π-π scattering and chiral Lagrangians

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We analyze results on π-π scattering using a framework based on nonlinear chiral Lagrangians both at the tree level and at one-loop order. Overall the data can be well understood up to energies of 0.7–0.8 GeV, as well as at threshold, and some of the Lagrangian parameters are well determined. As a side comment, we note that the Skyrmion solution is stable given this experimental fourth-order Lagrangian, but its mass is about 70% too high.

The reaction $\pi^\alpha+\pi^\beta\to\pi^\gamma+\pi^\delta$ is in many ways the purest and most fundamental hadronic process in QCD. It involves only the self-interactions of the lightest particle in the theory. Indeed, π-π scattering may be treated by the only rigorous methodology for low-energy QCD—chiral symmetry. The pion has a special role as the nearly Goldstone boson associated with the dynamically broken $SU(2)_L\times SU(2)_R$ chiral symmetry which would be an exact symmetry of QCD in the limit $m_u,m_d\to 0$. Since the masses are not far from this limit, chiral SU(2) is expected to be a very good symmetry, being almost as valid as isospin symmetry but with far more subtle dynamical consequences. The subject of π-π scattering[1] and its connection with chiral symmetry[2,3] has a long history. The modest purpose of this work is to update the phenomenological discussion in order to provide the best determination of some of the parameters of the effective chiral Lagrangian and to provide visual evidence for the compatibility of the data with chiral symmetry.

The modern discussion of chiral symmetry utilizes nonlinear effective Lagrangians.[3–5] These are organized in an expansion in terms of the energy, or eqivalently in terms of numbers of derivatives. One defines the SU(2) matrix Σ,

$$\Sigma=\exp\left(\frac{i\tau\cdot\pi}{F}\right) \tag{1}$$

transforming as

$$\Sigma\to L^\dagger\Sigma R \tag{2}$$

under $SU(2)_L\times SU(2)_R$ transformations, with π^i, $i=1,2,3$ being the pion field. The lowest-order effective Lagrangian involving Σ occurs at order E^2 (as both a ∂_μ and m_π count as one power of E=energy)

$$\mathcal{L}_2=\frac{F^2}{4}\mathrm{Tr}(\partial_\mu\Sigma\,\partial^\mu\Sigma^\dagger)+\frac{m^2F^2}{4}\mathrm{Tr}(\Sigma+\Sigma^\dagger)\ . \tag{3}$$

Working at this order one can identify F with $F_\pi=94$ MeV by consideration of the axial-vector current, and $m=m_\pi$. An expansion of $\mathcal{L}_2$ in powers of the pion field easily reproduces the Weinberg scattering lengths (see below). At this order, all of pion physics is uniquely specified in terms of F_π and m_π.

The effective Lagrangian at order E^4 has been worked out by Gasser and Leutwyler[3] (on whose work we rely in much of this paper). They find, in a slightly different notation

$$\mathcal{L}=\mathcal{L}_2+\mathcal{L}_4\ , \tag{4}$$

where the hadronic parts of $\mathcal{L}_4$ are

$$\begin{aligned}\mathcal{L}_4=&\frac{\alpha_1}{4}[\mathrm{Tr}(\partial_\mu\Sigma\,\partial^\mu\Sigma^\dagger)]^2\\&+\frac{\alpha_2}{4}\mathrm{Tr}(\partial_\mu\Sigma\,\partial_\nu\Sigma^\dagger)\mathrm{Tr}(\partial^\mu\Sigma\,\partial^\nu\Sigma^\dagger)\\&+\frac{\alpha_3}{4}[\mathrm{Tr}(m_\pi^2\Sigma)]^2+\frac{\alpha_4}{4}\mathrm{Tr}[\partial_\mu\Sigma\,\partial^\mu\Sigma^\dagger m_\pi^2(\Sigma+\Sigma^\dagger)]\\&+\frac{\alpha_5}{4}m_\pi^4[\mathrm{Tr}(\tau_3\Sigma)]^2\ ,\end{aligned} \tag{5}$$

and the α_i are dimensionless coefficients typically of order 10^{-3}–10^{-2}. The α_5 term arises from second-order isospin breaking, is extremely tiny, and will be dropped from now on. At low enough energies the effect of $\mathcal{L}_4$ is small compared to that of $\mathcal{L}_2$, while at high enough energies they become comparable and yet higher-order terms in the energy expansion are also equally important. However at low and moderate energies the use of only $\mathcal{L}_2$ and $\mathcal{L}_4$ seems sufficient. The parameters α_i encode the very-low-energy behavior of theories with dynamically broken chiral symmetry. Different underlying theories will predict different values of these parameters, and ultimately we should be able to obtain them as predictions of QCD.

