

## Phenomenology of a horizontal gauge boson in $e^+e^-$ collisions

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(Received 9 November 1987; revised manuscript received 12 May 1988)

The effects of an extra neutral gauge boson  $\tilde{Z}$  predicted by the  $Sp(8) \times U(1)$  model, are studied both on and off the  $Z$  resonance in  $e^+e^-$  collisions. Distinctive features are pointed out.

The family repetition of quarks and leptons strongly suggests that there should be a larger flavor group than the familiar  $SU_L(2) \times U_Y(1)$  group.<sup>1</sup> Given the six left-handed quarks and leptons, it would be desirable to include them in a single, six-dimensional representation  $\mathbf{6}$  of a simple flavor gauge group  $G_F$ . There are two obvious physical constraints on  $G_F$ . First, the theory must not be anomalous. Second, we must be able to embed the usual  $SU_L(2)$  in  $G_F$  such that  $\mathbf{6} \rightarrow 3 \times 2$ . Out of a few possible candidates for  $G_F$ , it was shown<sup>2</sup> that there is a unique extension of  $SU_L(2) \times U_Y(1)$  into the anomaly-free  $Sp_L(6) \times U_Y(1) = G_F$ . Under  $Sp_L(6)$ , the left-handed fermions (leptons and quarks) transform like  $\mathbf{6}$ , while all right-handed fermions are singlets. Note that this extension seems rather natural since  $Sp(2) \simeq SU(2)$ . A doublet of  $Sp_L(2)$  [ $SU_L(2)$ ], for one generation, is readily generalized to a sextet of  $Sp_L(6)$ , for three generations.  $Sp(6)$  can be naturally broken into  $[SU(2)]^3 = SU(2)_1 \times SU(2)_2 \times SU(2)_3$ , where  $SU(2)_i$  operates on the  $i$ th generation exclusively. Thus, the standard  $SU_L(2)$  is to be identified with the diagonal  $SU(2)$  subgroup of  $[SU(2)]^3$ . In terms of the  $SU(2)_i$  gauge boson  $\mathbf{A}_i$ , the  $SU_L(2)$  gauge bosons are given by  $\mathbf{A} = (1/\sqrt{3})(\mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3)$ . Of the other orthogonal combinations of  $\mathbf{A}_i$ , it was found that  $\mathbf{A}' = (1/\sqrt{6})(\mathbf{A}_1 + \mathbf{A}_2 - 2\mathbf{A}_3)$  has a mass scale bounded by  $\geq 1$  TeV. They would give rise to interesting physics at the TeV energy range.<sup>3</sup>

We turn now to the possible existence of a fourth generation of fermions. If they exist,  $G_F$  may be generalized to  $Sp_L(8) \times U_Y(1)$  (Ref. 4). In this case, we would have  $\mathbf{A} = \frac{1}{2}(\mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3 + \mathbf{A}_4)$ . There will again be additional gauge bosons, the lightest being  $\tilde{\mathbf{A}} = (1/\sqrt{12})(\mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3 - 3\mathbf{A}_4)$ . In this work we will concentrate on the neutral member,  $\tilde{Z}$ . We wish to analyze the effects of the presence of  $\tilde{Z}$  on  $e^+e^-$  collisions, both on and off the  $Z$  resonance.

Several articles have dealt with the effects of an additional neutral gauge boson in  $e^+e^-$  collisions.<sup>5</sup> In general an additional neutral gauge boson, here  $\tilde{Z}$ , will mix with the standard  $Z$ , resulting in physical states which are mixtures of  $Z$  and  $\tilde{Z}$ . Hence, the physical  $Z$  will have different mass and couplings, which can be revealed as deviations from the standard-model predictions on and

off the  $Z$  resonance in  $e^+e^-$  collisions.

