Natural seesaw mechanism, eV-keV-MeV-type neutrino spectrum, and cosmology

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We show that in left-right-symmetric models one can have $m_{\nu_e}:m_{\nu_\mu}:m_{\nu_\tau}\simeq$eV:keV:MeV without any fine-tuning of parameters. The ν_μ and ν_τ decay via Majoron emission with lifetimes short enough to avoid cosmological constraints. The key ingredient of the model is the breaking of D parity, present in SO(10), at a scale much higher than the scale of $SU(2)_R$ breaking. Using the Fritzsch ansatz for up-quark mass matrices and a diagonal form for the heavy neutrino masses, we predict $m_t<53$ GeV, 2.3 TeV $<m_{W_R}<10$ TeV, and $m_{\nu_e}\geq 0.2$ eV.

INTRODUCTION

There are two ranges for the neutrino masses which are of great phenomenological interest. In one case, the mass of ν_e is in the eV range and is, therefore, accessible to searches involving the tritium beta decay and neutrinoless double-beta decay. The other range of interest is given by $m_{\nu_e}\ll m_{\nu_\mu}\simeq 10^{-2}$ eV which, coupled with appropriate mixings, can provide a solution of the solar-neutrino puzzle via the Mikheyev-Smirnov-Wolfenstein matter oscillation mechanism. This value of m_{ν_e} is of course too small to be of interest to laboratory experiments. In either case, it is plausible to assume that the neutrino mass scales as the square of a charged fermion mass of the corresponding generation.[1,2] If this scaling law is to hold, then, for m_{ν_e} in the eV range, m_{ν_μ} and m_{ν_τ} will be in the keV and MeV ranges, respectively.[1] This kind of spectrum is interesting since it puts all the neutrino masses near their present upper limits. We will call this the eV-keV-MeV spectrum of the neutrinos.

Two questions arise in discussing the theoretical and phenomenological consistency of the eV-keV-MeV spectrum. Firstly, whether theoretical models such as $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ or SO(10) lead to such a spectrum without fine-tuning of parameters. Secondly, since the keV and the MeV masses lie outside the Cowsik-Mclelland bound on stable neutrinos, whether ν_μ and ν_τ decay fast enough to avoid these constraints. The questions are important since the eV-keV-MeV neutrino spectrum implies a W_R in the TeV range. Thus, if cosmology rules out this spectrum, $m_{W_R}\gtrsim 10^7$ GeV (Ref. 3), which is beyond the reach of experiments.

It was shown[4] that if the D-parity symmetry present in SO(10) holds [or parity in $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ models] is broken at a scale much higher than the scale of $SU(2)_R$ breaking, one can naturally obtain an eV-keV-MeV mass spectrum without fine-tuning of parameters. The key point is the following: when the seesaw mechanism is implemented in the left-right or the SO(10) models, one obtains the following neutrino mass terms:

$$(\bar{\nu}_L \;\; \bar{N}_L^c)\begin{bmatrix} m_{\nu\nu} & m_D \\ m_D^T & M \end{bmatrix}\begin{bmatrix} \nu_R^c \\ N_R \end{bmatrix}+\text{H.c.} \tag{1}$$

Here $m_{\nu\nu}$, m_D, and M are matrices in generation space. $M=fv_R$, v_R being the scale of $SU(2)_R\times U(1)_{B-L}$ breaking and f a Yukawa coupling matrix; m_D is the Dirac mass matrix whose elements are comparable to those of the charged fermion mass matrices, and $m_{\nu\nu}=\gamma f\kappa^2/v_R$, where κ is the scale of $SU(2)_L\times U(1)_Y$ breaking and γ is a combination of coupling parameters in the Higgs potential. Concentrating on a single generation for the moment, the diagonalization of the matrix in Eq. (1) gives

$$m_\nu\simeq m_{\nu\nu}-\frac{m_D^2}{fv_R} \tag{2}$$

for the light neutrino mass. The second term is the seesaw contribution whereas the first one is directly induced by the Yukawa couplings. Unless γ is unnaturally small, the first term dominates in Eq. (2). While this may be acceptable in cases where $v_R\gtrsim 10^{12}$ GeV [as in some SO(10) models], this is unacceptable for v_R in the TeV range. In any case, it ruins the seesaw picture which assumes $m_{\nu\nu}=0$. On the other hand, if D-parity symmetry in SO(10) models (or parity in left-right models) is broken at a scale $m_P\gg v_R$, minimization of the Higgs potential leads to[4] $m_{\nu\nu}\simeq\gamma\kappa^2 v_R/m_P^2$. For $m_P\gtrsim 10^5 v_R$, $m_{\nu\nu}\ll m_D^2/M$, restoring the seesaw picture. In particular, one can have M and hence v_R in the TeV region, leading to the eV-keV-MeV neutrino spectrum as well as a low-mass W_R boson. For SO(10) models, there are cosmological reasons[5] for m_P to be bigger than 10^{13} GeV, making the seesaw picture even more accurate.

