PHYSICAL REVIEW D VOLUME 38, NUMBER 8 15 OCTOBER 1988

# Black holes in string-generated gravity models

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(Received 10 May 1988)

Spherically symmetric solutions of $d$-dimensional Einstein-Maxwell theory with a Gauss-Bonnet term are classified. All spherically symmetric solutions of $d$-dimensional Einstein gravity coupled to the Gauss-Bonnet and Born-Infeld terms are derived, classified, and compared with the previous solutions. Thermodynamic properties of the black holes are discussed and the black-hole temperatures derived. Unlike the solutions of Einstein-Maxwell theory the solutions with a Born-Infeld term do not appear to have a stable end point with regard to thermal evaporation.

## I. INTRODUCTION

In their low-energy limit string theories give rise to effective models of gravity in higher dimensions which involve higher powers of the Riemann curvature in addition to the usual Einstein term.[1] It is hoped that the full low-energy theory will solve the problem of singularities in general relativity. However, in the absence of knowledge of the details of such a theory, attempts have been made to gain further insight by studying models which include only the leading-order corrections, such as the quadratic-order Gauss-Bonnet term. Black holes,[2-7] wave propagation,[2,8,9] dimensional reduction,[10-14] and cosmological models[3,10,15-20] have been a particular focus of research. Models involving dimensionally continued Euler densities[21,22] of higher than quadratic order have also been investigated,[3,8,10-13,23-27] although the relationship between such models and the low-energy limit of string theory has not been established. In any case, higher-dimensional models involving dimensionally continued Euler densities are of independent interest since they allow spontaneous compactification.[23,28]

Two somewhat different, but complementary, approaches have been taken to the study of black holes in string-generated gravity models. One approach[6,7] has been to take black-hole solutions of conventional Einstein theory, and to treat the additional leading-order string-generated terms as perturbations about these backgrounds. Such an approach is valid if the curvature is small compared with the scale set by the string tension $T=(2\pi\alpha')^{-1}$. A second approach[2-5] has been to study exact solutions of Einstein theory supplemented by a Gauss-Bonnet term. These two approaches are equivalent insofar as they relate to regions in which the first approach is valid. The second approach, by concentrating on exact solutions, accommodates a study of the global properties of solutions. However, this is by no means an advantage as far as string physics is concerned, since in regions of high curvature additional higher-order terms in the slope expansion will become important. The second approach has revealed some interesting properties, such as the existence of a second branch of asymptotically (anti−)de Sitter solutions in addition to the familiar asymptotically flat branch.[2] If the dilaton is included the asymptotically (anti−)de Sitter branch is excluded;[5] however, the extra branch could persist at the level of higher-order curvature corrections.

The exact nature of the effective low-energy theory is naturally of importance. Unfortunately there is as yet no unambiguous way of determining it. One method of obtaining the low-energy theory is to calculate string tree $N$-point amplitudes with massless external states, and then look for a conventional field-theory action which reproduces these amplitudes at the tree level. However, an ambiguity arises since one can make field redefinitions without changing the $S$ matrix.[29-32] At quadratic order one obtains terms of the form

$$\alpha \exp\left[\frac{-4\kappa\sigma}{d-2}\right](R_{ABCD}R^{ABCD}+aR_{AB}R^{AB}+bR^2)\,, \tag{1.1}$$

where $\sigma$ is a real scalar field (the dilaton) and $\alpha$ is a constant of dimension $L^{d-4}$ related to the slope parameter $\alpha'$. The coefficients $a$ and $b$ cannot be uniquely determined because of the field redefinition ambiguity. The effective action may be derived in other ways, in particular by investigation of world-sheet $\sigma$ models.[33,34] However, these schemes lead to a similar ambiguity.[29]

Some constraints on the coefficients of the higher-derivative terms in (1.1) can be made if one requires that the strong slope expansion be ghost-free, as the string itself is. Zwiebach has shown that this is indeed the case if one chooses $a=-4$ and $b=1$ (Ref. 35). The resulting combination is the Gauss-Bonnet term which is a topological invariant in four dimensions. The basis of Zwiebach's argument is that if one requires that the graviton propagator near flat space is not modified by the higher-derivative terms then one is uniquely led to the Gauss-Bonnet combination. It has been pointed out[36] that Zwiebach's criterion is in fact somewhat stronger than mere ghost freedom: one could add an arbitrary amount of $R^2$ without introducing ghosts.[37,38] The resulting theory would contain additional massive scalar fields (with positive norm). However, the Gauss-Bonnet term is the only possible combination for which the quadratic-order gravitational action describes a pure massless graviton at the linearized level.

 

In an earlier paper[4] black-hole solutions of $d$-dimensional Einstein-Maxwell theory with a Gauss-Bonnet term were derived. However, if one is to consider an electromagnetic field coupled to a gravitational action which includes string-generated corrections, then it is natural to consider string-generated corrections to the Maxwell action also. Such corrections arise if one considers the coupling of an Abelian gauge field to the open bosonic string[29,39–42] or the open superstring.[43] In fact, the effective electromagnetic action can be determined exactly at the tree level—a rather better state of affairs than in the case of the gravitational action. It coincides with the nonlinear action of Born and Infeld:[44]

$$\mathcal{L}_{\rm BI}=b^2\left\{(\,|\det g_{AB}\,|\,)^{1/2}-\left[\left|\det\left[g_{AB}+\frac{1}{b}F_{AB}\right]\right|\right]^{1/2}\right\}, \tag{1.2}$$

where $b$ is a constant of dimension $L^{-d/2}$, which is related to the slope parameter. [In (1.2) we have ignored the coupling to the dilaton and Kalb-Ramond fields.] The Born-Infeld Lagrangian coincides with the Maxwell Lagrangian to leading order in $1/b$. The Born-Infeld theory was originally formulated with the aim of describing apparently nonsingular static configurations such as electrons,[44] although it did not have success in this regard. It was later realized[45] that the Born-Infeld action admits nontrivial vortex solutions.[46] Fradkin and Tseytlin have pointed out that since the effective field theory derived from a fundamental string theory may therefore have classical stringlike solutions, there could perhaps be a kind of bootstrapping between the effective and full theories.[39]

Another way in which theories involving nonlinear electromagnetic terms can arise is through a Kaluza-Klein-type reduction of higher-dimensional theories which include dimensionally continued Euler densities. Müller-Hoissen[10,11] and Kerner[14] have investigated the reduction of the five-dimensional Einstein-Gauss-Bonnet theory. The resulting dimensionally reduced four-dimensional theory contains nonlinear electromagnetic terms, which do not correspond to the Born-Infeld term, however.

In this paper we shall derive solutions of a model which includes the Gauss-Bonnet and Born-Infeld terms. In doing so, we shall neglect the coupling of the dilaton and Kalb-Ramond fields. Our only reason for doing so is that it allows the field equations to be integrated exactly. Before deriving these solutions we shall, for the purpose of comparison, classify the "Schwarzschild"[2,3] and "Reissner-Nordström"[4] solutions—namely, the spherically symmetric solutions of the field equations derived from the action

$$S=\int d^4x\sqrt{g}\left[\frac{-R}{4\kappa^2}+\alpha(R_{ABCD}R^{ABCD}-4R_{AB}R^{AB}+R^2)-\tfrac{1}{4}F_{AB}F^{AB}\right]. \tag{1.3}$$

