Central charge, trace and gravitational anomalies in two dimensions

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The origin of the trace anomaly and of the central charge in the Virasoro algebra associated with the energy-momentum tensor of a bosonic two-dimensional model is considered. It is shown that both these quantities represent different aspects of the same gravitational anomaly.

I. INTRODUCTION

It is well known that scale-invariant σ models in two dimensions play a crucial role in string theories. After a classical action has been produced, the relevant quantity to consider in building up a string model is the central charge of the Virasoro algebra associated with the energy-momentum tensor. Equivalently, one is interested in the trace anomaly of the two-dimensional model.[1]

The fact that the central charge is strictly related to the trace anomaly is not obvious *a priori.* A first hint in this direction can be obtained by considering the two-point function of the components of the energy-momentum tensor.[2] The short-distance behavior of this correlation function together with the conservation of the energy-momentum tensor can be used to show[3] that the Virasoro coefficient is equal in fact to the multiplicative constant of the trace anomaly.

In the above approach it is hard to realize a possible geometrical origin of the trace anomaly and therefore of the central charge. The subtractions needed to define a regularized energy-momentum tensor mask the meaning of the anomaly.

Yet there are indications that a topological interpretation of the trace anomaly might be possible.[4,5] Alvarez has shown[5] that in certain two-dimensional models the trace anomaly can be evaluated by using a family index theorem. He has also pointed out the connection between the trace and the holomorphic anomalies.[5] However, the geometrical origin of the anomaly still remains unknown. The use of the family index theorem is in fact justified only *a posteriori.*

Another open problem is to find a closed expression for the trace anomaly and for the central charge of a generic (interacting) scale-invariant two-dimensional model. The only hope to solve this problem is to have a clear understanding of the origin of the trace anomaly and of the central charge even for the simple systems. For this reason, I consider in the present paper the bosonic model in two-dimensional space-time with Euclidean signature, with action

$$S=\int d^2x\tfrac{1}{2}\sqrt{g}\,g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^i \ . \tag{1.1}$$

The dynamical scalar fields $\phi^i(x)$, with $i=1,\dots,D$, are in presence of an external gravitational background; $g^{\mu\nu}$ and g indicate the inverse and the determinant of the classical metric $g_{\mu\nu}$. I shall use mainly functional methods to understand the origin of the trace anomaly and of the central charge for the model (1.1).

It turns out that both the trace anomaly and the central charge represent different aspects of a unique "true" anomaly of the model: a gravitational (Diff) anomaly. This result is not quite surprising, especially in view of the works of Alvarez-Gaumé and Witten[6] and of Bonora, Bregola, Pasti, and Tonin.[7,8] However, at least two aspects of the above result are particularly interesting: (i) the new direction suggested on the problem of the topological origin (if any) of the conformal anomalies; (ii) a clear setting for the problem of the computation of the trace anomaly (and central charge) for an interacting model.

The paper is organized as follows. A preliminary discussion of the general framework together with the basic definitions are contained in Sec. II. The computation of the Diff anomaly is reported in Secs. III and IV. The connection between the Diff anomaly and the trace anomaly is considered in Sec. V. In Sec. VI it is shown how the central charge is related to the Diff anomaly. Finally, a summary and the conclusions are contained in Sec. VII.

II. A CRUCIAL PROPERTY OF THE TWO DIMENSIONS

To display the anomalous behavior of the symmetry transformations, it is convenient to introduce the generating functional Γ defined by

$$e^{-\Gamma}=\int \mathcal{D}\phi^i e^{-S} \ , \tag{2.1}$$

where the action S is given in Eq. (1.1). Γ is a functional of the classical background of course, and one is interested in the behavior of Γ under a variation of the metric induced by an infinitesimal coordinate transformation

$$\Delta_V g_{\mu\nu}(x)=\nabla_\mu V_\nu(x)+\nabla_\nu V_\mu(x) \ , \tag{2.2}$$

where ∇_μ is the covariant derivative, and under Weyl transformations

$$\Delta_\sigma g_{\mu\nu}(x)=2\sigma(x)g_{\mu\nu}(x) \ . \tag{2.3}$$

What is the expected behavior of Γ under the transformations (2.2) and (2.3)? A variation (2.2) of the metric alone in general does not leave the action S invariant; but if in addition to (2.2) one also transforms the scalar field ϕ^i as

$$\Delta_v \phi^i = V^\mu(x) \nabla_\mu \phi^i(x) , \tag{2.4}$$

then the action is invariant. Since Γ is obtained by "integrating over ϕ^i," one naively expects that Γ is invariant under a transformation (2.2).

With the Weyl transformations (2.3) it is a different story. The action (1.1) depends on the classical background through the combination

$$\hat{g}^{\mu\nu} = \sqrt{g}\, g^{\mu\nu} . \tag{2.5}$$

Now, it is a pecularity of the *two dimensions* that $\hat{g}^{\mu\nu}$ is invariant under the transformations (2.3). So, not only the classical action S is invariant under Weyl transformations, but also any functional of $\hat{g}^{\mu\nu}$ is invariant. In particular, the generating functional Γ depends on $\hat{g}^{\mu\nu}$ only:

$$\Gamma = \Gamma[\hat{g}^{\mu\nu}] . \tag{2.6}$$

Therefore, Γ is invariant under Weyl transformations for the good reason that under a transformation (2.3) we have nothing to do on Γ.

The above argument shows that the possible geometrical origin of the anomaly in two dimensions lies on the structure of the group of $\hat{g}^{\mu\nu}$'s transformations

$$\Delta_V \hat{g}^{\mu\nu}(x) = V^\lambda(x) \partial_\lambda \hat{g}^{\mu\nu}(x) + \partial_\lambda V^\lambda(x) \hat{g}^{\mu\nu}(x) - \hat{g}^{\mu\lambda}(x) \partial_\lambda V^\nu(x) - \hat{g}^{\nu\lambda}(x) \partial_\lambda V^\mu(x) . \tag{2.7}$$

In the following, a transformation (2.7) will be called a Diff transformation. It is also assumed that the parameters $V^\mu(x)$ of the transformation vanish sufficiently fast at infinity, so that partial integrations can be performed without any contribution at infinity.