With the additional gauge boson  $\tilde{Z}$ , the neutral-current Lagrangian is generalized to contain an additional term

$$-L_{\text{NC}} = eJ_{\text{em}}^\mu A_\mu + g_Z J_Z^\mu Z_\mu + g_{\tilde{Z}} J_{\tilde{Z}}^\mu \tilde{Z}_\mu, \quad (1)$$

where  $g_Z = \sqrt{(1-x_W)/3}g_Z = g/\sqrt{3}$ ,  $x_W = \sin^2\theta_W$ , and  $g = e/\sin\theta_W$ . In this paper we use  $x_W = 0.23$ . The neutral currents  $J_Z$  and  $J_{\tilde{Z}}$  are given by

$$J_Z^\mu = \frac{1}{2} \sum_f \bar{\psi}_f \gamma^\mu (g_V^f + g_A^f \gamma_5) \psi_f, \quad (2)$$

$$J_{\tilde{Z}}^\mu = \frac{1}{2} \sum_f \bar{\psi}_f \gamma^\mu (\tilde{g}_V^f + \tilde{g}_A^f \gamma_5) \psi_f, \quad (3)$$

where  $g_V^f = (T_{3L} - 2x_W Q)_f$ ,  $g_A^f = (T_{3L})_f$ , and  $\tilde{g}_V^f = \tilde{g}_A^f = (T_{3L})_f$ . Here  $(T_{3L})_f$  and  $Q_f$  are the third component of weak isospin and electric charge of fermion  $f$ , respectively. Let  $\phi$  denote the mixing angle between  $Z$  and  $\tilde{Z}$ , then the physical (mass eigenstates) gauge bosons, denoted by  $Z_1$  and  $Z_2$  are given as linear combinations of  $Z$  and  $\tilde{Z}$ ,

$$\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} Z \\ \tilde{Z} \end{bmatrix}, \quad (4)$$

and the neutral-current Lagrangian reads

$$-L_{\text{NC}} = g_Z \sum_{i=1}^2 \left[ \sum_f \bar{\psi}_f \gamma_\mu (g_V^f + g_A^f \gamma_5) \psi_f \right] Z_i, \quad (5)$$

where  $g_V^f$  and  $g_A^f$  are the vector and axial-vector couplings of fermion  $f$  to physical gauge boson  $Z_i$ , respectively. In the  $Sp(8) \times U(1)$  model they are given by

$$g_{V_1, A_1}^f = \frac{1}{2} \left[ g_{V, A}^f \cos\phi + \frac{g_{\tilde{Z}}}{g_Z} \tilde{g}_{V, A}^f \sin\phi \right], \quad (6)$$

$$g_{V_2, A_2}^f = \frac{1}{2} \left[ -g_{V, A}^f \sin\phi + \frac{g_{\tilde{Z}}}{g_Z} \tilde{g}_{V, A}^f \cos\phi \right]. \quad (7)$$

The change of the fermions couplings provided by Eqs. (6) and (7) will affect measurements in  $e^+e^-$  collisions. Among the quantities that are sensitive to this change are the forward-backward and the left-right asymmetries. The forward-backward asymmetry  $A_{FB}$  is defined by

$$A_{FB} = \frac{\int_0^1 \frac{d}{d \cos \theta} \sigma(e^+ e^- \rightarrow \mu^+ \mu^-) d \cos \theta - \int_{-1}^0 \frac{d}{d \cos \theta} \sigma(e^+ e^- \rightarrow \mu^+ \mu^-) d \cos \theta}{\int_{-1}^1 \frac{d}{d \cos \theta} \sigma(e^+ e^- \rightarrow \mu^+ \mu^-) d \cos \theta}, \quad (8)$$

where  $\theta$  is the angle between the incident electron and the outgoing muon. On the other hand, the left-right asymmetry is defined by

