Non-Abelian gauge couplings at finite temperature in the general covariant gauge

Hisao Nakkagawa
Institute for Natural Science, Nara University, 1500 Misasagi-cho, Nara 631, Japan

Akira Niégawa and Hiroshi Yokota*
Department of Physics, Osaka City University, Sumiyoshi-ku, Osaka 558, Japan
(Received 22 February 1988)

By the use of the real-time formalism of field theory at finite temperature we carry out the one-loop off-shell calculation for massless non-Abelian gauge theories at finite temperature in the general covariant gauge, and study the properties of the temperature- and gauge-dependent couplings, $a(\mu,\xi\equiv T/\mu;\alpha)$. The effects of temperature manifest themselves as a powerlike behavior of the inverse of the coupling $a^{-1}(\mu,\xi;\alpha)$ on the parameter $\xi=T/\mu$, in sharp contrast to the logarithmic dependence on the momentum μ. The strong vertex dependence of the coupling is shown to come out irrespective of any choice of the gauge parameter α within the covariant gauge, and thus may invalidate the perturbative treatment of gauge theory at finite (especially at high) temperature.

I. INTRODUCTION

In recent years growing interest in phenomena such as the quark-gluon plasma to be soon produced in heavy-ion collisions and the evolution of the early Universe has stimulated particle physicists to study seriously the consequences of the real-time formalism[1] of field theories at finite temperature (FT^2). Extensive analyses of the infrared (or long-distance) structure of the gluon (gauge-boson) propagator in a gauge theory at $T\neq 0$ have been done[2–4] so far, which revealed the following fact:[3,4] In gauge theories at $T\neq 0$ thermal fluctuations (of the gluon) act to screen the electric field component of the gluon, through the development of temperature-dependent electric mass $m^2\sim g^2T^2$, where g is a gauge-coupling constant. As for magnetic screening, for an Abelian case it is known that the magnetic component is not screened, in fact the long-range interaction between purely Abelian magnetic fields is always that of a free field.[3] For a non-Abelian case, though the situation is still controversial, it is likely that the magnetic field is less screened, if at all, than the electric field. Therefore, interactions at long distance or at large T/μ (T=temperature of environment, μ^{-1}=distance from a charge) are expected to be magnetic dominant. It is worth noticing that magnetic screening in non-Abelian gauge theories has in perturbation theory been studied by considering only the propagator. It should be studied further by considering at least two- and three-point functions. This will be done in the present paper.

From the above observations, for analyses of high-energy processes that take place in a hot environment, it is necessary to neatly define an effective charge or a coupling constant (a parameter of a perturbative expansion) as a function of the temperature T of the environment and the energy scale μ that characterizes the process considered, without restricting ourselves to the infrared regions $T/\mu\gg 1$. For this purpose it is natural to employ the following renormalization procedure: In the calculation of a physical quantity characterized by a given distance μ^{-1}, any modes of thermal as well as quantum excitations, which correspond to shorter distances than μ^{-1}, are incorporated, i.e., renormalized into the effective charge. This renormalization procedure is nothing but the finite-temperature generalization of the momentum-space subtraction method extensively studied in ordinary QCD (Ref. 5) and also in QCD at finite density,[6] in conjunction with the renormalization-scheme ambiguity.[7] With the use of effective coupling thus defined, we expect to get "good" perturbative expansion for the physical quantity.

Let us here consider the theory renormalized at finite nonzero temperature (hereafter we denote[8] the renormalization temperature as $\tilde{T}$ and the renormalization momentum as μ) and the same theory renormalized at zero temperature $\tilde{T}=0$. Because of the fact[9,10] that the ultraviolet divergences in FT^2 appear only in the zero-temperature contributions, the coupling renormalized at nonzero temperature $\tilde{T}$, $a(\mu,\xi\equiv\tilde{T}/\mu)$, and the coupling renormalized at $\tilde{T}=0$, $\tilde{a}(\mu)$, differ only by a finite renormalization,

$$a(\mu,\xi)=z(\xi)\tilde{a}(\mu)\equiv Z_a^{-1}Z_{\tilde{a}}\tilde{a}(\mu)$$
$$=\tilde{a}(\mu)[1+v_1(\xi)\tilde{a}(\mu)+\cdots], \qquad (1.1)$$

where $z(\xi)\equiv Z_a^{-1}Z_{\tilde{a}}$ denotes the temperature-dependent *finite* renormalization constant. The coefficients $v_i(\xi)$ reflect the difference between the renormalization schemes used to define two couplings $a(\mu,\xi)$ and $\tilde{a}(\mu)$. The leading coefficient $v_1(\xi)$ is nothing but the leading one-loop term of $z(\xi)$:

$$v_1(\xi)=z^{(1)}(\xi)=-(Z_a^{(1)}-Z_{\tilde{a}}^{(1)}), \qquad (1.2)$$

where

$$z(\xi)=1+z^{(1)}(\xi)\tilde{a}(\mu)+\cdots. \qquad (1.3)$$

The temperature (ξ) dependence of the coupling $a(\mu,\xi)$

is completely specified by $z(\xi)$, or by the coefficients $v_i(\xi)$.

A more convenient as well as conventional way to keep track of the temperature ($\xi \equiv \tilde{T}/\mu$) and momentum (μ) dependences of the coupling is to integrate (generally speaking, a set of) renormalization-group equations (RGE's), which express the response of the coupling under the change of renormalization point $\tilde{T}$ and μ, with a suitable boundary condition[11,12] by introducing a μ-independent scale parameter Λ that depends, in general, on the temperature through the parameter $\xi \equiv \tilde{T}/\mu$ (see Secs. II and III). Integrating the RGE's satisfied by $a(\mu,\xi)$ and $\tilde{a}(\mu)$ with the same boundary condition, μ and ξ dependences of the couplings are expressed by (only the leading behaviors of interest are given)

$$a(\mu,\xi) \simeq 1/\{b \ln[\mu/\Lambda(\xi)]\} \ , \tag{1.4a}$$

$$\tilde{a}(\mu) \simeq 1/[b \ln(\mu/\Lambda)] \ , \tag{1.4b}$$

where $\Lambda \equiv \Lambda(\xi \equiv \tilde{T}/\mu = 0)$, and b is the leading coefficient of the RGE's (see Sec. II). Thus it is obvious that the logarithmic μ dependence of the coupling $a(\mu,\xi)$ is exactly the same as that of the familiar coupling $\tilde{a}(\mu)$ in the zero-temperature theory. If we confine our interest to the μ dependence, then the temperature-dependent scale parameter $\Lambda(\xi)$ sets the scale (that varies with the temperature) for the momentum μ from which the perturbation analysis in terms of the coupling $a(\mu,\xi)$ works, i.e.,

$$a(\mu,\xi) \ll 1 \quad \text{for } \mu \gg \Lambda(\xi) \ . \tag{1.5}$$

In the present method the temperature dependence of $a(\mu,\xi)$ can be specified if we can calculate the scale parameter $\Lambda(\xi)$.

Now let us remember the fact that as Celmaster and Gonsalves have shown[5] [and as is easily derived from Eqs. (1.1) and (1.4)] two scale parameters $\Lambda(\xi)$ and $\Lambda \equiv \Lambda(\xi=0)$ are *exactly* related with the leading one-loop coefficient $v_1(\xi)$ in Eq. (1.1) as

$$\Lambda(\xi)/\Lambda = \exp[v_1(\xi)/b] \ . \tag{1.6}$$

Then up to leading one-loop orders the temperature dependence of the coupling $a(\mu,\xi)$ can be determined by calculating the temperature-dependent finite renormalization constant $z(\xi) \equiv Z_a^{-1} Z_{\tilde{a}}$ at the one-loop level.

Up to now, most existing analyses are confined to investigations of the thermodynamic and effective potentials. In such analyses, on the basis of the fact[9] mentioned earlier that the renormalization of FT^2 can be completed with the counterterms determined at zero temperature, the coupling renormalized at $\tilde{T}=0$, $\tilde{a}(\mu)$, has been used, in which the temperature dependence is assumed to be reasonably taken into account by simply choosing[2] $\mu \simeq T$, the temperature of the environment. This is an as-yet-unproved assumption that will be proved or disproved to hold only when the relation between two couplings $\tilde{a}(\mu \simeq T)$ and $a(\mu,\xi)$ is clarified. Turning our eyes to analyses of dynamical processes taking place in a thermal reservoir, which are now becoming important issues, in these analyses we should employ $a(\mu,\xi)$ instead of $\tilde{a}(\mu \simeq T)$. This is especially important if two couplings $a(\mu \simeq T, \xi \simeq 1)$ and $\tilde{a}(\mu \simeq T)$ show different behaviors. In this case if we perform perturbative expansion in terms of the coupling $\tilde{a}(\mu \simeq T)$ for a physical quantity, for which the coupling $a(\mu \simeq T, \xi \simeq 1)$ is actually a good expansion parameter, then we apparently face the problem of large higher-order perturbation coefficients—the reason why it is important to use "good" coupling.

In a previous paper[13] we briefly reported, based on the Feynman-gauge calculation, that the last statement is in fact the case, i.e., the T dependences of $a(\mu \simeq T, \xi \simeq 1)$ and $\tilde{a}(\mu \simeq T)$ are completely different from each other. We also showed that the vertex dependence of the coupling is enormous, which may cause a difficult problem in the perturbative FT^2. However, it is still an open question whether or not these discouraging results come out *independent of the particular gauge choices.*

The purpose of the present paper is essentially to answer the above posed question. For this purpose we carry out a detailed and extended analysis of the temperature dependence of the gauge coupling in massless non-Abelian gauge theories at finite temperature in the general covariant gauge. In the actual physical processes such as the quark-gluon plasma and the evolution of the early Universe, the effect of finite baryon-number density is expected[14] not to be significant; thus in the present paper we study the case where chemical potential is negligibly small. Inclusion of the nonzero chemical potential will be discussed in a separate paper.

