

## Probing $\gamma WW$ couplings through $p\bar{p} \rightarrow W\gamma X$ at the Fermilab Tevatron

S.-C. Lee and Wang-Chang Su

*Institute of Physics, Academia Sinica, Taipei, Taiwan 11529, Republic of China*

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We investigated the possibilities of probing the  $\gamma WW$  couplings through  $p\bar{p} \rightarrow W\gamma X$  at the Fermilab Tevatron. Of the seven general form factors  $f_i$ ,  $i=1, \dots, 7$ , only four are independent for an on-shell photon. We may obtain bounds of order 0.5 and 0.1, respectively, on  $f_2$  and  $f_7$  when the integrated luminosity at the Tevatron reaches a few  $\text{pb}^{-1}$ . It is difficult to obtain useful bounds on  $f_6$  and  $\Delta f_3$ , the deviation of  $f_3$  from its standard-model value, even if the polarization of  $W$  or  $\gamma$  can be measured.

The  $W\gamma$  production of  $p\bar{p}$  colliders was first studied by Brown, Sahdev, and Mikaelian<sup>1</sup> and by Mikaelian, Samuel, and Sahdev.<sup>2</sup> They found that, for  $\bar{u}d \rightarrow W^+\gamma$ , the differential cross section predicted by the standard model vanishes exactly at  $\cos\theta = \frac{1}{3}$ , where  $\theta$  is the angle between  $W^+$  and  $u$  in the center-of-mass frame. This prompted several studies of the so-called "radiation zero" by several authors<sup>3</sup> in the following years. An anomalous  $\gamma WW$  coupling will remove this radiation zero. The effect of an anomalous magnetic moment as well as an anomalous electric quadrupole moment on the  $W\gamma$  production cross section was discussed by Brown *et al.*,<sup>3</sup> Cortès, Hagiwara, and Herzog,<sup>4</sup> Robinett,<sup>5</sup> Humpert,<sup>6</sup> Stroughair and Bilchak,<sup>7</sup> and Wallet.<sup>8</sup> In this Rapid Communication, we investigate the possibilities of probing the  $\gamma WW$  couplings through  $W\gamma$  production process at the Fermilab Tevatron, allowing for the most general  $\gamma WW$  couplings consistent with the Lorentz covariance as well as the electromagnetic gauge invariance while neglecting the masses of the quarks and the leptons.

For the general  $\gamma WW$  couplings, we use the convention of Hagiwara, Peccei, Zeppenfeld, and Hikasa.<sup>9</sup> Of the seven form factors  $f_i$ ,  $i=1, \dots, 7$ ,  $f_4$  and  $f_5$  can be expressed in terms of  $f_2$  and  $f_7$ , respectively, by requiring electromagnetic gauge invariance.<sup>9,10</sup> Denoting by  $\Delta f_i$  the deviation of  $f_i$  from its tree-level standard-model value, we have  $\Delta f_1=0$  for on-shell photon since the

charge of the  $W$  is fixed. Hence only  $\Delta f_2$ ,  $\Delta f_3$ ,  $\Delta f_6$ , and  $\Delta f_7$  will be probed through  $p\bar{p} \rightarrow W\gamma X$ .

Let  $\mathcal{M}(k_1\sigma_1, k_2\sigma_2; p\lambda, k_3\lambda_3)$  be the helicity amplitude for the collision of a down-type antiquark of momentum  $k_1$ , helicity  $\sigma_1$  with an up-type quark of momentum  $k_2$ , helicity  $\sigma_2$  into a  $W^+$  of momentum  $p$ , helicity  $\lambda$  and a photon of momentum  $k_3$ , helicity  $\lambda_3$ . The polarization density matrix  $\mathcal{P}$  is defined by

$$\mathcal{P}_{\lambda\lambda_3; \lambda'\lambda'_3} = \sum_{\sigma_1\sigma_2} \mathcal{M}(k_1\sigma_1, k_2\sigma_2; p\lambda, k_3\lambda_3) \times \mathcal{M}(k_1\sigma_1, k_2\sigma_2; p\lambda', k_3\lambda'_3)^* \quad (1)$$