To complete the argument one still has to consider an important task: the regularization. The point is whether the *regularized* generating functional depends only on $\hat{g}^{\mu\nu}$. Perhaps the regularization necessarily introduces a dependence on the determinant of the metric also.

The classical action depends on the background through $\hat{g}^{\mu\nu}$ which is Weyl invariant because, apart from the requirement of general covariance, the fields ϕ^i are massless and because we are in two dimensions. So, one needs a regularization which preserves both these two properties. Such a regularization exists; one can use for instance Schwinger's proper-time regularization.[9]

One formally has

$$\Gamma = \tfrac{1}{2} \mathrm{Tr} \ln \frac{\delta^2 S}{\delta\phi^i \delta\phi^j} , \tag{2.8}$$

and therefore the regularized generating functional is given by

$$\Gamma = -\frac{D}{2} \int_\epsilon^\infty \frac{d\tau}{\tau} \mathrm{Tr}(e^{-\tau H}) , \tag{2.9}$$

where

$$H = p_\mu \hat{g}^{\mu\nu}(q) p_\nu \tag{2.10}$$

and

$$p_\mu = -i \frac{\partial}{\partial q^\mu} . \tag{2.11}$$

The H operator (2.10) is Hermitian and positive, and under a Diff transformation (2.7) one obtains

$$\Delta_V H = i p_\lambda V^\lambda H - i H V^\lambda p_\lambda . \tag{2.12}$$

From the transformation law (2.12) one can easily see that Γ is naively Diff invariant; in fact

$$\Delta_V \ln \mathrm{Det} H = \mathrm{Tr}(H^{-1} \Delta_V H) = i\, \mathrm{Tr}([p_\lambda, V^\lambda]) = 0 .$$

For the regularized Γ one gets, instead,

$$\Delta_V \Gamma = i \frac{D}{2} \int_\epsilon^\infty d\tau\, \mathrm{Tr}[e^{-\tau H}(p_\lambda V^\lambda H - H V^\lambda p_\lambda)] = i \frac{D}{2} \mathrm{Tr}(e^{-\epsilon H}[p_\lambda, V^\lambda]) ,$$

which can be written as

$$\Delta_V \Gamma = \frac{D}{2} \mathrm{Tr}(\partial_\lambda V^\lambda e^{-\epsilon H}) . \tag{2.13}$$

The similarity of the expression (2.13) with the well-known analogous formula for the chiral anomalies should be noted.

In the $\epsilon \to 0$ limit, the right-hand side (RHS) of Eq. (2.13) has an expansion

$$\frac{1}{\epsilon} a_{-1} + a_0 + \epsilon a_1 + \cdots , \tag{2.14}$$

and as usual the anomaly, if it is present, is contained in the ϵ-independent part of the expansion: a_0. Apart from the transformation parameter factor $\partial_\lambda V^\lambda$, a_0 will be a local functional of $\hat{g}^{\mu\nu}$ with two space-time derivatives because of dimensional reasons.

One can also show at this stage that a_0 satisfies the consistency condition following from the structure of the Diff transformations group. The proof is very simple. The expression (2.12), being the variation of the regularized generating functional, does of course satisfy the consistency conditions. The point is that for *any* value of ϵ the consistency conditions are satisfied. So, each single term of the expansion (2.14) also does, in particular, a_0.

As a check, one can verify that the expression (2.13) indeed satisfies the consistency conditions for any value of ϵ. From Eq. (2.7) it follows that

$$\Delta_W \Delta_V - \Delta_V \Delta_W = \Delta_Y , \tag{2.15}$$

where

$$Y^\mu = V^\lambda \partial_\lambda W^\mu - W^\lambda \partial_\lambda V^\mu . \tag{2.16}$$

On the other hand, from the expression (2.13) and by means of Eq. (2.12) one finds

$$\Delta_W \Delta_V \Gamma = -\frac{\epsilon}{2} \int_0^1 d\alpha\, \mathrm{Tr}(e^{-\epsilon(1-\alpha)H} \partial_\mu W^\mu H e^{-\epsilon\alpha H} \partial_\nu V^\nu) - \frac{D}{2} \mathrm{Tr}(e^{-\epsilon H} W^\nu \partial_\nu \partial_\mu V^\mu) . \tag{2.17}$$

Hence

$$(\Delta_W\Delta_V-\Delta_V\Delta_W)\Gamma=\frac{D}{2}\mathrm{Tr}[e^{-\epsilon H}(V^\nu\partial_\nu\partial_\mu W^\mu-W^\nu\partial_\nu\partial_\mu V^\mu)]=\Delta_Y\Gamma\ , \qquad (2.18)$$

in agreement with Eq. (2.15).

The explicit computation of the first two terms of the expansion (2.14) is reported in the next section. I conclude this section with a few remarks on the algebraic origin of the anomaly.

The clue of the matter is the transformation law (2.12). If one exponentiates the infinitesimal variation (2.12), one obtains

$$H\mapsto H'=U^\dagger HU\ , \qquad (2.19)$$

where

$$U=e^{-iV^\nu p_\nu}\ . \qquad (2.20)$$

The U operator (2.20) just represents the action of a change of coordinates on the scalar fields. The anomaly can arise because if $\partial_\nu V^\nu\neq 0$ then U is not unitary; $\Delta_\nu\Gamma$ is in fact proportional to $\partial_\nu V^\nu$, Eq. (2.13).

One can imagine to avoid the Diff anomaly by reordering in some appropriate way the generator operator $-iV^\nu p_\nu$; for instance, by performing the substitution

$$-iV^\nu p_\nu\mapsto -i\{V^\nu,p_\nu\}\ . \qquad (2.21)$$

Exponentiating the anti-Hermitian operator shown in (2.21) one certainly gets a unitary operator and, as a consequence, the generating functional Γ will be invariant. Unfortunately, with the substitution (2.21) one does not generate the algebra (2.15) of the Diff transformations. It is the anomaly of the Diff transformations with precisely the algebra (2.15) on which we are interested in. On the other hand, it is immediately recognized that a general change of coordinates on a scalar field cannot be represented by a unitary operator. So, tricks such as that shown in Eq. (2.21) lead nowhere.