In terms of the standard Mandelstam variables

$$\begin{aligned}s&=(p_\alpha+p_\beta)^2\ ,\\t&=(p_\alpha-p_\gamma)^2\ ,\\u&=(p_\alpha-p_\delta)^2\ ,\end{aligned} \tag{6}$$

the π-π scattering amplitudes are determined by crossing symmetry in terms of a single function $A(s,t,u)$ as

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$$T_{\alpha\beta;\gamma\delta}(s,t,u)=A(s,t,u)\delta_{\alpha\beta}\delta_{\gamma\delta}+A(t,s,u)\delta_{\alpha\gamma}\delta_{\beta\delta}+A(u,t,s)\delta_{\alpha\delta}\delta_{\beta\gamma}\ . \tag{7}$$

They can be decomposed into amplitudes of definite isospin as follows:

$$\begin{aligned}T^0(s,t,u)&=2T(+-;+-)-2T(+0;+0)\\&\quad+T(00;00)\\&=3A(s,t,u)+A(t,s,u)+A(u,t,s)\ ,\\T^1(s,t,u)&=2T(+0;+0)-T(++,++)\\&=A(t,s,u)-A(u,t,s)\ ,\\T^2(s,t,u)&=T(00;00)+T(00,+-)\\&\quad+T(++,++)\\&=A(t,s,u)+A(u,t,s)\ .\end{aligned}\tag{8}$$

In turn, the partial-wave amplitudes can be projected out

$$T_l^I(s)=\frac{1}{64\pi}\int_{-1}^{1}d(\cos\theta)P_l(\cos\theta)T^I(s,t,u)\ . \tag{9}$$

These have the following form in the region below inelastic thresholds:

$$T_l^I=\left[\frac{s}{s-4m_\pi^2}\right]^{1/2}e^{i\delta_l^I}\sin\delta_l^I \tag{10}$$

in terms of the phase shift δ_l^I. In practice this form is useful up to about 1 GeV.

At lowest order, the Weinberg results[2] on π-π scattering can be obtained from

$$A(s,t,u)=\frac{s-m_\pi^2}{F_\pi^2}\ . \tag{11}$$

At tree level to order E^4, the on-shell amplitude can be worked out by expanding Eq. (5), and yields

$$\begin{aligned}A(s,t,u)&=\frac{s-m^2}{F^2}+C(s,t,u)\ ,\\C(s,t,u)&=\frac{1}{2F^4}\{4\alpha_1(s-2m^2)^2+\alpha_2[s^2+(t-u)^2]\}\ .\end{aligned}\tag{12}$$

In order to include pion loop effects one must specify how one regulates the theory. We will use the result of Gasser and Leutwyler,[3] who utilize dimensional regularization with a renormalization scale μ. The renormalized coefficients of the chiral Lagrangian then become functions of the choice of μ. In this case the pionic amplitude is given by

$$A(s,t,u)=\frac{s-m^2}{F^2}+B(s,t,u)+\tilde{C}(s,t,u)\ , \tag{13}$$

where

$$B(s,t,u)=\frac{1}{6F_\pi^4}\left\{3(s^2-m^4)\bar{J}(s)+[t(t-u)-2m^2t+4m^2u-2m^4]\bar{J}(t)+[u(u-t)-2m^2u+4m^2t-2m^4]\bar{J}(u)-\frac{1}{96\pi^2}[21s^2+5(t-u)^2+8m^2s-26m^4]\right\}\ , \tag{14}$$