Since the fermions are left handed and massless, the summation contains only one term so that we may write

$$\mathcal{P}_{\lambda\lambda_3; \lambda'\lambda'_3} = \mathcal{F}_{\lambda\lambda_3} \mathcal{F}_{\lambda'\lambda'_3}^*, \quad \mathcal{F}_{\lambda\lambda_3} = \frac{e^2 U_{li}^\dagger}{\sqrt{2x_W}} \mathcal{R}_{\lambda\lambda_3}, \quad (2)$$

where  $x_W = \sin^2\theta_W$ ;  $l, i$ , label up- and down-type fermions with  $U_{li}$  denoting the mixing matrix.  $\mathcal{R}_{\lambda\lambda_3}$  can be obtained from the corresponding quantity  $\hat{\mathcal{R}}_{\lambda\lambda_3}$  for  $W^+ \rightarrow \gamma f f'$ . Explicitly, we have

$$\mathcal{R}_{\lambda\lambda_3} = -\hat{\mathcal{R}}_{\lambda\lambda_3}^* |_{k_3 \rightarrow -k_3}. \quad (3)$$

The convention for the polarization vectors of the vector bosons follows the one in Ref. 11.  $\hat{\mathcal{R}}_{\lambda\lambda_3}$  was given in Ref. 12. We get, for example,

$$\mathcal{R}_{0+} = \frac{\sqrt{1+z}}{\sqrt{2z}} (z+\Delta) \left\{ -2 \left[ \frac{1}{z} + \frac{a_i}{z+\Delta} \right] + \frac{1}{2} \left[ \Delta f_3 - i\Delta f_4 + \left( 1 + \frac{4}{z} \right) \Delta f_5 + i\Delta f_6 \right] \right\}, \quad (4)$$

$$\mathcal{R}_{++} = \frac{1}{z} \sqrt{z^2 - \Delta^2} \left\{ - \left[ \frac{1}{z} + \frac{a_i}{z+\Delta} \right] + \frac{1}{2} \left[ \Delta f_3 + i\Delta f_4 + \left( 3 + \frac{4}{z} \right) \Delta f_5 + i\Delta f_6 \right] \right\}, \quad (5)$$

$$\mathcal{R}_{+-} = \frac{1}{z} (1+z) \sqrt{z^2 - \Delta^2} \left[ \frac{1}{z} + \frac{a_i}{z+\Delta} \right], \quad (6)$$

and the relations

$$\Delta f_4 = -i \frac{1}{2} z \Delta f_2, \quad \Delta f_5 = -iz \Delta f_7. \quad (7)$$

In the above formulas,

$$z = \frac{2p \cdot k_3}{p^2}, \quad \Delta = \frac{2p \cdot (k_2 - k_1)}{p^2}, \quad (8)$$

and  $a_i = 2Q_i$ ,  $a_l = 2Q_l$  with  $Q_i = -\frac{1}{3}$ ,  $Q_l = \frac{2}{3}$  for quarks and  $Q_i = -1$ ,  $Q_l = 0$  for leptons.

Let  $y_1, y_2$  be the rapidities of  $W^+$  and  $\gamma$ , respectively, in the hadron center-of-mass frame, and let  $\hat{s}$  be the invariant mass squared of the  $W\gamma$  pair and  $s$  be the invariant mass squared of  $p\bar{p}$ . We shall write

$$y = \frac{1}{2} (y_1 - y_2), \quad y_+ = \frac{1}{2} (y_1 + y_2) + \Delta_2, \quad (9)$$

where

$$\sinh 2\Delta_s = D \sinh 2y, \quad D = \frac{M_W^2}{\hat{s}}. \quad (10)$$

The differential cross section for  $p\bar{p} \rightarrow W^+ \gamma X$  is given by

$$\frac{d\sigma}{dy d\hat{s}} = \frac{1}{32\pi N_c s} J(\hat{s}, y) \int dy + \sum_{i,l} [f_i^p(x_a) f_l^{\bar{p}}(x_b) \mathcal{P}(\hat{s}, \theta) + f_i^{\bar{p}}(x_a) f_l^p(x_b) \mathcal{P}(\hat{s}, \pi - \theta)], \quad (11)$$

where  $N_c = 3$  is the color factor,

$$J(\hat{s}, y) = \frac{1 - D^2}{4(\cosh 2y + \cosh 2\Delta_s) \cosh 2\Delta_s} \quad (12)$$

is a Jacobian factor, and

$$x_a = \sqrt{\tau} \exp(y_+), \quad x_b = \sqrt{\tau} \exp(-y_+), \quad \tau = \hat{s}/s. \quad (13)$$

