Scaling and correlations of squeezed coherent distributions: Application to hadronic multiplicities

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It is shown that a k-mode squeezed-coherent-state distribution is the most general one in describing hadronic multiplicity distributions in particle collision processes. An exact expression for the k-mode squeezed coherent multiplicity distribution is derived. The properties of this distribution are compared with the Glauber-Lachs distribution and it is shown that pure squeezed states show asymptotic scaling. The correlation properties of this distribution are shown and its usefulness in pion-interferometry experiments is discussed. The domain of reach of these states is shown to be wider than that of the Glauber-Lachs distribution.

I. INTRODUCTION

Asymptotic scaling laws of various photon-counting distributions have become the focus of interest in view of their applicability to various physical phenomena. These include stochastic models of multiparticle production in high-energy collisions. The idea of applying stochastic methods developed for studying photon-counting statistics of light beams to particle production processes was used in the early days of high-energy collisions, when it was noted by Knox, Giovannini, and others that a formidably complex dynamical process such as a hadron-hadron collision can be explained by a simple statistical picture independent of dynamical details.[1] Among the experimentally observed properties that lent themselves to a stochastic interpretation were the multiplicity distributions of hadrons emerging from collisions, or, in quantum-optical language, the counting statistics of the hadrons and also the correlations of the outgoing pions in rapidity space. These correlations were similar to the Bose-Einstein correlations observed in light beams when the emitting sources are Gaussian,[2] the resulting counting distribution being a negative-binomial one. The scaling forms and the departure from scaling of various photon-counting distributions such as the thermal (negative-binomial) distribution have been studied only recently, as their usefulness in describing multiplicity distributions has been noted. A counting distribution P_n is said to admit asymptotic scaling if

$$\lim_{\substack{n\to\infty \\ \langle n\rangle\to\infty}} \langle n\rangle P_n=\psi(\hat{z}) \ , \tag{1}$$

where $\hat{z}=n/\langle n\rangle$ is a dimensionless quantity. This is known as Koba-Nielsen-Oleson (KNO) scaling in particle physics.[3]

Hadronic multiplicity distributions apart, asymptotic scaling forms are of interest in quantum optics. For a general photon-counting distribution, it implies that for a large number of photons, the shape of the counting distribution is independent of the variables upon which the number of photons is dependent. These days the significant violation of KNO scaling has led workers to study not only asymptotic scaling forms but also the general conditions under which scaling is violated systematically.

For complete knowledge of the dynamics of the radiation field responsible for the production of any kind of particle (photons, hadrons, pions, etc.), we require the counting distribution of the field and all its characteristics. Depending on the statistics of the counting distribution of the radiation field, we realize that a particular class of states will be more useful to work with in a given context. We also get some knowledge of the source of the radiation. For example, chaotic light possesses a geometric counting distribution whose statistics imply a noisy source. On the other hand, a coherent source is characterized by Poissonian counting statistics. These states (sources) are termed "classical" as they admit a Glauber P representation.

There are other nonclassical states which have different counting distributions exhibiting squeezing and antibunching such as binomial states, logarithmic states, and squeezed states. In this paper we are concerned with the properties and applications of squeezed coherent distributions. These do not arise from classical sources. They show purely quantum-mechanical properties such as antibunching and squeezing. These distributions have been seen in neutrino-induced and low-mass diffractive hadronic multiplicity distributions. It is the aim of this paper, to show that the most general distribution that characterizes e^+e^-, $p\bar{p}$, neutrino-induced, and low-mass diffractive collisions is the k-mode squeezed-coherent-state distribution. We show the evolution of this idea for multiparticle production in which the stochastic and the nonstochastic elements of the dynamics are clearly delineated. If we assume that the sources of hadronic production are squeezed-coherent-state sources rather than completely coherent or completely chaotic, then, not only can we explain the scaling violation at high energies in hadron-hadron collisions but a plethora of data in e^+e^-, hadron-nucleus, and neutrino-induced collisions by a single distribution—the generalized squeezed-coherent-state distribution. We may also mention that to the best of our knowledge the multiplicity distribution of k squeezed sources has not been derived earlier in the literature nor have their scaling properties and correlation properties been studied. We shall present the results of this model and try to give a physical interpretation for some of the parameters introduced in the model.