For this picture to be consistent with cosmology, the neutrinos ν_μ and ν_τ must decay. It is this question that we answer in this paper. One decay mode which has been discussed earlier[6] is the Higgs mediated $\nu_{\mu,\tau}\to 3\nu_e$ mode. In the scheme with high-scale D-parity breaking, the particles mediating such decays become superheavy ($\sim m_P$), implying negligible decay rates in the three-neutrino channel. Alternative decay modes must therefore be sought.

Such an alternative mode is $\nu_{\mu,\tau} \to \nu_e$ + Majoron. It was argued that[7,8] one needs to extend the minimal left-right-symmetric models in order to make room for a global symmetry which, when broken, gives rise to the Majoron J. This was done in Ref. 8. However, the implementation of the seesaw mechanism was unnatural there. Also, the stellar energy loss bounds require the right-handed scale to be about 50 TeV, beyond the reach of the proposed Superconducting Super Collider (SSC).

The purpose of this paper is threefold. First, we include the D-parity breaking into the work of Ref. 8 so that an eV-keV-MeV spectrum arises naturally. Second, we show that the Majoron decay mode in this model is strong enough to avoid cosmological constraints on the masses. These constraints arise from the requirement that the energy of the decay products at the present era, t_0, must be less than the the critical density of the Universe. Assuming the present value of the Hubble constant to be 60 km s^{-1}Mpc^{-1}, this gives[9,7,10]

$$m_\nu (\tau/t_0)^{1/2} < 30 \text{ eV} , \tag{3}$$

where m_ν is the mass of the decaying neutrino and τ is its lifetime. Finally, we discuss how this kind of spectrum is achieved in SO(10) models.

THE LEFT-RIGHT-SYMMETRIC MODEL

We consider the standard left-right symmetric model[1] with left [right] quarks and leptons transforming as doublets of $SU(2)_L$ [$SU(2)_R$]. The new feature of the model is the extended Higgs structure consisting of the multiplets $\phi(2,2,0)$, $\Delta_L(3,1,-2)$, $\Delta_R(1,3,-2)$, $\chi_L(2,1,-1)$, $\chi_R(1,2,-1)$, and a singlet $\eta(1,1,0)$ which is odd under parity. In the Higgs potential, first of all there are terms involving one type of multiplet taken at a time. We assume that the mass terms are all negative so that all the neutral fields have nonzero vacuum expectation values (VEV's). The exact magnitudes of the VEV's depend on the mixed terms as well. Of these, the terms involving ϕ and Δ are given in Ref. 1, and

$$\begin{aligned} V_{\phi\chi} &= \mu \chi_L^\dagger \phi \chi_R + \text{H.c.} \\ V_{\eta\Delta} + V_{\eta\chi} &= -\mu_\Delta \eta (\Delta_L^\dagger \Delta_L - \Delta_R^\dagger \Delta_R) \\ &\quad - \mu_\chi \eta (\chi_L^\dagger \chi_L - \chi_R^\dagger \chi_R) . \end{aligned} \tag{4}$$

The VEV's of different fields are then given by

$$\langle \eta \rangle = m_P, \quad \langle \Delta_R^0 \rangle = v_R, \quad \langle \chi_R^0 \rangle = \lambda_R ,$$

$$\langle \phi \rangle = \begin{bmatrix} \kappa & 0 \\ 0 & \kappa' \end{bmatrix}, \quad \langle \Delta_L^0 \rangle = v_L \simeq \gamma \frac{\kappa^2 v_R}{m_P^2} ,$$

$$\langle \chi_L^0 \rangle = \lambda_L \simeq \gamma \frac{\mu \kappa v_R}{m_P^2} .$$

Next we discuss the global symmetries of the model and their breaking. For $\mu=0$, the model has a global $U(1)_L \times U(1)_R$ symmetry under which $\chi_L \to e^{i\theta_L}\chi_L$ and $\chi_R \to e^{i\theta_R}\chi_R$, all other fields being invariant. For $\mu \neq 0$, these two symmetries combine to one $U(1)_\chi$ symmetry under which $\theta_L = \theta_R$. Since $\mu \to 0$ leads to a bigger symmetry, μ can be taken naturally small.