Finally, it remains the possibility of modifying the definition of the regularized generating functional

$$\Gamma[\hat g^{\mu\nu}]\mapsto\Gamma'[\hat g^{\mu\nu},b] \qquad (2.22)$$

by introducing an additional dependence of Γ' on a new field $b(x)$ with definite transformation laws under general coordinates and Weyl transformations. In this case the anomalous content of Γ' may differ drastically from that of Γ. It is precisely for this reason that I have taken care of using a regularization which does not introduce a spurious dependence of Γ on the determinant of the classical metric.

In general, the change (2.22) corresponds to consider different models, and the comparison of the two sets of anomalies does not make much sense. A comparison of the anomalies might be meaningful if Γ and Γ' differ by local functionals. The relation between the trace and the Diff anomalies, when one allows the possibility to add to Γ, Eq. (2.9), local functionals of $\hat g^{\mu\nu}$ and of the determinant of the metric, will be discussed in Sec. V.

III. DIRECT COMPUTATION OF THE Diff ANOMALY

In this section the computation of the $\epsilon\to 0$ limit of expression (2.13) is reported. Equation (2.13) can be written as

$$\Delta_V\Gamma=\frac{D}{2}\int d^2x\,\partial_\lambda V^\lambda(x)\langle x\,|\,e^{-\epsilon H(p,q)}\,|\,x\rangle\ . \qquad (3.1)$$

In deriving Eq. (3.1), I used

$$\int d^2x\,|\,x\rangle\langle x\,|=1\ , \qquad (3.2)$$

where

$$q^\mu\,|\,x\rangle=x^\mu\,|\,x\rangle\ . \qquad (3.3)$$

The ϵ expansion of the amplitude $\langle x\,|\exp[-\epsilon H(p,q)]\,|\,x\rangle$ can be obtained in several different ways. For instance, one can write

$$\langle x\,|\,e^{-\epsilon H(p,q)}\,|\,x\rangle=\langle 0\,|\,e^{ixp}e^{-\epsilon H(p,q)}e^{-ixp}\,|\,0\rangle=\langle 0\,|\,e^{-\epsilon H(p,q+x)}\,|\,0\rangle\ . \qquad (3.4)$$

Then, a Taylor expansion of $H(p,q+x)$ in powers of q^μ gives

$$H(p,q+x)=H_0(p;x)+H_I(p,q;x)\ , \qquad (3.5)$$

where

$$H_0(p;x)=p_\mu\hat g^{\mu\nu}(x)p_\nu \qquad (3.6)$$

and

$$H_I(p,q;x)=p_\mu q^\lambda\partial_\lambda\hat g^{\mu\nu}(x)p_\nu+\tfrac{1}{2}p_\mu q^\lambda q^\sigma\partial_\lambda\partial_\sigma\hat g^{\mu\nu}(x)p_\nu+\cdots\ . \qquad (3.7)$$

The amplitude (3.4) is evaluated by means of a perturbative expansion in H_I:

$$\langle 0\,|\,e^{-\epsilon(H_0+H_I)}\,|\,0\rangle=\langle 0\,|\,e^{-\epsilon H_0}\,|\,0\rangle-\epsilon\int_0^1 d\alpha\langle 0\,|\,e^{-\epsilon(1-\alpha)H_0}H_Ie^{-\epsilon\alpha H_0}\,|\,0\rangle+\epsilon^2\int_0^1\alpha\,d\alpha\int_0^1 d\beta\langle 0\,|\,e^{-\epsilon(1-\alpha)H_0}H_Ie^{-\epsilon\alpha(1-\beta)H_0}H_Ie^{-\epsilon\alpha\beta H_0}\,|\,0\rangle+\cdots\ . \qquad (3.8)$$

Now, each term of the expansion (3.8) can be easily computed. First, one notices that H_0 does not depend on the q^μ operators. Second, all the q^μ operators appearing in H_I can be moved (through commutators with the p^μ's) on the left or on the right. Finally, one uses the fact that

$$\langle 0\,|\,q^\mu=0\ \text{ or }\ q^\mu\,|\,0\rangle=0\ . \qquad (3.9)$$

Because of dimensional reasons, the relevant terms in the expansion (3.8) are those explicitly displayed. One obtains the following.

Zero order:

$$(0)=\langle 0|e^{-\epsilon H_0}|0\rangle=\int d^2k\langle 0|k\rangle\langle k|e^{-\epsilon H_0}|0\rangle=\frac{1}{(2\pi)^2}\int d^2k\, e^{-k_\mu k_\nu \hat{g}^{\mu\nu}(x)}=\frac{1}{4\pi\epsilon}\det{}^{-1}\hat{g}^{\mu\nu}(x)=\frac{1}{4\pi\epsilon}\ . \tag{3.10}$$

First order:

$$\begin{aligned}(\mathrm{I})&=-\epsilon\int_0^1 d\alpha\langle 0|e^{-\epsilon(1-\alpha)H_0}H_I e^{-\epsilon\alpha H_0}|0\rangle\\&=-\epsilon\int_0^1 d\alpha\langle 0|e^{-\epsilon(1-\alpha)H_0}[p_\mu q^\lambda\partial_\lambda\hat{g}^{\mu\nu}(x)p_\nu+\tfrac{1}{2}p_\mu q^\lambda q^\rho\partial_\lambda\partial_\rho\hat{g}^{\mu\nu}(x)p_\nu]e^{-\epsilon\alpha H_0}|0\rangle\ .\end{aligned} \tag{3.11}$$

