cdot k=0$ are regularized, the theory with the regularization parameter different from zero is not a gauge. Therefore, even if we were able to handle correctly the second-order derivatives, we would be faced with the problem of the definition of the product of two or more propagators.

For these reasons we can never claim that the planar gauge is a well-defined gauge of Yang-Mills theory. We thank that the problems encountered in the calculation of the gluon self-energy, i.e., singular terms, nonconservation, nonmultiplicative renormalization, . . . , have their roots in this point.

IV. PLANAR GAUGE WITH ANTI-BRS SYMMETRY

A. Lagrangian and Feynman rules

Let us now consider the second possibility to extend the planar gauge to non-Abelian theories. The Lagrangian is

$$\mathcal{L}=-\tfrac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu}_{\alpha}+(D_\mu S)_\alpha(D^\mu n\cdot A)^\alpha-\tfrac{1}{2}(D_\mu S)_\alpha(D^\mu S)^\alpha + \mathcal{L}_{\text{ghost}} , \tag{43}$$

where $\mathcal{L}_{\text{ghost}}$ is obtained by requiring the invariance not under BRS transformations because this is not possible but under anti-BRS transformations given by

$$\delta A^{\alpha}_{\mu}=(D_\mu\eta)^\alpha\overline{\delta\lambda},\quad \delta S_\alpha=gf_\alpha{}^{\beta\gamma}S_\beta\eta_\gamma\overline{\delta\lambda} ,$$
$$\delta\eta_\alpha=\frac{g}{2}f_\alpha{}^{\beta\gamma}\eta_\beta\eta_\gamma\overline{\delta\lambda},\quad \delta\xi_\alpha=S_\alpha\overline{\delta\lambda}+gf_\alpha{}^{\beta\gamma}\xi_\beta\eta_\gamma\overline{\delta\lambda} . \tag{44}$$

By invariance requirements,

$$\mathcal{L}_{\text{ghost}}=(D^\mu\xi)_\alpha(D_\mu n\cdot\partial\eta)^\alpha=(D^\mu\xi)_\alpha(D_\mu\eta')^\alpha , \tag{45}$$

where we have set

$$\eta'=n\cdot\partial\eta \tag{46}$$

in order to remove second-order derivatives from the Lagrangian. Here, this removal is done without the introduction of symmetry-breaking four-divergences. In order to discuss the perturbation theory (it is useful to remove the S field like in Sec. II), the Lagrangian becomes

$$\mathcal{L}'=-\tfrac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu}_{\alpha}+\frac{1}{2n^2}(D_\mu n\cdot A)_\alpha(D^\mu n\cdot A)^\alpha + (D^\mu\xi)_\alpha(D^\mu\eta')^\alpha , \tag{47}$$

where $n^2=1$ is reintroduced in order to allow the formal extension to spacelike values. The Feynman rules derived from (47) are (1) the gluon propagator is given by (1) with the principal-value prescription, (2) the ghost propagator is

$$D^a{}_b(k)=\frac{i\delta^a{}_b}{k^2+i\epsilon} , \tag{48}$$

and (3) the vertices are given in Fig. 1.

Let us remark that, in the three- and four-gluon vertices, the metric tensor $g_{\mu\nu}$ is replaced by

$$P_{\mu\nu}=g_{\mu\nu}-\frac{n_\mu n_\nu}{n^2} ,$$

which is a projection operator. Since

$$n^\mu P_{\mu\nu}=n^\nu P_{\mu\nu}=0 , \tag{49}$$

this replacement will be the source of numerous cancellations in the course of calculations. This operator $P_{\mu\nu}$ is already present in the free part of the Lagrangian, i.e., in the inverse of the propagator.

B. Gluon self-energy

In order to get some informations about the usefulness of (43) as a gauge, we computed the gluon self-energy at the one-loop level by using the standard rules for nonrelativistic gauges.[10,15] In order to take full advantages of the properties of $P_{\mu\nu}$, all the algebraic calculations were performed with the formal pole $1/(n\cdot p)$. We separately computed the singular $\delta(q\cdot n)$ term. Here, all such terms are multiplied by $(q\cdot n)^2$ and therefore do not contribute.