This paper is organized as follows. In Sec. II we explain how we performed the off-shell one-loop calculations at finite temperature. We briefly discuss the RGE's at finite temperature in non-Abelian gauge theories in the general covariant gauge, and give their explicit forms up to the one-loop level. Then in Sec. III these RGE's are explicitly integrated out, and the temperature-dependent scale parameter $\Lambda(\xi)$ is calculated, thus the effective coupling and the gauge parameter are determined. The behavior of the coupling with respect to temperature, to the different choices of vertices, and to the gauge parameter is also discussed extensively. Finally, in Sec. IV we give conclusions and some discussions on the problems that are revealed from the present analysis. In Appendix A we summarize the Feynman rules for massless non-Abelian gauge theories at finite temperature for the sake of completeness. In Appendix B we present exact one-loop results for various Z factors, together with several comments on the explicit calculation and renormalization procedures.

II. RENORMALIZATION-GROUP EQUATIONS AT FINITE TEMPERATURE IN THE GENERAL COVARIANT GAUGE

Let us consider non-Abelian gauge theories with massless fermions in the general covariant gauge, defined by the Lagrangian (all the counterterms are neglected)

$$\begin{aligned}\mathcal{L} = & -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{2(1-\alpha)}(\partial_\mu A^{a\mu})^2 \\ & + (\partial_\mu \bar{c}^{\,a})(\partial^\mu c^a + g f^{abd} A^{b\mu} c^d) \\ & + \sum_\psi \bar{\psi}(i\not{\partial} + g \not{A}^a T^a)\psi \ ,\end{aligned} \tag{2.1}$$

where

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+gf^{abd}A^b_\mu A^d_\nu ,$$

with f^{abd} the structure constant of the non-Abelian gauge group G which is assumed to be the compact Lie group. g is the coupling constant, A is a gauge field, c is the ghost field, ψ represents the massless fermion field, and α is the gauge parameter. Renormalization constants Z_i's are defined conventionally, e.g., Z_3 is the gauge-boson[15] wave-function renormalization constant, etc., and are constrained by the Ward-Takahashi identities

$$\frac{Z_1}{Z_3}=\frac{\tilde{Z}_1}{\tilde{Z}_3}=\frac{Z_1^F}{Z_2},\quad \frac{Z_4}{Z_3}=\left[\frac{Z_1}{Z_3}\right]^2 , \tag{2.2}$$

and

$$g=(Z_1^{-1}Z_3^{3/2})g_B,\quad 1-\alpha_B=Z_3(1-\alpha) , \tag{2.3}$$

where g_B and α_B are the bare coupling constant and the bare gauge parameter.

As mentioned in Sec. I our primary interest is to determine the scale parameter $\Lambda(\xi)$ at finite temperature. For this purpose it is enough to calculate various Z factors in the leading one-loop approximations. Feynman rules which are necessary are summarized in Appendix A.

Throughout this paper we use the coupling a defined by $a\equiv g^2/4\pi^2$ for convenience, and define the coupling renormalization constant Z_a by $a=Z_a^{-1}a_B$; then we have three expressions for Z_a's, i.e.,

$$Z_a(3G)=Z_1^2Z_3^{-3} , \tag{2.4a}$$

$$Z_a(gG)=\tilde{Z}_1^2Z_3^{-1}\tilde{Z}_3^{-2} , \tag{2.4b}$$

$$Z_a(fG)=(Z_1^F)^2Z_3^{-1}Z_2^{-2} , \tag{2.4c}$$

which correspond to couplings defined through the triple-"gluon" ($3G$), the ghost-"gluon" (gG), and the fermion-"gluon" (fG) vertices, respectively.[16]

In order to determine Z factors, we should specify explicitly how we regularize and renormalize the theory. In FT^2 Lorentz covariance is explicitly broken due to the existence of a heat bath that specifies a preferred frame to be the rest frame (center-of-mass frame) of the heat bath. Because of this fact when defining a renormalized theory, besides carefully specifying a subtraction momentum we should also be careful about the prescription of how we determine the subtraction part of each quantity (each self-energy and vertex), by taking notice of which component (temporal and/or spatial) of the quantity is involved. As for a subtraction momentum we choose the "static" spacelike momentum[17] for self-energies and the collinear momentum configuration with "static" spacelike momenta for vertices. As mentioned in Sec. I because it is known[3,4,18] that the interaction at long distance or at large $\xi\equiv T/\mu$ is dominated by the magnetic component, in this paper we choose a "magnetic" prescription[19] to determine the subtraction part. The present calculational scheme is the same as that of a previous paper,[13] and is summarized as follows [hereafter, we refer to the present renormalization scheme as the magnetic momentum subtraction scheme ($\overline{\text{MOM}}$)]: (a) dimensional regularization[20] $D=4-\varepsilon$; (b) momentum-space subtraction[5] at the subtraction point characterized by (1) the off-shell renormalization momentum[20] μ, (2) the "static" subtraction momentum[17] for a propagator $p=(0,\mathbf{p})$, (3) the collinear momentum configuration for a vertex $p_1+p_2\to p_3$, characterized[17] by

$$p_1=(0,0,0,\mu)=-p_2/2=-p_3 , \tag{2.5}$$

with p_2 attributed to a "gluon," and (4) the finite renormalization temperature $\tilde{T}(\equiv\xi\mu)$; (c) "magnetic" prescription for a propagator and a vertex. (See Appendix B for more detail.)

The exact one-loop results for the renormalization constants Z_i's, together with the explicit calculation and renormalization procedures, are given in Appendix B.

In non-Abelian gauge theories at finite temperature in the general covariant gauge, the renormalized gauge coupling $a(\mu,\xi)$ and the renormalized gauge parameter $\alpha(\mu,\xi)$ satisfy the following *coupled* RGE's (Refs. 21 and 22) under changes of the renormalization momentum μ and the renormalization temperature $\tilde{T}\equiv\xi\mu$,

$$\frac{\partial a}{\partial\ln\mu}=\beta_\mu(a,\alpha;\xi)=-ba^2[1+c_1(\xi,\alpha)a+\cdots] , \tag{2.6a}$$

$$\frac{\partial a}{\partial\ln\xi}=\beta_\xi(a,\alpha;\xi)=-\rho(\xi,\alpha)a^2[1+\eta_1(\xi,\alpha)a+\cdots] , \tag{2.6b}$$

$$\frac{\partial \alpha}{\partial\ln\mu}=\delta_\mu(a,\alpha;\xi)=-[\gamma_0(\alpha)a+\gamma_1(\xi,\alpha)a^2+\cdots] , \tag{2.6c}$$

$$\frac{\partial \alpha}{\partial\ln\xi}=\delta_\xi(a,\alpha;\xi)=-[\varepsilon_0(\xi,\alpha)a+\varepsilon_1(\xi,\alpha)a^2+\cdots] , \tag{2.6d}$$

where $a=a(\mu,\xi)$ and $\alpha=\alpha(\mu,\xi)$. β and δ functions are determined by following the standard procedure, e.g.,

$$\beta_\mu(a,\alpha;\xi)\equiv\left.\frac{\partial a}{\partial\ln\mu}\right|_{a_B,\alpha_B,\xi\ \text{fixed},\ \text{cutoff}}=-a\left.\frac{\partial\ln Z_a}{\partial\ln\mu}\right|_{a_B,\alpha_B,\xi\ \text{fixed},\ \text{cutoff}} . \tag{2.7}$$

It is to be noted that in the set of RGE's only the leading one-loop coefficient b in (2.6a) is a constant independent of the choice of renormalization scheme and of gauge.

When we are considering physical quantities, i.e., gauge-invariant Green's functions or S-matrix elements, a change in the gauge parameter can be reabsorbed by a change in the coupling and scale of the fields.[23] As a result, in this case the RGE's to be satisfied by the coupling parameter that governs the physical quantities are as follows:

$$\frac{\partial a}{\partial\ln\mu}\equiv\bar{\beta}_\mu(a,\alpha;\xi)\equiv\beta_\mu-\sigma\delta_\mu=-ba^2(1+ca+\cdots) , \tag{2.8a}$$

$$\frac{\partial a}{\partial\ln\xi}\equiv\bar{\beta}_\xi(a,\alpha,\xi)\equiv\beta_\xi-\sigma\delta_\xi=-\rho(\xi,\alpha)a^2[1+\bar{\eta}_1(\xi,\alpha)a+\cdots] , \tag{2.8b}$$

where $a=a(\mu,\xi)$, $\alpha=\alpha(\mu,\xi)$, and

$$\sigma=\sigma(a,\alpha;\xi)\equiv\left.\frac{\partial a}{\partial\alpha}\right|_{a_B,\mu,\xi \atop \text{cutoff fixed}}$$

$$=-a\left[\frac{\partial\alpha_B}{\partial\alpha}\frac{\partial\ln Z_a}{\partial\alpha_B}\right]_{a_B,\mu,\xi \atop \text{cutoff fixed}}$$

$$=-[\sigma_0(\xi,\alpha)a^2+\sigma_1(\xi,\alpha)a^3+\cdots]\ . \tag{2.9}$$

In this case the second-order coefficient c in Eq. (2.8a) is, besides the leading-order one b, a renormalization scheme and gauge invariant, and is related to the coefficient $c_1(\xi,\alpha)$ in Eq. (2.6a) by

$$c=c_1(\xi,\alpha)+\sigma_0(\xi,\alpha)\gamma_0(\alpha)/b\ . \tag{2.10}$$

In the RGE's (2.8) the gauge parameter α is nothing but a constant (not a running parameter) to be fixed at any value. It is also worth mentioning that in calculations up to leading one-loop orders, β functions appearing in Eqs. (2.6) and (2.8) are identical in their expressions, i.e.,

$$\bar\beta_\mu=\beta_\mu=-ba^2,\ \ \bar\beta_\xi=\beta_\xi=-\rho a^2\ , \tag{2.11}$$

which can be easily understood by noticing $\sigma\delta=O(a^3)$. There is, however, one important difference between them. In Eqs. (2.8) (in $\bar\beta$'s) the gauge parameter α is just a fixed constant, but in Eqs. (2.6) (in β's) α is a running parameter subject to the RGE's (2.6c) and (2.6d) coupled with Eqs. (2.6a) and (2.6b) even in the leading-one-loop-order calculations.