The first term on the RHS of Eq. (3.11) vanishes by parity; so, by moving $q^\lambda q^\rho$ on the left-hand side one finds

$$\begin{aligned}(\mathrm{I})=-\frac{\epsilon}{2}\int_0^1 d\alpha\langle 0|e^{-\epsilon(1-\alpha)H_0}[\,&2\epsilon(1-\alpha)\hat{g}^{\rho\lambda}(x)\partial_\rho\partial_\lambda\hat{g}^{\mu\nu}(x)p_\mu p_\nu\\&-4\epsilon^2(1-\alpha)^2 g^{\tau\lambda}(x)\hat{g}^{\rho\sigma}(x)\partial_\lambda\partial_\sigma\hat{g}^{\mu\nu}(x)p_\tau p_\mu p_\nu p_\rho\\&+4\epsilon(1-\alpha)\hat{g}^{\rho\lambda}(x)\partial_\lambda\partial_\mu\hat{g}^{\mu\nu}p_\rho p_\nu]e^{-\epsilon\alpha H_0}|0\rangle\ .\end{aligned} \tag{3.12}$$

At this point, one can go to the momentum basis and integrate over k_μ by using

$$\int d^2k\, e^{-\epsilon k_\rho k_\sigma \hat{g}^{\rho\sigma}(x)}k_{\mu_1}k_{\nu_1}\cdots k_{\mu_n}k_{\nu_n}=\frac{\pi}{2^n\epsilon^{n+1}}(\hat{g}_{\mu_1\nu_1}\cdots\hat{g}_{\mu_n\nu_n}+\text{permutations})\ , \tag{3.13}$$

where

$$\hat{g}_{\mu\nu}(x)=\frac{1}{\sqrt{g(x)}}g_{\mu\nu}(x)=\epsilon_{\mu\tau}\epsilon_{\nu\sigma}\hat{g}^{\tau\sigma}(x) \tag{3.14}$$

is the inverse of $\hat{g}^{\mu\nu}(x)$. The final result is

$$(\mathrm{I})=-\frac{1}{24\pi}[\partial_\mu\partial_\nu\hat{g}^{\mu\nu}(x)+\tfrac{1}{2}\hat{g}^{\mu\nu}(x)\partial_\mu\partial_\nu\hat{g}^{\rho\sigma}(x)\hat{g}_{\rho\sigma}(x)]\ . \tag{3.15}$$

Second order: At second order in H_I, the only nonvanishing term in the $\epsilon\to 0$ limit is

$$(\mathrm{II})=\epsilon^2\int_0^2\alpha\, d\alpha\int_0^1 d\beta\langle 0|e^{-\epsilon(1-\alpha)H_0}p_\mu q^\lambda\partial_\lambda\hat{g}^{\mu\nu}(x)p_\nu e^{-\epsilon\alpha(1-\beta)H_0}p_\rho q^\sigma\partial_\sigma\hat{g}^{\rho\tau}(x)p_\tau e^{-\epsilon\alpha\beta H_0}|0\rangle\ . \tag{3.16}$$

By using the same method illustrated before, one finds

$$(\mathrm{II})=\frac{1}{48\pi}[\partial_\tau\hat{g}^{\mu\nu}(x)\partial_\mu\hat{g}^{\tau\rho}(x)\hat{g}_{\nu\rho}(x)+\tfrac{1}{2}\partial_\tau\hat{g}^{\mu\nu}(x)\partial_\rho\hat{g}^{\lambda\sigma}(x)\hat{g}^{\tau\rho}(x)\hat{g}_{\mu\lambda}(x)\hat{g}_{\nu\sigma}(x)]\ . \tag{3.17}$$

From Eqs. (3.10), (3.15), and (3.17) one obtains, finally,

$$\langle x|e^{-\epsilon H(p,q)}|x\rangle|_{\epsilon\to 0}=\frac{1}{4\pi\epsilon}+\frac{1}{24\pi}[-\partial_\mu\partial_\nu\hat{g}^{\mu\nu}(x)+\tfrac{1}{2}\partial_\mu\hat{g}^{\nu\sigma}(x)\partial_\nu\hat{g}^{\mu\tau}(x)\hat{g}_{\sigma\tau}(x)+\tfrac{1}{4}\hat{g}^{\mu\nu}\partial_\mu\hat{g}^{\tau\sigma}(x)\partial_\nu\hat{g}_{\tau\sigma}(x)]\ , \tag{3.18}$$

and therefore, from Eq. (3.1),

$$\Delta_V\Gamma=\frac{D}{48\pi}\int d^2x\,\partial_\lambda V^\lambda(x)\mathcal{A}(x)\ , \tag{3.19}$$

where

$$\begin{aligned}\mathcal{A}(x)=&-\partial_\mu\partial_\nu\hat{g}^{\mu\nu}(x)+\tfrac{1}{2}\partial_\mu\hat{g}^{\nu\sigma}(x)\partial_\nu\hat{g}^{\mu\tau}(x)\hat{g}_{\sigma\tau}(x)\\&+\tfrac{1}{4}\hat{g}^{\mu\nu}(x)\partial_\mu\hat{g}^{\tau\sigma}(x)\partial_\nu\hat{g}_{\tau\sigma}(x)\ .\end{aligned} \tag{3.20}$$

There is no $1/\epsilon$ term in expression (3.19) because the $1/\epsilon$ factor in expression (3.18), when inserted in (3.1), multiplies the integral of the divergence $\partial_\lambda V^\lambda(x)$. With the appropriate behavior of $V^\lambda(x)$ at infinity, this integral vanishes. On the other hand, if the integral of $\partial_\lambda V^\lambda(x)$ is not vanishing, then the additional term

$$\frac{D}{8\pi\epsilon}\int d^2x\,\partial_\lambda V^\lambda(x) \tag{3.21}$$

just represents the *explicit* and not the anomalous noninvariance of Γ under Diff transformations:

$$\begin{aligned}\Delta_V\left[\frac{D}{2}\ln\mathrm{Det}H\right]&=\frac{D}{2}\mathrm{Tr}(\partial_\lambda V^\lambda)\\&=(\infty)\frac{D}{2}\int d^2x\,\partial_\lambda V^\lambda(x)\ .\end{aligned}$$

IV. CONSISTENCY CONDITIONS

In this section it is shown that the expression (3.19) satisfies the consistency conditions following from Eq. (2.15). It is also shown that $\Delta_V\Gamma[\hat{g}^{\mu\nu}]$ represents in fact an anomaly; i.e., it cannot be written as the variation of a local functional of $\hat{g}^{\mu\nu}$.