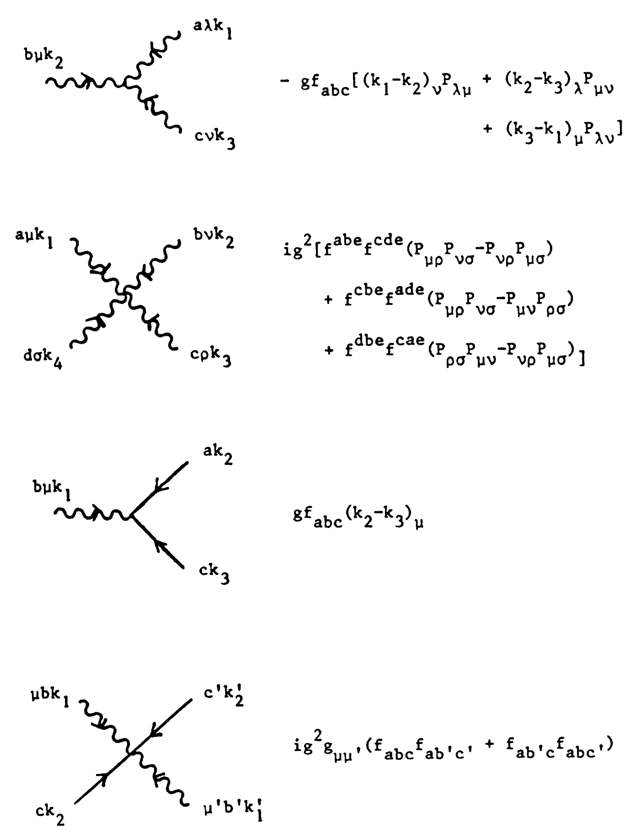

FIG. 1. Vertices of the planar gauge with anti-BRS symmetry. Wavy lines stand for gluons, straight lines for ghosts. All the particles are incoming and $P_{\lambda\mu}=g_{\lambda\mu}-n_\lambda n_\mu/n^2$.

In the same way, the formal identity $(n\cdot p)/(n\cdot p)=1$ which actually should be

$$\frac{(n\cdot p)^2}{(n\cdot p)^2+\alpha^2}=1-\frac{\alpha^2}{(n\cdot p)^2+\alpha^2} \tag{50}$$

can be used without additional contributing terms. Actually, it is even plausible but we did not check it that the negligible terms in $(q\cdot n)^2\delta(q\cdot n)$ are canceled by negligible contributions from the term in α^2 of Eq. (50).

The result for the pole part is

$$\pi^{bb'}_{\mu\mu'\text{pole}}(q)=\frac{g^2}{16\pi^2}f_a{}^{bc}f^a{}_{b'c}\left[10q_\mu q_{\mu'}-\tfrac{29}{3}q^2p_{\mu\mu'}+q_\mu q_{\mu'}\frac{(n\cdot q)^2}{n^2q^2-(n\cdot q)^2}-\tfrac{1}{4}(n_\mu q_{\mu'}+n_{\mu'}q_\mu)\frac{n\cdot q}{n^2}\frac{n^2q^2+(n\cdot q)^2}{n^2q^2-(n\cdot q)^2}\right] . \tag{51}$$

There is no particular appealing feature in this formula. We remark that the contribution is not of the metric tensor but of $P_{\mu\mu'}$. There is neither conservation nor orthogonality to n and the tensor structure is not compatible with multiplicative renormalization, so that the use of (43) as a gauge can also be questioned.

C. The invariance problem

Although there is, in the gluon self-energy, no problem with the singular $\delta(q\cdot n)$ term which vanishes, Eq. (51) does not seem to be accepted for Yang-Mills theory. There should again be a problem with the invariance of Lagrangian (43) when it contains only first-order derivatives. Actually, (43) with

$$\mathcal{L}_{\text{ghost}}=(D^\mu\xi)_\alpha(D_\mu\eta')_\alpha$$

is not invariant under (44) and the transformation for η'

$$\delta\eta'_\alpha=gf_\alpha{}^{\beta\gamma}\eta'_\beta\eta_\gamma\overline{\delta\lambda} . \tag{52}$$

Its variation is

$$\delta\mathcal{L}=(D_\mu S)_\alpha[D^\mu(\eta'-n\cdot\partial\eta)]^\alpha\overline{\delta\lambda} \tag{53}$$

and it is only when we set by hand

$$\eta'=n\cdot\partial\eta \tag{54}$$

that $\delta\mathcal{L}$ vanishes. Again, the anti-BRS transformations introduce a field η which is not present in the starting Lagrangian. It is impossible to introduce it inside a strictly invariant Lagrangian without the presence of second-order derivatives. If second-order derivatives are not wanted, only an invariance up to the validity of field equations [in particular Eq. (54)] is obtained by adding to $\mathcal{L}$ the term $u_\alpha(\eta'-n\cdot\partial\eta)^\alpha$.