$$\bar{J}(a)=\frac{1}{16\pi^2}\left[\left[1-\frac{4m^2}{a}\right]^{1/2}\ln\left[\frac{\left[1-\frac{4m_\pi^2}{a}\right]^{1/2}-1}{\left[1-\frac{4m_\pi^2}{a}\right]^{1/2}+1}\right]+2\right]\ , \tag{15}$$

and $\tilde{C}(s,t,u)$ has the same functional form as $C(s,t,u)$, Eq. (12); but with α_i replaced by $\tilde{\alpha}_i$ where the renormalized coefficients are given by

$$\alpha_1^{\text{ren}}=\tilde{\alpha}_1+\frac{1}{96\pi^2}\ln\frac{m_\pi^2}{\mu^2},\quad \alpha_2^{\text{ren}}=\tilde{\alpha}_2+\frac{1}{48\pi^2}\ln\frac{m_\pi^2}{\mu^2}\ . \tag{16}$$

Note that the scattering depends on α_3 and α_4 only implicitly in that they enter the relation between (F,m^2) and the physical values (F_π,m_π^2). This distinction makes very little difference in our results, as we get much of our information above the threshold region. Gasser and Leutwyler[3] have estimated

$$m=(1.01\pm0.01)m_\pi,\quad F=0.94F_\pi\ , \tag{17}$$

at order E^4, and we will accept these values. The tree-level amplitudes are then easy to work out. We find

$$T_0^0=\frac{1}{32\pi F^2}\left[7m_\pi^2+\frac{40m_\pi^4}{F^2}(\alpha_1+\alpha_2)+(s-4m^2)\left[2+\frac{8m_\pi^2}{F^2}(4\alpha_1+3\alpha_2)+\frac{(s-4m_\pi^2)^2}{3F^2}(22\alpha_1+14\alpha_2)\right]\right],$$

$$T_1^1=\frac{1}{96\pi F^2}(s-4m_\pi^2)\left[1+\frac{4m_\pi^2}{F^2}(\alpha_2-2\alpha_1)+(\alpha_2-2\alpha_1)\frac{s-4m_\pi^2}{F^2}\right],$$

$$T_0^2=\frac{-1}{32\pi F^2}\left[2m_\pi^2\left[1-\frac{8m_\pi^2}{F^2}(\alpha_1+\alpha_2)\right]+(s-4m_\pi^2)\left[1-\frac{4m_\pi^2}{F^2}(2\alpha_1+3\alpha_2)\right]-\frac{4}{3}\frac{(s-4m_\pi^2)^2}{F^2}(\alpha_1+2\alpha_2)\right],\tag{18}$$

$$T_2^0=\frac{2\alpha_2+\alpha_1}{240\pi F^4}(s-4m_\pi^2)^2,\quad T_2^2=\frac{2\alpha_1+\alpha_2}{480\pi F^4}(s-4m_\pi^2)^2 .$$

Contained within these formulas are the Weinberg results for the scattering lengths, defined in terms of the threshold behavior

$$\mathrm{Re}T_0^I=a_0^I+b_0^I\left[\frac{s-4m_\pi^2}{4}\right]+\cdots ,$$
$$\mathrm{Re}T_1^1=a_1^1\left[\frac{s-4m_\pi^2}{4}\right]+\cdots .\tag{19}$$

These are at lowest order

$$a_0^0=\frac{7m_\pi^2}{32\pi F_\pi^2},\quad b_0^0=\frac{1}{4\pi F_\pi^2},$$
$$a_0^2=\frac{-m_\pi^2}{16\pi F_\pi},\quad b_0^2=\frac{-1}{8\pi F_\pi^2},\tag{20}$$
$$a_1^1=\frac{1}{24\pi F_\pi^2}.$$

Note that the lowest-order results will violate the most basic consequence of unitarity,

$$|T_l^I|\le\left[\frac{s}{s-4m_\pi^2}\right]^{1/2},\tag{21}$$

at about 700 MeV for T_0^0 and 1 GeV for T_0^2. Tree-level amplitudes are necessarily real and do not respect unitarity. In order to generate the proper unitary phase shifts, one must include rescattering through real, physical intermediate states. These are valid low-energy processes, and are included in the energy expansion through loop effects, such as given in Fig. 1.