The leading one-loop coefficients of the RGE's (2.6) and (2.8) can be determined with the Z factors given in Appendix B. They are

$$b=\frac{11C_2(G)-4T(R)}{6}\ , \tag{2.12a}$$

$$\rho(\xi,\alpha)=-\sum_{k=0}^{2}\alpha^k\frac{d\Phi^{(k)}(\xi)}{d\ln\xi}\ , \tag{2.12b}$$

$$\gamma_0(\alpha)=(1-\alpha)[\tfrac{1}{4}(\tfrac{10}{3}+\alpha)C_2(G)-\tfrac{2}{3}T(R)]\ , \tag{2.12c}$$

$$\varepsilon_0(\xi,\alpha)=-(1-\alpha)\Bigg\{\tfrac{1}{4}C_2(G)\frac{\partial}{\partial\ln\xi}[3F_0(2\xi)+F_2(2\xi)]+\tfrac{1}{2}T(R)\frac{\partial}{\partial\ln\xi}[F_0(2\xi)-2F_0(\xi)+F_2(2\xi)-2F_2(\xi)]$$
$$+\tfrac{1}{4}\alpha C_2(G)\frac{\partial}{\partial\ln\xi}[-F_0(2\xi)+4G(2\xi)]+\tfrac{1}{16}\alpha^2C_2(G)\frac{\partial}{\partial\ln\xi}[F_0(2\xi)-2G(2\xi)]\Bigg\}\ , \tag{2.12d}$$

$$\sigma_0(\xi,\alpha)=-\sum_{k=1}^{2}k\alpha^{k-1}[C_2(G)B_1^{(k)}+T(R)B_2^{(k)}+C_2(R)B_3^{(k)}+\Phi^{(k)}(\xi)]\ , \tag{2.12e}$$

where $F_j(\xi)$ and $G(\xi)$ are given in Appendix B. The functions $\Phi^{(k)}(\xi)$ and the constants $B_i^{(k)}$ appearing in Eqs. (2.12b) and (2.12e) essentially determine the couplings renormalized at finite and zero temperatures, respectively, and are defined, for convenience, in the next section by Eqs. (3.8) and (3.10). As we already know the second-order coefficient c in (2.8a),

$$c=\frac{17C_2^2(G)-10C_2(G)T(R)-6C_2(R)T(R)}{2[11C_2(G)-4T(R)]}\ , \tag{2.13}$$

we can also get the coefficient $c_1(\xi,\alpha)$ through Eq. (2.10).

III. EFFECTIVE COUPLING AND GAUGE PARAMETER AT FINITE TEMPERATURE

In this section we first determine the coupling and gauge parameter by integrating the RGE's in Sec. III A, and then in Sec. III B study their properties, especially the behavior with respect to the temperature.

A. Determination of the effective coupling and gauge parameter

1. Gauge-invariant case

At first we discuss the case where we are considering physical quantities, i.e., gauge-invariant Green's functions or S-matrix elements. In this case the coupling $a(\mu,\xi;\alpha)$ satisfies the coupled RGE's (2.8), in which the gauge parameter α appears as a mere constant to be fixed at the beginning. This type of coupled RGE has been already extensively studied by two of the present authors[12] and how to integrate them has been shown. Therefore, we only give the result with a few comments. With an appropriate boundary condition (in the present case we adopt the condition in the manner of Stevenson[11,12]) we can integrate the coupled RGE's, thanks to the integrability condition,

$$\frac{\partial\bar\beta_\xi(a,\xi)}{\partial\ln\mu}=\frac{\partial\bar\beta_\mu(a,\xi)}{\partial\ln\xi}\ , \tag{3.1}$$

and get

$$\int_0^{a(\mu,\xi;\alpha)} \frac{dx}{\bar{\beta}_\mu(x,\xi;\alpha)} + \int_0^\infty \frac{dx}{bx^2(1+cx)} = \ln\frac{\mu}{\Lambda(\xi,\alpha)} . \tag{3.2}$$

The temperature-dependent scale parameter $\Lambda(\xi,\alpha)$ is defined by[12]

$$\ln\frac{\Lambda(\xi,\alpha)}{\Lambda(\xi=0,\alpha)} = -\frac{1}{b}\int_0^\xi \rho(y,\alpha)\frac{dy}{y} = -(Z_a^{(1)}-Z_{\bar{a}}^{(1)})/b , \tag{3.3}$$

where $\rho(\xi,\alpha)$ is the leading one-loop coefficient in Eq. (2.8b) and the second equality comes from Eqs. (2.6b), (1.2), and (1.3). As for the coupling $\bar{a}(\mu)$ renormalized at $\bar{T}=0$, the $\bar{\beta}_\xi$ function vanishes identically. *With the same boundary conditions as above* we get

$$\int_0^{\bar{a}(\mu;\alpha)} \frac{dx}{\bar{\beta}_\mu(x,\xi=0;\alpha)} + \int_0^\infty \frac{dx}{bx^2(1+cx)} = \ln\frac{\mu}{\Lambda} , \tag{3.4}$$

with $\bar{a}(\mu;\alpha)=a(\mu,\xi=0;\alpha)$, or

$$\Lambda \equiv \Lambda(\xi=0,\alpha) . \tag{3.5}$$

From Eqs. (3.2)–(3.5) we get in the one-loop approximation the results

$$a(\mu,\xi;\alpha) \simeq \{b\ln[\mu/\Lambda(\xi,\alpha)]\}^{-1} = (b\{\ln(\mu/\Lambda)-\ln[\Lambda(\xi,\alpha)/\Lambda]\})^{-1} , \tag{3.6a}$$

$$\bar{a}(\mu;\alpha) \simeq [b\ln(\mu/\Lambda)]^{-1} , \tag{3.6b}$$

thus the temperature dependence of the coupling $a(\mu,\xi;\alpha)$ can be completely determined by calculating the ratio of scale parameters $\Lambda(\xi,\alpha)/\Lambda$, which can be evaluated through Eq. (3.3).

We give here the exact expressions in the present renormalization scheme $\overline{\text{MOM}}$ for the ratio of scale parameters for three types of couplings a_{3G}, a_{gG}, and a_{fG} defined through the $3G$, gG, and fG vertices, respectively. They are

$$b\ln[\Lambda(\xi,\alpha)/\Lambda]_{\overline{\text{MOM}}} = \sum_{k=0}^{2} \alpha^k \Phi^{(k)}(\xi) , \quad \alpha = \text{gauge parameter} , \tag{3.7}$$

where

$$\Phi^{(k)}(\xi) = \sum_{i=1}^{N} A_i \left[\sum_{j=1}^{3} [\beta_{ij}^{(k)}F_0(\xi_j)+\gamma_{ij}^{(k)}F_2(\xi_j)+\delta_{ij}^{(k)}G(\xi_j)] + \tau_i^{(k)}\pi^2\xi^2 \right] \tag{3.8a}$$

$$\sim \pi^2 \sum_{i=1}^{N} A_i(\tau_i^{(k)}\xi^2+\eta_i^{(k)}\xi) + O(\ln\xi) \quad \text{for } \xi \gg 1 \tag{3.8b}$$

$$\sim \pi^2 \sum_{i=1}^{N} A_i\omega_i^{(k)}\xi^2 + O(\xi^4) \quad \text{for } \xi \ll 1 . \tag{3.8c}$$

In Eqs. (3.8) $N=2$ for the $3G$ and gG vertices whereas $N=3$ for the fG vertex; $A_1=C_2(G)$, $A_2=T(R)$, $A_3=C_2(R)$; $\xi_1=2\xi$, $\xi_2=\xi$, $\xi_3=\frac{1}{2}\xi$; and the functions $F_k(\xi)$ and $G(\xi)$ are defined in Eqs. (B9) and (B10). The calculable constants $\beta_{ij}^{(k)}$, $\gamma_{ij}^{(k)}$, $\delta_{ij}^{(k)}$, $\tau_i^{(k)}$, $\eta_i^{(k)}$, and $\omega_i^{(k)}$ in Eqs. (3.8) are given in Table I.

In order for readers to have ideas about the absolute values of scale parameters in the present $\overline{\text{MOM}}$ scheme, $\Lambda(\xi,\alpha)_{\overline{\text{MOM}}}$, we present here the ratios $\Lambda_{\overline{\text{MOM}}}/\Lambda_{\overline{\text{MS}}}$ ($\overline{\text{MS}}$ denotes the modified minimal subtraction scheme) for three types of vertices $3G$, gG, and fG, calculated at zero temperature. They can be expressed as

$$b\ln\Lambda_{\overline{\text{MOM}}}/\Lambda_{\overline{\text{MS}}} = C_2(G)B_1+T(R)B_2+C_2(R)B_3 , \tag{3.9}$$

where B_i $(i=1,2,3)$ are quadratic functions of the gauge parameter α,

$$B_i = \sum_{j=0}^{2} \alpha^j B_i^{(j)}, \quad \alpha = \text{gauge parameter} , \tag{3.10}$$

and the constants $B_i^{(j)}$ $(i=1-3,\ j=0-2)$ are given in Table II.

2. *Gauge-noninvariant general case*

Next we discuss briefly the case where we are considering general Green's functions. In this case the coupling a and the gauge parameter α satisfy the coupled RGE's (2.6). It is a hard task to solve these equations analytically, and here we only consider them in the leading-one-loop-order approximation. For convenience, however, the first equation (2.6a) is presented in its two-loop-order expression:

$$\frac{\partial a}{\partial\ln\mu} = -ba^2[1+c_1(\xi,\alpha)a] , \tag{3.11a}$$

$$\frac{\partial a}{\partial\ln\xi} = -\rho(\xi,\alpha)a^2 , \tag{3.11b}$$

$$\frac{\partial\alpha}{\partial\ln\mu} = -\gamma_0(\alpha)a , \tag{3.11c}$$

$$\frac{\partial\alpha}{\partial\ln\xi} = -\varepsilon_0(\xi,\alpha)a , \tag{3.11d}$$

where the coefficients b, ρ, γ_0, and ε_0 are given in Eqs. (2.12). These equations can be solved by a heuristic

TABLE I. Calculable constants $\beta_{ij}^{(k)}$, etc. [(a) $k=0$, (b) $k=1$, (c) $k=2$], in Eqs. (3.8). In all parts (a), (b), and (c), for rows with $i=1$ and 2, the first line corresponds to the $3G$ vertex, the second line to the gG vertex, and the third line to the fG vertex. The row with $i=3$ corresponds only to the fG vertex. It is also to be noted that in $\Phi^{(1)}$ there are no terms that are proportional to $T(R)$ and in $\Phi^{(2)}$ no terms proportional to $C_2(R)$ as well as $T(R)$; thus, corresponding constants that are all zero are not given in the table.