II. THE SQUEEZED-COHERENT-STATE DISTRIBUTION VERSUS OTHER DISTRIBUTIONS

The idea of using squeezed coherent states to describe pion radiation in hadron-hadron collisions was motivated by the fact that in pion interferometry experiments, correlations in rapidity space of pairs of identical bosons (pions) are measured. As is well known from condensed-matter physics in a system of identical bosons, correlated pairs of bosons can correspond to a situation when the "vacuum" (source) from which the bosons are emitted actually emits correlated pairs of particles, i.e., if $a^\dagger$ and a are the creation and annihilation operators of single bosons, the actual vacuum is described by the quadratic operators $(a)^2$, $(a^\dagger)^2$, and $aa^\dagger$. Such a vacuum (or source) is also, therefore, called a two-photon vacuum (source).

Previously, the technique used to describe these correlations was to assume that there was some coherence in the source (emitting the pions). If the source is completely coherent then the two-particle correlation function (defined later) $g^2(0)=1$, whereas if the source is completely noisy (thermal) $g^2(0)=2$, but in reality the source is a mixture of these extremes. The distribution describing such a source is the Glauber-Lachs distribution whose scaling properties have been extensively discussed by Carruthers and Shih.[4] This is also used to characterize pion condensates in nuclear matter.[5] There are, however, two cases out of the reach of such a distribution: $g^2(0)<1$ and $g^2(0)>2$. Phenomena admitting such correlation functions [in particular $g^2(0)<1$] have been observed experimentally and will be discussed later. Before discussing the distribution that will encompass this situation, let us review the scaling properties and domain of reach of the Glauber-Lachs distribution. In the case of a classical noisy source, the averaging over field fluctuations for a single source gives the Bose-Einstein distribution

$$P_n=\frac{\langle n\rangle^n}{(\langle n\rangle+1)^{n+1}} . \tag{2}$$

For k sources, we have the negative-binomial distribution

$$P_n^k=\frac{(n+k-1)!(\langle n\rangle/k)^n}{n!(k-1)!(1+\langle n\rangle/k)^{n+k}} . \tag{3}$$

Both these distributions show the asymptotic scaling law (1):

$$\lim_{\substack{n\to\infty\\ \langle n\rangle\to\infty}}\langle n\rangle P_n^k=\frac{k^k\hat{z}^{k-1}\exp(-k\hat{z})}{(k-1)!}=\psi(\hat{z}) , \tag{4}$$

where, for $k=1$,

$$\psi(\hat{z})=\exp(-\hat{z}) . \tag{5}$$

With the introduction of a coherent component S, the distribution of k partially noisy, partially coherent sources is the Glauber-Lachs distribution:

$$P_n^{\mathrm{GL}}=\frac{(N/k)^n\exp[S/(1+N/k)]L_n^{k-1}(-kS/N(1+N/k))}{(1+N/k)^{n+k}} , \tag{6}$$

where L_n^{k-1} is the generalized $(k-1)$th Laguerre polynomial of order n, N is the noise amplitude, and S is the signal amplitude.

Introducing a new parameter, $m=(S/N)$ allows one to interpolate between the multiple Poissonian distribution which is found to explain e^+e^- data till 20 GeV and the negative-binomial distribution which explains $p\bar{p}$ data.[6] Then the average multiplicity becomes $\langle n\rangle=N+S$ or $\langle n\rangle=N(1+m)$ so that $N=\langle n\rangle/(1+m)$ and $S=\langle n\rangle m/(1+m)$. In terms of m the expression (6) becomes

$$P_n^{\mathrm{GL}}=\frac{[\langle n\rangle/(1+m)k]^n\exp[m\langle n\rangle/(1+m+\langle n\rangle/k)]L_n^{k-1}(-[km(m+1)]/(m+1+\langle n\rangle/k))}{\{1+[\langle n\rangle/(1+m)k]\}^{n+k}} . \tag{7}$$

Initially the parameter k was identified as the number of jets. In the limit $m\to0$, since

$$L_n^{k-1}(0)=\frac{(n+k-1)!}{(n)!(k-1)!} ,$$

we have expression (3). The limit $m\to\infty$ gives the Poissonian (for $k=1$). This implies that e^+e^- collisions are mostly coherent and $p\bar{p}$ collisions are mostly noisy.