(We use a signature $+--\cdots-$, and conventions in which $g\equiv|\det g_{AB}|$, $R^A{}_{BCD}=\partial_C\Gamma^A_{BD}+\cdots$, $R_{AB}=R^C{}_{ACB}$, and $\kappa^2=4\pi G$ is the $d$-dimensional gravitational constant.) The solutions are given by

$$ds^2=\Delta dt^2-\frac{dr^2}{\Delta}-r^2d\Omega^2_{d-2} \tag{1.4a}$$

and

$$F=\frac{Q}{4\pi r^{2d-4}}dt\wedge dr\ , \tag{1.4b}$$

where

$$\Delta=1+\frac{r^2}{8\kappa^2\tilde\alpha}\left\{1\mp\left[1+16\kappa^2\tilde\alpha\left[\frac{2GM}{r^{d-1}}-\frac{GQ^2}{2\pi(d-2)(d-3)r^{2d-4}}\right]\right]^{1/2}\right\} \tag{1.4c}$$

and $\tilde\alpha\equiv(d-3)(d-4)\alpha$. The solutions have two branches. In the case of the upper branch the solutions are asymptotically flat and

$$\Delta\sim 1-\frac{2GM}{r^{d-3}}+\frac{GQ^2}{2\pi(d-2)(d-3)r^{2(d-3)}}\ , \tag{1.5}$$

as $r\to\infty$, which corresponds to the arbitrary-dimensional Reissner-Nordström solution.[47] The parameter $M$ used here is proportional to the mass $\tilde M$ of the solution, which is defined asymptotically by

$$\int d^{d-1}x\, T_{00}=\tilde M\ . \tag{1.6}$$

The two parameters are related by[48]

$$\tilde M=\frac{(d-2)\mathcal{A}_{d-2}M}{8\pi}\ , \tag{1.7}$$

where $\mathcal{A}_{d-2}=2\pi^{(d-1)/2}/\Gamma((d-1)/2)$ is the area of a $(d-2)$-sphere. For the lower branch,

$$\Delta\sim 1+\frac{2GM}{r^{d-3}}-\frac{GQ^2}{2\pi(d-2)(d-3)r^{2(d-3)}}+\frac{r^2}{4\kappa^2\tilde\alpha}\ , \tag{1.8}$$

as $r\to\infty$, which corresponds to a Reissner-Nordström-(anti-)de Sitter solution with negative gravitational mass, if $M>0$, and imaginary charge $Q$. These solutions are asymptotically anti-de Sitter if $\alpha>0$ (or de Sitter if $\alpha<0$).

The Schwarzschild- and Reissner-Nordström-type solutions are classified in Secs. II and III. In Sec. IV the solutions of the model including the Born-Infeld and Gauss-Bonnet terms are derived and classified. In Sec. V the thermodynamics of the black holes are discussed. Some concluding remarks are made in Sec. VI.

## II. CLASSIFICATION OF THE SCHWARZSCHILD-TYPE SOLUTIONS

The global properties of the solutions (1.4) may be readily determined. We shall assume that $d \geq 6$ since the properties of the solutions may differ slightly in the special case $d=5$. If $Q=0$ the structure of the singularities and horizons may be summarized as follows.

*(1) Asymptotically flat branch.* (a) If $M>0$ and $\alpha \geq 0$, or if $M=0$ and $\alpha<0$, then there is a singularity at the origin. The singularity is shielded by a regular horizon which is located at $r_{\mathcal{H}}$, where

$$r_{\mathcal{H}}^{d-3}+4\kappa^2\tilde{\alpha} r_{\mathcal{H}}^{d-5}-2GM=0 . \tag{2.1}$$

The spacetime thus has the global properties of the Schwarzschild solution. One may define Kruskal coordinates in the usual fashion to eliminate the apparent singularity at $r=r_{\mathcal{H}}$ (Ref. 3). The value of $r_{\mathcal{H}}$ is always less than that of the $\alpha=0$ case. (b) If $M>0$ and $\alpha_0<\alpha<0$, where

$$8\kappa^2(d-3)(d-4)\alpha_0=-(4GM)^{2/(d-3)} , \tag{2.2}$$

then there is a singularity at $r=r_0$, where

$$r_0^{d-1}+8\kappa^4\tilde{\alpha} M/\pi=0 , \tag{2.3}$$

which is shielded by a horizon at $r=r_{\mathcal{H}}$. The spacetime once again has the properties of the Schwarzschild solution. The value of $r_{\mathcal{H}}$ is always greater than that of the $\alpha=0$ case. (c) In the remaining cases there is a naked timelike singularity. If $M<0$ and $\alpha>0$, or if $M>0$ and $\alpha \leq \alpha_0$, then the singularity is located at $r=r_0$, where $r_0$ is given by (2.3). If $M<0$ and $\alpha<0$, or $M=0$ and $\alpha>0$, then the singularity is located at the origin.

*(2) Asymptotically (anti-)de Sitter branch.* (a) If $M<0$ and $\alpha<\alpha_1$, where

$$4\kappa^2(d-4)(d-5)\alpha_1=-\,|(d-5)GM|^{2/(d-3)} , \tag{2.4}$$

then there is a singularity at the origin which is shielded by two horizons. The horizons are given by solutions of (2.1). In the limiting case $\alpha=\alpha_1$ there is a single degenerate horizon. (b) If $M>0$ and $\alpha<\alpha_0$, then there is a timelike singularity at $r=r_0$, shielded by a horizon at $r=r_{\mathcal{H}}$. (c) In the remaining cases there is a naked singularity. If $M<0$ and $\alpha>0$, or if $M>0$ and $\alpha_0 \leq \alpha<0$, then the singularity is located at $r=r_0$. If $M>0$ and $\alpha \geq 0$, or if $M<0$ and $\alpha_1<\alpha \leq 0$, then the singularity is located at the origin.

## III. CLASSIFICATION OF THE REISSNER-NORDSTRÖM-TYPE SOLUTIONS

In general, the spacetimes (1.4) will have branch singularities at $r=r_0$, where

$$a(r_0) \equiv 1+16\kappa^2\tilde{\alpha}\left[\frac{2GM}{r_0^{d-1}}-\frac{GQ^2}{2\pi(d-2)(d-3)r_0^{2d-4}}\right]=0 . \tag{3.1}$$

[We will denote the argument of the square-root factor in (1.4c) by $a(r)$.] If $\alpha>0$ then (3.1) always has a solution. If $\alpha<0$ then (3.1) has solutions only if $M>0$ and $\alpha<\alpha_0$, where

$$16\kappa^2(d-3)(d-4)\alpha_0=-\left[\frac{d-2}{d-3}\right]\frac{1}{GM}\left[\frac{Q^2}{2\pi(d-1)(d-3)M}\right]^{(d-1)/(d-3)} . \tag{3.2}$$

If $\alpha_0 \leq \alpha \leq 0$ the solutions are defined for all $r>0$. The singularities may be shielded by horizons, which are given by solutions of the equation

$$f(r) \equiv \frac{1}{r^{2d-6}}(r^{d-3}-\tilde{r}_+^{d-3})(r^{d-3}-\tilde{r}_-^{d-3})+\frac{4\kappa^2\tilde{\alpha}}{r^2}=0 , \tag{3.3a}$$

where

$$\tilde{r}_\pm^{d-3}=GM\pm\frac{\kappa}{4\pi}(\kappa^2M^2-\tilde{Q}^2)^{1/2} \tag{3.3b}$$

denote the inner and outer horizons of the usual arbitrary dimensional Reissner-Nordström solution,[47] $\tilde{Q}$ being given by

$$\tilde{Q}^2 \equiv \frac{2Q^2}{(d-2)(d-3)} . \tag{3.3c}$$

The structure of the horizons and singularities may be determined by a careful analysis of the functions $a(r)$ and $f(r)$. In each distinct case one can determine the number of turning points of $f(r)$, and hence the number of possible zeros, since $f(r)$ is continuous on any domain on which $\Delta(r)$ is defined. In fact, $f(r)$ has a maximum of two zeros in any particular case. Furthermore, if $r_{\mathcal{H}}$ denotes a solution of (3.3) then it will correspond to a horizon of the upper branch if $1+8\kappa^2\tilde{\alpha}/r_{\mathcal{H}}^2>0$, or a horizon of the lower branch if $1+8\kappa^2\tilde{\alpha}/r_{\mathcal{H}}^2<0$. If $1+8\kappa^2\tilde{\alpha}/r_{\mathcal{H}}^2=0$ then $r_{\mathcal{H}}=r_0$ and we have a naked singularity. Let us define $r_{c\pm}$ to be the values of $r_{\mathcal{H}}$ in the critical case, i.e.,

$$r_{c\pm}^{d-3}=2GM\pm\frac{\kappa}{2\pi}(\kappa^2M^2-\tfrac{1}{2}\tilde{Q}^2)^{1/2} . \tag{3.4}$$

The various cases can be summarized as follows (for $d \geq 6$).

