Modification of the equivalence theorem due to loop corrections

York- Peng Yao and C.- P. Yuan
Physics Department, University of Michigan, Ann Arbor, Michigan 48109-1120
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We show that, because of mixing between longitudinal vector bosons and pseudo-Goldstone scalars and because of differences in external leg amputations, there are loop corrections to the equivalence theorem, which relates S-matrix elements of $W^{\pm}$ and Z to those of their respective pseudo-Goldstone partners. We derive general compact expressions in the form of multiplicative factors to account for these effects. An explicit one-loop calculation is performed to demonstrate the consistency of our formulation. In the limit when the physical Higgs and/or the top quark become very massive, we obtain a result that there is no m_H^2, $\ln m_H^2$, m_t^2, or $\ln m_t^2$ dependence in these factors if on-shell or finite-momentum subtractions are used for renormalization. However, we have not been able to, nor do we think it likely to, arrive at this conclusion for higher-order loops.

I. GENERAL DISCUSSION

There is considerable interest in the "equivalence theorem,"[1] which states that at high energy S-matrix elements involving longitudinal W and Z processes are equal to those with proper replacements by the corresponding *external* pseudo-Goldstone bosons *in the same theory.* There are physical circumstances which spur such a study. On the one hand, it is generally easier to calculate matrix elements with incoming and outgoing scalar bosons than with vector bosons. More importantly, in the event that one is interested in a strongly interacting Higgs system,[2] one may invoke this theorem and experimentally use longitudinal W and Z to probe the dynamics. Thereupon, it should be of importance to validate this theorem under quantum loop corrections. We shall show that the theorem as stated earlier should be somewhat modified.

We will work with the standard model. To begin with, we note that a simple way to derive the equivalence theorem is via gauge transformation, i.e., through Ward identities. Thus, let the gauge-fixing term be G. It follows easily that[3]

$$\langle G \text{ physical fields} \rangle = 0 \tag{1.1}$$

in the simplest case when we want to establish an equivalence theorem with only one external longitudinal vector boson.

A. $W^{\pm}$ Bosons

For example, the gauge-fixing term for the charged vector bosons may be[4]

$$G^{\pm} = \partial_\mu W^{\pm\mu} + \alpha_W H^{\pm} \phi^{\pm} , \tag{1.2}$$

where α_W is a gauge parameter and $H^{\pm}$ may be some nonlocal functions. Through this, one can relate the longitudinal $W^{\pm}$ to the pseudo-Goldstone boson $\phi^{\pm}$ at high energy. It is immediately obvious that in order to obtain S-matrix elements from Eqs. (1.1) and (1.2), one needs to invert all the external particle legs and project onto the appropriate mass shells. To put it differently, while the Ward identity of Eq. (1.1) deals with Green's functions, the equivalence theorem is a statement of S-matrix elements. To go from one to the other, one needs to take care of the external legs. There are two effects one will encounter along this procedure: the amputated longitudinal W propagators are different from the pseudo-Goldstone scalar propagators, and there is mixing between $W^{\pm}$ and $\phi^{\pm}$ under loop corrections. These are the structural issues we want to resolve in this paper. They are essential in normalizing amplitudes involving longitudinal vector bosons and relating them to amplitudes with pseudo-Goldstone bosons. Needless to say, the true dynamics must be studied on a process-by-process basis.

We can take care of the mixing between $W^{\pm}$ and $\phi^{\pm}$ by choosing $H^{\pm}$ to be some proper nonlocal functions so that mixing in fact does not exist in any loop order.[5] Also, by properly renormalizing α_W, which is tantamount to choosing a proper gauge parameter, Eq. (1.2) holds for renormalized quantities as well. Here, we write

$$\begin{aligned} W^{\pm\mu} &= Z_W^{1/2} (W^{\pm\mu})^{\text{ren}} , \\ \phi^{\pm} &= Z_\phi^{1/2} (\phi^{\pm})^{\text{ren}} , \\ \alpha_W &= Z_W (\alpha_W)^{\text{ren}} , \\ H^{\pm} &= Z_W^{-1/2} Z_\phi^{-1/2} H^{\pm\text{ren}} , \end{aligned} \tag{1.3}$$

where Z_W and Z_ϕ are the wave-function renormalizations for $W^{\pm}$ and $\phi^{\pm}$, respectively. Equation (1.2) then becomes

$$G^{\pm} = Z_W^{1/2} [\partial_\mu (W^{\pm\mu})^{\text{ren}} + \alpha_W^{\text{ren}} H^{\pm\text{ren}} \phi^{\pm\text{ren}}] . \tag{1.4}$$

Now, we decompose the vector propagators into transverse and longitudinal parts

$$\langle T(W^{\pm\mu}(x)W^{\mp\nu}(y))\rangle = \int \frac{d^4p}{(2\pi)^4}\frac{1}{i}e^{ip\cdot(x-y)} \times\left[\left[g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right]\frac{1}{A^\pm(p^2)}+\frac{p^\mu p^\nu}{p^2}\frac{\alpha_W}{B^\pm(p^2)}\right], \quad (1.5)$$

and write the scalar propagators as

$$\langle T(\phi^\pm(x)\phi^\mp(y))\rangle = \int\frac{d^4p}{(2\pi)^4}\frac{1}{i}e^{ip\cdot(x-y)}\frac{1}{C^\pm(p^2)} . \quad (1.6)$$

