Multiplicity distributions and Bose-Einstein correlations in high-energy multiparticle production in the presence of squeezed coherent states

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We generalize existing quantum-statistical approaches to multiparticle production processes in high-energy physics by studying the effects of squeezed states on second-order Bose-Einstein correlations and multiplicity distributions. The most important and surprising result is the appearance of oscillations in the multiplicity distributions. These oscillations as well as related effects of antibunching and enhanced bunching are investigated for various mixtures of squeezed coherent and chaotic distributions. The experimental situation is briefly discussed.

I. INTRODUCTION

Lack of fundamental understanding of the dressing mechanism of quarks into hadrons limits the theory of multiparticle production to phenomenology. Even if a QCD-based "dressing theory" is developed in the future, it will be practically difficult to apply it in processes where large numbers of particles are produced without invoking statistical methods. Furthermore, in many particle-production processes one deals with identical particles and this implies certain symmetry properties which are taken into account automatically by quantum statistics (QS). For these reasons quantum-statistical models have been extensively used in the study of multiparticle production (cf., e.g., Ref. 1).

There are two main directions into which the application of the methods of QS in particle physics have evolved: (1) determination of radii, lifetimes, and amount of coherence of sources through Bose-Einstein correlations; (2) study of multiplicity distributions $P(n)$ in analogy to photon counting in quantum optics and interpretation of these distributions in terms of hadronic sources and fields.

So far most of these applications were limited to what one could call[2] "standard quantum statistics," i.e., to states which can be constructed from standard coherent states $|\alpha\rangle$ defined by the relation

$$a\,|\alpha\rangle=\alpha\,|\alpha\rangle\ , \tag{1}$$

where a is the annihilation operator.

The density operator can be written in the so-called P representation as

$$\rho=\int \mathcal{P}(\alpha)\,|\{\alpha\}\rangle\langle\{\alpha\}\rangle d^2\alpha\ , \tag{2}$$

where $\mathcal{P}(\alpha)$ characterizes the distribution. Thus for a coherent-state distribution one has

$$\mathcal{P}(\alpha)=\delta(\alpha-\tilde{\alpha})\ , \tag{3}$$

leading to a second-order correlation function[3]

$$g^{(2)}\equiv\frac{\langle n(n-1)\rangle}{\langle n\rangle^2}=1\ , \tag{4}$$

while for a chaotic distribution

$$\mathcal{P}(\alpha)=\exp(-|\alpha|^2/\langle n\rangle)/(\pi\langle n\rangle)\ , \tag{5}$$

one has

$$g^{(2)}=2\ . \tag{4$'$}$$

The corresponding multiplicity distributions are (for a coherent state) the Poisson distribution

$$P(n)=e^{-\langle n\rangle}\frac{\langle n\rangle^n}{n!} \tag{6}$$

and (for a chaotic distribution with k cells) the negative binomial

$$P(n)=\frac{(n+k-1)!}{n!(k-1)!}\frac{\left[\dfrac{\langle n\rangle}{k}\right]^n}{\left[1+\dfrac{\langle n\rangle}{k}\right]^{n+k}}\ . \tag{7}$$

As long as one limits oneself in Eq. (2) to positive-definite functions $\mathcal{P}(\alpha)$, one can show that $1\le g^{(2)}\le 2$ and that correspondingly (6) and (7) are the narrowest and widest multiplicity distributions. The condition that $\mathcal{P}(\alpha)$ be positive definite can also be used to define "standard" QS.

On the other hand, it is by now well known that the standard coherent (Glauber) states are only a special case of a much wider class of coherent states called *squeezed* states,[4] which have recently attracted much attention because of their fundamental quantum-mechanical importance,[5,6] as well as the possible important practical applications they might lead to. The natural question is what, if any, are the modifications one would expect in the applications of this more general QS formalism to particle physics and in particular to the effects (1) and (2) mentioned above.

This question is not only of methodological interest, but has its own physical motivation. To see this it is enough to recall that while standard coherent states arise from one-particle annihilation operators, squeezed states arise from two- or more-particle annihilation operators. They appear naturally in the study of quadratic (or

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higher-order, nonlinear) Hamiltonians

$$H=\omega a^{\dagger}a+\sigma a^{\dagger 2}+\sigma^{*}a^{2} \tag{8}$$

which describe, e.g., superfluidity. (The vacuum of a superfluid is a squeezed state.) Given the important role played by condensates and by models inspired from superfluidity and superconductivity in the attempts to understand quark-gluon confinement,[7] the investigation of the implications of squeezed states for high-energy physics appears desirable. Moreover, as will be shown below, these new, more general, coherent states lead, among other things, to peculiar oscillations in the multiplicity distributions not found in standard QS.

So far no experimental evidence for this last effect has been reported either in the quantum-optical literature or in other fields, although other effects of squeezed states have been reported in quantum optics recently. The main purpose of this paper is to point out that such oscillations could in principle be detected in multiparticle production processes where hints of similar effects have been seen. We will also show that the existence of squeezed states opens up completely new possibilities with regard to the applications (1) and (2) of QS in particle physics, mentioned above.

In Sec. II we introduce the formalism of squeezed states and give the expressions for the correlation $g^{(2)}$ and the multiplicity distribution and exemplify the oscillating behavior of $P(n)$. We also mention the more general heuristic modifications introduced by these new states.

In Sec. III three different mixtures of squeezed and chaotic distributions are considered, and the corresponding formulas for $g^{(2)}$ and $P(n)$ are derived.

Section IV contains the conclusions of the paper and discusses some experimental implications.

