Fermion mass matrix, horizontal *CP* violation, and natural flavor conservation in electroweak theories with horizontal flavor chirality

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We show that with minimal Higgs systems both the $SU(2)_L \times U(1)_Y \times SU(3)_H^{VL}$ and the $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_H^{VL}$ theories establish the canonical mass matrices for both up- and down-type quarks. Interestingly, the latter, unlike the former, yields spontaneous *CP* violation in the horizontal interactions exclusively with superweak coupling $G_H \approx 10^{-8} G_F$. Furthermore, the latter suggests the top-quark mass in the range $69 \lesssim m_t \lesssim 110$ GeV.

I. INTRODUCTION

An outstanding problem of present-day particle physics is the understanding of the fermion family replication. This is perhaps the origin of the interrelation of the quark masses and the weak-interaction mixing parameters. The solution to the problem does not lie either in the standard $SU(2)_L \times U(1)_Y$ theory[1] or its viable alternative the left-right-symmetric theory[2] based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ but horizontal interactions[3] between the fermion families may provide some clue. In spite of the spectacular success of the standard theory in explaining a large number of experimental data at low energies the recent measurements of the *B*-meson lifetime,[4] the ratios $\Gamma(b \to ue\nu)/\Gamma(b \to ce\nu)$ (Ref. 5), and ϵ'/ϵ (Ref. 6) indicate a possible problem for the standard theory to explain the *CP* violation. This has made the left-right-symmetric theory more attractive with its unified treatment of spontaneous breaking of gauge and discrete symmetries such as parity and charge conjugation. Thus the left-right-symmetric model seems to offer a better chance for spontaneous *CP* violation.[7] However, the natural flavor conservation (NFC) cannot be achieved without the assumption of the *ad hoc* discrete symmetries.[8]

Several horizontal symmetries[9-12] have been incorporated in the standard theory to understand the problem of different generations of fermions and the spontaneous emergence of the Cabibbo-Kobayashi-Maskawa[13] (CKM) structure. It is shown that $SU(2)_H$ can lead[14] to calculable weak mixing angles and *CP* violation but the vectorial SU(3), $SU(3)_H^V$, fails.[15] Recently we discussed[16] the impact of horizontal symmetries on the operator analysis of nucleon decay in the standard and left-right-symmetric theories and showed that it is possible to discriminate among the different horizontal symmetries from a study of the nucleon decay modes. This motivated us to examine the influence of different horizontal symmetries on the fermion mass matrix and *CP* violation within the framework of the standard and left-right-symmetric electroweak theories. The fermion mass matrix is considered more fundamental than the CKM matrix as this should be given in the weak-eigenstate basis in which the gauge theories are formulated.

We have chosen the horizontal group to be $SU(3)_H^{VL}$, which is flavor chiral. Under $SU(3)_H^{VL}$, the left-handed fermions and antifermions transform as triplets, while the right-handed fermions and antifermions transform as antitriplets. A mirror set of fermions,[12] with a helicity-flip coupling, supplementing the basic set of fermions is assumed such that the anomalies of the basic and mirror fermions cancel each other. The mirror fermions couple to the charged gauge bosons generating familiar low-energy weak interactions through $V+A$ rather than $V-A$ projection and do not mix with the basic fermions. We investigate the possibility of having the real CKM matrix with *CP* violation in the horizontal gauge-boson interactions alone. This would relate the smallness of the *CP*-violation parameter to the horizontal coupling. We demonstrate that the Fritzsch-type mass matrices and calculable weak mixing angles are obtained in the standard and left-right-symmetric theories including the flavor chiral horizontal group $SU(3)_H^{VL}$. *CP* violation arises in the $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_H^{VL}$ theory from the horizontal sector alone where as this is not true, in general, for the $SU(2)_L \times U(1)_Y \times SU(3)_H^{VL}$ theory. Our calculation shows that, with $SU(3)_H^{VL}$ as the horizontal group, the left-right-symmetric theory seems to be preferred with *CP* violation exclusively in the horizontal interactions to the standard theory where *CP* violation is not restricted to the horizontal sector without specific ansatz.

The plan of the paper is as follows. We discuss the $SU(2)_L \times U(1)_Y \times SU(3)_H^{VL}$ theory in Sec. II. Section III contains the $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_H^{VL}$ theory. Some concluding remarks are given in Sec. IV.