However, since the above formula was introduced by Carruthers and independently by Biyajima the UA5 results changed the situation dramatically. It was found that there was systematic violation of KNO scaling so that the simple k-mode negative-binomial distribution with integral k no longer fits the $p\bar{p}$ data. Instead it is a systematic variation of k with energy in the following manner:

$$k^{-1}=a+b\ln(s) \tag{8}$$

with $a=-0.098\pm0.008$, $b=0.0282\pm0.009$, and s in GeV^2, which gives a good fit to $p\bar{p}$ data.[6]

The simple explanation of k as the number of jets was no longer applicable, nor did expression (6) neatly describe both the e^+e^- and $p\bar{p}$ distributions by one function. This led to the consideration of distributions other than the negative binomial or the more general Glauber-Lachs distribution which not only accommodated the systematic violation of KNO scaling but kept k integral and amenable to a physical interpretation. One such distribution is the generalized squeezed-coherent-state distribution.[7]

We go on to find the squeezed-coherent-state analogue of the Glauber-Lachs distribution. To do so we have to generalize the photon-counting distribution from the single-mode case to the multimode case. First, let us define a single-mode squeezed coherent state. It is obtained from a coherent state by the application of the unitary operator

$$S(\xi)=\exp\tfrac{1}{2}(\xi a^\dagger a^\dagger-\xi^* aa)\ .$$

Thus

$$[\alpha,\xi\rangle=D(\alpha)S(\xi)[0,0\rangle\ , \tag{9}$$

where $D(\alpha)=\exp(\alpha a^\dagger-\alpha^* a)$ and $|0\rangle$ is the vacuum state. It is seen that since $S(\xi)$ is quadratic in $a^\dagger$ and a, we can think of this vacuum as a coherent superposition of two photon states. For such a state, the average number of particles is

$$\langle n\rangle=|\alpha|^2+\sinh^2\xi\ . \tag{10}$$

Thus, squeezing adds more particles to a coherent state $|\alpha\rangle$. Alternatively, we can think of the squeezed-coherent-state vacuum as an eigenstate of an annihilation operator b, where b and $b^\dagger$ are related to a and $a^\dagger$ by

$$\begin{bmatrix} b \\ b^\dagger \end{bmatrix}=\begin{bmatrix} \cosh r & \exp(i\theta)\sinh r \\ -\exp(i\theta)\sinh r & \cosh r \end{bmatrix}\begin{bmatrix} a \\ a^\dagger \end{bmatrix}\ , \tag{11}$$

where $\xi=r\exp(i\theta)$.

As an aside, it is interesting to note that this transformation is the Bogolyubov transformation and squeezed coherent states may either be viewed as pairing between original bosons or as coherent states of Bogolyubov quasiparticles.[8,9]

The multiplicity distribution for the real source (i.e., $\theta=0$), $\xi=r$,

$$P_n=(x/2)^n(1-x^2)^{1/2}\exp[-\alpha^2(1+x)]\times(1/n!)H_n^2([\alpha^2(1+x)^2/2x]^{1/2})\ , \tag{12}$$

where $x=\tanh(r)=\sinh(r)/\cosh(r)$. Although it appears that P_n depends on both α and r, the relation

$$\langle n\rangle=\alpha^2+\sinh^2 r$$

enables us to express everything in terms of the parameter

$$(\sinh^2 r)/\alpha^2=s \quad\text{and}\quad \langle n\rangle\ .$$

The parameter s is analogous to the parameter m used in the previous discussion of the Glauber-Lachs distribution. It is convenient for brevity of expression to also define $q=1/(s+1)$, then in terms of s and $\langle n\rangle$ (and q) we have $\alpha^2=\langle n\rangle q$ and $\sinh^2 r=qs\langle n\rangle$ so that $x^2=sq\langle n\rangle/(1+sq\langle n\rangle)$.

Hence the expression (12) becomes

$$P_n=\frac{\exp(-\langle n\rangle q\{1+[sq\langle n\rangle/(1+s\langle n\rangle q)]^{1/2}\})(sq\langle n\rangle/4)^{n/2}}{n!(sq\langle n\rangle+1)}\times H_n^2([\langle n\rangle q\{1+[(1+sq\langle n\rangle)/4sq\langle n\rangle]^{1/2}+[sq\langle n\rangle/4(1+sq\langle n\rangle)]^{1/2}\}]^{1/2})\ , \tag{13}$$

s gives a measure of the relative squeezing to coherence and $\langle n\rangle$ is fixed experimentally for a given energy so that s is the only variable at a given $\langle n\rangle$. In general, we have two variables s and $\langle n\rangle$.