II. SQUEEZED COHERENT STATES

Let us consider a representation of SU(1,1) realized with the unitary operators

$$\begin{aligned} U_2(r,\theta,\lambda)&=\exp[-\tfrac{1}{4}re^{-i\theta}(a^{\dagger})^2+\tfrac{1}{4}re^{i\theta}a^2] \\ &\quad\times\exp(i\lambda a^{\dagger}a)\ , \\ U_2^{\dagger}U_2&=1,\quad [a,a^{\dagger}]=1\ , \end{aligned} \tag{9}$$

where r,θ,λ are real parameters, $r\geq 0$. The .squeezed coherent states[8] $|A;r\theta\lambda\rangle$ are defined as

$$|A,r\theta\lambda\rangle=U_2(r,\theta,\lambda)|A\rangle=U_2(r,\theta,\lambda)U_1(A)|0\rangle\ , \tag{10}$$

$$U_1(A)=\exp(Aa^{\dagger}-A^{*}a)\ . \tag{11}$$

From Eqs. (10) and (11) one gets

$$\begin{aligned} &U_2aU_2^{\dagger}=\mu a+\nu a^{\dagger}=b,\quad U_2a^{\dagger}U_2^{\dagger}=\nu^{*}a+\mu^{*}a^{\dagger}=b^{\dagger}\ , \\ &\mu=e^{-i\lambda}\cosh(\tfrac{1}{2}r),\quad \nu=e^{-i(\lambda+\theta)}\sinh(\tfrac{1}{2}r)\ , \\ &|\mu|^2-|\nu|^2=1\ . \end{aligned} \tag{12}$$

The operators $b,b^{\dagger}$ obey the boson commutation relations $[b,b^{\dagger}]=1$ and the transformation (12) is a Bogoliubov transformation. From Eq. (12) and the fact that U_2 is unitary one can prove for any function $f(a,a^{\dagger})$ the relations

$$U_2f(a,a^{\dagger})U_2^{\dagger}=f(b,b^{\dagger}),\quad U_2f(a,a^{\dagger})=f(b,b^{\dagger})U_2\ . \tag{13}$$

Equation (13) implies that $U_2a=bU_2$ and hence the $|A;r\theta\lambda\rangle$ are eigenstates of the destruction operator b:

$$b\,|A,r\theta\lambda\rangle=bU_2|A\rangle=U_2a|A\rangle=A|A;r\theta\lambda\rangle\ . \tag{14}$$

Note also that

$$\begin{aligned} |A;r\theta\lambda\rangle&=U_2\exp(Aa^{\dagger}-A^{*}a)|0\rangle \\ &=\exp(Ab^{\dagger}-A^{*}b)U_2|0\rangle \\ &=\exp(Ab^{\dagger}-A^{*}b)|0;r\theta\lambda\rangle\ . \end{aligned} \tag{15}$$

From (12), (14), and (15) we see that $|A;r\theta\lambda\rangle$ may be viewed as ordinary coherent states with respect to the operators $b,b^{\dagger}$, and as two-particle coherent states with respect to the operators $a,a^{\dagger}$.

The correlation $g^{(2)}$ defined by Eq. (4) can also be written[8]

$$\begin{aligned} g^{(2)}&\equiv\frac{\langle(a^{\dagger})^2a^2\rangle}{\langle a^{\dagger}a\rangle^2} \\ &=1+\frac{|A_1\mu-A_1^{*}\nu|^2-|A_1|^2+|\nu|^2+2|\nu|^4}{(|A_1|^2+|\nu|^2)^2}\ , \\ A_1&=\mu^{*}A-\nu A^{*}\ , \end{aligned} \tag{16}$$

where

$$\langle a^{\dagger}a\rangle\equiv\langle A;r\theta\lambda|a^{\dagger}a|A;r\theta\lambda\rangle=|A_1|^2+|\nu|^2\ .$$

We consider here the particular case in which $\lambda=\theta=0$ and A is a real positive number. Equation (16) simplifies now into

$$g^{(2)}=1+\frac{1}{\langle n\rangle}(e^{-r}-1)+\frac{1}{\langle n\rangle^2}(1+\sinh r)\sinh^2(\tfrac{1}{2}r)\ , \tag{17}$$

with

$$\langle n\rangle=A^2(\cosh\tfrac{1}{2}r-\sinh\tfrac{1}{2}r)^2+\sinh^2\tfrac{1}{2}r\ . \tag{18}$$

We see that in this particular case there are two parameters, A and r, which characterize a squeezed state. (In the following we shall use instead of A the mean multiplicity $\langle n\rangle$.) This situation is to be compared with the case of ordinary coherent states, where one parameter ($\langle n\rangle$) suffices to characterize the multiplicity distribution [cf. Eq. (6)].

In Fig. 1 we plot $g^{(2)}$ from Eq. (17) as a function of the squeezing parameter r for $\langle n\rangle=5$. We see that $g^{(2)}$ can take values less than 1 (antibunching), between 1 and 2 (bunching), or larger than 2 (enhanced bunching). Thus squeezed states generalize the "classical" distributions for which $1\leq g^{(2)}\leq 2$. Correspondingly, narrower distribu-

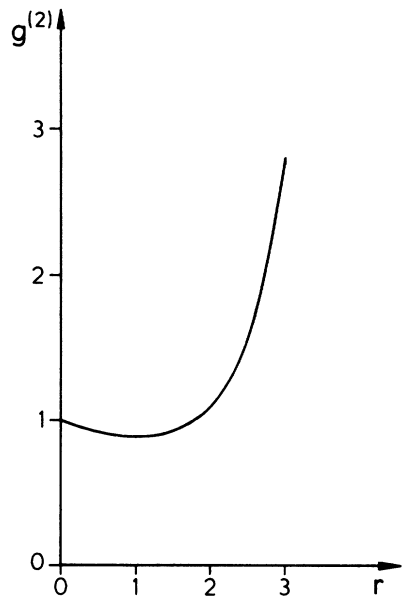

FIG. 1. Second-order correlation $g^{(2)}$ as a function of the squeezed parameter r for pure squeezed states with $\langle n \rangle = 5$.

tions than the Poisson distribution (sub-Poissonian) and broader than the negative binomial with $k = 1$ are compatible with quantum statistics. This means that the standard bounds[9] of multiplicity distributions represented by Eqs. (6) and (7) can be overcome.