From the transformation (2.7) one has

$$\Delta_W(\partial_\mu\partial_\nu\hat{g}^{\mu\nu})=\partial_\lambda(W^\lambda\partial_\mu\partial_\nu\hat{g}^{\mu\nu}-\hat{g}^{\mu\nu}\partial_\mu\partial_\nu W^\lambda)\ , \tag{4.1}$$

$$\begin{aligned}\Delta_W(\partial_\mu\hat{g}^{\nu\sigma}\partial_\nu\hat{g}^{\mu\tau}\hat{g}_{\sigma\tau})=&\ \partial_\lambda[W^\lambda(\partial_\mu\hat{g}^{\nu\sigma}\partial_\nu\hat{g}^{\mu\tau}\hat{g}_{\sigma\tau})]\\&+2\partial_\mu\hat{g}^{\mu\nu}\partial_\nu\partial_\lambda W^\lambda-2\partial_\lambda\hat{g}^{\mu\nu}\partial_\mu\partial_\nu W^\lambda\\&+2\hat{g}^{\mu\nu}\hat{g}^{\tau\sigma}\partial_\nu\partial_\tau W^\lambda\partial_\mu\hat{g}_{\lambda\sigma}\ ,\end{aligned} \tag{4.2}$$

and

$$\begin{aligned}\Delta_W(\hat{g}^{\mu\nu}\partial_\mu\hat{g}^{\tau\sigma}\partial_\nu\hat{g}_{\tau\sigma})=&\ \partial_\lambda(W^\lambda\hat{g}^{\mu\nu}\partial_\mu\hat{g}^{\tau\sigma}\partial_\nu\hat{g}_{\tau\sigma})\\&+4\hat{g}^{\mu\nu}\partial_\mu\hat{g}^{\rho\sigma}\hat{g}_{\rho\lambda}\partial_\nu\partial_\sigma W^\lambda\ .\end{aligned} \tag{4.3}$$

Therefore, from Eqs. (3.19), (4.1), (4.2), and (4.3) it follows that

$$\begin{aligned}\Delta_W\Delta_V\Gamma=-\frac{D}{48\pi}\int d^2x[&\mathcal{A}(x)W^\mu\partial_\mu\partial_\lambda V^\lambda\\&+\partial_\mu\partial_\lambda V^\lambda\hat{g}^{\mu\nu}\partial_\nu\partial_\sigma W^\sigma]\ .\end{aligned} \tag{4.4}$$

Finally, from Eq. (4.4) one gets

$$(\Delta_W\Delta_V-\Delta_V\Delta_W)\Gamma=\frac{D}{48\pi}\int d^2x\ \partial_\lambda Y^\lambda(x)\mathcal{A}(x)\ , \tag{4.5}$$

where Y^μ is given in Eq. (2.16). Equation (4.5) compared with Eq. (3.19) shows that the consistency conditions are indeed satisfied.

It remains to be seen that expression (3.19) represents a true anomaly. Since $\partial_\lambda V^\lambda$ is dimensionless (V^λ has the dimensions of a length), one must consider the most general local functional of $\hat{g}^{\mu\nu}$ with two derivatives and invariant under global rotations of the two-dimensional space-time:

$$\begin{aligned}\mathcal{F}=\int d^2x\ \Big[&B\partial_\mu\partial_\nu\hat{g}^{\mu\nu}+C\partial_\mu\hat{g}^{\nu\sigma}\partial_\nu\hat{g}^{\mu\tau}\hat{g}_{\sigma\tau}\\&+E\hat{g}^{\mu\nu}\partial_\mu\hat{g}^{\tau\sigma}\partial_\nu\hat{g}_{\tau\sigma}+G\partial_\mu\hat{g}^{\mu\tau}\partial_\nu\hat{g}^{\nu\sigma}\hat{g}_{\tau\sigma}\Big]\ .\end{aligned} \tag{4.6}$$

Terms with an odd number of antisymmetric tensors $\epsilon_{\mu\nu}$ are excluded, of course. Also, is should be noted that

$$\hat{g}^{\mu\nu}(x)\partial_\tau\hat{g}_{\mu\nu}(x)=0\ , \tag{4.7}$$

because $\hat{g}^{\mu\nu}(x)$ has determinant 1 for every value of x^μ. Under a Diff transformation (2.7), one obtains

$$\begin{aligned}\Delta_V\mathcal{F}=2\int d^2x\,[&(C-2E)\hat{g}^{\mu\nu}\hat{g}^{\tau\sigma}\partial_\nu\partial_\sigma V^\lambda\partial_\mu\hat{g}_{\lambda\tau}\\&+G\hat{g}^{\mu\nu}\hat{g}^{\tau\sigma}\partial_\mu\partial_\nu V^\lambda\partial_\sigma\hat{g}_{\lambda\tau}]\ .\end{aligned} \tag{4.8}$$

Clearly, regardless of the choice of the coefficients $\{B,C,E,G\},\Delta_V\mathcal{F}$ is never equal to the expression (3.19). So, Eq. (3.19) represents an anomaly.

By using the explicit form of the scalar curvature R,

$$\begin{aligned}R=&-\partial_\mu\partial_\nu g^{\mu\nu}+\tfrac{1}{2}\partial_\mu g^{\nu\sigma}\partial_\nu g^{\mu\tau}g_{\sigma\tau}\\&+\tfrac{1}{4}g^{\mu\nu}\partial_\mu g^{\tau\sigma}\partial_\nu g_{\tau\sigma}-g^{\mu\nu}\partial_\mu\partial_\nu\ln g\\&-\partial_\mu g^{\mu\nu}\partial_\nu\ln g-\tfrac{1}{4}g^{\mu\nu}\partial_\mu\ln g\,\partial_\nu\ln g\ ,\end{aligned} \tag{4.9}$$

Eqs. (3.19) and (3.20) can be rewritten in a more compact form as

$$\Delta_V\Gamma=\frac{D}{48\pi}\int d^2x\ \partial_\lambda V^\lambda\sqrt{g}\,(R+\nabla^2\ln\sqrt{g}\,)\ . \tag{4.10}$$

One easily recognizes that the result (4.10) is a particular case of the one-parameter family of functionals satisfying the combined consistency conditions for the Diff and Weyl transformations found by Bonora, Bregola, and Pasti with a different method.[8] How the trace anomaly is related to the Diff anomaly is discussed in the next section.