When loops are considered, one must change all factors of α_1,α_2 to $\tilde\alpha_1,\tilde\alpha_2$ and add to each amplitude a contribution B_l^I from the $B(s,t,u)$ term in Eq. (14). For example,

$$T_2^0=\frac{2\tilde\alpha_2+\tilde\alpha_1}{240\pi F^2}(s-4m_\pi^2)^2+B_2^0 .\tag{22}$$

The imaginary parts of B_l^I are calculable analytically, and are simply those values which will unitarize the lowest-order (order-E^2) amplitude. Specifically

$$\mathrm{Im}B_0^0=\frac{-\pi}{(32\pi F^2)^2}\left[\frac{s-4m_\pi^2}{s}\right]^{1/2}(2s-m_\pi^2)^2,$$
$$\mathrm{Im}B_1^1=\frac{-\pi}{(96\pi F^2)^2}\left[\frac{s-4m_\pi^2}{s}\right]^{1/2}(s-4m_\pi^2)^2,$$
$$\mathrm{Im}B_0^2=\frac{-\pi}{32\pi F^2}\left[\frac{s-4m_\pi^2}{s}\right]^{1/2}(s-2m_\pi^2)^2,\tag{23}$$
$$\mathrm{Im}B_2^0=\mathrm{Im}B_2^2=0 .$$

Because of the logarithmic factors in $B(s,t,u)$ the real parts of the partial-wave projection cannot be accomplished analytically. Instead, we have calculated it numerically, with results displayed in Fig. 2.

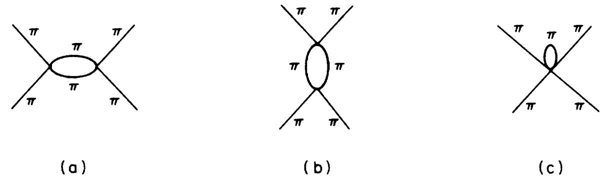

FIG. 1. The pion loop diagrams which contribute to the scattering amplitudes, renormalized to one-loop order.

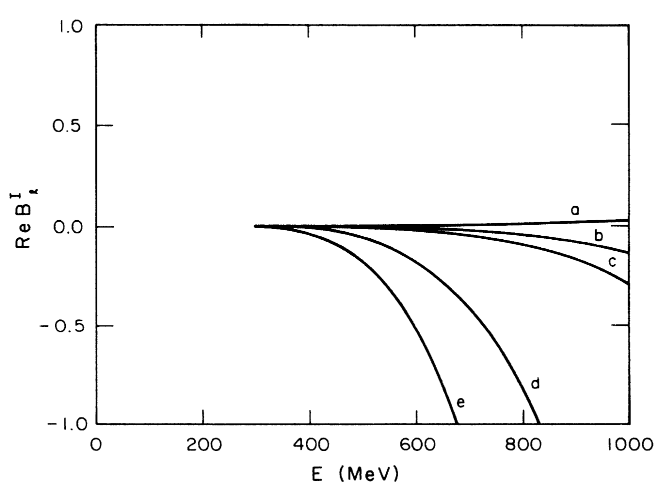

FIG. 2. The real parts of the loop corrections factor defined using Eqs. (14) and (22). The curves a, b, c, d, and e refer to T_1^1, T_2^2, T_2^0, T_0^2, and T_0^0 in order.

Data on π-π scattering come from a variety of sources. The most reliable is that from $K^- \to \pi^+\pi^- e^- \bar{\nu}$, where rescattering in the final state leads to an observation of $\delta_0^0 - \delta_1^1$, which is essentially pure δ_0^0 in this energy range. This information is available only at low energy (for obvious reasons) but is very free of theoretical problems. It is also traditional to obtain π-π scattering results from $\pi p \to \pi\pi p$ or $\pi p \to \pi\pi\Delta$ by an extrapolation to the pion pole in the t channel. The extrapolation required makes these measurements somewhat more problematic, and experiments will often disagree with each other outside of their quoted error bars. The constraints on the extrapolation at $t=0$ and the inclusion of absorption or lack of it have been developed pragmatically in this field, but have not been justified by solid theory to the best of our knowledge. Often the assumed details of the extrapolation are more significant than the quoted statistical errors. This seems at present to preclude a truly reliable determination of the scattering, aside from the low-energy K_{e4} data. Our procedure has been to take some of the good recent and older results which reflect the range of values found. For example, in T_0^0 there are other experiments which fill in the gap between the data of Refs. 6 and 7, but we do not know any way to favor one value or another in this range. However, the general trends in the data are reasonably clear, and it is fortunate that use of just these general features yields a surprisingly good determination of the chiral parameters. This is due to the constraints of five independent channels on only the two parameters α_1, α_2. That a good fit can be obtained at all is some important evidence for the chiral-Lagrangian framework.