II. $SU(2)_L \times U(1)_Y \times SU(3)_H^{VL}$ THEORY

We discuss an $SU(2)_L \times U(1)_Y \times SU(3)_H^{VL}$ theory with the left-handed quarks $\Psi_{iL}(2,\frac{1}{3},3) \equiv (u_i, d_i)_L$ and the right-handed quarks $U_{iR}(1,\frac{4}{3},\bar{3})$ and $d_{iR}(1,-\frac{2}{3},\bar{3})$ where the numbers in the parentheses denote the representation content appropriate for the respective quarks. There are three left-handed gauge bosons $W_L^\mu(3,0,1)$ and eight horizontal gauge bosons $H^\mu(1,0,8)$. We choose a minimal set of Higgs systems $\chi(2,1,6)$, $\chi'(2,1,\bar{6})$, and $\eta(1,0,3)$.

While χ and χ' generate the fermion and gauge-boson masses, η contributes to the horizontal gauge-boson mass only. The up-quark mass matrix vanishes in the absence of χ' and down quarks are massless without χ. Obviously, the Higgs field η does not couple to the fermions and, hence, does not contribute to their masses. The following choice of vacuum expectation values (VEV's) for the Higgs fields are made:

$$\langle \chi^{ij} \rangle = \begin{bmatrix} 0 \\ v_{ij} e^{i\phi_{ij}} \end{bmatrix} , \quad \langle \chi'^{i*j*} \rangle = \begin{bmatrix} 0 \\ v'_{ij} e^{i\phi'_{ij}} \end{bmatrix} , \tag{1}$$

and

$$\langle \eta^k \rangle = p_k \exp(i\gamma_k) . \tag{2}$$

The asterisk denotes the conjugate representation and i,j,k are the $SU(3)_H^{VL}$ indices $(i,j,k=1,2,3)$. The most general renormalizable Higgs potential in our model is given by

$$V = V_\chi + V_{\chi'} + V_\eta + V_{\chi\eta} + V_{\chi'\eta} + V_{\chi\chi'} + V_{\chi\chi'\eta} , \tag{3}$$

where

$$V_\chi = -f_1^2(\chi^\dagger\chi) + f_2(\chi^\dagger\chi)^2 , \tag{4}$$

$$V_{\chi'} = -f_3^2(\chi'^\dagger\chi') + f_4(\chi'^\dagger\chi')^2 , \tag{5}$$

$$V_\eta = -f_5^2(\eta^\dagger\eta) + f_6(\eta^\dagger\eta)^2 , \tag{6}$$

$$V_{\chi\eta} = f_7(\chi^\dagger\chi\eta^\dagger\eta) , \tag{7}$$

$$V_{\chi'\eta} = f_8(\chi'^\dagger\chi'\eta^\dagger\eta) , \tag{8}$$

$$V_{\chi\chi'} = f_9(\chi^\dagger\chi\chi'^\dagger\chi') , \tag{9}$$

$$V_{\chi\chi'\eta} = f_{10}(\chi'^\dagger\chi\eta\eta + \eta^\dagger\eta^\dagger\chi^\dagger\chi') . \tag{10}$$

It is to be noted that only gauge-invariant terms are written in Eqs. (4)–(10). The potential in Eq. (3) can be minimized by demanding that the first derivatives vanish and then from the positivity of the second derivatives of Eq. (3) one can show the existence of absolute minima for the following choice of parameters:

$$v_{11}=0, \quad v_{22}=0, \quad v_{13}=0, \quad v_{12}\neq 0, \quad v_{23}\neq 0, \quad v_{33}\neq 0 , \tag{11}$$

$$v'_{11}=0, \quad v'_{22}=0, \quad v'_{13}=0, \quad v'_{12}\neq 0, \quad v'_{23}\neq 0, \quad v'_{33}\neq 0 , \tag{12}$$

$$\phi'_{12}-\phi_{33}=\phi'_{33}-\phi_{12}=\gamma_1+\gamma_2 , \tag{13}$$

$$\phi'_{23}-\phi_{23}=2\gamma_1, \quad \phi'_{12}-\phi_{12}=2\gamma_3 . \tag{14}$$