(a)

i	$\beta_{i1}^{(0)}$	$\beta_{i2}^{(0)}$	$\beta_{i3}^{(0)}$	$\gamma_{i1}^{(0)}$	$\gamma_{i2}^{(0)}$	$\gamma_{i3}^{(0)}$	$\delta_{i1}^{(0)}$	$\delta_{i2}^{(0)}$	$\delta_{i3}^{(0)}$	$\tau_i^{(0)}$	$\eta_i^{(0)}$	$\omega_i^{(0)}$
1	$\frac{7}{6}$	$\frac{7}{12}$	0	$\frac{2}{3}$	$-\frac{5}{12}$	0	0	0	0	0	$\frac{35}{24}$	1
	$\frac{1}{4}$	$\frac{7}{4}$	0	$\frac{1}{4}$	$-\frac{3}{4}$	0	0	0	0	0	$\frac{9}{8}$	$\frac{5}{6}$
	0	$\frac{17}{4}$	-2	0	$\frac{3}{4}$	-2	0	0	0	0	$\frac{13}{8}$	$\frac{7}{6}$
2	$\frac{4}{3}$	$-\frac{7}{2}$	$\frac{5}{3}$	$\frac{4}{3}$	$-\frac{7}{2}$	$\frac{5}{3}$	0	0	0	0	0	0
	0	$\frac{1}{2}$	-1	0	$\frac{1}{2}$	-1	0	0	0	0	0	0
	0	$\frac{1}{2}$	-1	0	$\frac{1}{2}$	-1	0	0	0	0	0	0
3	0	$-\frac{9}{2}$	4	0	$-\frac{5}{2}$	4	0	0	0	-2	$-\frac{5}{4}$	$-\frac{8}{3}$

(b)

i	$\beta_{i1}^{(1)}$	$\beta_{i2}^{(1)}$	$\beta_{i3}^{(1)}$	$\gamma_{i1}^{(1)}$	$\gamma_{i2}^{(1)}$	$\gamma_{i3}^{(1)}$	$\delta_{i1}^{(1)}$	$\delta_{i2}^{(1)}$	$\delta_{i3}^{(1)}$	$\tau_i^{(1)}$	$\eta_i^{(1)}$	$\omega_i^{(1)}$
1	$-\frac{17}{48}$	$-\frac{1}{12}$	0	$\frac{1}{16}$	$-\frac{1}{4}$	0	$\frac{7}{3}$	$-\frac{4}{3}$	0	0	$-\frac{19}{48}$	$-\frac{11}{6}$
	$\frac{1}{8}$	$-\frac{3}{4}$	0	$-\frac{1}{4}$	1	0	$\frac{3}{4}$	0	0	0	$-\frac{1}{4}$	$-\frac{7}{12}$
	0	$-\frac{17}{16}$	1	0	$-\frac{13}{16}$	1	0	0	0	0	$-\frac{9}{32}$	$-\frac{1}{12}$
3	0	$\frac{7}{4}$	-2	0	$\frac{5}{4}$	-2	0	1	0	0	$\frac{3}{8}$	0

(c)

i	$\beta_{i1}^{(2)}$	$\beta_{i2}^{(2)}$	$\beta_{i3}^{(2)}$	$\gamma_{i1}^{(2)}$	$\gamma_{i2}^{(2)}$	$\gamma_{i3}^{(2)}$	$\delta_{i1}^{(2)}$	$\delta_{i2}^{(2)}$	$\delta_{i3}^{(2)}$	$\tau_i^{(2)}$	$\eta_i^{(2)}$	$\omega_i^{(2)}$
1	$\frac{7}{48}$	$-\frac{7}{48}$	0	$-\frac{1}{16}$	$\frac{1}{4}$	0	$-\frac{1}{4}$	$\frac{1}{8}$	0	0	$\frac{7}{96}$	$\frac{7}{24}$
	$-\frac{3}{32}$	$\frac{3}{16}$	0	$-\frac{1}{32}$	$\frac{1}{8}$	0	$-\frac{1}{8}$	$-\frac{1}{8}$	0	0	0	$\frac{1}{24}$
	0	$-\frac{1}{16}$	0	0	$\frac{3}{8}$	0	0	$-\frac{1}{8}$	0	0	$-\frac{1}{32}$	$-\frac{1}{8}$

method. Let us try to find a solution for the coupling $a=a(\mu,\xi;\alpha)$ having the same expression as (3.2), except that in the present case the gauge parameter α is a running parameter $\alpha=\alpha(\mu,\xi)$:

$$\frac{1}{a}-c\ln\left|\frac{1+ca}{ca}\right|=b\ln\frac{\mu}{\Lambda(\xi,\alpha)}\,, \tag{3.12}$$

or

$$a=\frac{1}{b\ln\mu/\Lambda(\xi,\alpha)}-\frac{c\ln[(b/c)\ln\mu/\Lambda(\xi,\alpha)]}{b^2\ln^2\mu/\Lambda(\xi,\alpha)}+\cdots\,. \tag{3.12$'$}$$

Substituting (3.12′) into Eqs. (3.11a) and (3.11b) and using Eqs. (3.11c), (3.11d), and (2.10), we get

$$\frac{\partial\ln\Lambda(\xi,\alpha)}{\partial\alpha}=-\sigma_0(\xi,\alpha)/b=-\frac{1}{b}\frac{\partial Z_a^{(1)}}{\partial\alpha}\,, \tag{3.13a}$$

$$\frac{\partial\ln\Lambda(\xi,\alpha)}{\partial\ln\xi}=-\rho(\xi,\alpha)/b=-\frac{1}{b}\frac{\partial Z_a^{(1)}}{\partial\ln\xi}\,. \tag{3.13b}$$

These equations can be easily integrated, giving the scale parameter $\Lambda(\xi,\alpha)$ as

$$b\,ln\,[\Lambda(\xi,\alpha)/\Lambda(\xi=0,\alpha=\alpha_0)]=-[Z_a^{(1)}(\xi,\alpha)-Z_a^{(1)}(\xi=0,\alpha=\alpha_0)]\,. \tag{3.14}$$

It is worth mentioning that the solution (3.12) satisfies Eq. (3.11a) up to the second order. The ratio (3.14) can be written as

$$b\ln\left[\frac{\Lambda(\xi,\alpha)}{\Lambda(\xi=0,\alpha=\alpha_0)}\right]_{\rm MOM}=b\left[\ln\frac{\Lambda(\xi,\alpha)}{\Lambda(0,\alpha)}+\ln\frac{\Lambda(0,\alpha)}{\Lambda(0,\alpha_0)}\right]_{\overline{\rm MOM}}\,, \tag{3.15}$$

where the first term on the right-hand side (RHS) is already given in Eqs. (3.7) and (3.8), while the second term

TABLE II. Constants $B_i^{(j)}$ in Eq. (3.10). It is to be noted that the constants $B_2^{(1)}$, $B_2^{(2)}$, and $B_3^{(2)}$ are all identically equal to zero and are not given in the table.

Vertex	$B_1^{(0)}$	$B_1^{(1)}$	$B_1^{(2)}$	$B_2^{(0)}$	$B_3^{(0)}$	$B_3^{(1)}$
$3G$	1.55	0.119	0.0208	-1.33	0	0
gG	0.988	-0.253	-0.0114	-0.0935	0	0
fG	1.39	0.594	-0.136	-0.0935	-2.18	0.758

is easily obtained from Eq. (3.9).

A solution for the gauge parameter $\alpha=\alpha(\mu,\xi)$ is, in principle, obtained by substituting the above solution for a into Eqs. (3.11c) and (3.11d) and then integrating them. Unfortunately, however, this integration cannot be analytically performed. In Sec. III B we give a qualitative discussion on the μ and ξ dependences of the gauge parameter α.

B. Behavior of the effective coupling and gauge parameter

1. Gauge-invariant case

In this case the gauge parameter α can be treated as a constant, and the μ and ξ dependences (as well as the initially fixed α dependence) of the coupling $a(\mu,\xi;\alpha)$ are described by Eq. (3.6a) with Eqs. (3.7) and (3.8). As is obvious from Eq. (3.6a) the μ dependence of the coupling $a(\mu,\xi;\alpha)$ is logarithmic, and thus is exactly the same as that of the familiar coupling $\bar{a}(\mu;\alpha)$ in the zero-temperature theory, Eq. (3.6b), except that the scale $\Lambda(\xi,\alpha)$ which measures the momentum μ is now ξ dependent. Therefore, we confine our interest to the ξ and α dependences of the coupling $a(\mu,\xi;\alpha)$.

First let us study the ξ dependence. All information on the ξ dependence of the coupling is stored in the scale parameter $\Lambda(\xi,\alpha)$, whose behavior we study. For the sake of convenience, we study hereafter the ratio $\ln\Lambda(\xi,\alpha)/\Lambda^{3G}$ normalized always by Λ^{3G} at the zero temperature instead of Eq. (3.7) itself. In massless QCD, $C_2(G)=3$, $C_2(R)=\frac{4}{3}$, and $T(R)=\frac{1}{2}n_F$ with n_F the number of quark flavors. In Fig. 1 we present the ξ dependence of scale parameters, Eqs. (3.7) and (3.8), at two typical values of the gauge parameter α: $\alpha=1$ and 10. In SU(5) GUT in a symmetric phase $C_2(G)=5$, $C_2(R)=\frac{12}{5}$ ($\frac{36}{5}$) for massless fermions transforming as **5** (**10***), and $T(R)=n_g$, the number of generations. For reasonable values of n_g the ξ dependence of Eqs. (3.7) and (3.8) is similar to the above QCD cases, Fig. 1. For other values of gauge parameter α the situation is the same.