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}, \quad (9)$$

where  $\sigma_L$  ( $\sigma_R$ ) is the cross section for scattering of a left- (right-) handed electron on an unpolarized positron. With the fermions couplings provided by Eqs. (6) and (7), the general expressions for the above asymmetries are written explicitly as

$$A_{FB} = \frac{3}{4D} \left[ 2 \sum_{j=1}^2 g_{A_j}^e g_{A_j}^e \operatorname{Re} \Delta_j + \sum_{j,k=1}^2 (g_{V_j}^e g_{A_k}^e + g_{A_j}^e g_{V_k}^e) (g_{V_j}^f g_{A_k}^f + g_{A_j}^f g_{V_k}^f) \operatorname{Re}(\Delta_j \Delta_k^*) \right], \quad (10)$$

$$A_{LR} = \frac{P_e}{D} \left[ 2 \sum_{j=1}^2 g_{V_j}^e g_{A_j}^e \operatorname{Re} \Delta_j + \sum_{j,k=1}^2 2g_{V_j}^e g_{A_k}^e (g_{V_j}^f g_{V_k}^f + g_{A_j}^f g_{A_k}^f) \operatorname{Re}(\Delta_j \Delta_k^*) \right], \quad (11)$$

where the superscript  $f$  refers to the final-state lepton,  $P_e$  is the degree of longitudinal polarization of the electron beam, and

$$D = 1 + 2 \sum_{j=1}^2 g_{V_j}^e g_{V_j}^e \operatorname{Re} \Delta_j + \sum_{j,k=1}^2 (g_{V_j}^e g_{V_k}^e + g_{A_j}^e g_{A_k}^e) (g_{V_j}^f g_{V_k}^f + g_{A_j}^f g_{A_k}^f) \operatorname{Re}(\Delta_j \Delta_k^*), \quad (12)$$

where

$$\Delta_j = \frac{s}{x_w(1-x_w)[(s-M_j^2)+iM_j\Gamma_j]}, \quad (13)$$

here  $M_j$  and  $\Gamma_j$  are the mass and total width of gauge boson  $Z_j$ , respectively.

On the  $Z_1$  resonance, the cross section is dominated by the pole in the  $Z_1$  propagator and the forward-backward asymmetry is approximated by

$$A_{FB}(\sqrt{s} = M_{Z_1}) = 3 \left[ \frac{g_{V_1}^e g_{A_1}^e}{(g_{V_1}^e)^2 + (g_{A_1}^e)^2} \right]^2, \quad (14)$$

where  $e$ - $\mu$ - $\tau$  universality provided by the  $\operatorname{Sp}(8) \times \operatorname{U}(1)$  model is assumed in deriving Eq. (14). With the same approximation, the left-right asymmetry on the  $Z_1$  resonance is given by

$$A_{LR}(\sqrt{s} = M_{Z_1}) = P_e \frac{2g_{V_1}^e g_{A_1}^e}{(g_{V_1}^e)^2 + (g_{A_1}^e)^2}. \quad (15)$$

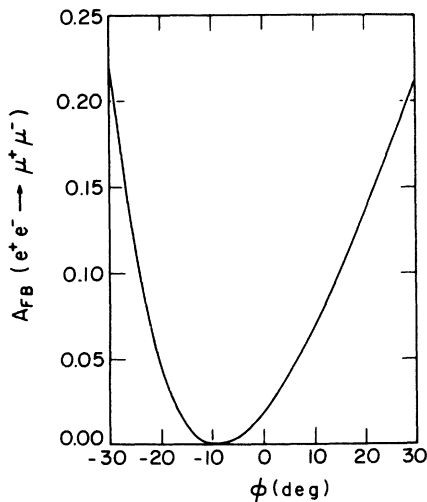


FIG. 1. The forward-backward asymmetry  $A_{FB}$  for  $e^+ e^- \rightarrow \mu^+ \mu^-$  at  $\sqrt{s} = M_{Z_1}$  as a function of the mixing angle  $\phi$ .