In our model the basic fermion–Higgs-boson interaction is given by

$$\mathcal{L} = \Gamma\overline{\Psi}^{i*}_{\alpha L} d_R^{j*} \chi_\alpha^{ij} \delta_{i*i}\delta_{j*j} + \Gamma'\overline{\Psi}^{i*}_{\alpha L} u_R^{j*} \tilde{\chi}'^{ij}_\alpha \delta_{i*i}\delta_{j*j} + \text{H.c.} \tag{15}$$

Here α is an $SU(2)_L$ index $(\alpha=1,2)$. The quark–Higgs-boson couplings Γ and Γ' are real or complex numbers independent of $SU(2)_L$ and $SU(3)_H^{VL}$ indices. We assume that *CP* violation arises due to spontaneous symmetry breaking and, hence, Γ and Γ' are real. The canonical form[17] of the quark mass matrices are given by

$$M^u = e^{i\phi'_{33}} \begin{bmatrix} 0 & a' & 0 \\ a' & 0 & b' \\ 0 & b' & c' \end{bmatrix} , \quad M^d = e^{i\phi_{33}} \begin{bmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{bmatrix} , \tag{16}$$

where

$$a=\Gamma v_{12} e^{i(\phi_{12}-\phi_{33})}, \quad b=\Gamma v_{23} e^{i(\phi_{23}-\phi_{33})}, \quad c=\Gamma v_{33} , \tag{17}$$

$$a'=\Gamma' v'_{12} e^{i(\phi'_{12}-\phi'_{33})}, \quad b'=\Gamma' v'_{23} e^{i(\phi'_{23}-\phi'_{33})}, \quad c=\Gamma' v'_{33} . \tag{18}$$

The complex symmetric mass matrices can be diagonalized by the following biunitary transformations:

$$U_L^{u\dagger} M^u U_R^u = \Gamma' U_L^{u\dagger} \langle \tilde{\chi}'^{ij} \rangle U_R^u = D^u , \tag{19}$$

$$U_L^{d\dagger} M^d U_R^d = \Gamma U_L^{d\dagger} \langle \chi^{ij} \rangle U_R^d = D^d , \tag{20}$$

with

$$U_L^u = U_R^{u*} \quad \text{and} \quad U_L^d = U_R^{d*} . \tag{21}$$

D^u and D^d are diagonalized mass matrices for the up and down quarks. U_L^u and U_L^d relate mass eigenstates $P_L \equiv (u,c,t)_L$, $N_L \equiv (d,s,b)_L$ to weak eigenstates P'_L and N'_L as follows:

$$P'_{L,R} = U^u_{L,R} P_{L,R} , \tag{22}$$

$$N'_{L,R} = U^d_{L,R} N_{L,R} , \tag{23}$$

with

$$U^{\text{KM}}_{L,R} = U^{u\dagger}_{L,R} U^d_{L,R} . \tag{24}$$

$U^{\text{KM}}_{L,R}$ refer to the CKM matrix for the left- and right-handed sector. Each element of the matrix is, in general, measurable as $W_{L,R}$ bosons couple to the weak quark current which is given by

$$(J_\mu)_{L,R} = \frac{g}{2\sqrt{2}} (u,c,t)\gamma_\mu(1\mp\gamma_5) U^{\text{KM}}_{L,R} \begin{bmatrix} d \\ s \\ b \end{bmatrix} . \tag{25}$$

In the standard model the right-handed gauge-boson triplet does not exist and right-handed currents too are absent. The unitary matrices can be written as

$$U_L^{u,d} = e^{-i\xi_L^{u,d}} \hat{O}_L^{u,d} , \tag{26}$$

where $e^{-i\xi_L^{u,d}}$ is a diagonal matrix consisting of pure phases with parameters $\xi_1^{u,d}$, $\xi_2^{u,d}$, $\xi_3^{u,d}$, and O is a KM-type real orthogonal matrix. Thus U_L^{KM} can be written as

$$U_L^{\rm KM}=U_L^{u\dagger}U_L^d=(O_L^u)^T e^{i(\xi_L^u-\xi_L^d)}O_L^d \ . \quad (27)$$