The comparison of the data with chiral symmetry is traditionally given in terms of the scattering lengths and slopes at threshold. In any given channel these are relatively poorly determined by a single experiment. However, by tying together different channels into the single function $A(s,t,u)$, and using analyticity assumptions, one can gain in power. This approach, the Roy equations,[1] uses data over a larger energy range in order to learn about the threshold parameters. The disadvantage of the method is that the rather complicated machinery involved obscures the dependence of the results on differing and often conflicting experimental inputs. We will instead simply compare the chiral predictions directly with the data over the full relevant energy range. This procedure must be compatible with the Roy equations, as the chiral amplitude must satisfy the analyticity properties, at least order by order in the energy expansion. If a good representation of the data is found throughout the region modestly above threshold, then the scattering lengths must also be in good agreement. In fact the full energy range gives more stringent constraints than just the scattering lengths. For example, the Gasser and Leutwyler values of α_1 and α_2, determined from scattering lengths, yield the shaded region in Fig. 3 at only the 1σ level. Small variations at threshold are amplified as the energy increases. Inspection of the formulas for T_J^I shows that a much fuller compatibility of the data with chiral symmetry is tested by using a larger energy range. In addition, the limits and strengths of the theory are much more transparent.

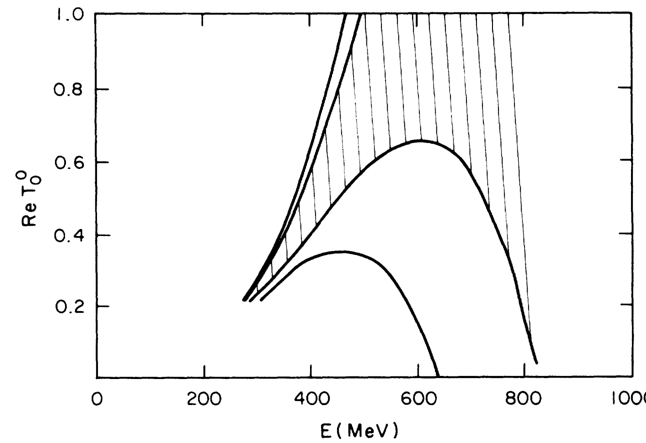

FIG. 3. The range of ReT_0^0 implied by the threshold determination of α_1 and α_2 in Ref. 3.

Some applications of chiral symmetry, such as Skyrmions, deal with tree-level Lagrangians. We may attempt to determine the best tree-level chiral Lagrangian. Such a method could never reproduce the imaginary part of a scattering amplitude. The only amplitude where the imaginary part is important is T_0^0. Here we fit the real