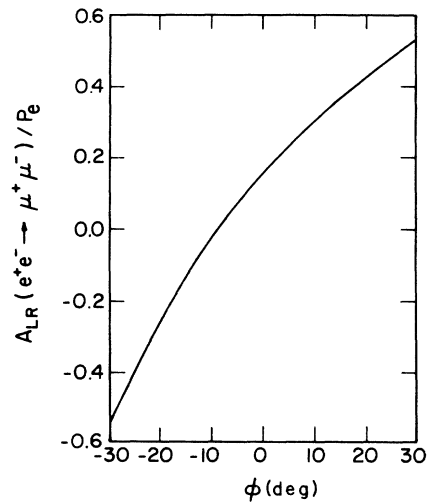


FIG. 2. The left-right asymmetry  $A_{LR}/P_e$  for  $e^+ e^- \rightarrow \mu^+ \mu^-$  at  $\sqrt{s} = M_{Z_1}$  as a function of the mixing angle  $\phi$ .

In general,  $P_e$  depends on the specific  $e^+e^-$  machine. For example, at the SLAC Linear Collider (SLC), beams with polarizations of  $\simeq 50\%$  will be available. Beam polarization at the CERN collider LEP is much less certain. In Figs. 1 and 2 we present the  $\text{Sp}(8)\times\text{U}(1)$  prediction for  $A_{FB}$  and  $A_{LR}/P_e$  at  $\sqrt{s}=M_{Z_1}$ , respectively, as functions of the  $Z-\tilde{Z}$  mixing angle  $\phi$ .

Measurements of the total width and branching fractions into known fermions at the  $Z_1$  peak provide another means of determining the mixing angle  $\phi$ . The total width is defined by

$$\begin{aligned} \Gamma(Z_1 \rightarrow \text{all}) &= \sum_f \Gamma(Z_1 \rightarrow f\bar{f}) \\ &= \sum_f \frac{2}{3} N_f \left[ \frac{GM_{Z_1}^3}{\sqrt{2}\pi} \right] [(g_{V_1}^f)^2 + (g_{A_1}^f)^2], \end{aligned} \quad (16)$$

where  $N_f$  is a color factor ( $N_f=3$  for quarks and  $N_f=1$  for leptons). With  $M_{Z_1}=92$  GeV, the total width and branching fractions for  $Z_1$  are shown, respectively, in Figs. 3 and 4 as functions of the mixing angle  $\phi$ . Figure 4 shows that the branching fractions are less sensitive to variations in  $\phi$ .

The  $Z_1$  factories, SLC at SLAC and LEP I at CERN, are proper places to look for the effects of the presence of  $\tilde{Z}$  in  $e^+e^-$  collisions at the  $Z_1$  peak. They can achieve a maximum collision energy of 100 GeV and will be optimized to run on the  $Z_1$  (Ref. 6). With the designed luminosities they will be capable of providing copious (up to  $10^6$  per year)  $Z_1$  events. This will allow precise, high-statistics studies on the  $Z_1$  resonance.

Constraints on the mass and the mixing angle of the additional neutral gauge bosons can be obtained from existing data on neutral-current experiments. In spite of the impressive agreement between the standard  $\text{SU}(2)\times\text{U}(1)$  electroweak model and experiment,<sup>7</sup> the experimental data allow for an extra neutral gauge boson. In Fig. 5 we present constraints on  $M_{Z_2}$  and  $\phi$  obtained from neutral-current data and from predicted measure-