The distribution $P_n(r, \langle n \rangle)$ corresponding to a squeezed state reads[8]

$$P_n(r,\langle n \rangle) = \frac{1}{n!\cosh\frac{1}{2}r}\left(\tfrac{1}{2}\tanh\tfrac{1}{2}r\right)^n H_n^2(Z_1)e^{Z_2} ,$$
$$Z_1 = \frac{|\langle n \rangle - \sinh^2(\frac{1}{2}r)|^{1/2}}{[\cosh\frac{1}{2}r - \sinh(\frac{1}{2}r)](\sinh r)^{1/2}} , \qquad (19)$$
$$Z_2 = \frac{\langle n \rangle - \sinh^2\frac{1}{2}r}{\cosh(\frac{1}{2}r)[\sinh\frac{1}{2}r - \cosh(\frac{1}{2}r)]} ,$$

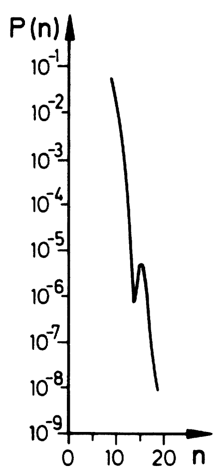

FIG. 2. Multiplicity distribution $P(n)$ for a pure squeezed state; $r = 0.5$ and $\langle n \rangle = 6$.

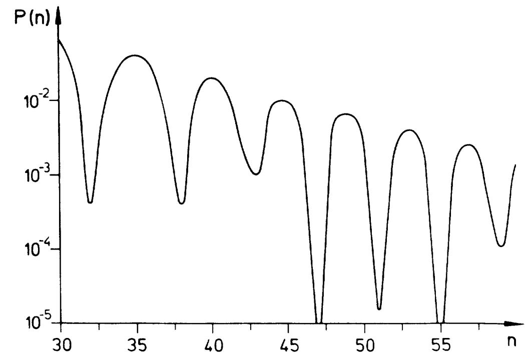

FIG. 3. Multiplicity distribution $P(n)$ for a pure squeezed state with $r = 3$, $\langle n \rangle = 30$.

where H_n are Hermite polynomials.

The remarkable feature of this multiplicity distribution which has been found only very recently,[10,11] is that for fixed values of the squeezing parameter r and of the mean multiplicity $\langle n \rangle$, $P(n)$ is an oscillating function of n. An example of this is given in Fig. 2.

Equation (19) gives the *exact* result for the counting distribution corresponding to squeezed states. However, it involves big cancellations between the Hermite polynomials and the factorials; for large values of n it is practically impossible to use this formula. An asymptotic expansion of the Hermitian polynomials is required. The authors of Ref. 10 have considered this problem. Using a semiclassical description of squeezed states in phase space, they derived an asymptotic formula for Hermitian polynomials, which is better than the one quoted in the literature. We inserted their formula in Eq. (19) and obtained the results of Fig. 3. As a matter of fact, it was shown by Schleich and Wheeler[10] that sufficient conditions for oscillations to appear are $r \gg 1$ and $\langle n \rangle \gg 1$. In this case an asymptotic expansion of the Hermite polynomials leads to

$$P_n \sim A_n \cos^2\psi_n , \qquad (20)$$

where A_n, ψ_n are functions of n given in Ref. 10.

III. MIXTURES OF CHAOTIC AND SQUEEZED-STATE DISTRIBUTIONS

One cannot always expect squeezing to appear in a pure coherent state. Our experience with standard coherent states teaches us that often mixtures of coherent and chaotic distributions appear. Thus in optics a laser near threshold is not a pure coherent state but rather a superposition of coherent and chaotic fields. Other mixtures are possible in terms of probabilities of the two distributions.

In the following three types of mixtures of squeezed and chaotic fields will be studied and the corresponding expressions for $g^{(2)}$ and $P(n)$ will be derived.

A. Superposition of squeezed and chaotic fields

As is well known the superpostion of coherent and chaotic fields leads to the Glauber-Lachs-Perina-Mollow formula (cf., e.g., Ref. 12)

$$P(n)=\exp[-\langle n_c\rangle/(1+\langle n_{\mathrm{ch}}\rangle)]\frac{\langle n_{\mathrm{ch}}\rangle^n}{(1+\langle n_{\mathrm{ch}}\rangle)^{n+1}}\times L_n\left[-\frac{\langle n_c\rangle}{\langle n_{\mathrm{ch}}\rangle(1+\langle n_{\mathrm{ch}}\rangle)}\right]. \tag{21}$$

Here $\langle n_c\rangle$ and $\langle n_{\mathrm{ch}}\rangle$ are the mean numbers of particles due to the coherent and chaotic sources and L_n is the Laguerre polynomial. Note that the total mean multiplicity is

$$\langle n\rangle=\langle n_c\rangle+\langle n_{\mathrm{ch}}\rangle . \tag{22}$$