Let us consider now the consequences of the anomaly on the energy-momentum tensor. As is well known, if a symmetry of the theory is anomalous, then the divergence of the associated current is nonvanishing. In our case, the Diff anomaly (4.10) means that the energy-momentum tensor $\Theta_{\mu\nu}$ is not conserved.[6]

If one defines

$$\Theta_{\mu\nu}=\frac{2}{\sqrt{g}}\frac{\delta S}{\delta g^{\mu\nu}}=\partial_\mu\phi^i\partial_\nu\phi^i-\tfrac{1}{2}g_{\mu\nu}g^{\rho\sigma}\partial_\rho\phi^i\partial_\sigma\phi^i\ , \tag{4.11}$$

then $\Theta_{\mu\nu}$ is traceless, symmetric, and depends on the classical metric $g_{\mu\nu}$:

$$\Theta_{\mu\nu}=\Theta_{\mu\nu}(g_{\rho\sigma})\ . \tag{4.12}$$

Under a Diff transformation, one has

$$\begin{aligned}\Delta_V\Gamma&=\int d^2x\ \Delta_V g^{\mu\nu}\Big\langle\frac{\delta S}{\delta g^{\mu\nu}}\Big\rangle\\&=-\int d^2x\sqrt{g}\,\nabla^\nu V^\mu\langle\Theta_{\mu\nu}\rangle\\&=\int d^2x\sqrt{g}\,V^\mu\nabla^\nu\langle\Theta_{\mu\nu}\rangle\ .\end{aligned} \tag{4.13}$$

Therefore, because of the anomaly (4.10), the energy-momentum tensor in general is not conserved.

V. TRACE-ANOMALY CONNECTION

We have seen in the previous sections that the generating functional $\Gamma[\hat{g}^{\mu\nu}]$ is not Diff invariant. As shown in Eq. (4.10), the group of general coordinate transformations has an anomaly. This means, by definition of anomaly, that whatever local functional of $\hat{g}^{\mu\nu}$ we add to Γ general covariance will never be exactly realized.

On the contrary, a local functional of the *entire* metric $g_{\mu\nu}$ can be found such that, under a transformation (2.2), it gives exactly the anomaly expression (4.10). The key property one has to note is the transformation law

$$\Delta_V\ln\sqrt{g}=V^\lambda\partial_\lambda\ln\sqrt{g}+\partial_\lambda V^\lambda\ . \tag{5.1}$$

Similarly to the variation of a scalar quantity, $\Delta_V\ln\sqrt{g}$ contains the first term $V^\lambda\partial_\lambda\ln\sqrt{g}$. In addition, a second term is present in Eq. (5.1), $\partial_\lambda V^\lambda$, which coincides with the transformation parameter factor appearing in the anomaly. Because of the inhomogeneous term in the transformation law (5.1), $\ln\sqrt{g}$ plays a role similar to that of the "pion" field in the construction of the Wess-Zumino term for the chiral anomaly.[10]

By using Eqs. (5.1) and (2.7), the anomaly (4.10) can be easily integrated. One obtains

$$\Gamma_{WZ}=\frac{D}{48\pi}\int d^2x\sqrt{g}\,(\ln\sqrt{g}\,R-\tfrac{1}{2}g^{\mu\nu}\partial_\mu\ln\sqrt{g}\,\partial_\nu\ln\sqrt{g}\,) \tag{5.2}$$

with

$$\Delta_V\Gamma_{WZ}=\frac{D}{48\pi}\int d^2x\,\partial_\lambda V^\lambda\sqrt{g}\,(R+\nabla^2\ln\sqrt{g}\,)\ . \tag{5.3}$$

So, at the price of introducing a new field, the determinant of the metric, in addition to $\hat{g}^{\mu\nu}$, one can define a Diff-invariant generating functional Γ':

$$\Gamma'[g_{\mu\nu}]=\Gamma[\hat{g}^{\mu\nu}]-\Gamma_{WZ}[\hat{g}^{\mu\nu},g] \tag{5.4}$$

with

$$\Delta_V\Gamma'=0\ . \tag{5.5}$$

However, Γ' is no longer invariant under Weyl transformations (2.3). In fact, one has

$$\Delta_\sigma\Gamma'=-\frac{D}{24\pi}\int d^2x\,\sigma\sqrt{g}\,R\ , \tag{5.6}$$

which is the well-known expression of the trace anomaly. I shall not elaborate here on the interplay between the trace and the Diff anomalies; a detailed discussion on this subject can be found in Refs. 7, 8, and 11.

The basic difference between the trace anomaly (5.6) and the Diff anomaly (4.10) should be noted. The expression (5.6) is meaningful only because it is related, through Γ_{WZ}, to the expression (4.10), which is the "true" anomaly of the theory.

Clearly, $\Gamma_{WZ}[\hat{g}^{\mu\nu},g]$ is not uniquely determined. One could add to the expression (5.2) any local Diff-invariant functional of $g_{\mu\nu}$ without spoiling the validity of Eq. (5.3). On the contrary, Eq. (5.6) would be accordingly modified.

Finally, if one sets

$$g_{\mu\nu}(x)=e^{\beta(x)}h_{\mu\nu}(x)\ , \tag{5.7}$$

where $h_{\mu\nu}$ is a reference metric, then from Eqs. (5.4) and (5.2) it follows that

$$\Gamma'[e^\beta h_{\mu\nu}]=\Gamma'[h_{\mu\nu}]+\frac{D}{48\pi}\int d^2x\sqrt{h}\,[\tfrac{1}{2}h^{\mu\nu}\partial_\mu\beta\partial_\nu\beta-\beta R(h)]\ , \tag{5.8}$$

where $R(h)$ is the scalar curvature of the metric $h_{\mu\nu}$.