ments of the forward-backward and left-right asymmetries in  $e^+e^-$  collisions. We show the 90% C.L. in the  $M_{Z_2}-\phi$  plane that results from a fit to existing data on parameters involved in neutral-current processes.<sup>8</sup> The neutral-current constraints allow large mixing and put a lower limit on  $M_{Z_2}$ ,  $M_{Z_2} \geq 103$  GeV. We also show boundaries in the  $M_{Z_2}-\phi$  plane expected from measurements of  $A_{FB}$  and  $A_{LR}$  at  $\sqrt{s}=M_{Z_1}$  with  $10^4$ ,  $10^5$ , and  $10^6$   $Z_1$ 's. The boundaries are almost independent of  $M_{Z_2}$ ; the bending of the boundaries near the bottom of the graph is due to the finite-width effects of  $M_{Z_2}$ . With  $10^6$   $Z_1$  events and a 1% systematic uncertainty, a measurement of  $A_{LR}$  in the  $\mu^+\mu^-$  channel will confine the mixing angle to within  $\Delta\phi \simeq \pm 1^\circ$ .

Now we would like to extend our investigations and consider the effects of the presence of  $\tilde{Z}$  in regions of energies off the  $Z_1$  resonance in  $e^+e^-$  collisions. First we note that the corresponding effects on top of the  $Z_1$  are demonstrated in Figs. 1–4 as deviations from the standard-model predictions. These deviations are sensitive to the  $Z-\tilde{Z}$  mixing angle and can be used to probe the gauge-boson mass via a comparison with the neutral-current constraints given in Fig. 5. However, it is possible that there is not much mixing between  $Z$  and  $\tilde{Z}$ . In this case, no such deviations will show up on the  $Z_1$  resonance. But, as we will show, pronounced effects can show up off the  $Z_1$  resonance regardless of the value of the mixing angle. As such, measurements of the forward-backward and left-right asymmetries off the  $Z_1$  resonance provide another means of testing new physics from the  $\text{Sp}(8)\times\text{U}(1)$  model.

In Fig. 6 we consider different values of  $M_{Z_2}$  and for each value we present the expected forward-backward asymmetry as a function of  $\sqrt{s}$  for the minimum and maximum values of the mixing angle allowed by constraints from neutral-current experiments. For comparison, we also present the forward-backward asymmetry predicted by the standard model. We find a distinctive modification of the standard-model predictions featured

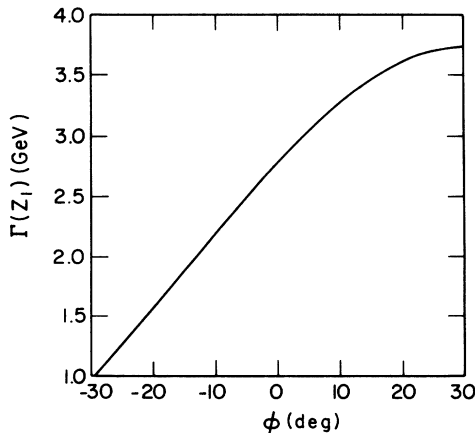


FIG. 3. The total width of the  $Z_1$  gauge boson as a function of the mixing angle  $\phi$ .

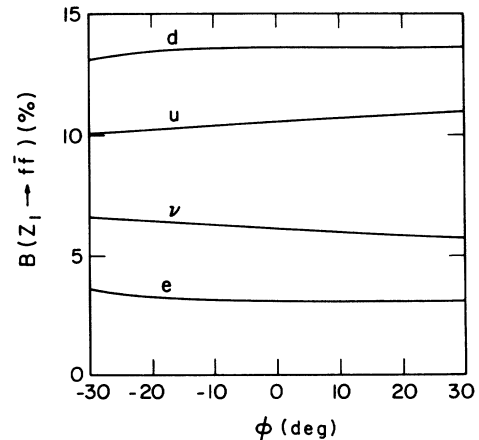


FIG. 4. The branching fraction of the  $Z_1$  gauge boson for  $u\bar{u}$ ,  $d\bar{d}$ ,  $e^+e^-$ , and  $\nu\bar{\nu}$  final states as a function of the mixing angle  $\phi$ .