Equation (21) describes a partially coherent laser distribution. This equation, as well as Eqs. (6) and (7) which are particular cases of it for $\langle n_{\mathrm{ch}}\rangle=0$ and $\langle n_c\rangle=0$, respectively, have found wide applications not only in optics and electronics[12] but more recently also in particle physics.[13]

We consider now the superposition of a squeezed coherent and a chaotic field. The corresponding distribution is known[14] in an analytic form for the case $A \gg |\sinh(r/2)|\exp(r/2)$, where A and r are defined in Sec. II. It is characterized by three parameters r, $\langle n_c\rangle$, and $\langle n_{\mathrm{ch}}\rangle$ or r, $\langle n\rangle$, and $\gamma=\langle n_c\rangle/\langle n_{\mathrm{ch}}\rangle$.

A numerical investigation of this distribution shows[11] that for $0\le\gamma\le 50$ and various values of r and $\langle n\rangle$ the characteristic oscillations found in a pure squeezed state disappear. This illustrates the important role which even a very weak chaotic background can have (for numerical reasons, values of $\gamma>50$ are difficult to handle), and is exemplified in Fig. 4. On the other hand, the nonclassical behavior of $g^{(2)}$ is already evident for $\gamma=50$, as can be seen in Fig. 5 where $g^{(2)}<1$. In other words there is no simple connection between the oscillation in $P(n)$ and the nonclassical values (less than one) of $g^{(2)}$.

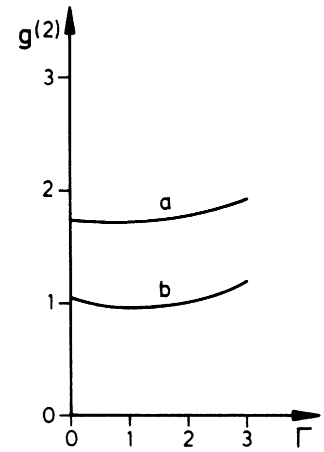

FIG. 5. Second-order correlation $g^{(2)}$ for a superposition of squeezed and chaotic fields with $\langle n_c\rangle=6$. a, $\gamma=1$; b, $\gamma=50$.

B. Convolution of squeezed and chaotic distributions

Convolutions of coherent distributions with chaotic ones have been widely used in classical approaches to optics[15] and have recently been applied to particle physics.[16]

Instead of considering a superposition of fields, chaotic and squeezed, which corresponds to the situation where the sources (and the corresponding fields) interfere, one could envisage a case when the sources act independently. In this situation the probability distribution $P(n)$ is a convolution of two distributions $P_1(n_1)$ and $P_2(n_2)$ referring to the chaotic and squeezed source, respectively:

$$P(n)=\sum_{n_1+n_2=n} P_1(n_1)P_2(n_2) , \tag{23}$$

where $P_1(n_1)$ is given by Eq. (7) with $k=1$, and $P_2(n_2)$ by Eq. (19).

The number of parameters is the same as in the previous case and the results are presented in Tables I–VI and Fig. 6. We found that, as in the previous case, the oscillations in $P(n)$ disappear for $0<\gamma<50$ (Tables I–III), but at $\gamma=1000$ they appear again (cf. Table IV). The $g^{(2)}$

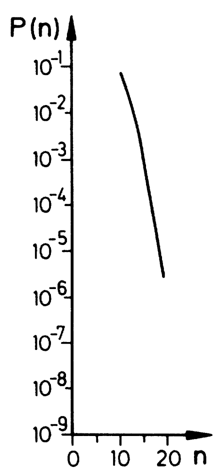

FIG. 4. Multiplicity distribution $P(n)$ for a superposition of squeezed and chaotic fields; $r=0.5$, $\langle n_c\rangle=6$, and $\gamma=50$.

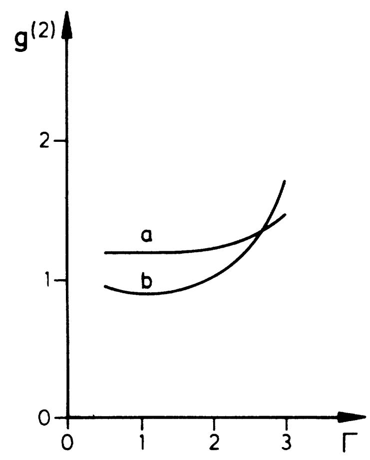

FIG. 6. $g^{(2)}$ for a convolution of squeezed and chaotic distributions with $\langle n_c\rangle=6$. a, $\gamma=1$; b, $\gamma=50$.

TABLE I. Multiplicity distributions $P(n)$ for a convolution [Eq. (23)] and superposition of chaotic and squeezed probabilities [Eq. (24)]; $\langle n_c \rangle = 6$, $\gamma = 1$, $r = 0.5$.