VI. CENTRAL CHARGE CONNECTION

Given a traceless, symmetric, and conserved energy-momentum tensor $\Theta_{\mu\nu}$ in two dimensions, one can define the associated Virasoro algebra.[2,12–14] The commutator of two Virasoro generators A and B is proportional to a third Virasoro generator C plus a term proportional to the identity operator. The properly normalized coefficient which multiplies the identity operator in the commutator algebra is called the central charge of the Virasoro algebra.

The existence of a nontrivial central charge in the Virasoro algebra means that the commutator of two components of the energy-momentum tensor contains, in addition to a canonical term, a Schwinger term.[2,15,16] In this section the connection between the central charge and the Diff anomaly is considered. In particular, I will show that expression (4.10) of the Diff anomaly implies that the commutator of two components of the energy-momentum tensor contains a Schwinger term which corresponds exactly to the well-known value of the central charge for a free bosonic model.

Let us introduce first some useful notations. The action of a free bosonic model in flat space-time [we use the Minkowski signature here, $\eta_{\mu\nu}=\mathrm{diag}(+,-)$] is

$$S_0=\int d^2x\,\tfrac{1}{2}(\partial_0\phi^i\partial_0\phi^i-\partial_1\phi^i\partial_1\phi^i)\ , \tag{6.1}$$

and the equations of motion are

$$\partial_-\partial_+\phi^i=0\ . \tag{6.2}$$

The light-cone coordinates are defined by

$$\begin{aligned}&x^\pm=x^0\pm x^1\ ,\\&d^2x=\tfrac{1}{2}dx^+dx^-\ ,\\&\partial_\pm\equiv\frac{\partial}{\partial x^\pm}=\tfrac{1}{2}(\partial_0\pm\partial_1)\ .\end{aligned} \tag{6.3}$$

The traceless and symmetric energy-momentum tensor is

$$\Theta_{\mu\nu}(\eta_{\rho\sigma})=\partial_\mu\phi^i\partial_\nu\phi^i-\tfrac{1}{2}\eta_{\mu\nu}\partial^\lambda\phi^i\partial_\lambda\phi^i\ . \tag{6.4}$$

From now on, $\Theta_{\mu\nu}(\eta_{\rho\sigma})$ will be denoted simply by $\Theta_{\mu\nu}$. The two independent components $\Theta_\pm$,

$$\Theta_\pm=\tfrac{1}{2}(\Theta_{00}+\Theta_{11}\pm2\Theta_{01})\ , \tag{6.5}$$

of the energy-momentum tensor can be written as

$$\Theta_\pm=2\partial_\pm\phi^i\partial_\pm\phi^i\ . \tag{6.6}$$

Regardless of the (infinite) constant which has to be subtracted from the classical expression (6.6) in order to get a regularized energy-momentum tensor, the equations of motion (6.2) imply that $\Theta_{\mu\nu}$ is conserved, i.e.,

$$\partial_-\Theta_+=\partial_+\Theta_-=0\ . \tag{6.7}$$

Note that Eq. (6.7) is not in contradiction with the conclusion drawn in Sec. IV on the effects of the Diff anomaly on the conservation of the energy-momentum tensor because here we are in flat space-time.

Having a traceless, symmetric, and conserved energy-momentum tensor, it makes sense to speak about the central charge of the Virasoro algebra. Equivalently, one is interested in the equal-time commutation relations (we concentrate on the left-moving sector)

$$[\Theta_+(x),\Theta_+(y)]_{\rm ET}=2i\partial_+\Theta_+(x)\delta(x^1-y^1)+4i\Theta_+(x)\delta'(x^1-y^1)-\frac{i\lambda}{6\pi}\delta'''(x^1-y^1)\ . \qquad (6.8)$$

The first two terms on the RHS of Eq. (6.8) follow from the canonical commutation relations:

$$[\partial_+\phi^i(x),\partial_+\phi^j(y)]_{\rm ET}=\frac{i}{2}\delta^{ij}\delta'(x^1-y^1)\ . \qquad (6.9)$$

The structure of the additional Schwinger term in the commutator (6.8) is fixed by dimensional reasons.

How to compute λ is not obvious *a priori*. Some care is needed in finding a consistent definition of the regularized energy-momentum tensor. Our strategy will be to determine λ by using the already computed expression of the Diff anomaly.

It is convenient first to extend the commutation relations (6.8) and (6.9) to any time. This can be done by using a "canonical" formalism in which x^- is interpreted as the "time" variable and x^+ as a "space" variable.[17] In fact, since Θ_+ and $\partial_+\phi^i$ only depend on x^+ [see Eqs. (6.2) and (6.7)], in this new formalism the extended commutation relations can be interpreted just as equal-"time" commutators.

In the light-cone basis (6.3), the variation of the action is

$$\delta S_0=\int dx^+dx^-\delta\phi^i(-2\delta_{ij}\partial_+)\partial_-\phi^j\ . \qquad (6.10)$$

According to Ref. 17, the symplectic matrix $F^{ij}(x^+,y^+)$ is given by

$$F^{ij}(x^+,y^+)=(-2\delta_{ij}\partial_+)^{-1}=-\tfrac{1}{2}\delta^{ij}\theta(x^+-y^+)\ . \qquad (6.11)$$

Then, the commutator of two operators X and Y (which do not contain derivatives in x^-) is defined to be[17]

$$[X,Y]=i\int dx^+dy^-F^{ij}(x^+,y^+)\frac{\delta X}{\delta\phi^i(x^+)}\frac{\delta Y}{\delta\phi^j(y^+)}\ . \qquad (6.12)$$

By using the rules (6.12) and (6.11) of the light-cone formalism, the commutation relations (6.8) and (6.9) take the covariant form

$$[\partial_+\phi^i(x^+),\partial_+\phi^j(y^+)]=\frac{i}{2}\delta^{ij}\delta'(x^+-y^+) \qquad (6.13)$$

and

$$[\Theta_+(x^+),\Theta_+(y^+)]=2i\Theta'_+(x^+)\delta(x^+-y^+)+4i\Theta_+(x^+)\delta'(x^+-y^+)-\frac{i\lambda}{6\pi}\delta'''(x^+-y^+)\ . \qquad (6.14)$$

Note that, if one sets $x^0=y^0$, then Eqs. (6.13) and (6.14) coincide with the Eqs. (6.9) and (6.8), respectively, as it should be.