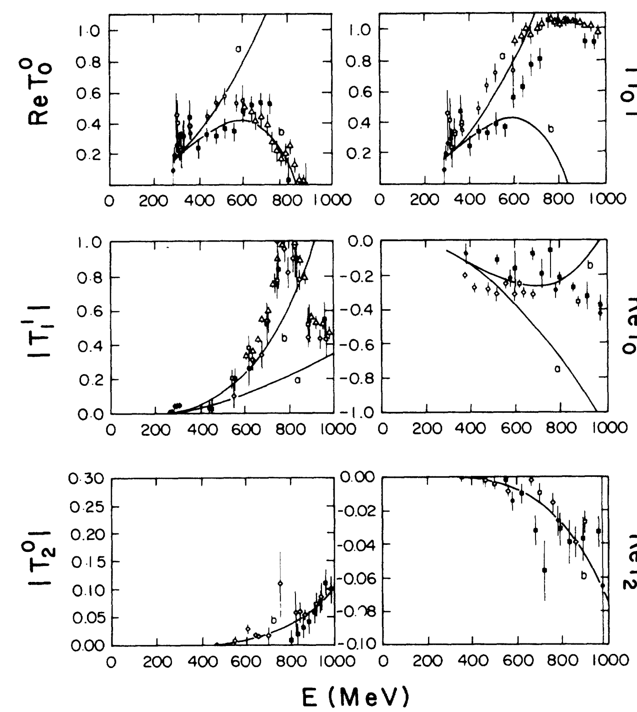

FIG. 4. Tree-level π-π scattering amplitudes. The key on the data is as follows: (i) For T_0^0, ■, Ref. 6; ○, Ref. 7; ◇, Ref. 8; ●, Ref. 9; △, Ref. 10; (ii) for T_1^1, ■, $\pi^-\pi^0$; ◇, $\pi^+\pi^0$; □, $\pi^+\pi^-$, from Ref. 11; △, Ref. 10; ●, Ref. 9; (iii) for T_0^2, □, Ref. 12; ■, Ref. 13; ◇, Ref. 14; ●, Ref. 15; (iv) for T_2^0, ◇, Ref. 11; ■, Ref. 16; (v) for T_2^2, ■, Ref. 11; ◇, Ref. 12; □, Ref. 17; ●, Ref. 15. Curve a is the lowest-order prediction, while curve b is the best fit, described in the text.

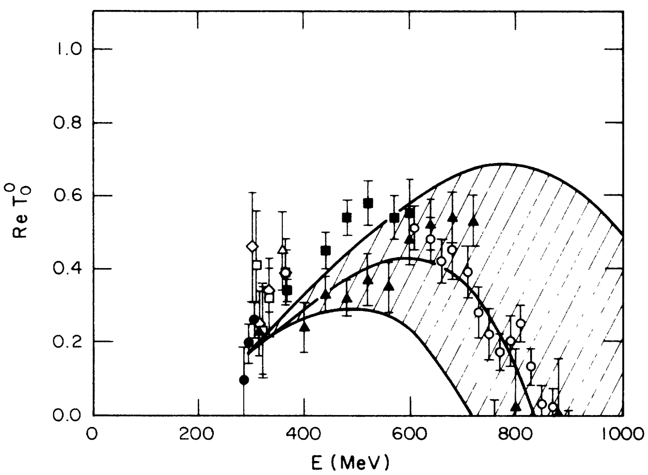

FIG. 5. The values of $\mathrm{Re}T_0^0$ which are spanned by varying the fit parameters α_1 and α_2 by 0.002.

part $\mathrm{Re}T_0^0$, but of course cannot reproduce the absolute value $|T_0^0|$. With this exception, a very satisfactory representation of the data is found with

$$\alpha_1=-0.0092, \quad \alpha_2=+0.0080 \tag{24}$$

as can be seen in Fig. 4.

When the data have obvious disagreements, it is hard to assign a well-defined error bar to these fits. However, shifts of these values by 0.002 produce demonstrably worse fits, as shown in Fig. 5, so we heuristically will assign error bars of this value. A similar exercise yields the solid line in Fig. 6 for the formulas including loops. The resulting parameters are

$$\tilde{\alpha}_1=-0.007, \quad \tilde{\alpha}_2=+0.013 \ . \tag{25}$$

Again a satisfactory fit is obtained up to energies of about 700 MeV. The renormalized coefficients $\alpha_1(\mu)$ are given at 1 GeV by