From Fig. 1 together with Eqs. (3.7) and (3.8) we can

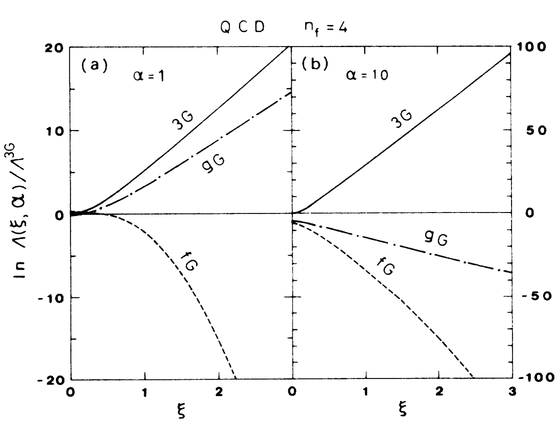

FIG. 1. The ξ dependence of the ratio $\ln\Lambda(\xi,\alpha)/\Lambda^{3G}$ [at (a) $\alpha=1$ and (b) $\alpha=10$] for QCD with four flavors. Solid, dot-dashed, and dashed curves correspond to the $3G$, gG, and fG vertices, respectively.

easily see the following important features. (i) At any value of α, $\ln\Lambda(\xi,\alpha)$ (or the inverse of coupling itself) behaves at large ξ as a power function of ξ, in sharp contrast to the above noted logarithmic dependence on μ. To be more precise, for $\xi\gtrsim 2$, it is a linear function of ξ for the $3G$ and gG vertices, whereas for the fG vertex it is a quadratic function of ξ. (ii) The vertex dependence of the scale parameter $\Lambda(\xi,\alpha)$ (or the coupling itself) is drastic especially at large values of ξ, irrespective of the choice of gauges. It is to be noted that the scale parameter determined through the fG vertex, $\Lambda^{fG}(\xi,\alpha)$, shows a completely opposite ξ dependence to that defined through the $3G$ vertex. It is worth mentioning that at $\xi\gg 1$, i.e., in the infrared regions the one-loop approximation may not be justified. In order to investigate the structure in this region, as was studied in Refs. 3 and 4, we need a calculational method beyond the one-loop approximation or even a nonperturbative method.

Next, let us study the gauge(-parameter) dependence. As is shown in Eq. (3.7), the logarithm of the scale parameters is a quadratic polynomial of α, i.e.,

$$b\ln[\Lambda(\xi,\alpha)/\Lambda^{3G}]_{\overline{\text{MOM}}}=\tilde{\Phi}^{(0)}(\xi)+\tilde{\Phi}^{(1)}(\xi)\alpha+\tilde{\Phi}^{(2)}(\xi)\alpha^2\,, \tag{3.7'}$$

where the coefficient of α^2, $\tilde{\Phi}^{(2)}(\xi)$, is always positive definite for the $3G$ vertex, whereas it is always negative definite for the gG and fG vertices. Moreover, we can show that $\ln[\Lambda(\xi,\alpha)/\Lambda]^{3G}$ is always larger than $\ln[\Lambda(\xi,\alpha)/\Lambda^{3G}]^{gG,fG}$ for $\xi\gtrsim 0.3$, and that the difference between them becomes larger as $\xi\to$large. To see the above facts more clearly we present in Fig. 2 the α dependence of the scale parameter for massless QCD with four flavors at two typical values of ξ, $\xi=0.2$ and 1.0. From these results we can understand that (i) at small values of ξ ($\lesssim 0.3$) there is a region of the gauge parameter α around $\alpha\simeq 0.8$, at which the vertex dependence of the scale parameter or of the coupling eventually disappear, while that (ii) at large values of ξ ($\gtrsim 0.3$) in any choice of gauges the strong vertex dependence of the coupling does survive. Even if we minimize at each value of ξ the above vertex dependence of the coupling by choosing some specific gauges α that depend on the value of ξ, such a "minimized" vertex dependence is still an increasing function of ξ. For example,

$$\min_{\{\alpha\}}|\Delta(3G-fG)|\equiv\min_{\{\alpha\}}|\ln[\Lambda(\xi,\alpha)/\Lambda^{3G}]^{3G}-\ln[\Lambda(\xi,\alpha)/\Lambda^{3G}]^{fG}|$$

in the four-flavor QCD, which vanishes for $\xi\lesssim 0.304$, shows as ξ increases a nearly quadratic increase which can be well reproduced by the following formula:

$$\min_{\{\alpha\}}|\Delta(3G-fG)|=-0.86+1.42\xi+6.3\xi^2\,. \tag{3.16}$$

Up to now we have treated the gauge parameter α as a constant fixed at the beginning because we are considering gauge-invariant physical quantities. However, in studying general gauge-noninvariant Green's functions, we should consider the gauge dependence of the coupling $a(\mu,\xi;\alpha)$ in which both the coupling a and the gauge parameter α are running parameters. Thus in the next subsection we go into this issue.

2. Gauge-noninvariant general case

Results for the coupling $a(\mu,\xi;\alpha)$ obtained in Sec. III A 2, Eq. (3.12) or (3.12′) with Eq. (3.14), are essentially the same as those in Sec. III A 1, except that in the present case the gauge parameter α is also a running parameter $\alpha=\alpha(\mu,\xi)$. Therefore, the primary interest in the present case is the behavior of the gauge parameter under changes in μ and ξ. With this information we can then answer the question of how the coupling depends on μ and ξ. The response of α under changes in μ and ξ is described by Eqs. (3.11c) and (3.11d), where for the coupling a we substitute the solution to the RGE's (3.11a) and (3.11b), i.e., Eq. (3.12′) with Eq. (3.14). As already mentioned in Sec. III A 2 these equations for α cannot be integrated in a compact form, thus we study the gross behavior of α only schematically.

First, we note that Eq. (3.11c) with Eq. (2.12c) can be written as

$$\frac{\partial\alpha}{\partial\ln\mu}=-\gamma_0(\alpha)a=-\tfrac{1}{4}C_2(G)(1-\alpha)(\alpha-\alpha_2)a\ , \quad (3.17a)$$

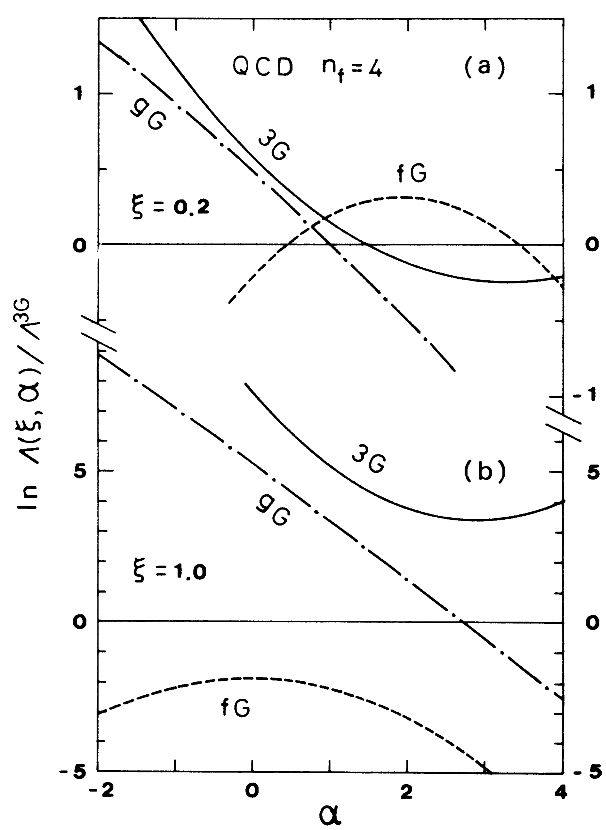

FIG. 2. The gauge parameter (α) dependence of the ratio $\ln\Lambda(\xi,\alpha)/\Lambda^{3G}$ at (a) $\xi=0.2$ and (b) $\xi=1.0$ for QCD with four flavors. Solid, dot-dashed, and dashed curves correspond to the $3G$, gG, and fG vertices, respectively.

where

$$\alpha_2=-\frac{10}{3}\left[1-\frac{4}{5}\frac{T(R)}{C_2(G)}\right]. \quad (3.17b)$$

The zero point at $\alpha=\alpha_1$ $(=1)$ is exact through all orders, whereas the other zero point at $\alpha=\alpha_2$ is exact only up to one-loop order. By noting that in most of the interesting cases α_2 is smaller than unity, $\alpha_2<\alpha_1=1$, we can easily see from Eq. (3.17a) that $\alpha=\alpha_2$ and ∞ are ultraviolet stable fixed points; as $\mu\to\infty$ with ξ kept fixed, (i) if we initially choose a gauge with $\alpha<1$, then α runs toward the value $\alpha=\alpha_2$, and (ii) if the initial choice is $\alpha>1$, then $\alpha\to\infty$. The Landau gauge $(\alpha=1)$ is the ultraviolet unstable fixed point.

Second, let us consider Eq. (3.11d), which can be rewritten as

$$\frac{\partial\alpha}{\partial\ln\xi}=-a(1-\alpha)(\varepsilon_0^{(0)}+\varepsilon_0^{(1)}\alpha+\varepsilon_0^{(2)}\alpha^2)\ , \quad (3.18)$$

where $\varepsilon_0^{(i)}$ are easily obtained from Eq. (2.12d). The RHS of this equation vanishes at the values of α

$$\alpha_1=1$$

and (3.19)

$$\alpha_\pm=\frac{-\varepsilon_0^{(1)}\pm[(\varepsilon_0^{(1)})^2-4\varepsilon_0^{(0)}\varepsilon_0^{(2)}]^{1/2}}{2\varepsilon_0^{(2)}}\ .$$

The first zero point at $\alpha=\alpha_1$ $(=1)$ is ξ independent and is exact through all orders. The one-loop exact zero points at $\alpha=\alpha_\pm$ are real zero points at small values of ξ $(\simeq 0)$:

$$\alpha_\pm\simeq 3\pm\sqrt{5}+\tfrac{16}{15}\pi^2\xi^2\left[1\pm\frac{1}{\sqrt{5}}\mp\frac{7}{\sqrt{5}}\frac{T(R)}{C_2(G)}\right]. \quad (3.20)$$

In Fig. 3 we depict the zero points α_1 and $\alpha_\pm$ as functions of ξ, and show schematically how the gauge parameter α

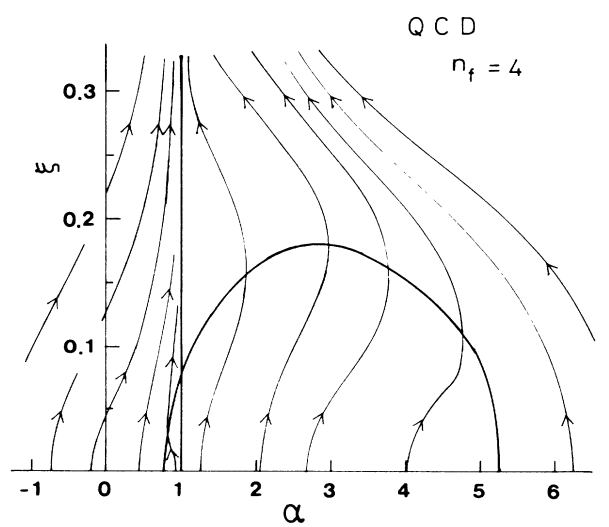

FIG. 3. Integral curve of Eq. (3.18) with fixed μ in the ξ-α plane.

flows under a change of ξ with μ kept fixed. From this figure we can see that at large ξ ($=\widetilde{T}/\mu$) the gauge parameter α runs always toward the value $\alpha_1=1$ (the Landau gauge), irrespective of the initial choice of gauges: namely, the Landau gauge is the unique stable fixed point in the limit of large ξ.