The connection between the Diff anomaly and the Schwinger term in Eq. (6.14) can be understood in the following way.

Let us consider the bosonic model in the presence of a nontrivial gravitational background specified by the classial metric

$$g_{\mu\nu}(x)=\begin{bmatrix}1-\alpha(x) & \alpha(x)\\ \alpha(x) & -1-\alpha(x)\end{bmatrix}_{\mu\nu}\ . \qquad (6.15)$$

Then, the action takes the form

$$S=S_0+\int d^2x\,\alpha(x)\Theta_+(x)\ . \qquad (6.16)$$

For small $\alpha(x)$, the metric (6.15) describes a small fluctuation around the flat space-time. The particular form of the metric has been chosen in such a way to select in Eq. (6.16) the expression $\Theta_+(x)$ of the energy-momentum tensor with respect to the flat space-time.

Let us consider now a Diff transformation on Γ:

$$\Delta_V\Gamma=\frac{1}{\int\mathcal{D}\phi^i e^{iS}}\int\mathcal{D}\phi^i e^{iS}\Delta_V S\ . \qquad (6.17)$$

If one chooses the parameters $V^\mu(x)$ of the transformation to be

$$V^0(x)=V^1(x)=\tfrac{1}{2}v(x) \qquad (6.18)$$

then, to first order in $\alpha(x)$, one has

$$\Delta_V S=\int d^2x[-\Theta_+\partial_-v+(v\partial_+\alpha-\alpha\partial_+v)\Theta_+]\ . \qquad (6.19)$$

By using Eqs. (6.16), (6.17), and (6.19), one finds (to first order in α)

$$\Delta_V\Gamma=\tfrac{1}{2}\int dx^+dx^-[v\langle\partial_-\Theta_+\rangle+(v\partial_+\alpha-\alpha\partial_+v)\langle\Theta_+\rangle]-\frac{i}{4}\int dx^+dx^-v(x)\langle\partial_-\Theta_+(x)\rangle\int dy^+dy^-\alpha(y)\langle\Theta_+(y)\rangle$$
$$-\frac{i}{4}\int dx^+dx^-dy^+dy^-T_-\langle[\partial_-v(x)\Theta_+(x)][\alpha(y)\Theta_+(y)]\rangle\ , \qquad (6.20)$$

where the expectation values have to be computed with respect to the flat space-time vacuum and T_- means (in the light-cone formalism) the "time" ordering with respect to x^-. By integrating by parts of the last term in Eq. (6.20) and by using the commutation relations (6.14) together with Eq. (6.7), one obtains, finally,

$$\Delta_V\Gamma=\frac{\lambda}{48\pi}\int dx^+dx^-(v\partial_+^3\alpha-\alpha\partial_+^3v)\ . \qquad (6.21)$$

On the other hand, we have already computed $\Delta_V\Gamma$ for any arbitrary background, Eqs. (3.19) and (3.20). By inserting the explicit form of the metric (6.15) and of the parameters (6.18) in Eq. (3.19), one finds, to first order in α (see also Ref. 18)

$$\Delta_V \Gamma = \frac{D}{48\pi} \int dx^+ dx^- (v\partial_+^3 \alpha - \alpha \partial_+^3 v) \ . \qquad (6.22)$$

Equation (6.22) compared with Eq. (6.21) gives

$$\lambda = D \ , \qquad (6.23)$$

which corresponds precisely to the standard value of the central charge for a free bosonic model.

VII. CONCLUSIONS

In this paper, the problem of the origin of the trace anomaly and of the central charge for a bosonic two-dimensional model has been considered. Beyond the different aspects of the conformal breakdown there is an essential fact: the bosonic model in gravitational background has a gravitational (Diff) anomaly.

The relation between the trace and the Diff anomalies is the following. The generating functional Γ is Weyl invariant because the action (1.1) of the two-dimensional bosonic model does not know about the determinant of the external metric $g_{\mu\nu}$. This is also the reason why the Diff anomaly is actually an anomaly.

In fact, one can introduce a new field and declare that it corresponds precisely to the determinant of $g_{\mu\nu}$. In terms of this new field, it becomes possible then to define a new Diff-invariant generating functional Γ' by adding to Γ an approriate local counterterm, Eq. (5.2). In this way, there is not a Diff anomaly, but the invariance under Weyl transformations is lost, and one has the trace anomaly.

The existence of a nontrivial central charge in the Virasoro algebra associated with the energy-momentum tensor is also an effect of the Diff anomaly. The Diff anomaly implies that the traceless and symmetric energy-momentum tensor of the model in gravitational background is not conserved. However, in flat space-time the energy-momentum tensor is conserved. Still, the theory in flat space-time knows the Diff anomaly.

In fact, the commutator of two components of the energy-momentum tensor contains a nontrivial Schwinger term. On one side, this Schwinger term must necessarily be present in order to reproduce the Diff anomaly in the presence of a nontrivial background. On the other side, the Schwinger term originates precisely the central charge in the Virasoro algebra.

All that is nice; even in the simple bosonic model considered in this paper the conformal anomalies show an interesting structure. But, what conclusions can be drawn in view of the open problems mentioned in the Introduction? The analysis of this paper suggests that if there are topological reasons for the existence of the trace anomaly, then these reasons have to be found in the group of the Diff transformations (2.7).

Finally, let us consider the case of an interacting scale-invariant model. Now it is clear what one has to do to find the trace anomaly and the central charge. One has to compute first the Diff anomaly. Perhaps, it is not easy to find a closed form of the Diff anomaly for a generic model. However, this is a well-defined problem and can be solved, for instance, order by order in perturbation theory.

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