n	$P(n)$ Eq. (23)	$P(n)$ $\alpha=0.99$ Eq. (24)	$P(n)$ $\alpha=0.5$ Eq. (24)
0	0.855×10^{-4}	0.202×10^{-2}	0.717×10^{-1}
1	0.860×10^{-3}	0.668×10^{-2}	0.640×10^{-1}
2	0.417×10^{-2}	0.248×10^{-1}	0.645×10^{-1}
3	0.130×10^{-1}	0.660×10^{-1}	0.779×10^{-1}
4	0.292×10^{-1}	0.126	0.102
5	0.511×10^{-1}	0.181	0.124
6	0.726×10^{-1}	0.200	0.129
7	0.871×10^{-1}	0.173	0.112
8	0.917×10^{-1}	0.118	0.804×10^{-1}
9	0.877×10^{-1}	0.635×10^{-1}	0.497×10^{-1}
10	0.790×10^{-1}	0.263×10^{-1}	0.284×10^{-1}
11	0.688×10^{-1}	0.814×10^{-2}	0.171×10^{-1}
12	0.592×10^{-1}	0.182×10^{-2}	0.120×10^{-1}
13	0.508×10^{-1}	0.354×10^{-3}	0.971×10^{-2}
14	0.435×10^{-1}	0.166×10^{-3}	0.825×10^{-2}
15	0.373×10^{-1}	0.147×10^{-3}	0.708×10^{-2}
16	0.320×10^{-1}	0.125×10^{-3}	0.607×10^{-2}
17	0.274×10^{-1}	0.105×10^{-3}	0.520×10^{-1}
18	0.235×10^{-1}	0.891×10^{-4}	0.445×10^{-2}
19	0.201×10^{-1}	0.764×10^{-4}	0.382×10^{-2}
20	0.173×10^{-1}	0.655×10^{-4}	0.327×10^{-2}
$g^{(2)}$	1.19	0.943	1.28

TABLE II. Same as Table I, for $\langle n_c \rangle = 6$, $\gamma = 50$, $r = 0.5$.

n	$P(n)$ Eq. (23)	$P(n)$ $\alpha=0.99$ Eq. (24)	$P(n)$ $\alpha=0.5$ Eq. (24)
0	0.534×10^{-3}	0.952×10^{-2}	0.447
1	0.497×10^{-2}	0.641×10^{-2}	0.506×10^{-1}
2	0.220×10^{-1}	0.239×10^{-1}	0.171×10^{-1}
3	0.611×10^{-1}	0.651×10^{-1}	0.334×10^{-1}
4	0.120	0.126	0.635×10^{-1}
5	0.175	0.180	0.911×10^{-1}
6	0.199	0.200	0.101
7	0.177	0.173	0.873×10^{-1}
8	0.125	0.118	0.596×10^{-1}
9	0.704×10^{-1}	0.632×10^{-1}	0.319×10^{-1}
10	0.310×10^{-1}	0.260×10^{-1}	0.131×10^{-1}
11	0.104×10^{-1}	0.788×10^{-2}	0.398×10^{-2}
12	0.255×10^{-2}	0.159×10^{-2}	0.805×10^{-3}
13	0.419×10^{-3}	0.161×10^{-3}	0.814×10^{-4}
14	0.456×10^{-4}	0.753×10^{-6}	0.380×10^{-6}
15	0.975×10^{-5}	0.540×10^{-5}	0.273×10^{-5}
16	0.454×10^{-5}	0.387×10^{-5}	0.196×10^{-5}
17	0.121×10^{-5}	0.802×10^{-6}	0.405×10^{-6}
18	0.156×10^{-6}	0.295×10^{-7}	0.149×10^{-7}
19	0.247×10^{-7}	0.882×10^{-8}	0.445×10^{-8}
20	0.125×10^{-7}	0.110×10^{-7}	0.554×10^{-8}
$g^{(2)}$	0.940	0.946	1.80

TABLE III. Same as Table I, for $\langle n_c \rangle = 9$, $\gamma = 1$, $r = 1$.

n	$P(n)$ Eq. (23)	$P(n)$ $\alpha=0.99$ Eq. (24)	$P(n)$ $\alpha=0.5$ Eq. (24)
0	0.254×10^{-6}	0.100×10^{-2}	0.500×10^{-1}
1	0.497×10^{-5}	0.947×10^{-3}	0.450×10^{-1}
2	0.466×10^{-4}	0.123×10^{-2}	0.407×10^{-1}
3	0.278×10^{-3}	0.307×10^{-2}	0.376×10^{-1}
4	0.119×10^{-2}	0.991×10^{-2}	0.375×10^{-1}
5	0.385×10^{-2}	0.281×10^{-1}	0.434×10^{-1}
6	0.988×10^{-2}	0.640×10^{-1}	0.586×10^{-1}
7	0.206×10^{-1}	0.116	0.824×10^{-1}
8	0.356×10^{-1}	0.169	0.107
9	0.519×10^{-1}	0.197	0.119
10	0.650×10^{-1}	0.181	0.109
11	0.715×10^{-1}	0.129	0.807×10^{-1}
12	0.712×10^{-1}	0.675×10^{-1}	0.481×10^{-1}
13	0.663×10^{-1}	0.231×10^{-1}	0.243×10^{-1}
14	0.601×10^{-1}	0.373×10^{-2}	0.132×10^{-1}
15	0.541×10^{-1}	0.208×10^{-3}	0.103×10^{-1}
16	0.487×10^{-1}	0.977×10^{-3}	0.966×10^{-2}
17	0.439×10^{-1}	0.995×10^{-3}	0.876×10^{-2}
18	0.396×10^{-1}	0.428×10^{-3}	0.765×10^{-2}
19	0.356×10^{-1}	0.148×10^{-3}	0.676×10^{-2}
20	0.321×10^{-1}	0.138×10^{-3}	0.609×10^{-2}
$g^{(2)}$	1.41	0.942	1.23

TABLE IV. Same as Table I, for $\langle n_c \rangle = 6$, $\gamma = 1000$, $r = 0.5$.