*(1) Asymptotically flat branch.* (a) If $M>0$ and $\alpha>0$ then there will be a timelike singularity at $r=r_0$, which is shielded by two horizons if $Q<Q_{ex}$, where $Q_{ex}$ is the critical value determined by $\Delta|_{r_{\mathcal{H}}}=(d\Delta/dr)|_{r_{\mathcal{H}}}=0$, i.e., it is the solution of the equation

$$r_{\rm ex}^{2(d-3)}+4\left[\frac{d-5}{d-3}\right]\kappa^2\tilde{\alpha}r_{\rm ex}^{2(d-4)}-\frac{GQ_{\rm ex}^2}{2\pi(d-2)(d-3)}=0\ , \tag{3.5a}$$

where

$$r_{\rm ex}^{d-3}=-\tfrac{1}{2}(d-5)GM+\left[\tfrac{1}{4}(d-5)^2G^2M^2+\frac{(d-4)GQ_{\rm ex}^2}{2\pi(d-2)(d-3)}\right]^{1/2}. \tag{3.5b}$$

If $Q=Q_{\rm ex}$ there is a single degenerate horizon at $r=r_{\rm ex}$. The spacetime therefore has the global properties of the nonextreme Reissner-Nordström solution if $Q<Q_{\rm ex}$, and of the extreme Reissner-Nordström solution if $Q=Q_{\rm ex}$. If $Q>Q_{\rm ex}$ the singularity is naked. (b) If $M\le 0$ and $\alpha\ge 0$ there is a naked timelike singularity at $r=r_0$. (c) If $M>0$ and $\alpha_0\le\alpha<0$, or if $M\le 0$ and $\alpha<0$, then there is a singularity at $r=0$. This singularity will be shielded by two horizons (or a single degenerate horizon in the extreme case) if (i) (3.3) has solutions $r_\pm$, $0<r_-\le r_+$, and (ii) $M>0$, $\tilde{Q}^2<2\kappa^2M^2$, and $r_+<r_{c+}$. The resulting spacetime will therefore have the properties of the Reissner-Nordström or extreme Reissner-Nordström solution. In the cases excluded by (i) and (ii) the singularity will be naked. (d) If $M>0$ and $\alpha<\alpha_0$ then (3.1) has two solutions: the $r_{0\pm}$, $r_{0-}<r_{0+}$. For an observer in the asymptotic region of the spacetime it is only the location of the outer singularity which is important. (The theory is undefined for $r_{0-}\le r\le r_{0+}$.) For $M>0$ and $\alpha<\alpha_0$ any turning point of $\Delta$ must lie at a value of $r$ less then $r_{0+}$. Consequently it is not possible for the outer singularity to be shielded by two horizons in this case. Thus the singularity is timelike and naked if $1+8\kappa^2\tilde{\alpha}/r_{0+}^2\ge 0$, i.e., if $r_{c-}\le r_{0+}\le r_{c+}$. If $r_{0+}>r_{c+}$, then the singularity is spacelike and shielded by a horizon. In the second case we have a Schwarzschild-type spacetime.

*(2) Asymptotically (anti-)de Sitter branch.* (a) If $\alpha>0$ then there is a naked branch singularity at $r=r_0$. (b) If $M>0$ and $\alpha_0\le\alpha<0$, or if $M\le 0$ and $\alpha<0$, then there is a singularity at $r=0$. This singularity will be shielded by two horizons (or a single degenerate horizon in the extreme case) if (i) (3.3) has solutions $r_\pm$, $0<r_-\le r_+$, and (ii) $M\le 0$, or $M>0$ and $\tilde{Q}^2\ge 2\kappa^2M^2$, or $M>0$, $\tilde{Q}^2<2\kappa^2M^2$ and $r>r_{c+}$. In the cases excluded by (i) and (ii) the singularity will be naked. (c) If $M>0$ and $\alpha<\alpha_0$ then there are branch singularities at $r=r_{0\pm}$. The outer singularity is timelike and shielded by a horizon if $r_{c-}<r_{0+}<r_{c+}$. If $r_{0+}\ge r_{c+}$ then we have a naked (spacelike) singularity.

## IV. SPHERICALLY SYMMETRIC SOLUTIONS WITH A BORN-INFELD TERM

We will now turn our attention to the action

$$S=\int d^dx\left[\frac{-\sqrt{g}R}{4\kappa^2}+b^2\left\{\sqrt{g}-\left[\left|\det\left[g_{AB}+\frac{1}{b}F_{AB}\right]\right|\right]^{1/2}\right\}+\alpha\sqrt{g}\,(R_{CDEF}R^{CDEF}-4R_{CD}R^{CD}+R^2)\right]. \tag{4.1}$$

Ideally the ratio of the coefficients $\alpha$ and $b^2$ appearing in front of the Gauss-Bonnet and Born-Infeld terms should be uniquely determined by string theory. However, to date this value has not been calculated.[42] We will leave $b^2$ arbitrary in this paper: it will not affect the classification of the spacetimes.

The usual Maxwell equation is replaced by

$$\partial_A(\sqrt{g}H^{AB})=0\ , \tag{4.2}$$

where

$$H^{AB}\equiv\frac{-2}{\sqrt{g}}\frac{\partial\mathcal{L}_{\rm BI}}{\partial F_{AB}}\ .$$

One still has the "Bianchi identity"

$$\partial_AF_{BC}+\partial_BF_{CA}+\partial_CF_{AB}=0\ . \tag{4.3}$$

We shall assume that the solutions are spherically symmetric, so that the metric takes the form

$$ds^2=e^{2\phi(t,r)}dt^2-e^{2\lambda(t,r)}dr^2-Y(t,r)^2d\Omega_{d-2}^2\ , \tag{4.4}$$

while the only nonzero components of the field $\mathbf{H}$ are given by

$$H_{\hat{t}\hat{r}}=\frac{Q}{4\pi Y^{d-2}} \tag{4.5}$$

in an orthonormal frame. Hence the only nonzero (frame) components of $\mathbf{F}$ are given by

$$F_{\hat{t}\hat{r}}=\frac{Q}{4\pi(Y^{2d-4}+\beta^2)^{1/2}}\ , \tag{4.6}$$

where $\beta^2\equiv Q^2/(16\pi^2b^2)$. If only electric fields are present, as is the case here, then

$$\mathcal{L}_{\rm BI}=b^2\sqrt{g}\left[1-\left[1+\frac{1}{2b^2}F_{CD}F^{CD}\right]^{1/2}\right]. \tag{4.7}$$

The gravitational-field equations which we will consider are therefore

$$R_{AB}-\tfrac{1}{2}g_{AB}R=2\kappa^2(T_{AB}^{bi}+T_{AB}^{gb})\ , \tag{4.8a}$$

where

$$T_{AB}^{bi}=\frac{b^2g_{AB}-F_{AC}F_B{}^C+\frac{1}{2}g_{AB}F_{CD}F^{CD}}{\left[1+\frac{1}{2b^2}F_{CD}F^{CD}\right]^{1/2}}-b^2g_{AB} \tag{4.8b}$$

and

$$T^{gb}_{AB}=4R_{ACDE}R_{B}{}^{CDE}-8R_{CD}R^{C}{}_{A}{}^{D}{}_{B}-8R_{AC}R^{C}{}_{B}+4RR_{AB}-g_{AB}(R_{CDEF}R^{CDEF}-4R_{CD}R^{CD}+R^2)\ . \tag{4.8c}$$