Hence, $U_L^{\rm KM}$ is real if $\xi_L^u-\xi_L^d=0$ or π. $U_L^{\rm KM}$ is determined by quark mass eigenvalues and two phases σ and τ which, according to Fritzsch,[18] are

$$\sigma=(\phi'_{12}-\phi_{12})-2(\phi'_{23}-\phi_{23})+(\phi'_{33}-\phi_{33}) \ , \quad (28)$$

$$\tau=(\phi'_{12}-\phi_{12})-(\phi'_{23}-\phi_{23}) \ . \quad (29)$$

Using Eqs. (13), (14), (28), and (29) we find that, in this model $\sigma=2(\gamma_2-\gamma_1)$ and $\tau=2(\gamma_3-\gamma_1)$. Furthermore, with $(\gamma_2-\gamma_1)=\pm\pi/2,0$, one obtains $\sigma=\pi,-\pi,0$ and the phase of the CKM matrix $\delta=0,2\pi,\pi$ where $\delta=\pi-\sigma$. With these values of δ the CKM matrix is real and then *CP* violation may arise in the horizontal interactions if $\sigma\neq\tau$; otherwise *CP* violation vanishes in the theory.[19] However, with $\sigma\neq0,\pm\pi$, *CP* violation occurs in the left-handed as well as horizontal interactions. Thus, with real CKM matrix *CP* violation is possible exclusively in the horizontal interactions provided the ansatz $\sigma\neq\tau$ holds. However, the condition does not follow naturally from the theory and, hence, the standard theory with $SU(3)_H^{VL}$ symmetry is not interesting for the present study.

III. $SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_H^{VL}$ THEORY

In the $SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_H^{VL}$ theory the left-handed $\Psi_{\alpha L}$'s transform as $(2,1,\frac{1}{3},3)$ and the right-handed $\Psi_{\alpha' R}$'s transform as $(1,2,\frac{1}{3},\bar{3})$. The model has three left-handed gauge bosons $W_L^\mu(3,1,0,1)$, three right-handed gauge bosons $W_R^\mu(1,3,0,1)$, and eight horizontal gauge bosons $H^\mu(1,1,0,8)$. We choose a minimal set of Higgs boson with $\omega(2,2,0,6)$, $\rho(1,1,0,3)$, $\Delta_L(3,1,2,1)$, and $\Delta_R(1,3,2,1)$. ω generates masses of the basic fermions and the left- and right-handed and horizontal gauge bosons, ρ gives masses to the horizontal gauge bosons, and Δ_L and Δ_R are introduced to break the left-right symmetry. We assume that $\langle\omega_{\alpha\alpha'}^{ij}\rangle$'s form complex symmetric sextet under $SU(3)_H^{VL}$ and the most general choice of VEV's[20] are

$$\langle\omega^{ij}\rangle=\begin{bmatrix}K_{ij}e^{i\theta_{ij}} & 0\\ 0 & K'_{ij}e^{i\theta'_{ij}}\end{bmatrix} \ , \quad (30)$$

$$\langle\rho^k\rangle=h_k\exp(i\beta_k) \ , \quad (31)$$

$$\langle\Delta_L\rangle=\begin{bmatrix}0 & 0\\ v_L & 0\end{bmatrix}, \quad \langle\Delta_R\rangle=\begin{bmatrix}0 & 0\\ v_R & 0\end{bmatrix} . \quad (32)$$

Here α' is an $SU(2)_R$ index and the other indices are explained before. The most general renormalizable Higgs potential which is invariant under $SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_H^{VL}$ is as follows:

$$V=V_\omega+V_\rho+V_\Delta+V_{\omega\Delta}+V_{\rho\Delta}+V_{\omega\rho} \ , \quad (33)$$

where

$$V_\omega=-\lambda_1^2\,{\rm Tr}(\omega^\dagger\omega)+\lambda_2[{\rm Tr}(\omega^\dagger\omega)]^2+\lambda_3\,{\rm Tr}(\omega^\dagger\omega\omega^\dagger\omega)+\lambda_4\,{\rm Tr}(\omega^\dagger\omega\tilde{\omega}^\dagger\tilde{\omega}) \ , \quad (34)$$

$$V_\rho=-\lambda_5^2(\rho^\dagger\rho)+\lambda_6(\rho^\dagger\rho)^2 \ , \quad (35)$$

$$V_\Delta=-\lambda_7^2({\rm Tr}\Delta_L^\dagger\Delta_L+{\rm Tr}\Delta_R^\dagger\Delta_R)+\lambda_8[({\rm Tr}\Delta_L^\dagger\Delta_L)^2+({\rm Tr}\Delta_R^\dagger\Delta_R)^2]+\lambda_9({\rm Tr}\Delta_L^\dagger\Delta_L)({\rm Tr}\Delta_R^\dagger\Delta_R)$$
$$+\lambda_{10}({\rm Tr}\Delta_L^\dagger\Delta_L\Delta_L^\dagger\Delta_L+{\rm Tr}\Delta_R^\dagger\Delta_R\Delta_R^\dagger\Delta_R) \ , \quad (36)$$