Then, Eq. (1.1) after amputation becomes

$$ip_\mu\langle (W^{\pm\mu})^{\rm ren}_{|\rm amp}\text{ physical fields}\rangle + \frac{H^{\pm\rm ren}B^{\pm\rm ren}}{C^{\pm\rm ren}}\langle \phi^{\pm\rm ren}_{|\rm amp}\text{ physical fields}\rangle = 0 \quad (1.7)$$

which is the properly modified equivalence theorem under loop corrections. Here

$$B^\pm = B^{\pm\rm ren},\quad (C^\pm)^{-1} = Z_\phi (C^{\pm\rm ren})^{-1} , \quad (1.8)$$

and the subscript |amp means that the indicated external legs have been amputated. At the tree level, $B^{\pm\rm ren}=C^{\pm\rm ren}=p^2+\alpha_W m_W^2$, and $H^{\pm\rm ren}=m_W$. Then Eq. (1.7) is just the usual equivalence theorem. Note that because of Eq. (1.4), the modified identity of Eq. (1.7) is renormalization covariant, in the sense that if we change a renormalization prescription, then Eq. (1.7) will be multiplied by a common wave-function renormalization factor. Obviously, there is no physical consequence. We shall verify explicitly in Secs. II and IV through explicit one-loop calculation that all the renormalized quantities are in fact finite. An important issue which we will investigate is the following: what is the limiting form of the modification factor $H^\pm B^\pm/C^\pm$ when the Higgs self-coupling or the Yukawa coupling for the top quark become strong, i.e., when m_H or $m_t \gg m,p$, where m is any other mass scale in the problem?

In this case, we are basically interested in the modification due to strong interactions. While one may argue that effects of the electroweak interaction may be only a few percent and that one can live without them in the modification factor, the same cannot be said of strong interactions. Here, the potential effects can be of order unity and it is important to account for them. We shall show that to the one-loop level, there is no m_H,$\ln m_H$ or m_t,$\ln m_t$ dependence at any gauge parameters α for the modification factor, if renormalizations are done on shell or at a finite momentum $p \ll m_H$ or m_t (Ref. 6). In this sense, it turns out that the equivalence theorem needs no modification. At this point, we should entertain the possibility that this may well be a one-loop result. We have not been able to put forth a power-counting or a symmetry argument to arrive at such a conclusion for higher-order loops.

Note that Eq. (1.7) is true for off-shell p of the vector-boson momentum as well. This is important when one wants to apply it to, say, a two-W fusion process.

We can simplify Eq. (1.7) somewhat by using a Ward identity. For the unrenormalized Green's functions, we have

$$p_\mu p_\nu\langle T(W^{+\mu}(x)W^{-\nu}(y))\rangle + \alpha_W^2 H^+H^-\langle T(\phi^+(x)\phi^-(y))\rangle = \frac{\alpha_W}{i}\delta(x-y) . \quad (1.9)$$

We write

$$B^\pm = p^2+\alpha_W m_W^2+\pi_L^\pm$$

and

$$C^\pm = p^2+\alpha_W H^+H^- + \tilde\pi^\pm , \quad (1.10)$$

in which we will strictly enforce the tadpole condition $\langle h\rangle=0$, where h is the physical Higgs field. This means $\tilde\pi^\pm(p^2=0)=0$. Equations (1.9) and (1.10) give

$$\frac{B^\pm}{C^\pm}=\frac{p^2}{p^2+\tilde\pi^\pm}$$

and

$$\frac{B^{\pm\rm ren}}{C^{\pm\rm ren}}=\frac{p^2}{p^2+\tilde\pi^\pm}Z_\phi^{-1} \quad (1.11)$$

in which $\pi_L^\pm$, which are the more complicated quantities to calculate, have been eliminated.

B. Z Boson

The treatment of longitudinal Z is complicated by its mixing with the photon field A. Nonetheless, a procedure can be devised to parallel that of the charged-boson case. Here, we shall make a simplifying assumption that Z and A mix with the pseudo-Goldstone boson ϕ_3, but not with the physical Higgs boson h. (This is true to the one-loop order and would be true to all orders if CP invariance were the case.)

Thus, we define a column vector field

$$V_\mu = \begin{bmatrix} Z_\mu \\ A_\mu \end{bmatrix} \quad (1.12)$$

which will be renormalized as

$$V_\mu = Z^{1/2}V_\mu^{\rm ren} , \quad (1.13)$$

where $Z^{1/2}$ is a 2×2 matrix. We now use as our gauge condition

$$G_0 = \partial_\mu V^\mu + \alpha_0 H_0\phi_3 , \quad (1.14)$$

where α_0 is a symmetric 2×2 matrix and

$$H_0 = \begin{bmatrix} H_Z \\ H_A \end{bmatrix} \quad (1.15)$$

is a column vector. ϕ_3 is a Hermitian component of the Higgs doublet

$$\phi = \begin{bmatrix}\phi^0 \\ \phi^-\end{bmatrix} = \begin{bmatrix}\frac{1}{\sqrt2}(v+h+i\phi_3) \\ \phi^-\end{bmatrix} , \quad (1.16)$$

H_Z and H_A are some nonlocal functions chosen so that there will be no mixing between Z and ϕ_3 and A and ϕ_3, respectively. The parameters are renormalized according to (t denotes transposed)

$$\alpha_0 = Z^{1/2}\alpha_0^{\text{ren}}(Z^{1/2})^t = \alpha_0^t \,, \qquad H_0 = (Z^{-1/2})^t H_0^{\text{ren}} Z_{\phi_3}^{-1/2} \,. \tag{1.17}$$

Then, the gauge condition of Eq. (1.14) is multiplicatively renormalized

$$G_0 = Z^{1/2}[\partial_\mu (V^\mu)^{\text{ren}} + \alpha_0^{\text{ren}} H_0^{\text{ren}} \phi_3^{\text{ren}}] \,. \tag{1.18}$$

We shall choose α_0^{ren} such that it is diagonal.