The solution of the field equations is directly parallel to the case of the Reissner-Nordström-type solutions.[4] If $Y_{,B}$ is spacelike ($Y^{,B}Y_{,B}<0$) one may make the choice $Y=r$. The independent field equations are then

$$\tfrac{1}{2}(d-2)(d-3)\psi+(d-2)\frac{1}{r}\lambda_{,r}e^{-2\lambda}-16\pi Gb^2\left[\frac{(r^{2d-4}+\beta^2)^{1/2}}{r^{d-2}}-1\right]=-4\kappa^2\tilde{\alpha}(d-2)\psi\left[\frac{2}{r}\lambda_{,r}e^{-2\lambda}+\tfrac{1}{2}(d-5)\psi\right], \tag{4.9a}$$

$$(d-2)\frac{1}{r}\lambda_{,t}e^{-(\phi+\lambda)}=-8\kappa^2\tilde{\alpha}(d-2)\frac{1}{r}\lambda_{,t}\psi e^{-(\phi+\lambda)}\ , \tag{4.9b}$$

$$-\tfrac{1}{2}(d-2)(d-3)\psi+(d-2)\frac{1}{r}\phi_{,r}e^{-2\lambda}+16\pi Gb^2\left[\frac{(r^{2d-4}+\beta^2)^{1/2}}{r^{d-2}}-1\right]=4\kappa^2\tilde{\alpha}(d-2)\psi\left[\frac{-2}{r}\phi_{,r}e^{-2\lambda}+\tfrac{1}{2}(d-5)\psi\right], \tag{4.9c}$$

where $\psi\equiv r^{-2}(1-e^{-2\lambda})$. From (4.9b) we deduce that $\lambda_{,t}=0$ since the alternative solution $1+8\kappa^2\tilde{\alpha}\psi=0$ is incompatible with the remaining field equations. Thus

$$\lambda=\lambda(r)\ . \tag{4.10}$$

By adding (4.9a) and (4.9c) we obtain the result $\phi_{,r}+\lambda_{,r}=0$ since once again the alternative solution is $1+8\kappa^2\tilde{\alpha}\psi=0$. Therefore

$$\phi(t,r)=-\lambda(r)+f(t)\ , \tag{4.11}$$

where $f$ is arbitrary. Equations (4.9a) and (4.9c) are therefore equivalent and may be written as

$$\frac{d}{dr}\left[r^{d-1}\psi(1+4\kappa^2\tilde{\alpha}\psi)-\frac{GQ^2}{\pi(d-1)}\left[\frac{r}{(d-2)[r^{d-2}+(r^{2d-4}+\beta^2)^{1/2}]}+\int_0^r\frac{dz}{(z^{2d-4}+\beta^2)^{1/2}}\right]\right]=0\ . \tag{4.12}$$

If we make the usual redefinition of the time variable by the replacement $\int e^{f(t)}dt\rightarrow t$, the metric then takes the form

$$ds^2=\Delta\, dt^2-\frac{dr^2}{\Delta}-r^2d\Omega^2_{d-2}\ , \tag{4.13a}$$

where

$$\Delta=1+\frac{r^2}{8\kappa^2\tilde{\alpha}}(1\mp\sqrt{a}\,)\ , \tag{4.13b}$$

$$a(r)\equiv1+16\kappa^2\tilde{\alpha}\left[\frac{2GM+U}{r^{d-1}}\right], \tag{4.13c}$$

and

$$U(r)\equiv\frac{GQ^2}{\pi(d-1)}\left[\frac{r}{(d-2)[r^{d-2}+(r^{2d-4}+\beta^2)^{1/2}]}+\int_0^r\frac{dz}{(z^{2d-4}+\beta^2)^{1/2}}\right]. \tag{4.13d}$$

If $\alpha=0$, then

$$\Delta=1-\frac{2GM}{r^{d-3}}-\frac{GQ^2}{\pi(d-1)r^{d-3}}\left[\frac{r}{(d-2)[r^{d-2}+(r^{2d-4}+\beta^2)^{1/2}]}+\int_0^r\frac{dz}{(z^{2d-4}+\beta^2)^{1/2}}\right]. \tag{4.14}$$

If a $d$-dimensional cosmological constant $\Lambda$ is added to the model the right-hand side (RHS) of Eq. (4.13c) is modified[49] by the addition of the term $32\kappa^2\tilde{\alpha}\Lambda/[(d-1)(d-2)]$ (and the RHS of (4.14) is modified by the addition of the term $-2\Lambda r^2/[(d-1)(d-2)]$).

The spacetimes (4.13) and (4.14) are incomplete and may be extended in the usual fashion. If $Y_{,B}$ is timelike the roles of $t$ and $r$ are interchanged in the above calculation and we obtain the region of the extended spacetime in which $\partial/\partial r$ is a timelike vector. In the case that $Y_{,B}$ is null there exist Robinson-Bertotti-type solutions, i.e., solutions with the topology $(\mathrm{AdS})_2 \times S^{d-2}$. These solutions are derived in the Appendix, both directly from the field equations and also by a limiting procedure from the extreme black holes. We have thus generalized Birkhoff's theorem to the model with Born-Infeld and Gauss-Bonnet terms: the only spherically symmetric solutions are the static solutions (4.13) and (A1) and (A3).

The solutions (4.13) and (4.14) are a straightforward generalization of the four-dimensional solution found by Demianski.[50] If $d=4$ one may perform the integration in (4.14) exactly and express $\Delta$ in terms of elliptic functions, but this is not possible in general. The standard Reissner-Nordström solution[47] is obtained from (4.14) in the limit $1/b \to 0$. Demianski has claimed[50] that (for $d=4$) the solution given by (4.13a) and (4.14) is regular everywhere if $M=0$. This claim is incorrect, however, since near $r=0$ the invariant

$$R_{ABCD}R^{ABCD}=2(d-2)(d-3)\frac{1}{r^4}(1-\Delta)^2+2(d-2)\left[\frac{\Delta'}{r}\right]^2+(\Delta'')^2 \tag{4.15}$$

diverges as $1/r^4$ (or $1/r^{2d-4}$ for arbitrary $d$).

We will now classify the spacetimes (4.13). As in the previous models there are two branches of solutions. The upper branch is asymptotically flat and tends to the solution (4.14) as $r \to \infty$. The solutions corresponding to the lower branch are once again asymptotically anti–de Sitter (de Sitter) if $\alpha>0$ ($\alpha<0$). In some cases there will be branch singularities at $r=r_0$, where

$$a(r_0)\equiv 1+16\kappa^2\tilde{\alpha}\left[\frac{2GM+U(r_0)}{r_0^{d-1}}\right]=0 . \tag{4.16}$$

Horizons are given by solutions of the equation

$$f(r)\equiv 1-\frac{2GM+U(r)}{r^{d-3}}+\frac{4\kappa^2\tilde{\alpha}}{r^2}=0 . \tag{4.17}$$

Many of the features of the analysis of the last section carry over to the present model. For example, $f(r)$ has at most two zeros, and these correspond to horizons of the upper branch (lower branch) if $1+8\kappa^2\tilde{\alpha}/r_{\mathcal{H}}^2>0$ ($1+8\kappa^2\tilde{\alpha}/r_{\mathcal{H}}^2<0$). In the critical case $r_{\mathcal{H}}=r_0=r_{c\pm}$, where

$$r_{c\pm}^{d-3}-4GM-2U(r_{c\pm})=0 \tag{4.18}$$

and $r_{c-}\le r_{c+}$.