With the above observation that as $\xi\to\infty$ the gauge parameter α always tends to the value of the Landau gauge ($\alpha=1$), together with the fact that in the gauge-noninvariant general case the gauge dependence of the coupling is essentially the same as the gauge-invariant case [see Eqs. (3.7), (3.8), (3.14), and (3.15)], we recognize that the strong vertex dependence of the coupling is essential: namely, it is beyond control with any choice of gauges and becomes more and more severe as ξ becomes large.

IV. CONCLUSIONS AND DISCUSSION

In this paper, by the use of real-time formalism of field theory at finite temperature, we have carried out an off-shell one-loop calculation for massless non-Abelian gauge theories at finite temperature in the general covariant gauge. From the achievement of previous works,[2-4] calculation was done in the finite-temperature generalization of the momentum-space subtraction scheme[5,6] supplemented with the "magnetic" prescription.[4,18,19] With the results obtained we calculated the renormalization-group-improved effective coupling $a(\mu,\xi;\alpha)$ renormalized at nonzero finite temperature and studied its consequences.

The main conclusions of the present paper are as follows.

(i) The inverse of coupling $a^{-1}(\mu,\xi;\alpha)$ strongly depends on the temperature through $\xi\equiv T/\mu$ and behaves at large ξ ($\gtrsim 1$) almost as a power (linear or quadratic) function of ξ in accord with the fact revealed through the analyses[3,4] of the infrared structure of gauge theories at $T\neq 0$. This is in sharp contrast to the familiar logarithmic dependence on the renormalization momentum μ.

(ii) At finite temperature the vertex dependence of the coupling a, or the corresponding scale parameter $\Lambda(\xi,\alpha)$, becomes drastic and severe. In considering the scale parameter, the logarithm of it defined through the fG vertex, $\ln\Lambda^{fG}(\xi,\alpha)$, *decreases* as a quadratic function of ξ, whereas $\ln\Lambda^{3G}(\xi,\alpha)$ defined through the $3G$ vertex *increases* as a linear function of ξ. The one defined through the gG vertex, $\ln\Lambda^{gG}(\xi,\alpha)$, is a linear function of ξ that increases (decreases) as $\xi\to\infty$ for the gauge parameter α smaller (larger) than $\frac{9}{2}$. These facts are clearly shown in Fig. 2. From these facts we can understand that the temperature-dependent radiative corrections are quite large at high temperature, and that *there is no choice of the coupling which can be defined so as to absorb such radiative corrections simultaneously.* At small ξ ($\simeq 0$) the ξ-dependent part of the scale parameter $\Lambda(\xi,\alpha)$ itself behaves as a quadratic function of ξ, and $\Lambda(\xi,\alpha)$ smoothly approaches the zero-temperature value.

The strong vertex dependence of the coupling renormalized at finite temperature mentioned above is essential: namely, it is beyond control with any choice of gauges. In fact the gauge dependence of the scale parameter is as follows: At any value of ξ, $\ln\Lambda^{3G}(\xi,\alpha)/\Lambda^{3G}$ is a quadratic polynomial of α convex to below, whereas $\ln\Lambda^{gG}(\xi,\alpha)/\Lambda^{3G}$ and $\ln\Lambda^{fG}(\xi,\alpha)/\Lambda^{3G}$ are quadratic polynomials of α convex to above. Even if we minimize differences among them over α, they increase as quadratic functions of ξ as $\xi\to\infty$. However, when ξ is small enough there is a choice of gauges with α nearly equal to 0.8, in which three scale-parameter ratios eventually coincide, see Fig. 2(a).

(iii) In the covariant-gauge calculation, the gauge parameter is, in general, a running parameter $\alpha=\alpha(\mu,\xi)$. The μ and ξ dependences of $\alpha(\mu,\xi)$ are, just as the coupling $a(\mu,\xi)$, completely different. Considering the μ dependence with ξ kept fixed,

$$\alpha=\alpha_2=-\frac{10}{3}\left[1-\frac{4}{5}\frac{T(R)}{C_2(G)}\right]$$

and $\alpha=\infty$ are shown to be the ultraviolet (UV) stable fixed point, whereas $\alpha=1$ (Landau gauge) is the UV unstable fixed point, which separates the gauge-parameter space into two regions in the sense of gauge-parameter flow. Turning to the ξ dependence, the Landau gauge ($\alpha=1$) is the unique stable fixed point at large ξ ($=T/\mu$). With this fact we can recognize that at large ξ the analysis in the Landau gauge can clarify the essential feature of the coupling, especially of its vertex dependence.

The powerlike strong ξ ($\equiv T/\mu$) dependence of the coupling $a(\mu,\xi;\alpha)$ is a reflection of large temperature-dependent corrections. With this fact in mind we can say that perturbation analyses of processes taking place in a thermal reservoir at T ($\neq 0$), which are becoming more and more important, should be carefully done by taking into account the present results. Most of the existing analyses on the thermodynamic and effective potentials, which are carried out in terms of the coupling renormalized at $T=0$ in the MS and $\overline{\text{MS}}$ scheme, $\tilde{a}_{\text{MS}}(\mu)$, where temperature dependence is taken into account by assuming $\mu\simeq T$, may be completely inaccurate. In such analyses with the coupling $\tilde{a}_{\text{MS}}(\mu\simeq T)$, temperature-dependent next-order corrections might be quite large and thus may modify in a nontrivial way the results obtained.[24]

The above observations make us recognize the presence of several difficulties in the perturbative analyses of non-Abelian gauge theories at finite temperature.

(1) The strong vertex dependence of the coupling $a(\mu,\xi;\alpha)$ causes the difficulty that *we cannot specify the coupling in terms of which perturbative calculation is carried out.* Consider that we start our calculation of a given Feynman diagram that includes several types of vertices, making use of a coupling, say, a_{3G}. Then contributions (to the final result) coming from "vicinities" of the fG and gG vertices in this diagram apparently contain large logarithmic factors such as $\ln\Lambda^{fG}/\Lambda^{3G}$ and $\ln\Lambda^{gG}/\Lambda^{3G}$, which might become uncontrollable as ξ becomes large. Thus, we face the problem of (uncontrollable) large higher-order corrections. We should be faithfully anxious about whether or not perturbation analysis at finite temperature, especially at large $\xi=T/\mu$, works.

(2) Let us for the moment forget about the difficulty (1), and consider the coupling renormalized through the 3G vertex. As repeatedly noted $a^{-1}(\mu,\xi;\alpha)$ shows essentially *the power behavior with respect to* ξ, in contrast with *the logarithmic behavior on* μ, and varies quite sensitively to the exact value of ξ. This means, first of all, that in a given situation a very exact estimation of the explicit value of ξ is required to define the "good" coupling. This might be really a hard task. Second, the enormously large ratio of the scale parameter at finite temperature to that at zero temperature means that the domain where perturbation analysis in FT^2 works, i.e., $a(\mu,\xi;\alpha)\ll 1$, is severely pushed up toward the large-μ regions, $\mu \gg \Lambda(\xi,\alpha)$.

As a matter of course, problems (difficulties) (1) and (2) do not imply that perturbation analysis does not work at all for any high-energy processes taking place in a thermal reservoir. For processes with $\mu \lesssim T$ or $\xi \gtrsim 1$, (1) and (2) become real difficulties, while for those with small ξ, say $\xi \le 0.3$, these problems do not exist: Not only is the numerical value of $\ln\Lambda(\xi,\alpha)/\Lambda$ small but there is also a region of the gauge parameter α around $\alpha \simeq 0.8$, where the three couplings a_{3G}, a_{gG} and a_{fG} coincide, thus perturbative treatment makes sense without any trouble.

We give here a comment on the μ and T dependences of the coupling $a(\mu,\xi;\alpha)$. Throughout this paper we stressed that the inverse of the coupling shows the logarithmic μ dependence, while the temperature dependence through the parameter $\xi \equiv T/\mu$ is powerlike. However, as is evident from the relation

$$\ln\mu/\Lambda(\xi)=\ln T/\hat{\Lambda}(\xi), \quad \hat{\Lambda}(\xi)=\xi\Lambda(\xi) , \tag{4.1}$$

we can also say that the inverse of the coupling shows the logarithmic T dependence, while the momentum dependence through the parameter $\xi = T/\mu$ is powerlike. Thus we should be careful about which parameter (or variable) is kept fixed when studying the renormalization point (μ and T) dependence of the coupling.