$$V_{\omega\Delta}=\lambda_{11}({\rm Tr}\Delta_L^\dagger\Delta_L+{\rm Tr}\Delta_R^\dagger\Delta_R)\,{\rm Tr}(\omega^\dagger\omega)+\lambda_{12}({\rm Tr}\omega\omega^\dagger\Delta_L^\dagger\Delta_L+{\rm Tr}\omega^\dagger\omega\Delta_R^\dagger\Delta_R)$$
$$+\lambda_{13}({\rm Tr}\tilde{\omega}\tilde{\omega}^\dagger\Delta_L^\dagger\Delta_L+{\rm Tr}\tilde{\omega}^\dagger\tilde{\omega}\Delta_R^\dagger\Delta_R)+\lambda_{14}({\rm Tr}\Delta_L^\dagger\omega\Delta_R\omega^\dagger+{\rm Tr}\omega\Delta_R^\dagger\omega^\dagger\Delta_L) \ , \quad (37)$$

$$V_{\rho\Delta}=\lambda_{15}({\rm Tr}\Delta_L^\dagger\Delta_L+{\rm Tr}\Delta_R^\dagger\Delta_R)(\rho^\dagger\rho) \ , \quad (38)$$

$$V_{\omega\rho}=\lambda_{16}[{\rm Tr}(\omega^\dagger\omega)](\rho^\dagger\rho) \ . \quad (39)$$

Now the minimization of the Higgs potential leads to

$$K_{11}=0,\quad K_{22}=0,\quad K_{33}\neq0,\quad K_{12}\neq0,\quad K_{13}=0,\quad K_{23}\neq0 \ , \quad (40)$$

$$K'_{11}=0,\quad K'_{22}=0,\quad K'_{33}\neq0,\quad K'_{12}\neq0,\quad K'_{13}=0,\quad K'_{23}\neq0 \ , \quad (41)$$

and

$$\theta_{ij}-\theta'_{ij}=0,\pi \ . \quad (42)$$

Interestingly, the phase difference π can be removed by a chiral rotation on all up- or down-type quarks. The most general basic quark-Higgs interaction is given by

$$\mathcal{L}=\Gamma_{LR}\bar{\Psi}_{\alpha L}^{i*}\psi_{\alpha' R}^{j*}\omega_{\alpha\alpha'}^{ij}\delta_{i*i}\delta_{j*j}+{\rm H.c.} \quad (43)$$

The coupling Γ_{LR} is independent of $SU(2)_L$, $SU(2)_R$, and $SU(3)_H^{VL}$ indices and $\Gamma_{LR}=\Gamma_{LR}^\dagger$ implies Γ_{LR} is a real number. In this model we obtain the canonical form of the mass matrices for both up and down quarks which are given by

$$M^u = e^{i\theta_{33}} \begin{pmatrix} 0 & Ae^{i\delta_1} & 0 \\ Ae^{i\delta_1} & 0 & Be^{i\delta_2} \\ 0 & Be^{i\delta_2} & C \end{pmatrix} ,$$
$$M^d = e^{i\theta_{33}} \begin{pmatrix} 0 & A'e^{i\delta_1} & 0 \\ A'e^{i\delta_1} & 0 & B'e^{i\delta_2} \\ 0 & B'e^{i\delta_2} & C' \end{pmatrix} , \tag{44}$$

where

$$A = \Gamma_{LR}K_{12}, \quad B = \Gamma_{LR}K_{23}, \quad C = \Gamma_{LR}K_{33} , \tag{45}$$

$$A' = \Gamma_{LR}K'_{12}, \quad B' = \Gamma_{LR}K'_{23}, \quad C' = \Gamma_{LR}K'_{33} , \tag{46}$$

and

$$\delta_1 = \theta_{12} - \theta_{33}, \quad \delta_2 = \theta_{23} - \theta_{33} . \tag{47}$$