The global properties of the spacetimes are in general somewhat different from the corresponding cases in the last section. The distinct cases may be summarized as follows (for $d\ge 6$).

*(1) Asymptotically flat branch.* (a) If $M\ge 0$ and $\alpha\ge 0$ then there is singularity at the origin which is shielded by a regular horizon at $r=r_{\mathcal{H}}$. The spacetime therefore has the global properties of the Schwarzschild solution. (b) If $M<0$ and $\alpha>0$ then there is a timelike branch singularity at $r=r_0$, which is shielded by two horizons if $Q<Q_{\rm ex}$, where $Q_{\rm ex}$ is the critical value such that $\Delta|_{r_{\mathcal{H}}}=(d\Delta/dr)|_{r_{\mathcal{H}}}=0$, i.e.,

$$\tfrac{1}{2}(d-2)(d-3)r_{\rm ex}^2+2(d-2)(d-5)\kappa^2\tilde{\alpha}-8\pi Gb^2r_{\rm ex}^4\left[\frac{(r_{\rm ex}^{2d-4}+\beta^2)^{1/2}}{r_{\rm ex}^{d-2}}-1\right]=0 , \tag{4.19}$$

where $r_{\rm ex}$ is given by (4.17) with $Q=Q_{\rm ex}$. (c) For all values of $M\ge 0$ and $\alpha<0$ there is a branch singularity at $r=r_0$. There will also be (two) branch singularities when $M<0$ and $\alpha<\alpha_0$, where (for fixed $M$, $Q$, and $b^2$) $\alpha_0$ is the critical value such that $a|_{r_0}=(da/dr)|_{r_0}=0$, i.e.,

$$(d-1)(d-2)+64(d-3)(d-4)\kappa^2\alpha_0 b^2\left[\frac{(r_0^{2d-4}+\beta^2)^{1/2}}{r_0^{d-2}}-1\right]=0 , \tag{4.20a}$$

where

$$2GM+U(r_0)-\frac{16\pi Gb^2r_0[(r_0^{2d-4}+\beta^2)^{1/2}-r_0^{d-2}]}{(d-1)(d-2)}=0 . \tag{4.20b}$$

We will take $r_0$ to denote the outer singularity if $M<0$ and $\alpha<\alpha_0$. The singularity at $r_0$ is timelike and naked if $r_{c-}\le r_0\le r_{c+}$, or spacelike and shielded by a horizon if $r_0>r_{c+}$. In the second case the spacetime has the properties of the Schwarzschild solution. (d) If $M<0$ and $\alpha_0\le\alpha<0$ then there is a singularity at the origin, which is shielded by two horizons (or a single degenerate horizon in the extreme case), if (i) (4.17) has solutions $r_\pm$, $0<r_-\le r_+$, (ii) (4.18) has solutions $r_{c\pm}$, $0<r_{c-}<r_{c+}$, and (iii) $r_+<r_{c+}$. The resulting spacetime will therefore have the properties of the Reissner-Nordström or extreme Reissner-Nordström solution. In the cases excluded by (i), (ii), and (iii) the singularity will be naked.

*(2) Asymptotically (anti-)de Sitter branch.* (a) If $\alpha>0$ then there is a naked singularity at the origin. (b) If $M\ge 0$ and $\alpha<0$, or if $M<0$ and $\alpha<\alpha_0$, then there is a branch singularity at $r=r_0$. (In the second case we take $r_0$ to be the outer singularity.) The singularity is timelike and shielded by a horizon if $r_{c-}<r_0<r_{c+}$, or spacelike and naked if $r_0\ge r_{c+}$. (c) If $M<0$ and $\alpha_0\le\alpha<0$ then there is a (spacelike) singularity at the origin. The singularity will be shielded by two horizons (or a single degenerate horizon in the extreme case) if (i) (4.17) has solutions $r_\pm$, $0<r_-\le r_+$, and (ii) (4.18) has no solutions or

TABLE I. Summary of the global properties of the asymptotically flat spacetimes (for $d \geq 6$): (I) Einstein + Gauss-Bonnet, (II) Einstein + Maxwell + Gauss-Bonnet, and (III) Einstein + Born-Infeld + Gauss-Bonnet. The quantities $\alpha_0$, $r_0$, and $r_{c\pm}$ are different for the three models (see text). The solutions in the "Reissner-Nordström" column have the properties of the Reissner-Nordström solution, the extreme Reissner-Nordström solution, or of a naked singularity depending on the relative values of the parameters of the particular model.

| | Naked singularity | Schwarzschild | Reissner-Nordström |
|---|---|---|---|
| (I) | $M>0,\ \alpha \leq \alpha_0$,<br>or $M<0$,<br>or $M=0,\ \alpha>0$ | $M>0,\ \alpha \geq 0$,<br>or $M>0,\ \alpha_0<\alpha<0$,<br>or $M=0,\ \alpha<0$ | |
| (II) | $M \leq 0,\ \alpha>0$,<br>or $M>0,\ \alpha<\alpha_0,\ r_{c-} \leq r_0 \leq r_{c+}$ | $M>0,\ \alpha<\alpha_0,\ r_0>r_{c+}$ | $M_{>}0,\ \alpha \geq 0$<br>or $M>0,\ \alpha \leq \alpha<0$,<br>or $M \leq 0,\ \alpha<0$ |
| (III) | $M \geq 0,\ \alpha<0,\ r_{c-} \leq r_0 \leq r_{c+}$,<br>or $M<0,\ \alpha<\alpha_0,\ r_{c-} \leq r_0 \leq r_{c+}$ | $M \geq 0,\ \alpha \geq 0$,<br>or $M \geq 0,\ \alpha<0,\ r_0>r_{c+}$,<br>or $M<0,\ \alpha<\alpha_0,\ r_0>r_{c+}$ | $M<0,\ \alpha>0$,<br>or $M<0,\ \alpha_0 \leq \alpha<0$ |

(4.18) has solutions $r_{c\pm}$, $0<r_{c-}<r_{c+}$, and $r_+>r_{c+}$. In the cases excluded by (i) and (ii) the singularity will be naked.

The case with $M>0$, $\alpha>0$ and the asymptotically flat branch is presumably of most physical interest. (Boulware and Deser[2] have argued that the asymptotically anti–de Sitter branch is unstable, and this should also be the case here.) The properties of the asymptotically flat branch are summarized in Table I for the three models we have considered.

The most important difference between the models with Born-Infeld and Maxwell terms is that in the Born-Infeld case the spacetime has the global properties of the Schwarzschild solution rather than the Reissner-Nordström solution (if $M>0$ and $\alpha \geq 0$). In the case of the ordinary Reissner-Nordström solution the charge $Q$ is a "central charge" in the sense of supersymmetry and the inequality $Q^2 \leq 2\pi(d-2)(d-3)\kappa^2 M^2$ is a Bogomolny-type inequality which is saturated in the case when the supersymmetric extension of the theory admits a Killing spinor.[51] The same could well be true of the Reissner-Nordström-type solution of the Einstein-Maxwell theory with a Gauss-Bonnet term (if a supersymmetric extension exists). However, for the model with a Born-Infeld term a Schwarzschild-type spacetime is obtained for *all* values of $Q^2$ and $b^2$, so there is no similar Bogomolny-type bound. Consequently one would expect the physics of the two models to be rather different. This is indeed the case, as we shall see in the next section.