The Ward identity in a simple case with only one G_0 of Eq. (1.14) is

$$\langle \partial_\mu V^\mu \text{ physical fields} \rangle + \alpha_0 H_0 \langle \phi_3 \text{ physical fields} \rangle = 0 \,. \tag{1.19}$$

Now we decompose the matrix vector propagator as

$$\langle T(V_\mu(x) V_\nu^t(y)) \rangle = \int \frac{d^4p}{(2\pi)^4} \frac{1}{i} e^{ip\cdot(x-y)} \times \left[\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right] \frac{1}{T(p^2)} + \frac{p_\mu p_\nu}{p^2} \frac{1}{L(p^2)} \right] \tag{1.20}$$

and the scalar propagator as

$$\langle T(\phi_3(x)\phi_3(y)) \rangle = \int \frac{d^4p}{(2\pi)^4} \frac{1}{i} e^{ip\cdot(x-y)} \frac{1}{C_3(p^2)} \,. \tag{1.21}$$

The equivalence theorem, which is again renormalization covariant, takes the form

$$ip_\mu \langle (V^\mu)^{\text{ren}}_{|\text{amp}} \text{ physical fields} \rangle + \frac{L^{\text{ren}} \alpha_0^{\text{ren}} H_0^{\text{ren}}}{C_3^{\text{ren}}} \langle \phi^{\text{ren}}_{3|\text{amp}} \text{ physical fields} \rangle = 0 \,. \tag{1.22}$$

At this point, one must sort out from our mixed vector propagator the correct combinations to form the physical Z and A. A similar problem of ω,ϕ mixing[7] was treated before and the methodology there can be transferred to our problem. Briefly, we solve for the eigenvector $(\cos\theta_Z, \sin\theta_Z)$ of the matrix

$$\lim_{(p^2+m_Z^2)\to 0} \left[(p^2+m_Z^2) \frac{1}{T^{\text{ren}}(p^2)} \right] ,$$

then the physical Z is

$$(Z^\mu_{\text{physical}})^{\text{ren}} = \cos\theta_Z (Z^\mu)^{\text{ren}} + \sin\theta_Z (A^\mu)^{\text{ren}} \,. \tag{1.23}$$

Similarly, we find the eigenvector $(-\sin\theta_A, \cos\theta_A)$ of the matrix

$$\lim_{p^2\to 0} \left[p^2 \frac{1}{T^{\text{ren}}(p^2)} \right] ,$$

then the physical photon is

$$(A^\mu_{\text{physical}})^{\text{ren}} = -\sin\theta_A (Z^\mu)^{\text{ren}} + \cos\theta_A (A^\mu)^{\text{ren}} \,. \tag{1.24}$$

In Secs. III and IV we shall explicitly carry out a one-loop calculation to verify that the procedure outlined here is correct. In particular, all the renormalized quantities will be shown to be indeed finite. The rotations of Eqs. (1.23) and (1.24) are not necessary to this order.

As with the $W^\pm$ case, we can use a Ward identity

$$\langle T[(\partial_\mu V^\mu + \alpha_0 H_0 \phi_3)(x)(\partial_\nu V^\nu + \alpha_0 H_0 \phi_3)^t(y)] \rangle = 0 \tag{1.25}$$

to obtain

$$\frac{p^2}{L} + \alpha_0 H_0 \otimes H_0^t \alpha_0 \frac{1}{C_3} = \alpha_0 \tag{1.26}$$

which is used to eliminate L. Specifically, if we parametrize

$$C_3 = p^2 + H_0^t \alpha_0 H_0 + \tilde{\pi}_3$$

then

$$\frac{L\alpha_0 H_0}{C_3} = \frac{p^2}{(p^2 + \tilde{\pi}_3 + H_0^t \alpha_0 H_0) I - \alpha_0 H_0 \otimes H_0^t} \,. \tag{1.27}$$

II. ONE-LOOP BOSONIC CORRECTIONS FOR $W^\pm$

In this section, we will give the one-loop corrections for the factor H^+B^+/C^+ in Eq. (1.7). We will separately give the fermion contributions in Sec. IV. First, we will examine the infinities of the counterterms and show that the renormalized quantities are indeed finite. Then, we will present the same factors in the heavy-Higgs-boson limit. Note that the Feynman rules to this order are just those in R_ξ gauges. Because the calculation is a standard one,[8] we do not belabor the details.

We will fix our counterterms by adopting on-shell subtractions, by which we mean that the constants

$$W_\mu^+ = (1 + \tfrac{1}{2}\delta Z_W)(W_\mu^+)^{\text{ren}} \,, \qquad \phi^+ = (1 + \tfrac{1}{2}\delta Z_\phi)(\phi^+)^{\text{ren}} \,, \qquad m_W^2 = (1 + \delta Z_{m_W})(m_W^2)^{\text{ren}} \,, \tag{2.1}$$

are determined from the transverse vector propagator A^+ and the scalar propagator C^+ at $p^2 = -m_W^2$. We write

$$H^+ = m_W + \tilde{m}^+ \,, \tag{2.2}$$

where $\tilde{m}^+$ is chosen to completely absorb the mixing between ϕ^+ and W^+ for an arbitrary external momentum p.