In Eq. (11),  $\theta$  is the angle between  $W^+$  and the positive- $z$  axis in the parton center-of-mass frame. The positive- $z$  direction is chosen to be the direction of motion of the proton. The  $x$ - $z$  plane is chosen to be the interaction plane and the positive  $x$  direction is chosen to be the direction of transverse motion of the photon.  $\cos\theta$  can be expressed in terms of  $y$  and  $\hat{s}$  as

$$\cos\theta = \frac{\cosh 2y - \cosh 2\Delta_s}{\beta^2 \sinh 2y}, \quad \beta^2 = 1 - \frac{M_W^2}{\hat{s}}. \quad (14)$$

We omitted the indices  $\lambda\lambda_3, \lambda'\lambda'_3$  on  $d\sigma$  and  $\mathcal{P}$  in Eq. (11).

We shall employ the rapidity cut  $|y_1| \leq Y, |y_2| \leq Y$ . The integration range for  $y_+$  is then given by

$$\max \left[ -\frac{1}{2} \ln \frac{1}{\tau}, |y| - Y + \Delta_2 \right] \leq y_+ \leq \min \left[ \frac{1}{2} \ln \frac{1}{\tau}, Y - |y| + \Delta_s \right]. \quad (15)$$

Each element of the polarization density matrix  $\mathcal{P}_{\lambda\lambda_3, \lambda'\lambda'_3}$  is a quadratic polynomial in  $\Delta f_i$  and so are the various differential cross sections. We consider only  $d\sigma_{\lambda\lambda_3, \lambda'\lambda'_3}/dy$  in this Rapid Communication and compute all the coefficients of the corresponding quadratic polynomials.

In our numerical work, we choose  $Y = 2, \sqrt{s} = 2$  TeV,  $x_W = 0.22, M_Z = 94$  GeV, the fine-structure constant  $\alpha = \frac{1}{128}$ , and the following values for the quark mixing matrix elements:  $U_{ud} = 0.975, U_{us} = 0.222, U_{ub} = 0.00954, U_{cd} = 0.221, U_{cs} = 0.9644, U_{cb} = 0.1455, U_{td} = 0.0231, U_{ts} = -0.144, U_{tb} = 0.9893$ . We use Eichten, Hinchliffe, Lane, and Quigg<sup>13</sup> (EHLQ) for the parton distribution function.  $\Delta f_i$ 's are taken to be constant. Besides the rapidity cut, we also require that the photon energy  $E_\gamma$  in the parton center-of-mass frame be greater than  $E_0$ .  $E_0$  values ranging from 10 GeV to 200 GeV are investigated.

We shall present the results for the sum over all polarization channels of the differential cross section first and comment on the polarization dependence later. The differential cross section can be written as

$$\frac{d\sigma}{dy} = c_0 + c_1(\Delta f_2' + \delta_1) + c_2(\Delta f_7')^2 + c_3(\Delta f_3' + \delta_2)^2 + c_4(\Delta f_6')^2, \quad (16)$$

where

$$\begin{aligned} \Delta f_2' &= \Delta f_2 - \epsilon \Delta f_3, & \Delta f_3' &= \epsilon \Delta f_2 + \Delta f_3, \\ \Delta f_6' &= \Delta f_6 - \epsilon' \Delta f_7, & \Delta f_7' &= \epsilon' \Delta f_6 + \Delta f_7. \end{aligned} \quad (17)$$