## V. BLACK-HOLE THERMODYNAMICS

It is by now well established that there is a close link between event horizons and thermodynamics in conventional black-hole physics. The question of how thermodynamic properties are altered by the presence of string-generated terms is of natural importance. It is not straightforward to derive the "laws of black-hole mechanics," since in the usual case the dominant energy condition[52] is needed in the proofs. This condition is violated by the energy-momentum tensor of the Gauss-Bonnet term. If a full set of laws of black-hole mechanics can be derived for the present theory it would require quite a considerable modification of the usual arguments—perhaps through replacing the dominant energy condition by some weaker condition, such as positivity of energy. For the present models the surface gravity is constant on the event horizon, as a consequence of spherical symmetry, and thus a black-hole temperature can be defined. We will evaluate the temperature of the asymptotically flat solutions of Secs. II–IV, and use the results to give qualitative arguments about the evaporation of the holes through Hawking radiation.[53]

The temperature of the black holes may be determined in each case by noting that if one analytically continues the metric to imaginary time, $t \to i\tau$, then the resulting manifold is regular if $\tau$ is identified with period

$$\beta = 4\pi \left[ \left. \frac{d\Delta}{dr} \right|_{r_H} \right]^{-1} .$$

Since the coordinate $\tau$ is periodic the analytically continued Hartle-Hawking propagator[54] has all the properties

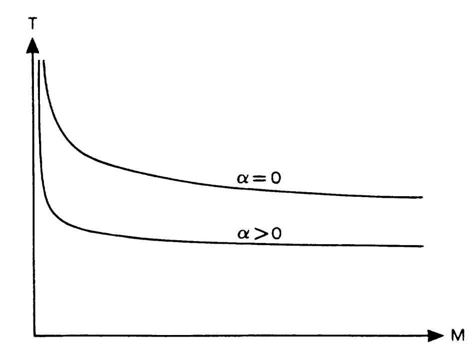


FIG. 1. Temperature of the "Schwarzschild" black hole.

of a thermal Green's function,[55] and thus the black hole has a temperature $T=1/\beta$. In the case of the Schwarzschild-type solution:

$$T=\frac{1}{\beta}=\frac{(d-5)GM+r_{\mathcal{H}}^{d-3}}{2\pi r_{\mathcal{H}}(4GM-r_{\mathcal{H}}^{d-3})}\ , \tag{5.1}$$

where $r_{\mathcal{H}}$ is given by the solution of (2.1). This temperature is always less than the temperature $T_0$ of the $\alpha=0$ solution:

$$T_0=\frac{(d-3)}{4\pi(2GM)^{1/(d-3)}}\ , \tag{5.2}$$

as can be seen from Fig. 1. However, the temperature becomes arbitrarily large as $M\rightarrow 0$, just as in the $\alpha=0$ case. This should be contrasted with the results of Callan, Myers, and Perry[6] and Myers,[7] who determined the temperature for Schwarzschild-type solutions is a model containing the dilaton plus curvature-squared string corrections. In their case the temperature is once again lower than that of the usual Schwarzschild solution. However, instead of increasing indefinitely as the mass decreases, it increases to a maximum and then decreases to zero. It is therefore possible that one could be left with a stable zero-temperature soliton, thereby circumventing one of the problems posed by Hawking radiation in conventional black-hole physics.[6] It would appear that the difference between the models of Refs. 6 and 7 and the one we have considered is essentially due to the dilaton coupling and the long-range scalar forces it introduces.

In the case of the Reissner-Nordström-type solution the temperature is similarly given by

$$T=\frac{(d-5)\kappa^2 Mr_{\mathcal{H}}^{d-3}-(d-4)G\tilde{Q}^2+4\pi r_{\mathcal{H}}^{2(d-3)}}{4\pi r_{\mathcal{H}}(2\kappa^2 Mr_{\mathcal{H}}^{d-3}-G\tilde{Q}^2-2\pi r_{\mathcal{H}}^{2(d-3)})}\ , \tag{5.3}$$

where $r_{\mathcal{H}}$ denotes the outer horizon, as given by (3.3). If $\alpha=0$, then

$$T=\frac{d-3}{2\pi}\left[G^2M^2-\frac{G\tilde{Q}^2}{4\pi}\right]^{1/2}\left[GM+\left[G^2M^2-\frac{G\tilde{Q}^2}{4\pi}\right]^{1/2}\right]^{-(d-2)/(d-3)}\ . \tag{5.4}$$

The isotherms for the $\alpha=0$ solution in the $|\tilde{Q}|/\kappa$ vs $M$ plane are displayed in Fig. 2(a). The pattern is the same as in the four-dimensional case.[56] For fixed $\tilde{Q}$ the temperature is a maximum when $|\tilde{Q}|=\kappa M\sqrt{2d-5}/(d-2)$. This is because the specific heat is negative for $|\tilde{Q}|<\kappa M\sqrt{2d-5}/(d-2)$ but positive for $|\tilde{Q}|>\kappa M\sqrt{2d-5}/(d-2)$. For fixed $\tilde{Q}$ and $M$ the value of $T$ for finite $\alpha$ ($>0$) is lower than in the $\alpha=0$ case. The temperature has the same qualitative features as that of the usual Reissner-Nordström solution, as may be seen by comparing Fig. 2(a) with Fig. 2(b). The curve on which the specific heat changes sign is now a rather complex function of $\alpha$, $Q$, and $M$. As in the $\alpha=0$ case the extreme black holes have zero temperature.

In the case of the model with Gauss-Bonnet and Born-Infeld terms the temperature is given by

$$T=[4\pi r_{\mathcal{H}}(r_{\mathcal{H}}^2+8\kappa^2\tilde{\alpha})]^{-1}\left[(d-3)r_{\mathcal{H}}^2+4(d-5)\kappa^2\tilde{\alpha}-\frac{16\pi Gb^2 r_{\mathcal{H}}^4}{d-2}\left[\frac{(r_{\mathcal{H}}^{2d-4}+\beta^2)^{1/2}}{r_{\mathcal{H}}^{d-2}}-1\right]\right]\ , \tag{5.5}$$

where $r_{\mathcal{H}}$ is a solution of (4.17). The isotherms are sketched in Fig. 3 for $\alpha=0$ and $\alpha>0$. They resemble the isotherms of the Reissner-Nordström-type solution, insofar as for fixed $|\tilde{Q}|$, and decreasing $M$, the temperature increases to a maximum and then decreases. The fact that the specific heat changes sign is not apparent from Fig. 3(a). However, this is merely due to the scale used in the graph—the specific heat changes sign very close to the $M=0$ axis, as can be verified numerically. If $\alpha>0$ then for fixed $Q$ and $b^2$ the specific heat changes sign at a greater value of $M$ than in the $\alpha=0$ case. The curve on which the specific heat changes sign is in general a rather complex function of $M$, $\tilde{Q}$, $\alpha$, and $b^2$. In other respects Figs. 2 and 3 are rather different, however. For $M\geq 0$ and $\alpha\geq 0$ the Born-Infeld model has no configuration corresponding to the extreme black holes, and in fact the $M=0$ solutions with $Q>0$ have a finite temperature. This could have interesting physical consequences in connection with the evaporation of the black holes.

The qualitative evolution of an evaporating black hole in the $|\tilde{Q}|/\kappa$ vs $M$ plot depends largely on whether the elementary fields carry the charge $Q$. If they do then the hole will lose charge through preferential emission of particles with charge of the same sign,[57] provided that $|e\Phi_{\mathcal{H}}|>M$, where $e$ and $m$ are the charge and mass of the elementary field and $\Phi_{\mathcal{H}}$ is the electrostatic potential of the horizon. In such a case the path of an evaporating black hole would be towards the origin of the $\tilde{Q}/\kappa$ vs $M$ plot, at which point the black-hole temperature diverges and strong quantum effects become important.

On the other hand, if the elementary fields do not carry the charge then the path of an evaporating hole will be horizontal in the $\tilde{Q}/\kappa$ vs $M$ plot. This is certainly the case for the ordinary Reissner-Nordström solution in four dimensions, for which the charge $Q$ is a "central charge" in the context of $N=2$ supergravity, i.e., it appears as the commutator of supercharges.[51,58] In that case the end point of thermal evaporation would be the zero-temperature extreme black hole. Similar remarks could well apply to $d$-dimensional Einstein-Maxwell theory with (or without) a Gauss-Bonnet term, since there is an extreme configuration with zero temperature in all cases.