As for the invariance up to a four-divergence, this situation is unacceptable in quantum field theory. Here, this incompatibility can be manifested by computing

$$\{Q,\pi_\eta\}$$

with

$$Q=\int d^3x\left[\pi^k_\alpha(D_k\eta)^\alpha+\pi^0_\alpha\eta'^\alpha+gf^\alpha{}_{\beta\gamma}\pi^0_\alpha A^\beta_0\eta^\gamma +gf^\alpha{}_{\beta\gamma}\pi_{S\alpha}S^\beta\eta^\gamma+\pi^\alpha_\xi S_\alpha+gf^\alpha{}_{\beta\gamma}\pi_{\xi\alpha}\xi^\beta\eta^\gamma +gf^\alpha{}_{\beta\gamma}\pi_{\eta'\alpha}\eta'^\beta\eta^\gamma+\frac{g}{2}f^\alpha{}_{\beta\gamma}u_\alpha\eta^\beta\eta^\gamma\right] . \tag{55}$$

We indeed get

$$\{Q,\pi^\alpha_\eta\}=D^k\pi^{k\alpha}+gf^\alpha{}_{\beta\gamma}\pi^{0\beta}A^\gamma_0+gf^\alpha{}_{\beta\gamma}\pi^\beta_S S^\gamma +gf^\alpha{}_{\beta\gamma}\pi^\beta_\xi\xi^\gamma+gf^\alpha{}_{\beta\gamma}\pi^\beta_{\eta'}\eta'^\gamma+gf^\alpha{}_{\beta\gamma}u^\beta\eta^\gamma , \tag{56}$$

which is to be compared with

$$\delta\pi^\alpha_\eta=\delta u^\alpha=gf^\alpha{}_{\beta\gamma}\pi^\beta_\eta\eta^\gamma\overline{\delta\lambda} . \tag{57}$$

V. SUMMARY AND CONCLUSIONS

We reexamined the problem of the planar gauge mainly from the point of view of BRS symmetry. We noted that BRS and anti-BRS symmetries cannot be realized simultaneously, a property which allows us to consider two possible theories with the same propagator. The first one is BRS invariant. The second one is anti-BRS invariant. In both cases, the strict invariance imposes the presence of second-order derivatives in the Lagrangian, a fact which is incompatible with usual perturbative methods. This may be the source of the difficulties encountered when the self-energy is computed perturbatively in both cases. Indeed, the gluon self-energy is not conserved and its tensor structure is such that multiplicative renormalization does not work. In both cases, the perturbative theory is obtained after a removal of second-order derivatives which breaks BRS invariance. Therefore, there is no reason that the result so obtained will correspond to a gauge of Yang-Mills theory.

We also stressed an internal difficulty in the BRS-invariant "gauge." In which order should the operations of regularizing the poles and decomposing the pole product $[1/(n\cdot p)]\{1/[n\cdot(p+q)]\}$ be made? There is a difference according to which operation is first made, at least in the BRS-invariant "gauge." For anti-BRS-invariant "gauge," the difference can be neglected at least in the gluon self-energy calculation.

Usually, one first regularizes the poles and, only after that, the decomposition is made. In such a procedure, the theory is usually a gauge before to take the limit for, in our case, $\alpha \rightarrow 0$. This appears not to be the case for the planar "gauge" which is a possible gauge only in the limit $\alpha \rightarrow 0$.

Our paper is in apparent contradiction with a recent paper by Kummer.[16] Without Faddeev-Popov ghosts and without BRS symmetry, he proved renormalization and gauge independence in the planar gauge. His result is however based on the Lagrangian (22) without the ghost term used in path-integral formalism. Our analysis shows ghost decoupling and BRS invariance of (22). Therefore, (22) is a gauge for Yang-Mills theory and there is no conflict with Kummer's result. The difficulties come when one tries to make a perturbative calculation with this Lagrangian. All the problems with the gluon propagator are present and the true question is the following: what is the gluon propagator associated with the Lagrangian (22)? Our paper criticizes the choice (1) which comes not from (22) but from (33) which is not BRS invariant.

As a last remark, let us mention another approach to the planar gauge.[17] It involves ghosts and a BRS invariance extended to variations of the gauge parameters.

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