A. The infinite parts

We have

$$\delta Z_W=\frac{g^2}{8\pi^2}\frac{1}{n-4}\left[\frac{-25}{6}+\frac{1}{2}\alpha_W+\frac{c^2}{2}\alpha_Z+\frac{s^2}{2}\alpha_A\right],$$

$$\delta Z_\phi=\frac{g^2}{8\pi^2}\frac{1}{n-4}\left[\frac{-3}{2}-\frac{3}{4c^2}+\frac{1}{2}\alpha_W+\left[-s^2+\frac{1}{4c^2}\right]\alpha_Z+s^2\alpha_A\right],$$

$$\delta Z_{m_W}=\frac{g^2}{8\pi^2}\frac{1}{n-4}\left[\frac{17}{3}-\frac{3}{4c^2}-\frac{1}{2}\alpha_W-\frac{1}{4c^2}\alpha_Z\right],$$

$$\tilde{m}^{\,+}=\frac{g^2}{8\pi^2}\frac{1}{n-4}m_W\left[\frac{3}{4c^2}-\frac{1}{4}\alpha_W+\left[\frac{1}{2}-\frac{3}{4}c^2\right]\alpha_Z-\frac{3}{4}s^2\alpha_A\right], \tag{2.3}$$

$$\tilde{\pi}^{\,+}=\frac{g^2}{8\pi^2}\frac{1}{n-4}p^2\left[\frac{3}{4c^2}+\frac{3}{2}-\frac{1}{2}\alpha_W+\left[s^2-\frac{1}{4c^2}\right]\alpha_Z-s^2\alpha_A\right],$$

$$\pi_L^+=\frac{g^2}{8\pi^2}\frac{1}{n-4}\alpha_W m_W^2\left[\frac{-3}{2}+\frac{3}{4c^2}+\left[\frac{1}{4c^2}-\frac{1}{2}c^2\right]\alpha_Z-\frac{1}{2}s^2\alpha_A\right],$$

where $c=\cos\theta_W$ and $s=\sin\theta_W$. Using these infinite parts, we see that the inverse propagators

$$\begin{aligned}C^{+\text{ren}}&=p^2(1+\delta Z_\phi)+\alpha_W^{\text{ren}}(m_W^{\text{ren}})^2(1+\delta Z_W+\delta Z_{m_W}+\delta Z_\phi)+2\alpha_W^{\text{ren}}m_W^{\text{ren}}\tilde{m}^{\,+}+\tilde{\pi}^{\,+}\,,\\ B^{+\text{ren}}&=p^2+\alpha_W^{\text{ren}}(m_W^{\text{ren}})^2(1+\delta Z_W+\delta Z_{m_W})+\pi_L^+\,,\end{aligned} \tag{2.4}$$

and

$$H^{+\text{ren}}=m_W^{\text{ren}}(1+\tfrac{1}{2}\delta Z_W+\tfrac{1}{2}\delta Z_\phi+\tfrac{1}{2}\delta Z_{m_W})+\tilde{m}^{\,+}$$

are finite.

B. Heavy-Higgs-boson limit

Here, we keep terms proportional to $m_H, \ln m_H^2$, and discard all others. We have expressed the strong coupling $\lambda=g^2m_H^2/4m_W^2$. It is the dependence on it that we are principally interested in. We have

$$\delta Z_W=\frac{g^2}{8\pi^2}\frac{1}{24}\ln m_H^2\,,$$

$$\delta Z_\phi=\frac{g^2}{8\pi^2}\left[-\frac{1}{16}\frac{m_H^2}{m_W^2}-\tfrac{3}{8}\ln m_H^2+\tfrac{1}{4}\alpha_W\ln m_H^2\right],$$

$$\delta Z_{m_W}=\frac{g^2}{8\pi^2}\left[-\frac{1}{16}\frac{m_H^2}{m_W^2}-\tfrac{5}{12}\ln m_H^2\right],$$

$$\tilde{m}^{\,+}=\frac{g^2}{8\pi^2}m_W\left[\frac{1}{16}\frac{m_H^2}{m_W^2}+\tfrac{3}{8}\ln m_H^2-\tfrac{1}{8}\alpha_W\ln m_H^2\right], \tag{2.5}$$

$$\tilde{\pi}^{\,+}=\frac{g^2}{8\pi^2}p^2\left[\frac{1}{16}\frac{m_H^2}{m_W^2}+\tfrac{3}{8}\ln m_H^2-\tfrac{1}{4}\alpha_W\ln m_H^2\right],$$

$$\pi_L^+=\frac{g^2}{8\pi^2}\alpha_W m_W^2\left[\frac{1}{16}\frac{m_H^2}{m_W^2}+\tfrac{3}{8}\ln m_H^2\right].$$

Again, substituting these into Eq. (2.4), we find that

$$\begin{aligned}C^{+\text{ren}}&=[p^2+\alpha_W^{\text{ren}}(m_W^{\text{ren}})^2][1+O(g^2)]\,,\\ B^{+\text{ren}}&=[p^2+\alpha_W^{\text{ren}}(m_W^{\text{ren}})^2][1+O(g^2)]\,,\\ H^{+\text{ren}}&=m_W^{\text{ren}}[1+O(g^2)]\,,\end{aligned} \tag{2.6}$$

and thus

$$\frac{B^{+\text{ren}}H^{+\text{ren}}}{C^{+\text{ren}}}=m_W^{\text{ren}}[1+O(g^2)]\,. \tag{2.7}$$

There is complete cancellation of the heavy mass effects between the counterterms fixed by on-shell subtractions and the π's and $\tilde{m}$ to give rise to this result. One can do subtractions at a finite $p \ll m_H$ and retain this cancellation, but minimal subtraction scheme would spoil it. An interesting question is whether this can hold to all orders. One can easily show that[9] naively the number of powers in external momentum p to which one can expand at N loop before the Higgs-boson effects go away is $2(N+1)$. Therefore, unless there is a dynamical argument, it seems unlikely that these m_H^2 (and $\ln m_H^2$) in the modification factor $B^\pm H^\pm/C^\pm$ can be absorbed by wave-function and physical parameter renormalizations at higher-order loops. A two-loop calculation may provide a partial answer.