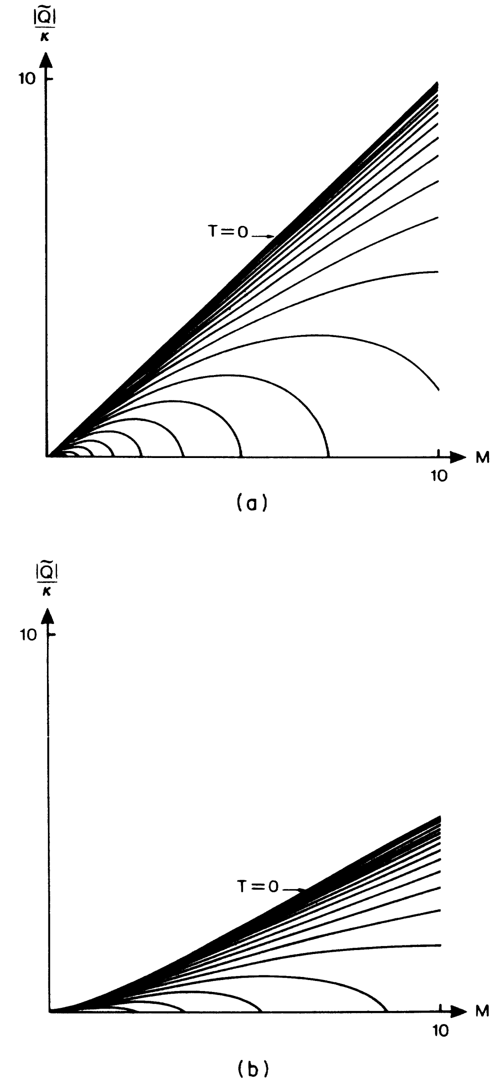


FIG. 2. (a) Isotherms for Einstein-Maxwell model ($\alpha=0$). (b) Isotherms for Einstein-Maxwell + Gauss-Bonnet model ($\alpha>0$).

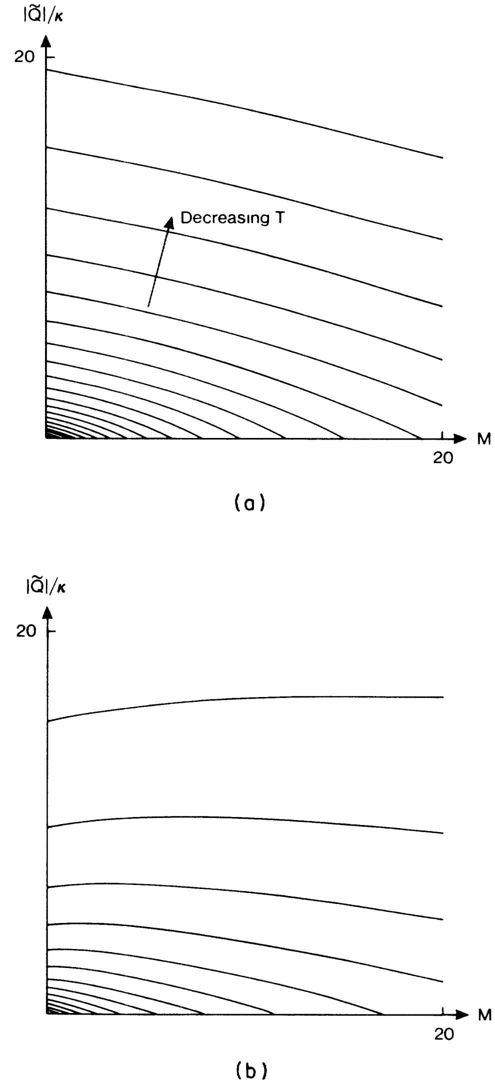


FIG. 3. (a) Isotherms for Einstein + Born-Infeld model ($\alpha=0$). (b) Isotherms for Einstein + Born-Infeld + Gauss-Bonnet model ($\alpha>0$).

In the case of a model with a Gauss-Bonnet term, however, the effects of evaporation through emission of particles which do not carry the charge $Q$—whether or not it was a "central charge" in the sense of supersymmetry—seem to be catastrophic. The holes would appear to evolve to the $M=0$ configuration (with finite $Q$). However, this configuration has a finite temperature and would not be stable. Furthermore, since solutions with regular horizons exist even for $M<0$ if $\alpha\geq 0$, it is conceivable that the mass of the solution could become negative as a result of evaporation. This would indicate that the theory has a rather unusual quantum instability. Clearly one cannot make definite statements without explicit calculations, and this point deserves further investigation.

## VI. CONCLUSION

We have classified the spherically symmetric exact solutions of various string-generated gravity models in $d$ dimensions, and have discussed their thermodynamic properties. These properties are similar to those of corresponding models in four dimensions. The model with a Born-Infeld term seems to offer the unusual possibility that the black-hole mass may become negative as a result of thermal evaporation. The thermodynamic properties of the solutions investigated here differ considerably from the models of Refs. 6 and 7, which included a dilaton field. It is quite possible that addition of the dilaton to the model with the Born-Infeld term could eliminate the possibility of "catastrophic evaporation" discussed above.

One important remaining problem is the investigation of the classical stability of the spacetimes we have classified. Boulware and Deser have argued that the asymptotically anti–de Sitter branch (with $M>0$, $\alpha>0$) is unstable.[2] Ideally this should be checked by performing an analysis of the type used by Regge and Wheeler.[59] In practice such an analysis is hampered by the fact that the perturbation equations are extremely complex: if one considers linearized perturbations of the metric

$$g_{AB} \rightarrow g'_{AB} = g_{AB} + h_{AB}, \quad |h_{AB}| \ll |g_{AB}| , \tag{6.1}$$

then in the case of the model with the Einstein and Gauss-Bonnet terms the equations one must solve are

$$\delta G_{AB} = 2\kappa^2 \alpha \delta T^{gb}_{AB} , \tag{6.2a}$$

where

$$\delta G_{AB} = -\tfrac{1}{2}\Box h_{AB} + R_{(A}{}^{C}h_{B)D} - R_{ACBD}h^{CD} - \tfrac{1}{2}Rh_{AB} + \tfrac{1}{2}g_{AB}R_{CD}h^{CD} + \tfrac{1}{4}g_{AB}\Box h , \tag{6.2b}$$

$$\begin{aligned}\delta T^{gb}_{AB} = {} & 2T^{1}_{(AB)} + 4R_{CADB}(\Box h^{CD} + 2R^{CEDF}h_{EF}) - 4R_{ACDE}R_{BF}{}^{DE}h^{CF} \\ & -2R_{AB}(\Box h + 2R_{CD}h^{CD}) - 2R(\Box h_{AB} - 2R_{(A}{}^{C}h_{B)D} + 2R_{ACBD}h^{CD}) \\ & -2g_{AB}[4R^{CDEF}h_{CF;DE} - R^{CDEF}R^{G}{}_{DEF}h_{CG} - RR^{CD}h_{CD} - \tfrac{1}{2}R\Box h + 2R^{CD}(\Box h_{CD} + 2R_{CDEF}h^{EF})] \\ & -h_{AB}(R_{CDEF}R^{CDEF} - 4R_{CD}R^{CD} + R^2) , \end{aligned} \tag{6.2c}$$

$$T^{1}_{AB} \equiv 2R^{CD}(h_{AB;CD} + h_{CD;AB} - 2h_{CA;BD}) - 4R_{A}{}^{CDE}(h_{DB;EC} + h_{EC;DB}) + 4R_{AC}(\Box h^{C}{}_{B} - R^{CD}h_{BD} + 2R^{C}{}_{DBE}h^{DE}) , \tag{6.2d}$$

$h \equiv h^{A}{}_{A}$, and we have chosen a de Donder gauge, $(h^{AB} - \frac{1}{2}g^{AB}h)_{;B} = 0$. Clearly the use of a powerful computer symbolic manipulation program is essential to solve these equations in a background such as (1.4). Such an investigation is currently is progress.[60]

An alternative approach to the question of stability would be to establish a positive-energy theorem. It seems plausible that the model with Einstein and Gauss-Bonnet terms has a supersymmetric extension[61] (at least for $M > 0$, $\alpha > 0$) and therefore Witten-type arguments might apply. The Born-Infeld theory also has a supersymmetric extension in flat space in four dimensions,[62] but it is not clear whether the combined theory with Einstein, Gauss-Bonnet, and Born-Infeld terms does.