In our model $\tau = 0$ and $\sigma = 0$ as $\theta_{ij} - \theta'_{ij} = 0$ and the parameters S_1, S_2, S_3, and the phase δ of the CKM matrix are

$$\delta = \pi - \sigma = \pi , \tag{48}$$

$$S_1 = \left| \left[\frac{m_d}{m_s} \right]^{1/2} + e^{i\delta} \left[\frac{m_u}{m_c} \right]^{1/2} \right| , \tag{49}$$

$$S_2 = \frac{\left[\frac{m_d}{m_s} \right]^{1/2}}{S_1} \gamma , \tag{50}$$

$$S_3 = \frac{\left[\frac{m_u}{m_c} \right]^{1/2}}{S_1} \gamma , \tag{51}$$

where

$$\gamma = \left| \left[\frac{m_c}{m_t} \right]^{1/2} + \left[\frac{m_s}{m_b} \right]^{1/2} e^{-i(\tau+\delta)} \right| . \tag{52}$$

We have calculated the CKM matrix elements with a different choice of quark masses and find that they are consistent with the present experiments[21] for the following choice of quark masses:

$$m_u \approx 5 \text{ MeV}, \quad m_d \approx 11 \text{ MeV}, \quad m_c \approx 1.4 \text{ GeV} ,$$
$$m_s \approx 150 \text{ MeV}, \quad m_b \approx 4.5 \text{ GeV}, \quad m_t \approx 90 \text{ GeV} . \tag{53}$$

With this choice we obtain $\gamma \approx 0.0579$. The constraints on m_t in our model comes from $|U_{cb}|$ alone as the CP-violating strength parameter in the left-handed sector $\text{Re}\epsilon = 3s_2c_2s_3 \sin\delta$ vanishes with $\delta = \pi$. The range of m_t allowed in our model is 69 GeV $\lesssim m_t \lesssim$ 110 GeV corresponding to $0.04 \le |U_{cb}| \le 0.07$. The fact $\delta = \pi$ implies U_L^{KM} is real and $U_R^{\text{KM}} = U_L^{\text{KM}*}$ is also real. Thus CP violation does not arise in the left- or right-handed sector. With the choice of quark masses in Eq. (53) the CKM matrix elements in our model are given by

$$|U_{ud}| \simeq 0.9775, \quad |U_{us}| \approx 0.211, \quad |U_{ub}| \approx 0.0035 ,$$
$$|U_{cd}| \simeq 0.2104, \quad |U_{cs}| \approx 0.9759, \quad |U_{cb}| \approx 0.0583 ,$$
$$|U_{td}| \approx 0.0157, \quad |U_{ts}| \approx 0.0563, \quad |U_{tb}| \approx 0.9983 .$$

The unitary matrices $U^u_{L,R}$ and $U^d_{L,R}$ can be written as

$$U^{u,d}_{L,R} = e^{-i\xi^{u,d}_{L,R}} \hat{O}^{u,d}_{L,R} . \tag{54}$$

The fact that the CKM matrix is real implies $\xi^u_{iL} = \xi^d_{iL}$ and $\xi^u_{iR} = \xi^d_{iR}$. Furthermore, Eq. (21) implies $\xi^{u,d}_{iL} = -\xi^{u,d}_{iR}$. The mass matrices are given by

$$M^{u,d}_{ij} = e^{-i(\xi^{u,d}_{iL} - \xi^{u,d}_{jR})} m^{u,d}_{ij} , \tag{55}$$

where $m^{u,d} = (O^{u,d}_L) D^{u,d} (O^{u,d}_R)^T$ is a real matrix, each element of which is a linear combination of m_u, m_c, m_t (or m_d, m_s, m_b). The interaction between the quarks and the horizontal gauge bosons H^μ is given in terms of quark-mass eigenstates by

$$\mathcal{L}_H = g_H [\bar{P}^{i*}_{\alpha L} \gamma_\mu (O^u_L)^T (\mathcal{H}^\mu)^{ij*} (O^u_L) P^j_{\alpha L} \delta_{i*i} \delta_{j*j} - (L \leftrightarrow R) + (P \leftrightarrow N)] + \text{H.c.} , \tag{56}$$

where

$$(\mathcal{H}^\mu)^{ij*} = e^{i(\xi_{iL} - \xi_{jL})} \left[\frac{\lambda^a}{2} \right]^{ij*} H^\mu_a ,$$