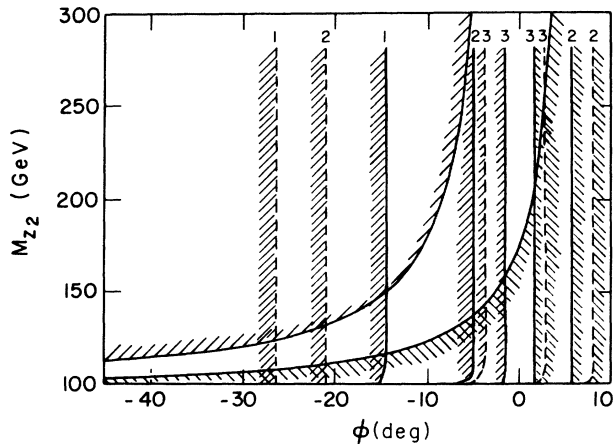


FIG. 5. The 90%-C.L. region in the  $M_{Z_2}$ - $\phi$  plane obtained from constraints derived from existing neutral-current data (solid curve), and from predicted measurements of  $A_{FB}$  and  $A_{LR}$  at  $\sqrt{s} = M_{Z_1}$  with  $10^4$ ,  $10^5$ , and  $10^6$   $Z_1$ 's (regions bounded by curves 1, 2, and 3, respectively; the dashed curves are for  $A_{FB}$  and the solid curves are for  $A_{LR}$ ).

in the existence of a dip due to cancellation among different contributions. The location of the dip is about 10% below the  $Z_2$  threshold. For fixed  $M_{Z_2}$ , we found that, while the location of the dip is insensitive to variations in  $\phi$ , the dip deepens for larger mixings. The effect

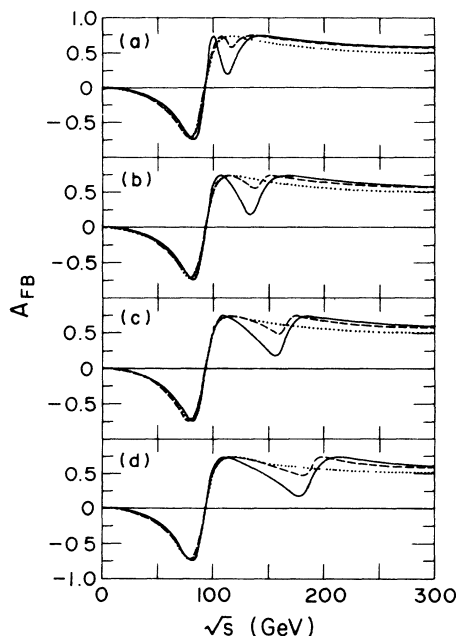


FIG. 6. The forward-backward asymmetry  $A_{FB}$  for  $e^+e^- \rightarrow \mu^+\mu^-$  as a function of  $\sqrt{s}$  for (a)  $M_{Z_2} = 125$  GeV and  $\phi = -0.4$  (solid curve) and  $-0.15$  (dashed curve); (b)  $M_{Z_2} = 150$  GeV and  $\phi = -0.25$  (solid curve) and  $-0.04$  (dashed curve); (c)  $M_{Z_2} = 175$  GeV and  $\phi = -0.18$  (solid curve) and  $0.0$  (dashed curve); (d)  $M_{Z_2} = 200$  GeV and  $\phi = -0.15$  (solid curve) and  $0.02$  (dashed curve). The dotted curve for all cases is the standard-model predictions without  $\tilde{Z}$ .