The $U(1)_\chi$ symmetry gets broken at the scale λ_R. The resulting Goldstone boson J has the following components:

$$J = a_1 \text{Im}\chi_L^0 + a_2 \text{Im}\chi_R^0 + a_3 \text{Im}\Delta_L^0 + a_4 \text{Im}\Delta_R^0 + a_5 \text{Im}\phi_1^0 , \tag{5}$$

where we have assumed, for simplicity, $\kappa'=0$. However, we emphasize later that the final conclusions of the paper are independent of this assumption. Among the coefficients, a_1 and a_2 are unimportant for our subsequent discussion. Putting $\lambda_L, v_L \ll \kappa \ll \lambda_R, v_R$ in the results of Ref. 8 and setting $\lambda_R = v_R$ for simplicity, we get

$$a_3 \simeq -\frac{1}{\sqrt{5}} \frac{v_L}{v_R}, \quad a_4 \simeq -\frac{1}{\sqrt{5}} . \tag{6}$$

The couplings of the Goldstone boson to the charged fermions arise from the ϕ component in it, which is

$$a_5 \simeq \frac{2}{\sqrt{5}} \frac{\lambda_L^2 - v_L^2}{\kappa v_R} \simeq \frac{2}{\sqrt{5}} \gamma^2 \frac{\kappa v_R}{m_P^4} (\mu^2 - \kappa^2) . \tag{7}$$

Even if μ is as large as m_P, this is very small for $m_P \gtrsim 10^5 v_R$ and there is no significant constraint from stellar energy loss rates[11] on the mass of the right-handed W_R boson. Its mass could therefore be in a range accessible to the SSC.

Thus, for a large enough m_P, we can set $\lambda_L = v_L = 0$ for all practical purposes. For one thing, it assures that neglecting κ' was not a bad assumption anyway, since, even if the Majoron contained a component of the ϕ_0', it would have been negligible under the same assumption. In the mass matrix of Eq. (1), we can now put $m_{\nu\nu}=0$, so that the masses of the light neutrinos arise entirely from the seesaw mechanism. Denoting the coupling of the neutrinos to J by

$$(\bar{\nu}_L \quad \bar{N}_L^c) \mathcal{G} J \begin{bmatrix} \nu_R^c \\ N_R \end{bmatrix} ,$$

we obtain

$$\mathcal{G} = \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{a_4}{v_R} M \end{bmatrix} . \tag{8}$$

In order to obtain couplings to the light neutrino eigenstates, we first block diagonalize the mass matrix $\mathcal{M}$ of Eq. (1). In other words, we look for an orthogonal matrix $\mathcal{O}$ such that $\mathcal{O}^T \mathcal{M} \mathcal{O} = \mathcal{D}$, where $\mathcal{D}$ is a matrix where there is no cross term between the light eigenstates and the heavy ones. Then the upper left block of $\mathcal{O}^T \mathcal{G} \mathcal{O}$ gives the coupling of the Goldstone boson to the light neutrinos.

To this end, we introduce the matrix $\rho = m_D M^{-1}$. Since ρ consists of small numbers, we can carry out the diagonalization perturbatively in ρ. To fourth order, we get

$$\mathcal{O}=\begin{bmatrix} 1-\frac{1}{2}\rho\rho^T+A_4 & \rho+C_3 \\ -\rho^T-C_3^T & 1-\frac{1}{2}\rho^T\rho+B_4 \end{bmatrix}, \tag{9}$$

where the terms up to $O(\rho^2)$ were obtained by Kanaya[12] and we get

$$\begin{aligned} A_4&=\tfrac{3}{8}\rho\rho^T\rho\rho^T+\rho M^{-1}\rho^T\rho M\rho^T , \\ B_4&=\tfrac{3}{8}\rho^T\rho\rho^T\rho+\rho^T\rho M\rho^T\rho M^{-1} , \\ C_3&=-\tfrac{1}{2}\rho\rho^T\rho-\rho M\rho^T\rho M^{-1} . \end{aligned} \tag{10}$$

Using this, the mass and the coupling matrices for the light neutrinos can be obtained as

$$m^{\text{light}}=-\rho M\rho^T+\tfrac{1}{2}\delta m \tag{11}$$

and

$$g^{\text{light}}=-\frac{a_4}{v_R}(m^{\text{light}}+\delta m) , \tag{12}$$

where

$$\delta m=\rho\rho^T\rho M\rho^T+\rho M\rho^T\rho\rho^T . \tag{13}$$

We observe that up to $O(\rho^2)$, m^{light} and g^{light} are proportional to each other, leading to no off-diagonal coupling.[13] The mismatch occurs at $O(\rho^4)$, leading to off-diagonal couplings of the light neutrinos at this order. This is responsible for the neutrino decays, whose rates we now estimate.