λ being the generators ($a = 1, \ldots, 8$). The phases $\xi_1 - \xi_2$, $\xi_1 - \xi_3$, and $\xi_2 - \xi_3$ appear in the horizontal interaction of the quarks and gauge bosons and lead to CP nonconservation. It is to be noted that these phases can be expressed in terms of the phases appearing in the VEV's of the Higgs field which gives masses to the quarks. Thus CP symmetry is spontaneously broken and our model predicts CP violation in K^0-$\bar{K}^0$, D^0-$\bar{D}^0$, B^0-$\bar{B}^0$, and T^0-$\bar{T}^0$ transitions. The four-fermion coupling G_H due to horizontal interactions is defined to be $G_H/\sqrt{2} = g_H^2/8M_H^2$. With $G_H \sim 10^{-8} G_F$ the horizontal mass scale is estimated to be $M_H \sim 1-10$ TeV corresponding to $g_H \sim (10^{-3}-10^{-2})\, g_W$.

Finally, we discuss the problem of the natural flavor conservation (NFC) in our model based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_H^{VL}$. In the absence of the horizontal symmetry the simplest way to satisfy NFC[8] is to arrange things so that positively and negatively charged quarks receive contribution to their masses from the VEV's of different neutral Higgs fields. In our model $\langle \omega^{ij}_{11} \rangle$'s generate the up-quark masses and $\langle \omega^{ij}_{22} \rangle$'s contribute to the down-quark masses. The quark–Higgs-boson coupling is a real number and, hence, it is not affected by the biunitary transformation which diagonalizes the mass matrix. Thus NFC is ensured in the quark–Higgs-boson coupling. However, NFC is spoiled by the horizontal symmetry and the flavor-changing neutral currents (FCNC's) arise due to exchange of horizontal gauge bosons. It is to be noted that $G_H \approx 10^{-8} G_F$ and, hence, FCNC's are suppressed by the superweak horizontal interactions, which are responsible for CP violation.

IV. CONCLUSION

We have discussed the standard and the left-right-symmetric electroweak theories including the $SU(3)_H^{VL}$ horizontal symmetry and investigated quark mass matrices and the possibility of *CP* violation due to horizontal interactions alone. We have also obtained estimates of quark masses leading to the phenomenologically viable CKM matrix.

Our study shows that the standard theory with $SU(3)_H^{VL}$ symmetry can lead to phenomenologically viable quark mass matrices if both the Higgs bosons $\chi(2,1,6)$ and $\chi'(2,1,\overline{6})$ are considered. In this theory *CP* violation can occur, in general, in the left-handed as well as horizontal interactions. The theory leads to *CP* violation exclusively in the horizontal interactions with an ansatz, $\sigma \neq \tau$ where σ and τ characterize the Fritzsch-type mass matrix. Furthermore, there exists a possibility of the CKM matrix being real and $\sigma = \tau$ so that *CP* violation is completely absent in the theory.

$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_H^{VL}$ theory with a minimal Higgs system guarantees the canonical mass matrix for both up and down quarks ensuring the calculability of charged current weak mixing angles in terms of quark mass ratios. The model admits spontaneous *CP* violation due to horizontal interactions. NFC is achieved in Higgs-boson coupling but FCNC and lepton nonconserving effects arise due to horizontal gauge bosons. However, these effects are suppressed by the superweak horizontal four-fermion coupling constant $G_H \sim 10^{-8} G_F$. Some suggestions of the model include the mirror fermions with mass $\sim 100-200$ GeV, the horizontal gauge bosons with mass $\sim 1-10$ TeV and the top-quark mass in the range 69 GeV $\lesssim m_t \lesssim$ 110 GeV.

Our study indicates that, of the standard and left-right-symmetric theories with $SU(3)_H^{VL}$ symmetry, the latter, unlike the former leads always to spontaneous *CP* violation exclusively in the horizontal interactions.

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teractions if the condition $\sigma \neq \tau$ holds.

[20]The phenomenology at low energy requires $M^2_{W_L} \ll M^2_{W_R}$ and $\tan\zeta \ll 1$ (where ζ is the W_L-W_R mixing angle) so that W_L-W_R mixing is suppressed. These restrictions are satisfied with appropriate choice of $|K_{ij}|^2$ and $|K'_{ij}|^2$ (see, for example, G. W. Beall, Ph.D. thesis, University of California, Irvine, California). In the present left-right-symmetric model with $SU(3)^{VL}_H$ symmetry these restrictions imply

$$v_R \gg K_{ij}, K'_{ij} \ .$$

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