Let us digress to elaborate somewhat on minimal subtractions, which correspond to setting $\delta Z_W=\delta Z_\phi=\delta Z_{m_W}=0$ in Eq. (2.5). Then one has

$$\frac{B^{+\text{ren}}H^{+\text{ren}}}{C^{+\text{ren}}}\ \text{(minimal subtraction)}=m_W^{\text{ren}}\left[1+\frac{g^2}{8\pi^2}\frac{1}{8}\alpha_W\ln m_H^2+O(g^2)\right].$$

We see that there is strong Higgs effects $\ln m_H^2$ left behind in the modification factor, except in the Landau gauge $\alpha_W=0$.

III. ONE-LOOP BOSONIC CORRECTIONS FOR Z

We write the wave-function renormalization and the renormalized gauge parameters, respectively, as

$$Z^{1/2}=\begin{bmatrix} 1+\frac{a^{11}}{2} & \frac{a^{12}}{2} \\ \frac{a^{21}}{2} & 1+\frac{a^{22}}{2} \end{bmatrix}, \tag{3.1}$$

$$\alpha_0^{\rm ren}=\begin{bmatrix} \alpha_Z^{\rm ren} & 0 \\ 0 & \alpha_A^{\rm ren} \end{bmatrix}, \tag{3.2}$$

in which a^{ij} are all of one-loop order g^2. Note that $Z^{1/2}$ is not symmetric, i.e., $a^{12}\neq a^{21}$. The bare gauge parameters are found to be

$$\begin{aligned}\alpha_0&=Z^{1/2}\alpha_0^{\rm ren}(Z^{1/2})^t\\ &\equiv\begin{bmatrix} \alpha_Z & \alpha_{ZA} \\ \alpha_{ZA} & \alpha_A \end{bmatrix}\\ &=\begin{bmatrix} \alpha_Z^{\rm ren}(1+a^{11}) & \frac{1}{2}(a^{21}\alpha_Z^{\rm ren}+a^{12}\alpha_A^{\rm ren}) \\ \frac{1}{2}(a^{21}\alpha_Z^{\rm ren}+a^{12}\alpha_A^{\rm ren}) & \alpha_A^{\rm ren}(1+a^{22}) \end{bmatrix}.\end{aligned} \tag{3.3}$$

We express the nonlocal function H_0 as

$$H_0=\begin{bmatrix} m_Z \\ 0 \end{bmatrix}+\tilde m_0=\begin{bmatrix} m_Z \\ 0 \end{bmatrix}+\begin{bmatrix} \tilde m_Z \\ \tilde m_A \end{bmatrix}, \tag{3.4}$$

where $\tilde m_Z$ and $\tilde m_A$ are so chosen as to cancel out the $(Z-\phi_3)$ and the $(A-\phi_3)$ transitions completely. The inverse scalar and longitudinal vector propagators are parametrized as

$$C_3=p^2+\alpha_Z m_Z^2+2\alpha_Z m_Z\tilde m_Z+\tilde\pi_3\,,$$
$$L=\begin{bmatrix} \alpha_Z^{-1}(p^2+\alpha_Z m_Z^2+\pi_L^Z) & \pi_L^{ZA}-p^2\frac{\alpha_{ZA}}{\alpha_Z\alpha_A} \\ \pi_L^{ZA}-p^2\frac{\alpha_{ZA}}{\alpha_Z\alpha_A} & \alpha_A^{-1}(p^2+\pi_L^A) \end{bmatrix}, \tag{3.5}$$

for which the Ward identity of Eq. (1.26) gives

$$p^2\pi_L^Z-p^22\alpha_Z m_Z\tilde m_Z+\alpha_Z m_Z^2\tilde\pi_3=0\,,$$
$$\pi_L^A=0\,,$$

and

$$-\pi_L^{ZA}+m_Z\tilde m_A=0\,. \tag{3.6}$$

The finiteness of the renormalized quantities to be checked are

$$L^{\rm ren}=(Z^{1/2})^tLZ^{1/2}=\begin{bmatrix} (\alpha_Z^{\rm ren})^{-1}[p^2+\alpha_Z^{\rm ren}(m_Z^{\rm ren})^2(1+\alpha^{11}+\delta Z_{m_Z})+\pi_L^Z] & \pi_L^{ZA}+\frac{a^{12}}{2}m_Z^2 \\ \pi_L^{ZA}+\frac{a^{12}}{2}m_Z^2 & (\alpha_A^{\rm ren})^{-1}p^2 \end{bmatrix},$$

$$H_0^{\rm ren}=\begin{bmatrix} \left[1+\frac{a^{11}}{2}+\frac{\delta Z_{\phi_3}}{2}+\frac{\delta Z_{m_Z}}{2}\right]m_Z^{\rm ren}+\tilde m_Z \\ \frac{a^{12}}{2}m_Z+\tilde m_A \end{bmatrix}, \tag{3.7}$$

and

$$C_3^{\rm ren}=p^2(1+\delta Z_{\phi_3})+\alpha_Z^{\rm ren}(m_Z^{\rm ren})^2(1+\delta Z_{\phi_3}+a^{11}+\delta Z_{m_Z})+2\alpha_Z m_Z\tilde m_Z+\tilde\pi_3\,.$$

The product which forms the modification factor in Eq. (1.22) is

$$\frac{L^{\rm ren}\alpha_0^{\rm ren}H_0^{\rm ren}}{C_3^{\rm ren}}=\frac{1}{C_3^{\rm ren}}\begin{bmatrix} [p^2+\alpha_Z^{\rm ren}(m_Z^{\rm ren})^2(1+a^{11}+\delta Z_{m_Z})+\pi_L^Z]\left[\left[1+\frac{a^{11}}{2}+\frac{\delta Z_{\phi_3}}{2}+\frac{\delta Z_{m_Z}}{2}\right]m_Z^{\rm ren}+\tilde m_Z\right] \\ \alpha_Z m_Z\left[\pi_L^{ZA}+\frac{a^{12}}{2}m_Z^2\right]+p^2\left[\frac{a^{12}}{2}m_Z+\tilde m_A\right] \end{bmatrix}. \tag{3.8}$$

For the modified equivalence theorem of Z, we need only the upper component of Eq. (3.8). Also, because the lower component of this equation is already of one-loop order, so is θ_Z, we need not rotate to obtain the equivalence theorem for the physical Z.