$c_i$ 's,  $\delta_1, \delta_2, \epsilon$ , and  $\epsilon'$  are functions of  $y$ . We plot  $c_0, c_1$ , and  $c_2$  for  $y$  between  $-1.0$  and  $1.0$  in Fig. 1. When the cut  $E_0$  is increased from 10 to 50 GeV,  $c_0$  is reduced by one order of magnitude while  $c_1$  and  $c_2$  hardly change. For  $E_0 = 100$  GeV,  $c_0$  is at most of the order of 0.1 pb while  $c_2$  is reduced by 10% or so and  $c_1$  remains almost the same.  $\epsilon$  and  $\epsilon'$  are at most of the order of 0.1 so that we can neglect them.  $c_3$  and  $c_4$  peak at  $y = 0$  when they are 0.95 and 0.5, respectively, for  $E_0 = 10$  GeV.  $|\delta_2| \lesssim 0.5$  over the whole range of  $y$ . Hence we can ignore the last two terms in Eq. (11) for  $\Delta f_i$ 's in the region of interest  $|\Delta f_i| \lesssim 1$ .  $\delta_1$  is also small and is plotted in Fig. 2. Choosing  $E_0 = 50$  GeV and carrying out the integration over the region  $-1.0 \leq y \leq 1.0$ , we find

$$\sigma \approx 1.1 + 32.0(\Delta f_2)^2 + 249.6(\Delta f_7)^2 \quad (18)$$

in units of picobarns. Terms of the order of 1 pb in Eq. (18) have been neglected except the constant term which gives the standard-model prediction. For an integrated luminosity of  $1 \text{ pb}^{-1}$  at the Fermilab Tevatron, we may obtain bounds of order 0.5 and 0.1 on  $\Delta f_2$  and  $\Delta f_7$ , respectively. To avoid the QCD background, one may have to limit oneself to events in which  $W$  decays into leptons. Then we may need a few  $\text{pb}^{-1}$  of integrated luminosity to

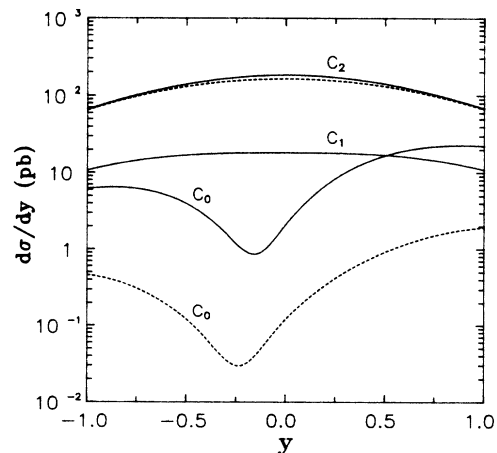


FIG. 1. The coefficients  $c_0, c_1$ , and  $c_2$  in Eq. (16) are plotted against  $y$  in units of picobarns. The solid and dotted lines are for  $E_\gamma$  greater than 10 and 50 GeV, respectively.  $c_1$  and  $c_2$  are insensitive to the  $E_\gamma$  cuts.

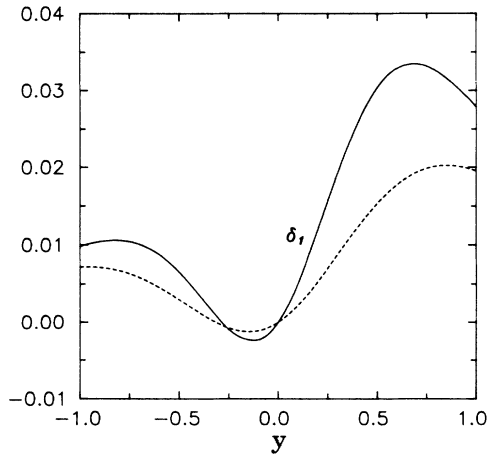


FIG. 2. As in Fig. 1 but for the coefficient  $\delta_1$ .

get the same bounds.

From formulas (4)–(7), one can see that the anomalous contributions to the helicity amplitudes are dominated by the contributions from  $\Delta f_2$  and  $\Delta f_7$  so that it is difficult to get useful information on  $\Delta f_3$  and  $\Delta f_6$  even if the decay angular distribution of  $W$  or the polarization of the photon is measured. This is confirmed by our numerical works. Useful bounds on  $\Delta f_3$  and  $\Delta f_6$  may be obtained from observing radiative decays of polarized  $W$  (Ref. 12).

Upon completion of this work, we received a paper by Baur and Zeppenfeld which discusses the probing of the  $\gamma WW$  vertex at the CERN Large Hadron Collider and the Superconducting Super Collider allowing also for the most general  $\gamma WW$  couplings.<sup>14</sup>

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