The relative dispersion in the multiplicities are given by the first moment of this distribution:

$$\left[\frac{\Delta n}{\langle n\rangle}\right]^2=\frac{\langle a^\dagger a a^\dagger a\rangle-(\langle a^\dagger a\rangle)^2}{(\langle a^\dagger a\rangle)^2}\ . \tag{14}$$

From expression (12) we have

$$\left[\frac{\Delta n}{\langle n\rangle}\right]^2=\frac{(1-x^2)\{2x^2+[\alpha(1-x)]^2\}}{\{(1-x^2)\alpha^2[\alpha^2(1-x^2)+2x^2]\}+x^4}\ .$$

For $x\ll\alpha^2$, i.e., $s<1$,

$$\left[\frac{\Delta n}{\langle n\rangle}\right]^2=\frac{1-x}{\alpha^2(1+x)}=\left[\frac{1}{\alpha^2}\right]-\left[\frac{2x}{\alpha^2}\right]$$

to first order in x. For a Poissonian $(\Delta n)/\langle n\rangle=1/\alpha$, thus, depending on the sign of x we can have sub-Poissonian or super-Poissonian statistics. Recalling that $x=\tanh(r)$ for $r>0$ we have a distribution narrower than the Poissonian and for $r<0$ we have a distribution broader than the Poissonian. Figure 1 shows this effect. For $x\gg\alpha^2$, i.e., $s>1$,

$$\left[\frac{\Delta n}{\langle n\rangle}\right]^2=\frac{2(1-x^2)(1+2\alpha^2)}{x^2}\ .$$

So to first order the distribution width is independent of

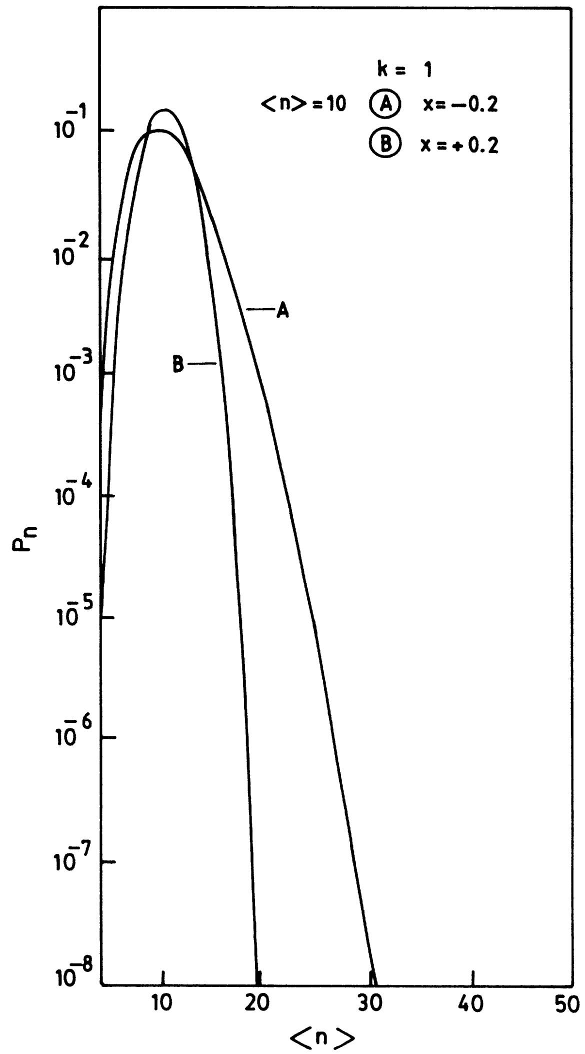

FIG. 1. The relative behavior of the multiplicity distribution of a squeezed-coherent-state source for negative and positive values of the squeezing parameter x. The curve A shows sub-Poissonian statistics and corresponds to $x=-0.2$, $\langle n\rangle=10$ and curve B shows super-Poissonian statistics $x=0.2$ and $\langle n\rangle=10$.

α. The generating function for a single-mode squeezed state is

$$Q(\lambda)=\exp[-\alpha^2(1+x)](1-x^2)[1-(1-\lambda)^2x^2]^{-1/2}$$
$$\times\exp\{[\alpha^2(1+x)(1-\lambda)x]/2[1+(1-\lambda)x]\}\;,$$

which gives (14) by the formula

$$\left.\frac{\partial^n Q(\lambda)}{\partial\lambda^n}\right|_{\lambda=0}=\langle a^{\dagger n}a^n\rangle\;.$$

An interesting property of the pure squeezed coherent state, i.e., $\alpha=0$ is that it admits asymptotic scaling. For $\alpha=0$ expression (12) becomes

$$P_n=\frac{(1-x^2)^{1/2}(x/2)^n H_n^2(0)}{n!},\quad n\text{ even}$$
$$P_n=0,\quad n\text{ odd} \tag{15}$$

and, since $\langle n\rangle=x^2/(1-x^2)$,

$$\langle n\rangle P_n=\frac{\langle n\rangle}{2^n}\left[\frac{\langle n\rangle}{1+\langle n\rangle}\right]^{n/2}\frac{n!}{(1+\langle n\rangle)^{1/2}(n/2)!(n/2)!} \tag{16}$$

which scales as

$$\lim_{\substack{n\to\infty\\ \langle n\rangle\to\infty}}\langle n\rangle P_n=\frac{C\exp(-\hat{z}/2)}{(\hat{z})^{1/2}}\;, \tag{17}$$

$C=$const. From the comparison of this with expression (5), we see that a squeezed state shows asymptotic scaling behavior but that the scaling form is much steeper than the classical thermal source.