NEUTRINO DECAY

In order to have a concrete example, we take m_D, defined in Eq. (1), in the Fritzsch form. This form is known to fit well with the observed mixings in the quark sector and can be obtained by imposing discrete horizontal symmetries on the Yukawa couplings. The phenomenological consequences of a neutrino mass matrix of this form have also been discussed by several authors.[14] Inspired by these, we take

$$m_D=\begin{bmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{bmatrix} . \tag{14}$$

At ordinary energies, this is smaller by about a factor of 3 compared to the mass matrix of the up-type quarks in simple SO(10) models. Motivated by that, we identify a, b, and c in terms of the quark masses:

$$a\simeq\tfrac{1}{3}\sqrt{m_u m_c},\quad b\simeq\tfrac{1}{3}\sqrt{m_c m_t},\quad c\simeq\tfrac{1}{3}m_t . \tag{15}$$

For the Majorana masses of the right-handed neutrinos, we take $M=\text{diag}(M_1,M_2,M_2)$. This gives the following mass eigenvalues for the light neutrinos:

$$m_{\nu_e}\simeq\frac{a^4c^2}{b^4M_1},\quad m_{\nu_\mu}\simeq\frac{b^4}{c^2M_2},\quad m_{\nu_\tau}\simeq\frac{c^2}{M_2} . \tag{16}$$

We now find the matrix U such that $U^T(\rho M\rho^T)U$ is diagonal. The off-diagonal elements of $U^T\delta m U$ then give the leading contribution to the off-diagonal Majoron couplings. We obtain

$$g_{\nu_e\nu_\mu J}\simeq\frac{a_4}{v_R}\frac{3ab^2c}{M_2^3},\quad g_{\nu_\mu\nu_\tau J}\simeq\frac{a_4}{v_R}\frac{bc^3}{M_2^3} . \tag{17}$$

Taking $M_2=f_2v_R$ and using Eqs. (6) and (15), the bound on ν_μ lifetime in Eq. (3) translates to the correlated bound:

$$M_2^7<7f_2^2m_t^4\times10^{19}\ \text{GeV}^3 . \tag{18}$$

However, using Eq. (16) and imposing the experimental limit[15] of $m_{\nu_\tau}<50$ MeV, we obtain the additional bound:

$$m_t^2<0.45\ \text{GeV}M_2 . \tag{19}$$

In Fig. 1 we summarize the bounds mentioned in Eqs. (18) and (19) for various values of f_2. Using $m_t>40$ GeV, as indicated by experiments,[16] we see that we need $f_2\geq\frac{1}{4}$ to satisfy both the constraints. Since $f_2\leq1$ in the perturbative region and $m_{W_R}=gM_2/f_2$, Fig. 1 gives the limits $m_t\leq53$ GeV, 2.3 TeV $\leq m_{W_R}\leq10$ TeV. Using Eq. (16), we now conclude that $m_{\nu_e}\geq0.2$ eV.

Similar analysis can be carried out for the decay of ν_τ as well. Using Eq. (17), we obtain

$$\tau(\nu_\tau\to\nu_\mu+J)\simeq2\times10^3\ \text{sec}f_2^7\left[\frac{v_R}{10\ \text{TeV}}\right]^9\left[\frac{50\ \text{GeV}}{m_t}\right]^9 . \tag{20}$$

If $m_{\nu_\tau}>1$ MeV, ν_τ can also decay into $\nu_e e^+e^-$ with strength $G_F m_u^{1/2}m_c m_t^{3/2}/v_R^3$, leading to the partial lifetime

$$\tau(\nu_\tau\to\nu_e e^+e^-)\simeq6\times10^6\ \text{sec}(10\ \text{MeV}/m_{\nu_\tau})^5 .$$

This clearly is much slower than the Majoron decay mode. Thus, within the range of allowed parameters, ν_τ decay is consistent with cosmological bounds from mass density as well as nucleosynthesis.[17] It is also consistent