$$\alpha_1(1\text{ GeV})=-0.011, \quad \alpha_2(1\text{ GeV})=0.0046 \ ,$$

or at $\frac{1}{2}$ GeV by

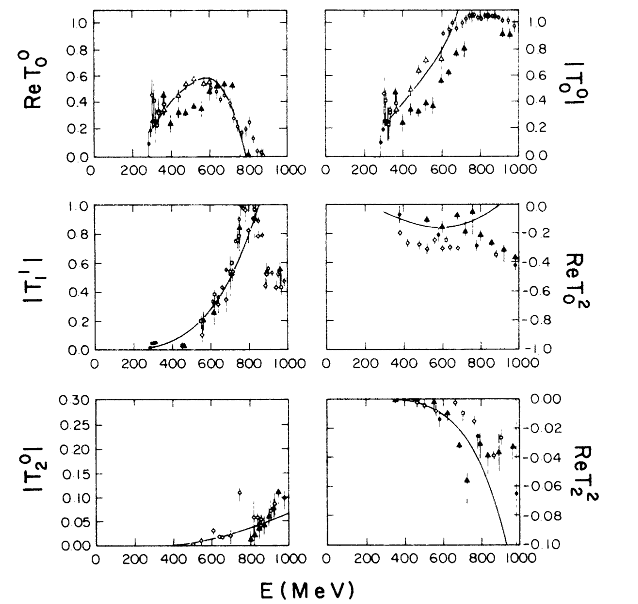

FIG. 6. The best fit to pion scattering at the one-loop level.

$$\alpha_1(500\text{ MeV})=-0.010, \quad \alpha_2(500\text{ MeV})=0.0075 \ .$$

The latter values are very similar to the tree-level parameters.

If we go to threshold, we can also compare with the low-energy behavior found in the K_{e4} experiments. Figure 7 displays these results in a clearer fashion. The curves display the various solutions discussed above. We see that the threshold behavior is reasonable, with a slight flavoring of the results based on the one-loop calculations.

We can also address some issues which are somewhat related to the present problem. One is the often used procedure of keeping only the leading nonanalytic terms from loops.[19] In our case this consists of keeping the logarithmic factor in J, Eq. (15), and setting $\alpha_1^{\text{ren}}=\alpha_2^{\text{ren}}=0$, such that

$$\tilde{\alpha}_1=\frac{1}{96\pi^2}\ln\left[\frac{\mu^2}{m_\pi^2}\right], \quad \tilde{\alpha}_2=\frac{1}{48\pi^2}\ln\left[\frac{\mu^2}{m_\pi^2}\right] .$$

Specifically the leading nonanalytic (LNA) loop correction is of the form

$$(B+C)_{\text{LNA}}=\frac{1}{6F^4}\left[3(s^2-m^4)K(s)+[t(t-u)-2m^2t+4m^2u-2m^4]K(t)\right.$$
$$\left.+[u(u-t)-2m^2u+4m^2t-2m^4]K(u)+\frac{1}{16\pi^2}(6s-7m^2)m^2\ln\frac{m^2}{\mu^2}\right] ,$$

where

$$K(a)=\frac{1}{16\pi^2}\left[\left[1-\frac{4m^2}{a}\right]^{1/2}\ln\frac{\left[1-\frac{4m_\pi^2}{a}\right]^{1/2}-1}{\left[1-\frac{4m_\pi^2}{a}\right]^{1/2}+1}-\ln\left[\frac{m_\pi^2}{\mu^2}\right]\right] .$$

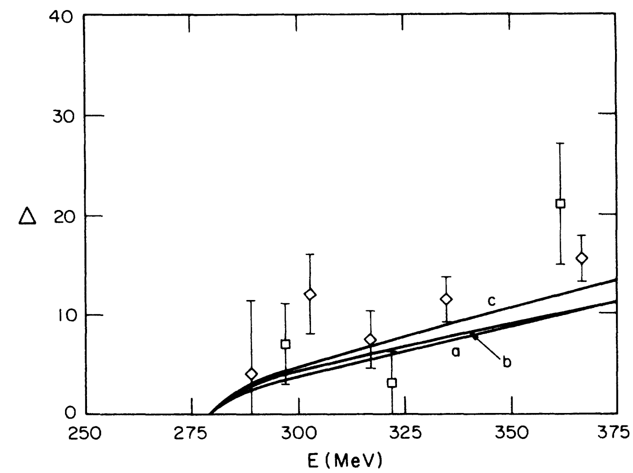

FIG. 7. Comparison of our fits to the phase differences $\Delta \equiv \delta_0^0 - \delta_1^1$ obtained from K_{e4} data (◇, Ref. 8; □, Ref. 18). The curve a is the lowest-order chiral prediction, curve b is the tree-level fit, while curve c is the full one-loop-level calculation.