n	$P(n)$ Eq. (23)	$P(n)$ $\alpha=0.99$ Eq. (24)	$P(n)$ $\alpha=0.5$ Eq. (24)
0	0.595×10^{-3}	0.105×10^{-1}	0.497
1	0.548×10^{-2}	0.551×10^{-2}	0.572×10^{-2}
2	0.239×10^{-1}	0.238×10^{-1}	0.120×10^{-1}
3	0.655×10^{-1}	0.651×10^{-1}	0.329×10^{-1}
4	0.126	0.126	0.634×10^{-1}
5	0.182	0.180	0.911×10^{-1}
6	0.201	0.200	0.101
7	0.175	0.173	0.873×10^{-1}
8	0.120	0.118	0.596×10^{-1}
9	0.641×10^{-1}	0.632×10^{-1}	0.319×10^{-1}
10	0.265×10^{-1}	0.260×10^{-1}	0.131×10^{-1}
11	0.807×10^{-2}	0.788×10^{-2}	0.398×10^{-2}
12	0.165×10^{-2}	0.159×10^{-2}	0.805×10^{-3}
13	0.172×10^{-3}	0.161×10^{-3}	0.814×10^{-4}
14	0.178×10^{-5}	0.753×10^{-6}	0.380×10^{-6}
15	0.543×10^{-5}	0.540×10^{-5}	0.273×10^{-5}
16	0.392×10^{-5}	0.387×10^{-5}	0.196×10^{-5}
17	0.828×10^{-6}	0.802×10^{-6}	0.405×10^{-6}
18	0.346×10^{-7}	0.295×10^{-7}	0.149×10^{-7}
19	0.906×10^{-8}	0.882×10^{-8}	0.445×10^{-8}
20	0.111×10^{-7}	0.110×10^{-7}	0.554×10^{-8}
$g^{(2)}$	0.937	0.947	1.87

TABLE V. Same as Table I, for $\langle n_c \rangle = 9$, $\gamma = 50$, $r = 1$.

n	$P(n)$ Eq. (23)	$P(n)$ $\alpha = 0.99$ Eq. (24)	$P(n)$ $\alpha = 0.5$ Eq. (24)
0	0.216×10^{-5}	0.848×10^{-2}	0.424
1	0.405×10^{-4}	0.134×10^{-2}	0.647×10^{-1}
2	0.363×10^{-3}	0.614×10^{-3}	0.101×10^{-1}
3	0.206×10^{-2}	0.237×10^{-2}	0.268×10^{-2}
4	0.824×10^{-2}	0.926×10^{-2}	0.491×10^{-2}
5	0.248×10^{-1}	0.275×10^{-1}	0.139×10^{-1}
6	0.581×10^{-1}	0.635×10^{-1}	0.321×10^{-1}
7	0.108	0.116	0.585×10^{-1}
8	0.161	0.169	0.853×10^{-1}
9	0.193	0.197	0.993×10^{-1}
10	0.184	0.181	0.915×10^{-1}
11	0.138	0.129	0.651×10^{-1}
12	0.786×10^{-1}	0.672×10^{-1}	0.339×10^{-1}
13	0.316×10^{-1}	0.229×10^{-1}	0.116×10^{-1}
14	0.781×10^{-2}	0.350×10^{-2}	0.177×10^{-2}
15	0.119×10^{-2}	0.235×10^{-5}	0.119×10^{-5}
16	0.859×10^{-3}	0.791×10^{-3}	0.400×10^{-3}
17	0.840×10^{-3}	0.828×10^{-3}	0.418×10^{-3}
18	0.366×10^{-3}	0.278×10^{-3}	0.140×10^{-3}
19	0.668×10^{-4}	0.128×10^{-4}	0.646×10^{-5}
20	0.243×10^{-4}	0.164×10^{-4}	0.831×10^{-5}
$g^{(2)}$	0.940	0.946	1.80

TABLE VI. Same as Table I, for $\langle n_c \rangle = 9$, $\gamma = 1000$, $r = 1$.

n	$P(n)$ Eq. (23)	$P(n)$ $\alpha = 0.99$ Eq. (24)	$P(n)$ $\alpha = 0.5$ Eq. (24)
0	0.252×10^{-5}	0.991×10^{-2}	0.496
1	0.471×10^{-4}	0.135×10^{-3}	0.444×10^{-2}
2	0.418×10^{-3}	0.418×10^{-3}	0.250×10^{-3}
3	0.234×10^{-2}	0.234×10^{-2}	0.118×10^{-2}
4	0.929×10^{-2}	0.926×10^{-2}	0.468×10^{-2}
5	0.276×10^{-1}	0.275×10^{-1}	0.139×10^{-1}
6	0.638×10^{-1}	0.635×10^{-1}	0.321×10^{-1}
7	0.117	0.116	0.585×10^{-1}
8	0.170	0.169	0.853×10^{-1}
9	0.198	0.197	0.993×10^{-1}
10	0.183	0.181	0.915×10^{-1}
11	0.131	0.129	0.651×10^{-1}
12	0.684×10^{-1}	0.672×10^{-1}	0.339×10^{-1}
13	0.235×10^{-1}	0.229×10^{-1}	0.116×10^{-1}
14	0.371×10^{-2}	0.350×10^{-2}	0.177×10^{-2}
15	0.355×10^{-4}	0.235×10^{-5}	0.119×10^{-5}
16	0.792×10^{-3}	0.791×10^{-3}	0.400×10^{-3}
17	0.836×10^{-3}	0.828×10^{-3}	0.418×10^{-3}
18	0.286×10^{-3}	0.278×10^{-3}	0.140×10^{-3}
19	0.154×10^{-4}	0.128×10^{-4}	0.646×10^{-5}
20	0.166×10^{-4}	0.164×10^{-4}	0.831×10^{-5}
$g^{(2)}$	0.937	0.946	1.87

behaves similarly as in the previous case of field superposition. For $\gamma = 1$ it is limited to the classical domain, while for $\gamma = 50$ it takes nonclassical values (Fig. 6).