As a side remark, we will fix a^{12} so that

$$\left[\pi_L^{ZA}+\frac{a^{12}}{2}m_Z^2\right]_{p^2=0}=0\,. \tag{3.9}$$

Then, it follows from Eq. (1.22) that

$$p_\mu \langle (A^\mu)^{\rm ren}_{|\rm amp} \text{ physical fields} \rangle_{p^2=0} = 0 \; . \qquad (3.10)$$

Here we need a mass-shell requirement for this result to hold for the amputated photon amplitude, in contradistinction to pure electrodynamics.

We now determine the wave-function and mass renormalizations, a^{11} and δZ_{m_Z}, by on-shell subtractions ($p^2=-m_Z^2$) from the transverse vector propagator T^{ZZ}. Similarly, the wave-function renormalization δZ_{ϕ_3} is fixed from the inverse scalar propagator C_3, also at $p^2=-m_Z^2$. The constants a^{12}, a^{21}, and a^{22} are not needed for our present purpose and their values will not be written down; suffice to remark that we have checked their consistency throughout the procedure.

A. Infinite parts

We have

$$
\begin{aligned}
a^{11} &= \frac{g^2}{8\pi^2}\frac{1}{n-4}\left[-\frac{1}{3}-4c^2+\frac{1}{6c^2}+c^2\alpha_W\right] ,\\
\delta Z_{\phi_3} &= \frac{g^2}{8\pi^2}\frac{1}{n-4}\left[-\frac{3}{2}-\frac{3}{4c^2}+\tfrac{1}{2}\alpha_W+\frac{1}{4c^2}\alpha_Z\right] ,\\
\delta Z_{m_Z} &= \frac{g^2}{8\pi^2}\frac{1}{n-4}\left[-\frac{7}{6}+7c^2-\frac{11}{12c^2}-\tfrac{1}{2}\alpha_W-\frac{1}{4c^2}\alpha_Z\right] ,\\
\tilde{m}_Z &= \frac{g^2}{8\pi^2}\frac{1}{n-4}m_Z\left[\frac{3}{2}+\frac{3}{4c^2}-\tfrac{3}{2}c^2-\frac{c^2}{2}\alpha_W\right] ,\\
\tilde{\pi}_3 &= \frac{g^2}{8\pi^2}\frac{1}{n-4}p^2\left[\frac{3}{4c^2}+\tfrac{3}{2}-\tfrac{1}{2}\alpha_W-\frac{1}{4c^2}\alpha_Z\right] ,\\
\pi_L^Z &= \frac{g^2}{8\pi^2}\frac{1}{n-4}\alpha_Z m_Z^2\left[\frac{3}{2}-3c^2+\frac{3}{4c^2}+(\tfrac{1}{2}-c^2)\alpha_W+\frac{1}{4c^2}\alpha_Z\right] .
\end{aligned}
\qquad (3.11)
$$

It follows that the quantities in Eq. (3.7) are finite.

B. The heavy-Higgs-boson limit

Here we have

$$
\begin{aligned}
a^{11} &= \frac{g^2}{8\pi^2}\frac{1}{24c^2}\ln m_H^2 ,\\
\delta Z_{\phi_3} &= \frac{g^2}{8\pi^2}\frac{1}{c^2}\left[-\frac{1}{16}\frac{m_H^2}{m_Z^2}-\tfrac{3}{8}\ln m_H^2+\tfrac{1}{4}\alpha_Z\ln m_H^2\right] ,\\
\delta Z_{m_Z} &= \frac{g^2}{8\pi^2}\frac{1}{c^2}\left[-\frac{1}{16}\frac{m_H^2}{m_Z^2}-\tfrac{5}{12}\ln m_H^2\right] ,\\
\tilde{m}_Z &= \frac{g^2}{8\pi^2}\frac{m_Z}{c^2}\left[\frac{1}{16}\frac{m_H^2}{m_Z^2}+\tfrac{3}{8}\ln m_H^2-\tfrac{1}{8}\alpha_Z\ln m_H^2\right] ,\\
\tilde{\pi}_3 &= \frac{g^2}{8\pi^2}p^2\left[\frac{1}{16}\frac{m_H^2}{m_Z^2}+\frac{3}{8c^2}\ln m_H^2-\frac{1}{4c^2}\alpha_Z\ln m_H^2\right] ,\\
\pi_L^Z &= \frac{g^2}{8\pi^2}\frac{\alpha_Z m_Z^2}{c^2}\left[\frac{1}{16}\frac{m_H^2}{m_Z^2}+\tfrac{3}{8}\ln m_H^2\right] .
\end{aligned}
\qquad (3.12)
$$