Expression (13) shows that although a squeezed-coherent-state source is sub-Poissonian the distribution is symmetric since the Hermite polynomials are symmetric. However, the $p\bar{p}$ collision multiplicity distribution is asymmetric, sub-Poissonian with a long tail. We thus generalize this distribution to k sources. The k-mode squeezed states are a subset of the generalized coherent states of $\mathrm{Sp}(2k,R)$ in the sense of Perelomov's definition of generalized coherent states.[10] They are the direct products of the single-mode squeezed coherent states of each mode:

$$|\alpha,r\rangle=|\alpha_1,r_1\rangle*|\alpha_2,r_2\rangle*\cdots|\alpha_k,r_k\rangle\;. \tag{18}$$

For deriving the k-mode distribution it is convenient to consider the generating function

$$Q_k(\lambda)=\prod_{i=1}^{k}Q_i(\lambda)\;. \tag{19}$$

It is a well-known fact that the product of k generating functions leads to a new distribution.

The form of the multiplicity distribution, i.e., the probability of k sources emitting n particles is thus given by

$$P_n^k=P_{\sum_i n_i=n}(n_1,n_2,\ldots,n_k)=\prod_{\substack{i=1\\ \sum_i n_i=n}}^{k}P(n_i)$$
$$=\exp[-k\alpha^2(1+x)](1-x^2)^{k/2}(x/2)^n\prod_{i=1}^{k}(1/n_i!)H_{n_i}^2([\alpha^2(1+x)^2/2x]^{1/2})\;. \tag{20}$$

The above expression for P_n^k could be greatly simplified by using

$$\sum_{\sum_i n_i = n} \frac{H_{n_1}^2(y) \cdots H_{n_k}^2(y)}{n_1! \cdots n_k!} = \sum_{m=0}^{[n/2]} \gamma_m \frac{H_{n-2m}^2(\sqrt{k}y) 2^{2m}}{m!(n-2m)!} , \tag{21}$$

where

$$\gamma_m = (\gamma+1) \cdots [\gamma+(m-1)], \quad \gamma_0 = 1 ;$$

$$\gamma = (k-1)/2 .$$

The expression for P_n^k becomes

$$P_n^k = \exp[-k\alpha^2(1+x)](1-x^2)^{k/2}(x/2)^n \times \sum_{m=0}^{[n/2]} \gamma_m H_{n-2m}^2(\sqrt{k}y) 2^{2m}/m!(n-2m)! , \tag{22}$$

where $y=[\alpha^2(1+x)^2/2x]^{1/2}$. This can be expressed in terms of s and $\langle n \rangle$ in a similar fashion to expression (13). For $k=1$, $\gamma=0$, and only $\gamma_0=1$ contributes to the summation, reducing Eq. (22) to (12).

III. COMPARISON WITH DATA

The distribution (22) is the most general one in our description of the observed multiplicity distribution for the following reasons: For various values of the squeezing parameter r this distribution shows both sub-Poissonian and super-Poissonian statistics. Hadronic distributions in $p\bar{p}$ collisions show broad sub-Poissonian statistics with a long multiplicity tail which gets broader and broader with the increase of energy. Figure 2 shows the $k=3$ squeezed-coherent-state distribution for $\langle n \rangle = 13.6, x=-0.20$ and $\langle n \rangle = 26.1, x=-0.35$, respectively [recall that $x=\tanh(r)$], along with the corresponding ISR (62.2 GeV) and UA5 (540 GeV) data; α for each of these is thus fixed by Eq. (10). The departure of the squeezed-coherent-state distribution from the Poissonian one is completely parametrized by the parameter s and $\langle n \rangle$ (which is experimentally observable). As strong-interaction dynamics (hard processes) becomes more dominant, the squeezing parameter changes to make the distributions broader. The skewness of the counting distribution rests on the number of sources k and the width is related to the moment $\Delta n / \langle n \rangle$. The single-mode squeezed-coherent-state distribution has been used by Shih and Carruthers to fit neutrino-induced collisions in which the distribution is super-Poissonian ($\Delta n / \langle n \rangle < 1$). However, for low-energy events they remark that even the squeezed-coherent-state distribution for $r > 0$ is not narrow enough to describe the data well.[11,12] A close reexamination of the neutron-induced hadronic multiplicity data presented in Ref. 11 shows that the multiplicity curve is asymmetric, thus the single-mode squeezed-coherent-state distribution will not fit the data. Contrary to Shih's analysis that the squeezed-coherent-state distribution can only describe high-$\langle n \rangle$ data, a generalization to k modes enables us to fit the entire range of data. Figure 3 shows the $k=3, x=0.50$ fit to the same data as Ref. 12 and verifies our conjecture. This is also the case for high-mass diffractive events where again the Poissonian is too wide to fit the data.[11] When the model was initially proposed in Ref. 7, to explain $p\bar{p}$ data, the physical interpretation given for k was that each $q\bar{q}$ pair acts as an independent squeezed-coherent-state source. However, this subsequent comparison of e^+e^-, $p\bar{p}$, and νp data at 29