C. Superposition of squeezed and chaotic probabilities

This corresponds to a situation where in each event *either* a chaotic *or* a squeezed distribution is found. In high-energy physics one could conceive of such a possibility in the following way: At a given center-of-mass energy multiparticle production events differ one from another by the inelasticity, i.e., by the energy available for particle production, because the leading particles take in each event a different amount of energy. Suppose that there exists a threshold energy below which the phenomenon of squeezing or the phenomenon of thermal equilibrium, leading to a chaotic distribution, cannot occur (instead of inelasticity one can imagine the centrality of the collision as being the decisive factor which triggers a certain reaction). In that case in some events a squeezed coherent distribution will emerge while in other events a chaotic distribution emerges. The overall multiplicity distribution is then given by

$$P(n) = \alpha P_1(n_1) + (1-\alpha) P_2(n_2), \quad 0 \le \alpha \le 1 , \tag{24}$$

with

$$n = n_1 + n_2$$

and P_1 and P_2 defined by Eqs. (7) with $k = 1$ and (19), respectively. α represents the fraction of events in which the distribution P_1 is realized. We have now one parameter more as compared with the previous case: namely α.

A numerical study of this case shows that the oscillating behavior of $P(n)$ appears already at $\gamma = 50$, for $\alpha = 0.99$ (Table II). The corresponding values of the correlation $g^{(2)}$ as a function of r are represented in Fig. 7.

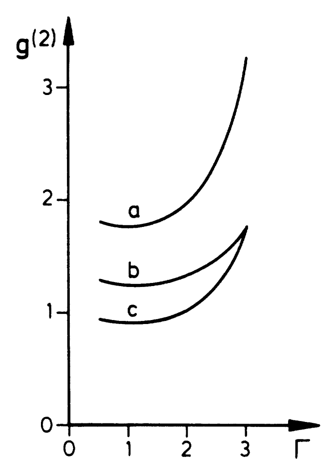

FIG. 7. $g^{(2)}$ for a superposition of chaotic and squeezed probabilities [Eq. (24)] as a function of r; $\langle n_c \rangle = 6$; a, $\gamma = 50$, $\alpha = 0.5$; b, $\gamma = 1$, $\alpha = 0.5$; c, $\gamma = 1$, and $\gamma = 50$, $\alpha = 0.99$. In case c, because of the value of $\alpha = 0.99$, both $\gamma = 1$ and $\gamma = 50$ give approximately the same results.

From these examples follows, among other things, that the oscillations can be quenched by chaotic fields and that the observation or nonobservation of oscillations is very sensitive to a chaotic perturbation.

Ideally, in order to detect experimentally squeezed states through these oscillations, one would have to choose specific experimental conditions where squeezed states are expected to appear in their clearest form. The observation that oscillations appear at large $\langle n \rangle$ [cf. Eq. (20)] (we have checked that at large $\langle n \rangle$ and n the oscillations become very frequent) may be useful in this respect.

On the other hand, it would be important to gain more physical insight into the relation between the squeezing parameter r and some physical observable. We consider this as the most important goal of future theoretical research along these lines.

IV. DISCUSSION

Although the importance of the detection of coherent states in high-energy physics is recognized,[17,1] the experimental evidence for these states is yet far from being confirmed.[1] Several different physical phenomena are sensitive to these states, but there are serious difficulties in disentangling various effects which could mimic coherence. Thus in Bose-Einstein correlations the fact that in almost all experiments $g^{(2)} < 2$, does not yet necessarily imply the existence of a coherent state, because resonances, e.g., would also contribute to a lowering of the Bose-Einstein correlation and this effect is not easy to estimate. Analogously, the observation that in certain reactions (e.g., e^+e^-) the multiplicity distribution $P(n)$ is almost Poissonian, does not prove that it arises from a coherent state, because any independent production mechanism would also lead to a Poisson distribution.

For squeezed coherent states a similar ambiguous situation has prevailed. Thus sub-Poissonian distributions have been observed in particle physics for decades, but they usually appear at small c.m. energies where conservation-law constraints are important and most probably they can be attributed to these trivial effects. Therefore, although squeezed states provide an elegant formalism for sub-Poissonian statistics (cf. Ref. 18), nobody could claim that these states have really been seen in high-energy physics. As a matter of fact, even in quantum optics the experimental proof of the existence of squeezed states has been given only very recently,[19] through the reduction of noise below the level corresponding to a classical coherent state.