Note that if we set $c=1$ and $m_W=m_Z$, the results for Z are identical to those for W in this limit. It follows that

$$
\begin{aligned}
p^2+\alpha^{\rm ren}(m_Z^{\rm ren})^2(1+a^{11}+\delta Z_{m_Z})+\pi_L^Z &= [p^2+\alpha_Z^{\rm ren}(m_Z^{\rm ren})^2][1+O(g^2)] ,\\
C_3^{\rm ren} &= [p^2+\alpha_Z^{\rm ren}(m_Z^{\rm ren})^2][1+O(g^2)] ,
\end{aligned}
$$

and

$$\left[1+\frac{a^{11}}{2}+\frac{\delta Z_{\phi_3}}{2}+\frac{\delta Z_{m_Z}}{2}\right]m_Z^{\rm ren}+\tilde{m}_Z = m_Z^{\rm ren}[1+O(g^2)] \; . \qquad (3.13)$$

Therefore, the upper component of the modification factor in Eq. (3.8), which is pertinent to the longitudinal Z equivalence theorem, is $m_Z^{\rm ren}[1+O(g^2)]$. Again, because of on-shell or finite-momentum subtractions, the theorem is valid without strong Higgs-boson effects, at least to the one-loop order, in any of the R_ξ gauges.

IV. ONE-LOOP FERMIONIC CORRECTIONS

We have deliberately separated out the fermionic corrections, because we want to study the heavy top limit, i.e., the dependence on strong Yukawa couplings.

A. $W^\pm$ Bosons

For W^+, let us denote

$$F=\sum_{\rm families}\left[\frac{1}{2}\frac{m_\nu^2}{m_W^2}+\frac{1}{2}\frac{m_e^2}{m_W^2}+\frac{3}{2}\frac{m_u^2}{m_W^2}+\frac{3}{2}\frac{m_d^2}{m_W^2}\right] , \qquad (4.1)$$

where a color factor of 3 had been inserted for each quark. We have also put in a mass for the neutrinos for convenience. The infinite parts are

$$
\begin{aligned}
\delta Z_{W^+} &= \frac{g^2}{8\pi^2}\frac{1}{n-4}\sum_{\rm families}\tfrac{4}{3} ,\\
\delta Z_{\phi^+} &= \frac{g^2}{8\pi^2}\frac{1}{n-4}F ,\\
\delta Z_{m_W} &= \frac{g^2}{8\pi^2}\frac{1}{n-4}\left[F+\sum_{\rm families}(-\tfrac{4}{3})\right] ,\\
\tilde{m}^{\,+} &= \frac{g^2}{8\pi^2}\frac{1}{n-4}m_W(-F) ,\\
\tilde{\pi}^{\,+} &= \frac{g^2}{8\pi^2}\frac{1}{n-4}p^2(-F) ,\\
\pi_L^+ &= \frac{g^2}{8\pi^2}\frac{1}{n-4}\alpha_W m_W^2(-F) .
\end{aligned}
\qquad (4.2)
$$

We see that the renormalized quantities of Eq. (2.4) are finite, as they should be.

In the heavy top limit, with $\sum_d V_{td}V_{dt}^*$ understood, where V_{ij} are the Kobayashi-Maskawa matrix elements, we have

$$
\begin{aligned}
\delta Z_{W^+} &= \frac{g^2}{8\pi^2}\tfrac{1}{2}\ln m_t^2 \,, \\
\delta Z_{\phi^+} &= \frac{g^2}{8\pi^2}\left[\frac{1}{n-4}\frac{3}{2}\frac{m_t^2}{m_W^2} - \frac{3}{8}\frac{m_t^2}{m_W^2} + \ln m_t^2\left[\frac{3}{4}\frac{m_t^2}{m_W^2} + \frac{3}{4}\frac{m_d^2}{m_W^2}\right]\right] , \\
\delta Z_{m_W} &= \frac{g^2}{8\pi^2}\left[\frac{1}{n-4}\frac{3}{2}\frac{m_t^2}{m_W^2} - \frac{3}{8}\frac{m_t^2}{m_W^2} + \ln m_t^2\left[-\frac{1}{2} + \frac{3}{4}\frac{m_t^2}{m_W^2} + \frac{3}{4}\frac{m_d^2}{m_W^2}\right]\right] , \\
\tilde{m}^{+} &= \frac{g^2}{8\pi^2} m_W\left[\frac{1}{n-4}\left[-\frac{3}{2}\frac{m_t^2}{m_W^2}\right] + \frac{3}{8}\frac{m_t^2}{m_W^2} + \ln m_t^2\left[-\frac{3}{4}\frac{m_t^2}{m_W^2} - \frac{3}{4}\frac{m_d^2}{m_W^2}\right]\right] , \\
\tilde{\pi}^{+} &= \frac{g^2}{8\pi^2} p^2\left[\frac{1}{n-4}\left[-\frac{3}{2}\frac{m_t^2}{m_W^2}\right] + \frac{3}{8}\frac{m_t^2}{m_W^2} + \ln m_t^2\left[-\frac{3}{4}\frac{m_t^2}{m_W^2} - \frac{3}{4}\frac{m_d^2}{m_W^2}\right]\right] , \\
\pi_L^{+} &= \frac{g^2}{8\pi^2}\alpha_W m_W^2\left[\frac{1}{n-4}\left[-\frac{3}{2}\frac{m_t^2}{m_W^2}\right] + \frac{3}{8}\frac{m_t^2}{m_W^2} + \ln m_t^2\left[-\frac{3}{4}\frac{m_t^2}{m_W^2} - \frac{3}{4}\frac{m_d^2}{m_W^2}\right]\right] .
\end{aligned}
\tag{4.3}
$$

When we substitute these into Eq. (2.4), we see that m_t^2 and $\ln m_t^2$ drop out. We thus arrive at the same conclusion as in Eqs. (2.6) and (2.7). In words, the equivalence theorem is true without heavy top effects in the form of m_t^2 or $\ln m_t^2$ to the one-loop order. Just as with the bosonic case, naive power counting does not suggest that this result can be inferred for higher-order loops.[10]