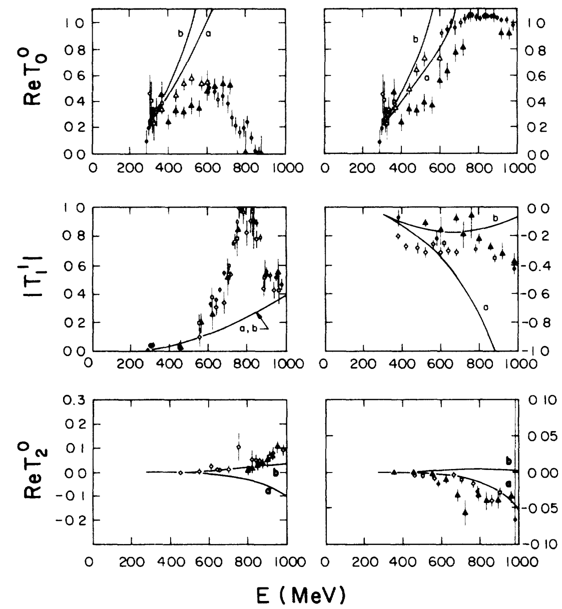

FIG. 8. The leading nonanalytic corrections to the one-loop amplitude do not yield a good fit by themselves. The curve a is for $\mu = 0.5$ GeV, while curve b is $\mu = 1$ GeV.

Notice that these formulas are finite in the chiral limit, $m_\pi \to 0$. The resulting amplitudes are displayed in Fig. 8 for $\mu = 1$ and $\frac{1}{2}$ GeV. Neither provides a reasonable description to the data.

Another issue consists of soliton solutions to this effective action. In the study of Skyrmion models[20,21] of the proton, one looks for soliton solutions for the chiral field of the form

$$\Sigma = \exp[i\tau \cdot \hat{r} F(r)] \; . \tag{26}$$

Often researchers will use a completely unjustified truncation of the chiral Lagrangian to include only one four-derivative Lagrangian, rather than the two which appear in reality, and will treat its coefficient as a free parameter. Instead one should take the full Lagrangian, with the parameters given to us by nature, and ask about the existence and properties of a soliton solution. The parameter space for this more complete soliton has been explored.[21,22] To convert $\mathcal{L}_4$ to the standard parametrizations of the Skyrmion community

$$\mathcal{L}_4 = \frac{1}{32e^2}\mathrm{Tr}\{[(\partial_\mu\Sigma)\Sigma^\dagger, (\partial_\nu\Sigma)\Sigma^\dagger]^2\} + \frac{\gamma}{8e^2}[\mathrm{Tr}(\partial_\mu\Sigma\,\partial^\mu\Sigma^\dagger)]^2 \; , \tag{27}$$

we note that

$$\frac{1}{32e^2} = \frac{\alpha_2}{8} = 0.0010 \; ,$$

$$\frac{\gamma}{8e^2} = \frac{\alpha_1 + \alpha_2}{4} = -0.0003, \quad \gamma = -0.075 \; ,$$

where the numbers follow from our tree-level fit. This is the appropriate usage, as loop corrections have not yet been added to Skyrmion calculations. There is an instability[22] in the Skyrme soliton for positive $\gamma > 0.12$. However, our small negative value avoids this. For this value, an expansion in γ, such as given in Ref. 21, is feasible. Using this formalism, the nucleon mass is

$$m_p = M + \frac{3}{8\lambda} \; ,$$

$$M = \frac{73F_\pi}{e}(1 - 0.77\gamma) \approx 1.3 \text{ GeV} \; ,$$

$$\lambda^{-1} = \frac{e^3 F_\pi}{53.3}(1 + 1.1\gamma) \approx 0.3 \text{ GeV} \; ,$$

$$m_p \approx 1.6 \text{ GeV} \; .$$

This value is about 70% high.

Overall, we have displayed the quality of chiral-symmetry predictions in π-π scattering. The effect of nonleading chiral Lagrangians is clearly visible and highly constrained by the data. The limits to these predictions become obvious at energies of around 0.7–0.8 GeV. The understanding of threshold and moderate-energy scattering amplitudes forms a complex test of chiral symmetry.

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