The observation[10,11] that squeezed coherent states induce oscillations in the multiplicity distribution may facilitate the investigation of these new states, both in quantum optics and in particle physics. The results of the present quantitative investigation of the role of squeezed states in multiplicity distributions show that (i) if the squeezing parameter and the multiplicity are large enough, oscillations in $P(n)$ occur, (ii) the amplitude of these oscillations depends on the magnitude of the chaotic background, and (iii) it depends also whether the *amplitude* or the *probabilities* of the squeezed coherent and

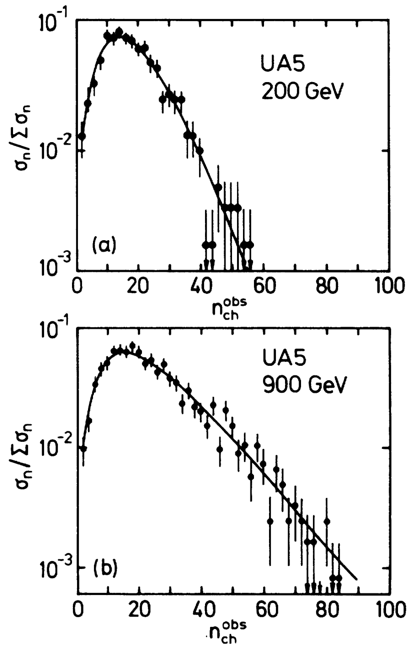

FIG. 8. Multiplicity distributions at 200 and 900 GeV corrected for secondary interactions, gamma conversion, and short-lived decays, but not for geometrical acceptance or trigger efficiency. The solid line is the result of a fit to the observed data of a negative binomial (from Ref. 21).

chaotic fields add. In the last case, it depends also whether or not the two sources (coherent and chaotic) act simultaneously.

From these results one may conclude that while the nonobservation of oscillations does not yet preclude the existence of squeezed states, the detection of such oscillations would constitute evidence for these states. Unfor-

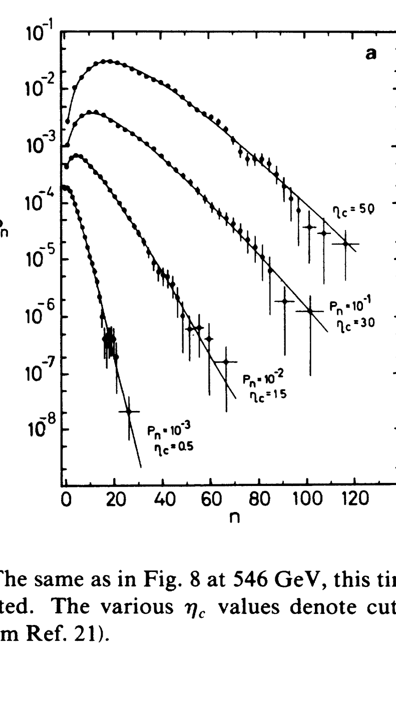

FIG. 9. The same as in Fig. 8 at 546 GeV, this time, however, fully corrected. The various η_c values denote cuts in pseudorapidity (from Ref. 21).

tunately, it seems that this would still not be a "proof" for the existence of squeezed states, because there are other mechanisms which could also produce oscillations in $P(n)$. Thus the superposition of several classical sources with very different mean multiplicities would also lead to oscillations. As a matter of fact, in particle physics such oscillations were predicted to occur as a consequence of multiple Pomeron scattering.[20] However, both these alternatives are very interesting, too.

Experimentally, the situation is far from being clear. Although it is true that no statistically significant oscillations have been reported so far in the literature, it is also probably true, that, with present techniques, they could not have been detected. The measurement of multiplicity distributions at high energies is affected by systematic errors which are not easy to control. Thus, e.g., in the UA5 measurements,[21] the observed distributions are usually corrected via Monte Carlo simulations in order to compensate for the limitations in the geometrical acceptance and the contamination of primary tracks by secondaries. In this correction procedure a smoothness constraint is imposed[21] which would hardly permit any existing oscillations to survive in the final result called "true" distribution. Even with this caveat in mind the published distributions present some structure which can be seen by the naked eyed. To illustrate this, in Fig. 8 partially corrected distributions obtained by the UA5 Collaboration are shown and in Fig. 9 fully corrected distributions. Note that even in the fully corrected distribution small oscillations can be seen, although we cannot comment about their statistical significance. There is, anyway, clear evidence in this figure for at least one shoulder around $n=80$, which could easily mask oscillations.

A systematic and careful investigation of multiplicity distribution with special emphasis on oscillations appears to be a rewarding task for future experiments. From the theoretical point of view, a dynamical understanding of the conditions under which squeezing is expected to occur in particle physics is highly desirable.

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[2]In the quantum-optical literature the term "classical light" is used because most of the properties derived from quantum optics can also be derived by using only classical considerations.

[3]$g^{(2)}$ is the intercept of

$$\frac{\partial^2\sigma}{\partial \mathbf{p}_1 \partial \mathbf{p}_2} \Big/ \frac{1}{\sigma}\frac{\partial\sigma}{\partial \mathbf{p}_1}\frac{\partial\sigma}{\partial \mathbf{p}_2}$$

at $|\mathbf{p}_1|=|\mathbf{p}_2|$ measured in Hanburg-Brown–Tuiss interferometry.

[4]For a review, cf., e.g., D. F. Walls, Nature (London) **306**, 141 (1986).

[5]It can be shown that, while for ordinary coherent states one has

$$\langle \alpha | (\Delta a_1)^2 | \alpha \rangle = \langle \alpha | (\Delta a_2)^2 | \alpha \rangle = \tfrac{1}{4} ,$$

for a two-photon squeezed coherent state (to be defined below), $\langle A, r\theta\lambda | (\Delta a_1)^2 | A, r\theta\lambda | \rangle$ can be smaller than $\frac{1}{4}$ [at the expense of $\langle A, r\theta\lambda | (\Delta a_2)^2 | A, r\theta\lambda \rangle$ which then exceeds $\frac{1}{4}$]. Here $\Delta a_{1,2}$ denote the dispersion of

$$a_1 = \frac{a^\dagger + a}{\sqrt{2}} \text{ and } a_2 = i\frac{a^\dagger - a}{\sqrt{2}} ,$$

where a is the annihilation operator.

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