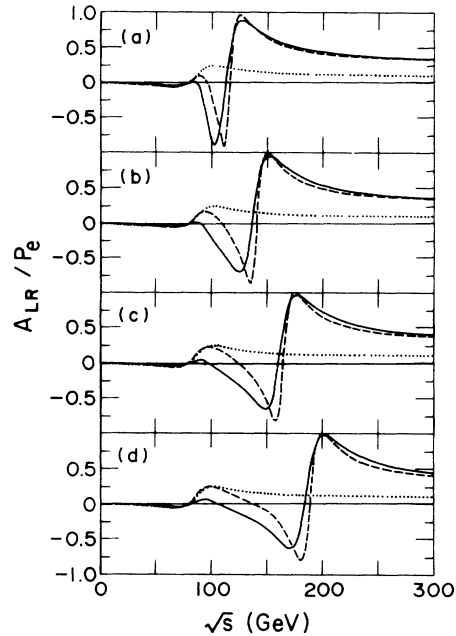


FIG. 7. The left-right asymmetry  $A_{LR}/P_e$  for  $e^+e^- \rightarrow \mu^+\mu^-$  as a function of  $\sqrt{s}$  for the same masses and mixing angles considered in Fig. 6. The dotted line for all cases is the standard-model predictions without  $\tilde{Z}$ .

of  $Z_2$  on the left-right asymmetry is found to be even more pronounced. Figure 7 shows the left-right asymmetry as a function of  $\sqrt{s}$  for the same values of  $M_{Z_2}$  and  $\phi$  considered in Fig. 6. A common feature is a sharp dip below  $Z_2$  threshold followed by a sharp peak on the  $Z_2$ . For fixed  $M_{Z_2}$ , larger mixing shifts the dip towards higher  $\sqrt{s}$ .

Finally, we calculate the cross section for the production of  $\tilde{Z}$  (assuming  $\phi = 0$ ) in  $p\bar{p}$  collisions using the quark distribution functions of Eichten, Hinchliffe, Lane, and

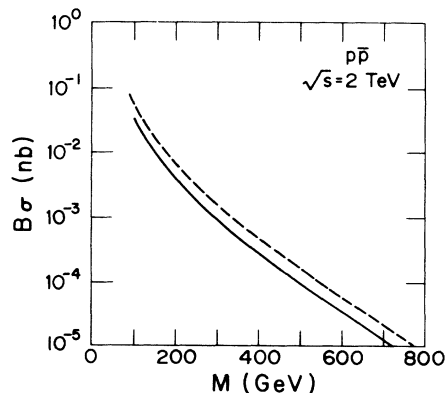


FIG. 8. Values of leptonic branching ratio  $B$  times gauge-boson production cross section  $\sigma$ , in  $p\bar{p}$  collisions at  $\sqrt{s} = 2$  TeV, as a function of gauge-boson mass  $M$ . The solid line is for  $\tilde{Z}$  and the dashed line is for a gauge boson  $Z$  with standard-model couplings but with mass a free parameter.

Quigg.<sup>9</sup> The  $\tilde{Z} \rightarrow e^+e^-$  production rate in  $p\bar{p}$  collisions at  $\sqrt{s}=2$  TeV is presented in Fig. 8 as a function of the gauge-boson mass which is taken as free parameter.<sup>8</sup> For comparison, we also present the corresponding rate for a gauge boson with couplings identical to the standard  $Z$  but with mass a free parameter. At a level of  $B\sigma \geq 10^{-3}$  nb (a reasonable lower limit at the Fermilab Tevatron), the accessible mass of  $\tilde{Z}$  is  $\leq 300$  GeV.

In conclusion, the effects of an additional neutral gauge boson suggested by the  $Sp(8) \times U(1)$  model are studied both on and off the  $Z_1$  resonance in  $e^+e^-$  collisions. On

the resonance, lepton asymmetries and total width of  $Z_1$  are found to be sensitive to the  $Z$ - $\tilde{Z}$  mixing angle and can be used to probe the gauge-boson mass. Off the resonance, the forward-backward and left-right asymmetries showed pronounced effects. Experimental tests of these effects are feasible in the near future.

One of us (A.A.B.) would like to thank Frederick J. Gilman for useful discussions. This work was supported in part by the United States Department of Energy.

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<sup>8</sup>Our calculations are identical to Bagnoid and Kuo (Ref. 4) except that we use the corrected couplings provided by Eq. (1).

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