B. Z Boson

For Z, the infinities are

$$
\begin{aligned}
a^{11} &= \frac{g^2}{8\pi^2}\ln m_t^2\left[\frac{-10}{9} + \frac{8c^2}{9} + \frac{17}{36c^2}\right] , \\
Z_{\phi_3} &= \frac{g^2}{8\pi^2}\frac{1}{n-1}F \,, \\
\delta Z_{\phi_3} &= \frac{g^2}{8\pi^2}\left[\frac{1}{n-4}\frac{3}{2c^2}\frac{m_t^2}{m_Z^2} + \ln m_t^2\left[\frac{3}{4c^2}\frac{m_t^2}{m_Z^2}\right]\right] , \\
\tilde{m}_z &= \frac{g^2}{8\pi^2}\frac{1}{n-4} m_Z(-F) \,, \\
\tilde{\pi}_3 &= \frac{g^2}{8\pi^2}\frac{1}{n-4} p^2(-F) \,, \\
\pi_L^Z &= \frac{g^2}{8\pi^2}\frac{1}{n-4}\alpha_Z m_Z^2(-F) \,.
\end{aligned}
\tag{4.4}
$$

They cancel once more when substituted in Eq. (3.7).

In the heavy top limit, we have

$$
\begin{aligned}
a_{11} &= \frac{g^2}{8\pi^2}\ln m_t^2\left[\frac{-10}{9} + \frac{8c^2}{9} + \frac{17}{36c^2}\right] , \\
\delta Z_{\phi_3} &= \frac{g^2}{8\pi^2}\left[\frac{1}{n-4}\frac{3}{2c^2}\frac{m_t^2}{m_Z^2} + \ln m_t^2\left[\frac{3}{4c^2}\frac{m_t^2}{m_Z^2}\right]\right] , \\
\delta Z_{m_Z} &= \frac{g^2}{8\pi^2}\left[\frac{1}{n-4}\frac{3}{2c^2}\frac{m_t^2}{m_Z^2} + \ln m_t^2\left[\frac{3}{4c^2}\frac{m_t^2}{m_Z^2} + \frac{10}{9} - \frac{8c^2}{9} - \frac{17}{36c^2}\right]\right] ,
\end{aligned}
\tag{4.5}
$$

$$
\begin{aligned}
\tilde{m}_z &= \frac{g^2}{8\pi^2} m_z\left[\frac{1}{n-4}\left[\frac{-3}{2c^2}\frac{m_t^2}{m_Z^2}\right] + \ln m_t^2\left[\frac{-3}{4c^2}\frac{m_t^2}{m_Z^2}\right]\right] , \\
\tilde{\pi}_3 &= \frac{g^2}{8\pi^2} p^2\left[\frac{1}{n-4}\left[\frac{-3}{2c^2}\frac{m_t^2}{m_Z^2}\right] + \ln m_t^2\left[\frac{-3}{4c^2}\frac{m_t^2}{m_Z^2}\right]\right] , \\
\pi_L^Z &= \frac{g^2}{8\pi^2}\alpha_Z m_Z^2\left[\frac{1}{n-4}\left[\frac{-3}{2c^2}\frac{m_t^2}{m_Z^2}\right] + \ln m_t^2\left[\frac{-3}{4c^2}\frac{m_t^2}{m_Z^2}\right]\right] .
\end{aligned}
$$

Once again, m_t^2 and $\ln m_t^2$ cancel in the renormalized quantities of Eq. (3.7).

V. CONCLUSION

We have shown that because of loop corrections the equivalence theorem, which relates S-matrix elements involving longitudinal $W^{\pm}$ and Z bosons to those with their corresponding unphysical pseudo-Goldstone scalars, needs some modification from how it was understood. This modification comes in the form of multiplicative factors, one for each external longitudinal vector boson.

We have explicitly shown that the renormalization properties of the gauge conditions used by us to arrive at the equivalence theorem and at the same time to take care of vector scalar mixing are consistent, at least to the one-loop order.

Furthermore, we have obtained a result in the heavy-Higgs-boson and/or heavy top limit that there is no modification in powers of m_H^2, $\ln m_H^2$, m_t^2, or $\ln m_t^2$, again to the one-loop order. However, we have not been able to find an argument to show this decoupling of heavy mass effects for higher-order loops by naive power counting. We consider it an interesting exercise to perform a two-loop calculation to render a partial answer.

We must emphasize that even though the equivalence theorem needs to be appended with these multiplicative modifiers, its usefulness as a calculational tool is by no means tarnished. In the case of strongly interacting

Higgs bosons or tops, where perturbative approach is helpless (unless the modifiers can be shown not to carry large mass effects), one must learn to isolate them so that results from longitudinal vector-boson experiments can be properly normalized and interpreted to unravel the true strong interaction dynamics of the matter-scalar sector.

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[5]We do not imply that one cannot state mathematically the equivalence theorem in a gauge where there is vector-scalar mixing (i.e., $\langle T[A_\mu(x)\phi(y)]\rangle \neq 0$). However, the theorem, which deals with S-matrix elements, does demand external leg amputations. In this sense, it behooves one to work in a basis where so to speak the vector and the scalar fields are "diagonal," i.e., there is no mixing, so that proper legs are more readily identified. The choice of a proper gauge, as we have proposed, is a vehicle to expedite this. If one insists on choosing a gauge in which there is vector-scalar mixing, then one has to specify what one means by the physical longitudinal vector bosons and the pseudo-Goldstone bosons to extract probability amplitudes from the one-particle-irreducible Green's functions. After all is done, we believe that our gauge prescription is simpler in bookkeeping.

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