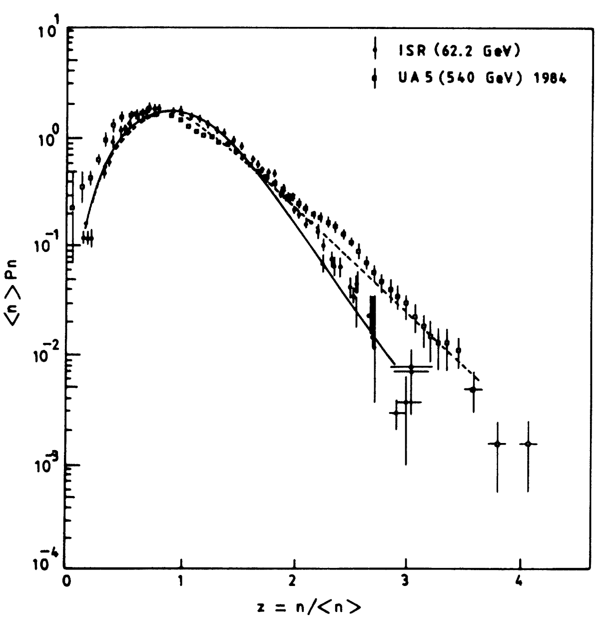

FIG. 2. The comparison of the CERN ISR 62.2-GeV data and the UA5 540-GeV data with the squeezed-coherent-state distribution for $x=-0.20$ and $\langle n \rangle = 13.6$ (narrower curve) and $x=-0.35$ and $\langle n \rangle = 26.1$ (broader curve). The value of α is determined by Eq. (10), $k=3$.

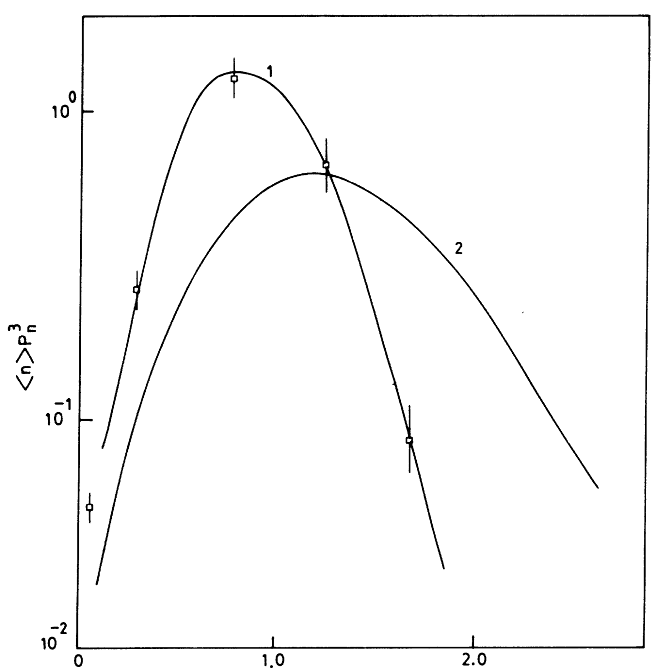

FIG. 3. The νp data for $\langle n \rangle = 4.18$ and the squeezed-coherent-state fit for $k=3$ and $x=+0.5$. The second curve shows the same distribution for $k=3$, $x=-0.5$.