Next, we give several comments on the infrared ($\xi \gg 1$) behavior of the present result in conjunction with the "screening" of effective charge. (1) First, we should note that explicitly temperature-dependent contributions from fermion loops to any Z factor of self-energy and vertex vanish in the infrared limit in contrast with other contributions that are at least of $O(\xi)$, in accord with previous works.[3,4] (2) Temperature-dependent self-energy correction behaves as follows: $\delta Z_3(\xi)\sim a\xi$, $\delta\tilde{Z}_3(\xi)\sim a\xi$, and $\delta Z_2(\xi)\sim -a\xi^2$, where $\delta Z_i \equiv Z_i - 1$ and $a \equiv g^2/4\pi^2$ is the coupling. This fact implies that, although fermion self-energy (or temperature-dependent mass) acts to screen the effective charge, gluon and ghost self-energy acts to anti-"screen." It is to be noted that for gluon and ghost δZ is linearly proportional to T and thus may not be interpreted as a temperature-dependent mass. (3) The temperature-dependent vertex correction shows the following behavior: $\delta Z_1(\xi)\sim a\xi$, $\delta Z_1^F(\xi) \sim a\xi^2$, independent of the choice of gauges, while $\delta\tilde{Z}_1(\xi) \sim A(\alpha)a\xi$ with $A(\alpha)\sim\alpha^2+4\alpha-8$ which changes its sign depending on the gauge parameter α. Thus, corrections to the 3G (fG) vertex always act to "screen" (screen) the effective charge. (4) Infrared behavior of temperature-dependent corrections to the coupling is as follows: $\delta Z_{3G}(\xi)\sim -a\xi$ and $\delta Z_{fG}(\xi)\sim a\xi^2$, namely, the effective charge defined through the 3G (fG) vertex is anti-"screened" (screened). As for $\delta Z_{gG}(\xi)$, its behavior depends on the choice of gauge, i.e., $\delta Z_{gG}(\xi)\sim B(\alpha)a\xi$ with $B(\alpha)\sim\alpha-\frac{9}{2}$, indicating that the effective charge $a_{gG}(\xi)$ is "screened" (anti-"screened") for $\alpha >$ ($<$) $\frac{9}{2}$. At $\alpha=\frac{9}{2}$, $a_{gG}(\xi)$ has no temperature-dependent effects. (5) Finally, we should mention that all the above observations (2)–(4) are gauge dependent and thus may not be considered to be physical consequences as they are. However, it is also worth mentioning that the existence of vertex dependence concerning the "screening" of the effective charge (coupling) has never been pointed out so far, because in previous works this phenomenon has only been studied by considering the propagator. Further study on this point should be done.

Finally, we give some comments on a closely related work. Recently, Fujimoto and Yamada[25] (FY) carried out almost the same analysis as our previous work based on the Feynman-gauge calculation. The only difference between the two works exists in the choice of subtraction point. We made the subtraction at the collinear momentum configuration the same as in the present work, while FY subtracted at the symmetric configuration. Essential results in FY, e.g., the powerlike strong ξ ($\equiv T/\mu$) dependence and the strong vertex dependence of the coupling, reconfirmed our previous results. There is, however, one important difference. Their coupling defined through the 3G vertex shows behavior similar to our coupling defined through the fG vertex. This is a quite surprising result. Here we come to face one more problem in order to carry out perturbative analysis in non-Abelian gauge theories at finite temperature.

The computation of the diagrams was done by computer using the algebraic manipulation program REDUCE2.

ACKNOWLEDGMENT

We thank V. Soni who kindly made us aware of his works.

APPENDIX A: THE FEYNMAN RULES

In this appendix we summarize the Feynman rules which are necessary to carry out the finite-temperature calculation. Because we are interested in the leading one-loop calculation it is not necessary to have complete matrix expressions of propagators in FT^2; thus, in this appendix we give only the component required for the present purpose. They are ($\beta = 1/T$) (1) the gauge-boson ("gluon") propagator

$$\Delta^{ab}_{\mu\nu}(k) = -i\delta^{ab}\left[g_{\mu\nu}-\alpha k_\mu k_\nu \frac{\partial}{\partial \mathbf{k}^2}\right]\left[\frac{1}{k^2}-\frac{2\pi i\delta(k^2)}{e^{\beta|k_0|}-1}\right],$$

(2) the ghost propagator

$$\Delta^{ab}(k) = i\delta^{ab}\left[\frac{1}{k^2}-\frac{2\pi i\delta(k^2)}{e^{\beta|k_0|}-1}\right],$$

and (3) the fermion propagator

$$S^{AB}_{\alpha\beta}(k)=i\delta^{AB}(\not k)_{\alpha\beta}\left[\frac{1}{k^2}+\frac{2\pi i\delta(k^2)}{e^{\beta|k_0|}+1}\right].$$

In the leading one-loop calculation the rules for the interaction vertices are completely the same as those in the zero-temperature theories and thus are not reproduced. From these facts it is obvious that we get the same one-loop results irrespective of any real-time formalism.[1]

APPENDIX B: RENORMALIZATION CONSTANTS AT FINITE TEMPERATURE IN THE ONE-LOOP APPROXIMATION

In this appendix we give the exact one-loop results for the renormalization constants Z_i's. Before giving them, however, it is worth giving several comments on the explicit calculation and renormalization procedures.

First, for the ghost-"gluon" vertex (the definition of the Z factor $\tilde{Z}_1$ is the same as in Ref. 5), a little care should be taken. In order to identify the subtraction part the calculation of the finite-temperature contribution in the collinear configuration, Eq. (2.5), should be carried out as follows: First, perform the calculation at a configuration that deviates from the collinear one, Eq. (2.5), by an infinitesimal angle $\Delta\theta$; then, after getting the final results set the angle $\Delta\theta\to 0$. As for the zero-temperature counterparts there is no need to use such a technique.

Second, in determining Z_3, the "gluon" wave-function renormalization constant, we subtracted only those terms that are proportional to $g_{\mu\nu}$; namely, we did not include those terms that are proportional to $n_\mu n_\nu$ $[n_\mu=(1,\mathbf{0})]$ into the subtraction part; see Enqvist and Kajantie.[18] Third, in the calculation of vertex renormalization constants, Z_1, $\tilde{Z}_1$, and Z_1^F, we subtracted at the subtraction point given by Eq. (2.5), only those terms having the same tensorial structures as the bare vertices.

It is worth explaining a little more explicitly our subtraction procedures for the $3G$ vertex. They are as follows: First, we express the $3G$ vertex as

$$f^{a_1a_2a_3}(\{g_{\mu_1\mu_2}[(p_1-p_2)_{\mu_3}A(p_1^2,p_2^2,p_3^2)+q^{(12)}_{\mu_3}B(p_1^2,p_2^2,p_3^2)]+\text{c.p.}\}$$
$$+\{n_{\mu_1}n_{\mu_2}[(p_1-p_2)_{\mu_3}C(p_1^2,p_2^2,p_3^2)+q^{(12)}_{\mu_3}D(p_1^2,p_2^2,p_3^2)]+\text{c.p.}\}+\cdots), \quad \text{(B1)}$$

where a_i and μ_i are internal and Lorentz indices of the gluon with momentum p_i ($p_{i0}=0$), and $q^{(12)}$ is a momentum constructed out of p_1 and p_2 so as to be orthogonal to p_1-p_2. At our collinear momentum subtraction point (2.5) $q^{(12)}$ and $q^{(23)}$ automatically vanish and also $B(p_3^2,p_1^2,p_2^2)$ vanishes. Furthermore, the terms indicated by the ellipses in Eq. (B1) add up to be zero. Next, we subtract out at the subtraction point (2.5) the terms in the first set of curly brackets in Eq. (B1). This is possible because at our collinear momentum configuration p_3-p_1 vanishes and $A(p_1^2,p_2^2,p_3^2)=A(p_2^2,p_3^2,p_1^2)$ and thus the terms in the first set of curly brackets are completely subtracted. Those terms in the second set of curly brackets, which are not included in the subtraction part, are the only contributions that remain after subtraction.

With the above comments in mind, we give the exact one-loop results for the Z factors.

(a) Wave-function renormalization constants. The subtraction point is $p=(0,\mathbf{p})$ and ξ_p is defined as $\xi_p\equiv\tilde{T}/|\mathbf{p}|$:

$$\begin{aligned}Z_3\mu^\epsilon-1={}&\tfrac14 aC_2(G)\left[\frac{10}{3\hat\epsilon}+\tfrac{31}{9}-\tfrac53\ln\frac{\mathbf{p}^2}{\mu^2}+3F_0(2\xi_p)+F_2(2\xi_p)\right.\\&\left.+\alpha\left[\frac{1}{\hat\epsilon}-1-\tfrac12\ln\frac{\mathbf{p}^2}{\mu^2}-F_0(2\xi_p)+4G(2\xi_p)\right]+\tfrac14\alpha^2[1+F_0(2\xi_p)-2G(2\xi_p)]\right]\\&-\tfrac12 aT(R)\left[\frac{4}{3\hat\epsilon}+\tfrac{10}{9}-\tfrac23\ln\frac{\mathbf{p}^2}{\mu^2}-F_0(2\xi_p)+2F_0(\xi_p)-F_2(2\xi_p)+2F_2(\xi_p)\right],\end{aligned}\quad\text{(B2)}$$

$$\tilde{Z}_3\mu^\epsilon-1=\tfrac14 aC_2(G)\left[\frac{1}{\hat\epsilon}+1-\tfrac12\ln\frac{\mathbf{p}^2}{\mu^2}+F_0(2\xi_p)+\alpha\left[\frac{1}{2\hat\epsilon}-\tfrac14\ln\frac{\mathbf{p}^2}{\mu^2}+G(2\xi_p)\right]\right],\quad\text{(B3)}$$

$$Z_2\mu^\epsilon-1=-\tfrac14 aC_2(R)\left[\frac{2}{\hat\epsilon}+1-\ln\frac{\mathbf{p}^2}{\mu^2}+2F_0(\xi_p)+2\pi^2\xi_p^2-\alpha\left[\frac{2}{\hat\epsilon}+\tfrac43-\ln\frac{\mathbf{p}^2}{\mu^2}+F_0(\xi_p)+2G(\xi_p)\right]\right].\quad\text{(B4)}$$