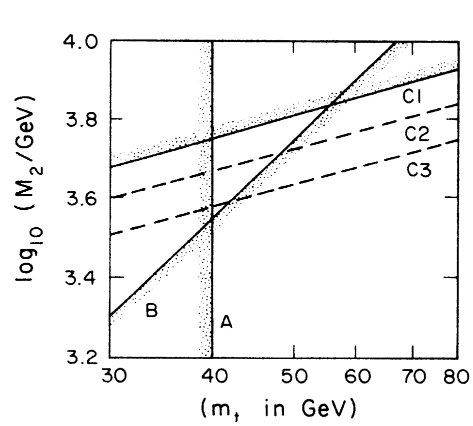

FIG. 1. The allowed region in m_t vs M_2 plane for the model discussed in the text. The line A denotes the lower bound on m_t discussed in Ref. 16. The line B is from Eq. (19). The regions above the lines $C1$, $C2$, and $C3$ are excluded by Eq. (18) for $f_2=1$, $\frac{1}{2}$, and $\frac{1}{4}$, respectively.

with the recent bounds[18] from SN1987A because those bounds apply only when the principal decay mode for ν_τ is $e^+e^-\nu_e$, which is not the case in our model.

We now address a few related questions. Firstly, we look at flavor changing processes involving the charged leptons, e.g., processes such as $\mu \rightarrow e + J$. There is a tree diagram for this process, but the tree-level coupling to the Majoron is suppressed by the ϕ component of the Majoron, as discussed in the context of Eq. (7). The leading contribution to the process, rather, comes from the one-loop diagram of Fig. 2. However, notice that there must be an overall helicity flip in that diagram. But only the left-handed components of the leptons couple to the W. Hence the diagram must be proportional to the charged-lepton masses. Moreover, in the inner legs, we encounter the same problem with the helicity flip, so that we obtain a proportionality to the neutrino masses as well. Thus, the dominant contribution to the effective coupling must be given by

$$g_{\bar{e}\mu J} \simeq \frac{1}{16\pi^2} g^2 g_{\nu_e \nu_\mu J} \frac{m_\mu m_{\nu_\mu}}{m_W^2} , \qquad (21)$$

g being the gauge coupling constant. This is of order $10^{-11} g_{\nu_e \nu_\mu J}$ or smaller, and the process is therefore negligible. Conclusions regarding similar processes involving the τ are also similar.

Secondly, if one assumes that the galaxies are formed from gravitational density perturbations, the decay products have to be nonrelativistic at the era of galaxy formation. The resulting bounds are more stringent than Eq. (3) and cannot be satisfied with the mass matrices of the present section. So, in order for this scenario to work, the galaxies will have to be formed by some explosive mechanism.

Thirdly, although we used a simplified form for the heavy neutrino mass matrix, our bounds cannot be significantly altered in the general case because of the sensitive dependence on the neutrino lifetimes on v_R.

EXTENSION TO SO(10) MODEL

We can easily extend this model to the case of SO(10) grand-unified model. The fermions in this case transform as 16-dimensional spinors of the gauge group and the Higgs multiplets are chosen to be $H_{1,2}\{10\}$, $\chi\{16\}$, $\Delta\{126\}$, $\Sigma\{210\}$. We choose the $U(1)_\chi$ symmetry in this case so that $\chi \rightarrow \chi e^{i\theta}$ whereas all other fields are invariant. Thus, it forbids the couplings $\chi\chi H$ and $\chi\chi\Delta$ but allows the $\chi^\dagger \Sigma \chi$ couplings. As was shown in Ref. 19, Σ has a D-parity odd component, which enables us to have a low-mass W_R.[20]

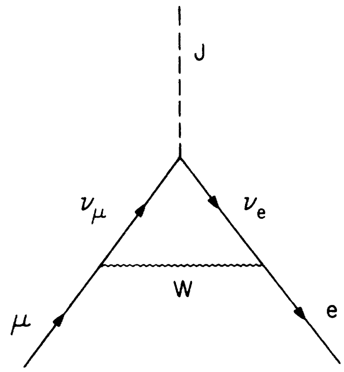

FIG. 2. The dominant diagram for the decay $\mu \rightarrow e + J$.

CONCLUSION

We have shown that an eV-keV-MeV mass spectrum[21] predicted by left-right models with the seesaw mechanism is fully consistent with cosmological constraints without any fine-tuning of parameters for m_{W_R} in the TeV range. This is a way out of the lower bound on m_{W_R} discussed by Harari and Nir[3] of 5×10^3 TeV. In view of the tremendous interest in a low-mass W_R in the TeV range (from theoretical considerations of CP violation as well as experimental considerations concerning possible detection at the SSC), we believe that our work should be of interest since it discusses how constraints on neutrino masses in a low-mass W_R model can be satisfied in a model without arbitrary fine-tuning of parameters.

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