GeV from recent experiments shows that for the same energy although the e^+e^- data points show a distribution narrower than $p\bar{p}$ and νp (see Fig. 4), the skewness of the distribution is similar to $p\bar{p}$ collisions and $k=3$ fits this data for various values of s. Thus our earlier interpretation is unlikely as it requires that $k=1$ should fit e^+e^- data. While the $k=1$ squeezed-coherent-state distribution is symmetric, preliminary data show that these distributions are asymmetric.[13,14] Thus our earlier interpretation of k is unlikely, rather it seems more likely that for central collisions s is the only projectile-dependent variable. Although we have achieved our aim in keeping k integral (and interpreted as the number of sources) and energy independent, the actual issue of the physical interpretation of k can only be resolved when higher-energy e^+e^- and $p\bar{p}$ data are available. If at higher energy the e^+e^- distributions persist in showing the skewness similar to $p\bar{p}$ distributions at the same energy, the issue can be resolved. It is very important to note here from the motivational point of view that the negative-binomial distribution which is used as the universal empirical distribution to describe all data has $\Delta n/\langle n\rangle = 1/\langle n\rangle + 1/k$, and hence can never be super-Poissonian ($\Delta n/\langle n\rangle < 1/\langle n\rangle$) as k can never be negative.

IV. BOSE-EINSTEIN CORRELATIONS IN THE SQUEEZED-COHERENT-STATE MODEL

The squeezed-coherent-state model also explains the two-particle correlations seen in pion interferometry ex-

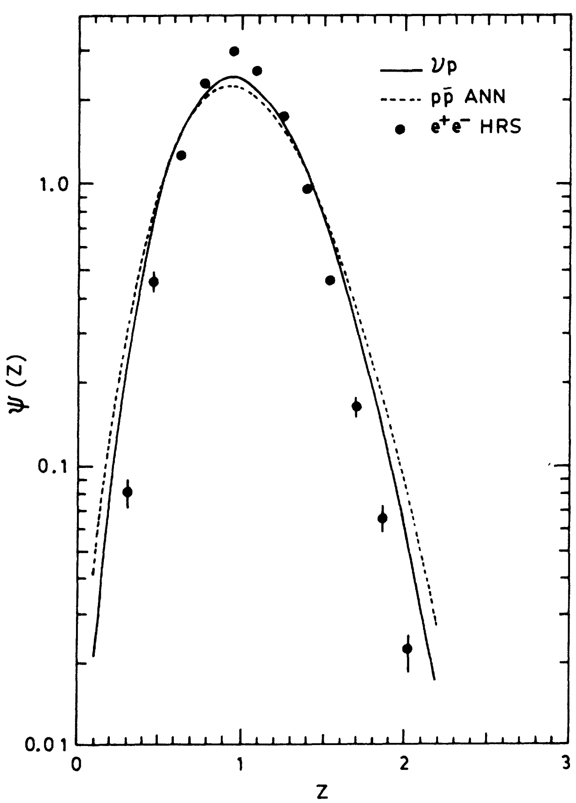

FIG. 4. The relative multiplicity curves at 29 GeV for $p\bar{p}$, e^+e^-, and νp data. The relative widths are to be noted.

periments in a consistent manner. The quantity which determines these correlations is the second-order correlation function given by

$$g^2(0)=\frac{\langle a^\dagger a^\dagger aa\rangle}{(\langle a^\dagger a\rangle)^2}$$

which can be written as

$$g^2(0)=\frac{\langle a^\dagger a a^\dagger a\rangle - \langle a^\dagger a\rangle}{(\langle a^\dagger a\rangle)^2} \tag{23}$$

and thus can be seen to be related to the moment $\Delta n/\langle n\rangle$ derived earlier. A radiation field is said to be antibunched if $g^2(0)<1$, which means that the probability of detecting a coincident pair of particles is less than that from a coherent field described by a coherent state which has the Poissonian distribution. Antibunching is considered to be a "clear demonstration of the quantum nature of the radiation" since it means anticorrelation in particle detection.[15] When $g^2(0)$ is not equal to 1, the states are said to have nonzero Hanbury-Brown–Twiss effect.[16] Those familiar with quantum-optical techniques will recall that in general g^2 is a function of time interval Δt between the arrival at the detector of two photons; whereas in particle physics the relevant variable is pseudorapidity y and the stationary situation $\Delta y=0$ is the pseudorapidity plateau. For a squeezed coherent state $g^2(0)$ is given by

$$g^2(0)=1+\frac{2\sinh^4(r)+\sinh^2(r)(2\alpha^2+1)-\alpha^2\sinh 2r}{(\alpha^2+\sinh^2 r)^2} . \tag{24}$$