(b) Vertex renormalization constants. Subtraction configuration is given by Eq. (2.5) and ξ is defined as $\xi\equiv\tilde{T}/\mu$:

$$\begin{aligned}Z_1\mu^\epsilon-1={}&\tfrac{1}{12}aC_2(G)\left[\frac{4}{\hat\epsilon}-\tfrac73\ln2+\tfrac{13}{3}+2F_0(2\xi)+F_0(\xi)-F_2(2\xi)+4F_2(\xi)\right.\\&+\tfrac18\alpha\left[\frac{36}{\hat\epsilon}-52\ln2-14-7F_0(2\xi)-8F_0(\xi)-3F_2(2\xi)+12F_2(\xi)-16G(2\xi)+112G(\xi)\right]\\&\left.+\tfrac18\alpha^2[8-F_0(2\xi)+10F_0(\xi)+3F_2(2\xi)-12F_2(\xi)-12G(\xi)]\right]\\&-\tfrac16 aT(R)\left[\frac{4}{\hat\epsilon}-\tfrac{28}{5}\ln2+\tfrac{79}{30}+F_0(2\xi)-6F_0(\xi)+8F_0\left[\frac{\xi}{2}\right]+F_2(2\xi)-6F_2(\xi)+8F_2\left[\frac{\xi}{2}\right]\right],\end{aligned}\quad\text{(B5)}$$

$$\tilde{Z}_1\mu^{\epsilon}-1=-\tfrac{1}{8}aC_2(G)\left[\frac{2}{\hat{\epsilon}}-\tfrac{8}{3}\ln 2+\tfrac{8}{3}-F_0(2\xi)+4F_0(\xi)+F_2(2\xi)-4F_2(\xi)\right.$$
$$+\tfrac{1}{2}\alpha\left[-\frac{4}{\hat{\epsilon}}+\tfrac{16}{3}\ln 2-\tfrac{7}{3}+F_0(2\xi)-4F_0(\xi)-2F_2(2\xi)+8F_2(\xi)+2G(2\xi)-8G(\xi)\right]$$
$$\left.+\tfrac{1}{8}\alpha^2[-\tfrac{16}{3}\ln 2+\tfrac{4}{3}-3F_0(2\xi)+4F_0(\xi)-F_2(2\xi)+4F_2(\xi)-4G(2\xi)]\right], \tag{B6}$$

$$Z_1^F\mu^{\epsilon}-1=-\tfrac{1}{4}aC_2(G)\left\{\frac{2}{\hat{\epsilon}}-\tfrac{2}{3}\ln 2+\tfrac{8}{3}+7F_0(\xi)-4F_0\left[\frac{\xi}{2}\right]+F_2(\xi)-4F_2\left[\frac{\xi}{2}\right]\right.$$
$$+\tfrac{1}{8}\alpha\left[-\frac{4}{\hat{\epsilon}}+\tfrac{56}{3}\ln 2+\tfrac{10}{3}-13F_0(\xi)+16F_0\left[\frac{\xi}{2}\right]-13F_2(\xi)+16F_2\left[\frac{\xi}{2}\right]-16G(\xi)\right]$$
$$\left.+\tfrac{1}{4}\alpha^2[-\tfrac{4}{3}\ln 2-\tfrac{2}{3}-F_0(\xi)+3F_2(\xi)]\right\}$$
$$+\tfrac{1}{4}aC_2(R)\left\{-\frac{2}{\hat{\epsilon}}+\tfrac{16}{3}\ln 2-\tfrac{1}{3}+7F_0(\xi)-8F_0\left[\frac{\xi}{2}\right]+5F_2(\xi)-8F_2\left[\frac{\xi}{2}\right]+2\pi^2\xi^2\right.$$
$$\left.+\tfrac{1}{2}\alpha\left[\frac{4}{\hat{\epsilon}}-\tfrac{16}{3}\ln 2+\tfrac{10}{3}-5F_0(\xi)+8F_0\left[\frac{\xi}{2}\right]-5F_2(\xi)+8F_2\left[\frac{\xi}{2}\right]\right]\right\}. \tag{B7}$$

In the above equations

$$\frac{1}{\hat{\epsilon}}=\frac{1}{\epsilon}-\tfrac{1}{2}(\gamma_E-\ln 4\pi), \tag{B8}$$

where γ_E is the Euler constant, and the functions $F_j(\xi)$ $(j=0,2)$ and $G(\xi)$ are

$$F_j(\xi)=\xi^{j+1}\int_0^{\infty}dx\frac{x^j}{e^x-1}\ln\left|\frac{1+\xi x}{1-\xi x}\right|-j\frac{\pi^2}{6}\xi^2 \quad (j=0,2), \tag{B9}$$

$$G(\xi)=\xi^2\int_0^{\infty}dx\frac{x}{e^x-1}\frac{\mathcal{P}}{\xi^2x^2-1}, \tag{B10}$$

where $\mathcal{P}$ denotes the principal part. The coupling renormalization constants Z_a's are easily determined through Eqs. (2.4a)–(2.4c) and are not reproduced.

---

*Present address: Institute for Natural Science, Nara University, 1500 Misasagi-cho, Nara 631, Japan.

[1]J. Schwinger, J. Math. Phys. **2**, 407 (1961); L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1965)]; Y. Takahashi and H. Umezawa, Collect. Phenom. **2**, 55 (1975); A. J. Niemi and G. W. Semenoff, Ann. Phys. (N.Y.) **152**, 105 (1984); K.-C. Chou, Z.-B. Su, B.-L. Hao, and L. Yu, Phys. Rep. **118**, 1 (1985); N. P. Landsman and Ch. G. van Weert, *ibid.* **145**, 141 (1987).

[2]For an extensive review of the physics of the quark-gluon plasma, see, e.g., L. McLerran, Rev. Mod. Phys. **58**, 1021 (1985).

[3]T. Appelquist and R. D. Pisarski, Phys. Rev. D **23**, 2305 (1981).

[4]K. Kajantie and J. Kapusta, Ann. Phys. (N.Y.) **160**, 477 (1985); U. Heinz, K. Kajantie, and T. Toimela, *ibid.* **176**, 218 (1987); see also Landsman and van Weert (Ref. 1).

[5]W. Celmaster and R. J. Gonsalves, Phys. Rev. Lett. **42**, 1435 (1979); Phys. Rev. D **20**, 1420 (1979).

[6]V. Soni, Nucl. Phys. **B216**, 244 (1983); **B216**, 267 (1983).

[7]See, e.g., D. W. Duke and R. G. Roberts, Phys. Rep. **120**, 275 (1985), and references therein. For the case of finite-density QCD at $T=0$, see Ref. 6.

[8]In the actual calculation we usually set $\tilde{T}=T$, the temperature of the environment, and $\mu\simeq Q$, the energy scale that characterizes the process considered.

[9]M. B. Kisslinger and P. D. Morley, Phys. Rev. D **13**, 2771 (1976); H. Matsumoto, I. Ojima, and H. Umezawa, Ann. Phys. (N.Y.) **152**, 348 (1984).

[10]From this fact we can understand that the coupling defined in the variety of the minimal subtraction schemes (MS, $\overline{\text{MS}}$, etc.) does not depend on the temperature at all. In such schemes, $z(\xi)$ in Eq. (1.1) is identically equal to unity.

[11]P. M. Stevenson, Phys. Rev. D **23**, 2916 (1981).

[12]H. Nakkagawa and A. Niégawa, Phys. Lett. **119B**, 415 (1982); Prog. Theor. Phys. **70**, 511 (1983).

[13]H. Nakkagawa and A. Niégawa, Phys. Lett. B **193**, 263 (1987); **196**, 571(E) (1987).

[14]The reduced chemical potential $c_B\equiv\mu_B/T$, where μ_B is the chemical potential that couples to the baryon number and T is the temperature of the environment, takes the following values at some physically interesting situations:

$$c_B\sim\begin{cases}0.15 \text{ at the QCD phase transition point at } T\simeq 200\text{ MeV},\\ 10^{-11}n_A \text{ in the early Universe},\end{cases}$$

where n_A in the total number of degrees of freedom of all the active particles in the considered era. In the case of QCD quark-gluon plasma phase, the more the temperature increases, the more c_B decreases rapidly.

[15]Hereafter, we call a gauge boson a "gluon" for brevity.

[16]As Celmaster and Gonsalves (Ref. 5) showed, even in the zero-temperature case, three Z factors (2.4) determined explicitly in the momentum-space subtraction method do not satisfy the Ward-Takahashi identities (2.2). Thus if we choose, say, the "3G prescription" (2.4a) to define the coupling, then the remaining two vertex renormalization factors $\tilde{Z}_1$ and Z_1^F should be determined so as to satisfy the identities (2.2).

[17]In FT^2 we must separately specify the timelike and spacelike components of the subtraction momentum. See Soni (Ref. 6).

[18]K. Enqvist and K. Kajantie, Mod. Phys. Lett. A **2**, 479 (1987).

[19]By "magnetic" prescription we mean the prescription in which, when determining the subtraction part, Lorentz indices of various propagators and vertices are restricted to spatial indices. In the temporal axial gauge and in the Landau gauge this prescription allows us to single out the magnetic effects in the static limit and thus is really the magnetic prescription. No such simple interpretation is possible in an arbitrary covariant gauge. See, e.g., Kajantie and Kapusta (Ref. 4).

[20]In the dimensional regularization method one is led to introduce an arbitrary mass parameter $\tilde{\mu}$. In this paper we set [following Celmaster and Gonsalves (Ref. 5)] $\tilde{\mu}=\mu$ where μ is the subtraction momentum defined in Eq. (2.5). The second equation in Eq. (2.3) should read $1-\alpha_B=Z_3\mu^{\epsilon}(1-\alpha)$ in this regularization method.

[21]H. Matsumoto, Y. Nakano, and H. Umezawa, Phys. Rev. D **29**, 1116 (1984).

[22]The first calculation of explicitly external-parameter-(chemical-potential-) dependent two-point functions with the use of the renormalization group was done in the Coulomb gauge by Soni [Ref. 6, and also, *Progress in Nuclear and Particle Physics* (Pergamon, Ericé, 1983), p. 585]. In this case, however, the chemical potential is treated as a fixed parameter, and the RGE is not a coupled equation but corresponds essentially to our Eq. (2.6a).

[23]D. J. Gross and F. Wilczek, Phys. Rev. D **8**, 3633 (1973); D. J. Gross, in *Methods in Field Theory*, edited by R. Balian and J. Zinn-Justin (North-Holland, Amsterdam, 1976), p. 141.

[24]K. Funakubo and M. Sakamoto, Prog. Theor. Phys. **76**, 490 (1986).

[25]Y. Fujimoto and H. Yamada, Phys. Lett. B **195**, 231 (1987); **200**, 167 (1988).