*Note added.* The temperature and entropy of the Schwarzschild-type solution of a model containing dimensionally continued Euler densities of arbitrary order has recently been obtained by Whitt,[63] and independently by Myers and Simon.[64] Further exact solutions of such models have also been derived by Lorentz-Petzold.[65]

## ACKNOWLEDGMENTS

I would like to thank Gary Gibbons for helpful discussions, and Professor Abdus Salam, the International Atomic Energy Agency, and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste, Italy. I would also like to thank Brian Whitt for some helpful comments and for pointing out a mistake in an earlier version of the manuscript. This work was partly supported by the Royal Society.

## APPENDIX: ROBERTSON-BERTOTTI-TYPE SOLUTIONS

In the case $Y^{,B}Y_{,B} = 0$ we may introduce coordinates $u$ and $v$ in which the metric takes the form

$$ds^2 = 2H(u,v)du\,dv - Y(u,v)^2 d\Omega^2_{d-2} . \tag{A1}$$

The condition $Y^{,B}Y_{,B} = 0$ now implies that either $Y_{,u} = 0$ or $Y_{,v} = 0$. We will assume that $Y = Y(u)$. The independent field equations then reduce to

$$\tfrac{1}{2}(d-2)Z + \tfrac{1}{2}(d-2)(d-3)\frac{1}{Y^2} - 16\pi G b^2 \left[\frac{(Y^{2d-4} + \beta^2)^{1/2}}{Y^{d-2}} - 1\right] = -4\kappa^2\tilde{\alpha}(d-2)\frac{1}{Y^2}\left[Z + \tfrac{1}{2}(d-5)\frac{1}{Y^2}\right] , \tag{A2a}$$

$$-\tfrac{1}{2}(d-2)Z = 4\kappa^2\tilde{\alpha}(d-2)\frac{Z}{Y^2} , \tag{A2b}$$

$$\frac{H_{,u}H_{,v}}{H^3} - \frac{H_{,uv}}{H^2} - \tfrac{1}{2}(d-3)(d-4)\frac{1}{Y^2} - 16\pi G b^2\left[\frac{(Y^{2d-4}+\beta^2)^{1/2}}{Y^{d-2}} - 1\right]$$
$$= 4\kappa^2\tilde{\alpha}\frac{1}{Y^2}\left[2\left[\frac{-H_{,u}H_{,v}}{H^3} + \frac{H_{,uv}}{H^2}\right] + \tfrac{1}{2}(d-5)(d-6)\frac{1}{Y^2}\right] , \tag{A2c}$$

where

$$Z \equiv \frac{1}{HY}\left[\frac{H_{,u}Y_{,u}}{H} - Y_{,uu}\right] .$$

One may readily see that there is no spherically symmetric solution with $Y_{,u} \neq 0$. If $Y$ is constant we obtain the solution

$$H=\frac{2}{(u-v)^2}\left[1+\frac{8\kappa^2\tilde{\alpha}}{r_{\mathcal{H}}^2}\right]\left[\frac{(d-3)(d-4)}{2r_{\mathcal{H}}^2}+\frac{2(d-5)(d-6)\kappa^2\tilde{\alpha}}{r_{\mathcal{H}}^4}+8\pi Gb^2\left[1-\frac{r_{\mathcal{H}}^{d-2}}{(r_{\mathcal{H}}^{2d-4}+\beta^2)^{1/2}}\right]\right]^{-1}, \tag{A3a}$$

where $Y^2$ is determined by the equation

$$\tfrac{1}{2}(d-2)(d-3)Y^2+2(d-2)(d-5)\kappa^2\tilde{\alpha}-8\pi Gb^2Y^4\left[\frac{(Y^{2d-4}+\beta^2)^{1/2}}{Y^{d-2}}-1\right]=0\ . \tag{A3b}$$

The structure of the spacetime is therefore the direct product of a two-dimensional anti–de Sitter space with a $(d-2)$-sphere—a type of Robinson-Bertotti solution.

One should note that Eq. (A3b) is the same as (4.19), with $r_{\rm ex}$ replaced by $Y$. This is due to the fact that the solution (A3) can be obtained from (4.13) by a limiting procedure, which is a generalized version of that used by Carter[66] to show that the Robinson-Bertotti solution is equivalent to the extreme Reissner-Nordström solution in the neighborhood of the "throat" at $r=r_{\mathcal{H}}$: suppose one has a metric of the general form

$$ds^2=\Delta\, dt^2-\frac{dr^2}{\Delta}-r^2d\Omega_{d-2}^2\ , \tag{A4}$$

where $\Delta(r)$ has a double zero, such that

$$\Delta(r)=(r-r_{\mathcal{H}})^2P(r)$$

and (A5)

$$P_{\mathcal{H}}\equiv P(r_{\mathcal{H}})=\frac{1}{2}\frac{d^2\Delta}{dr}\bigg|_{r=r_{\mathcal{H}}}>0\ .$$

In the neighborhood of the horizon one can set $r=r_{\mathcal{H}}+v$. One then finds that

$$d\tilde{s}^2\equiv\lim_{v\to0}ds^2=P_{\mathcal{H}}dt^2-\frac{dv^2}{P_{\mathcal{H}}v^2}-r_{\mathcal{H}}^2d\Omega_{d-2}^2\ . \tag{A6}$$

If one transforms to coordinates $u=P_{\mathcal{H}}t+1/v$ and $v=P_{\mathcal{H}}t-1/v$ then (A6) becomes

$$d\tilde{s}^2=\frac{4\,du\,dv}{(u-v)^2P_{\mathcal{H}}}-r_{\mathcal{H}}^2d\Omega_{d-2}^2\ . \tag{A7}$$

One may verify by explicit calculation that $2P_{\mathcal{H}}^{-1}$ is indeed given by (A3a) in the present case.

For the corresponding solutions of the model with a Maxwell term one has[4]

$$P_{\mathcal{H}}^{-1}=\left[1+\frac{8\kappa^2\tilde{\alpha}}{Y^2}\right]\left[\frac{(d-3)^2}{Y^2}+\frac{4\kappa^2\tilde{\alpha}(d-4)(d-5)}{Y^4}\right]^{-1}, \tag{A8a}$$

and $Y^2$ is determined by the equation

$$Y^{2(d-3)}+4\kappa^2\tilde{\alpha}\left[\frac{d-5}{d-3}\right]Y^{2(d-4)}-\frac{GQ^2}{2\pi(d-2)(d-3)}=0\ . \tag{A8b}$$

For the model with a Born-Infeld term the spacetimes (4.13) have the global properties of the Schwarzschild solution for values of the parameters of physical interest, viz., $M\geq0$, $\alpha\geq0$. The Robertson-Bertotti-type solutions derived here correspond to solutions derived by the limiting procedure from extreme black holes with $M<0$ (unlike the solutions for the model with a Maxwell term). Application of the limiting procedure to the spherically symmetric solution of the model with Einstein, Gauss-Bonnet, and cosmological terms yields a generalization of the Nariai solution.[67]

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