The state $|\alpha,r\rangle$ is bunched only if the numerator of the second term in Eq. (24) is positive. Calling this $f(\alpha)$ the roots of $f(\alpha)$ are

$$\alpha_1,\alpha_2=\frac{\pm[\sinh r(1+2\sinh^2 r)]^{1/2}}{[2(\cosh r-\sinh r)]^{1/2}} ,$$

which in terms of x are

$$\alpha_1,\alpha_2=\frac{\pm[x(1+x^2)]^{1/2}}{[2(1-x)(1-x^2)]^{1/2}} . \tag{25}$$

For $r>0$ the roots are real and distinct and the coefficient of α^2 is negative. For $\alpha>\alpha_2$ and $\alpha<\alpha_1$ we have antibunching, i.e., super-Poissonian statistics; for $\alpha_1<\alpha<\alpha_2$ we have bunching. For $r<0$, $f(\alpha)$ is always positive so we have $g^2(0)>1$ and sub-Poissonian statistics. Thus, as our analysis of data has shown, νp (and $\bar{\nu}p$) interactions cannot be described by classical distributions such as the negative-binomial distribution. For a pure squeezed state $\alpha=0$,

$$g^2(0)=2+1/x^2 . \tag{26}$$

The above distribution is to be compared with the pure Poissonian distribution which has $g^2(0)=1$ and no Bose-Einstein correlations, whereas for a single negative-binomial distribution, $g^2(0)=2$ and the Glauber-Lachs distributions which have $g^2(0)=2-[m^2/(1+m)^2]$.

Thus, the two-particle Bose-Einstein correlations are much stronger in squeezed light than in thermal light for particular values of the squeezing parameter. We may also interpolate between coherent and thermal light for other values of this parameter. For a k-mode squeezed coherent state,

$$g_k^2(0)=1+(1/k)[g^2(0)-1] , \quad (27)$$

where $g^2(0)$ is given by Eq. (24). Thus, for a k-mode squeezed state

$$g_k^2(0)=1+\frac{1}{k}\frac{2\sinh^4 r+(2\alpha^2+1)\sinh^2 r-\alpha^2\sinh 2r}{(\alpha^2+\sinh^2 r)^2} . \quad (28)$$

Thus we see that for a value of the squeezing parameter $1+\coth^2 r=k$ the squeezed coherent state shows the same bunching properties as that of thermal light. It is interesting to note that whereas for a single-mode state, no real value of r shows $g^2(0)=2$, for a k-mode squeezed state, a finite value of r shows the characteristics of a classical source. What we have seen from this discussion is that the multiplicity distribution in hadronic collisions is similar to a laser and the width is characterized by the "quantum noise" in the system not the classical noise as is inferred by the negative-binomial and the Glauber-Lachs distributions, since only quantum sources can show $g^2(0)<1$.

V. CONCLUSION

In hadronic production processes, purely classical distributions such as the Glauber-Lachs distribution have been a successful phenomenological tool.[9] However, we feel that particle creation in high-energy collisions is an inherently quantum process and therefore it is more appropriate to use a set of intrinsically quantum states such as squeezed states. Moreover, since the ingredients of strong interaction dynamics are completely parameterized by the squeezing parameter r, or the ratio of squeezing to coherence s, we can link up various processes such as hadron-proton, proton-proton, electron-electron, and neutrino-induced collisions, and isolate those aspects of these collisions that are universal and projectile independent. It has been noted that highly nonlinear, nonquadratic Hamiltonians generate these states. The possibility of nonlinear, nonquadratic interaction is stronger in quantum chromodynamics where, unlike the case of photons (having no self-coupling), three-gluon self-coupling is very strong and dynamically important.

It has been shown by Biyajima in an elegant fashion that the generalized Glauber-Lachs distribution for k sources is a general solution for a Fokker-Planck–type equation with the pseudorapidity playing the role of time for this the negative-binomial distribution is the stationary solution.[17] We are attempting a similar formulation for the k-mode squeezed-coherent-state distribution with the nonstochastic elements characterized by the squeezing parameter the results of which will be published later: then the connection between strong-interaction dynamics and squeezing will be clearer.

In conclusion, we would like to state that stochastic classical models are unable to accommodate the role of energy and momentum in hadronic processes, whereas in the quantum squeezed-coherent-state model we can easily do so (through the squeezing parameter), without having to sacrifice the conservation laws of strong interactions such as charge and color. To do so however, we must generalize these squeezed coherent states to SU(2) and SU(3) so as to enable us to impose global charge- and color-conservation constraints. Leading laser physicists have presented academic discussions of SU(2) squeezed coherent states.[18] A generalization to SU(3) and its subsequent use in hadronic production processes in order to give a physical interpretation to the squeezing parameter is the subject of future investigations.

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