Covariant operators and the classical limit of the Dirac equation with a color field

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The covariant operators of the position, spin, and color in Dirac quantum mechanics with an external color field are constructed. Their expectation values have good transformation properties with respect to the Poincaré group in the classical limit. This set of operators solves the problem of a relativistic generalization of the classical equations of motion for a colored, spinning particle obtained as the classical limit of Dirac quantum mechanics.

I. INTRODUCTION

Wong's equations for color, classical particles in an external color field[1] are the non-Abelian counterpart of the Lorentz equations for charged particles. These equations turned out to be very useful in investigations of several theoretical problems.[2] Quite recently the interest in Wong's equations has grown, especially in the context of the description of a quark-gluon plasma.[3] At present there are many approaches both to the classical mechanics of color particles in an external color field and to the derivation of dynamical equations for such mechanics.

In this paper we shall concentrate on an approach to the derivation of classical equations for color particles based on the classical limit in the Ehrenfest sense for the Dirac equation.[4] In the first-quantized theory a quark—natural prototype of the color particle is described by the Dirac equation for the particle belonging to the fundamental representation of the SU(N) gauge group. Therefore, the derivation of the classical equations through the classical limit seems to be the most natural. The equations we obtain are the dynamical equations for expectation values of certain operators in a state of the form of a wave packet. In order to avoid troubles with the physical interpretation of the phenomena such as *Zitterbewegung* and Klein paradoxes, the wave packets are built out of positive-energy states.[5] In this case only so-called even operators contribute to expectation values. This is why the Foldy-Wouthuysen (FW) representation[16] is essential in this approach. There is a clear distinction between the positive- and negative-energy states and even and odd operators in the FW representation. The price we must pay for using the FW representation is a nonrelativistic form of the classical equation we obtain. Indeed, the FW Hamiltonian in the Dirac equation is known only in the form of a nonrelativistic expansion in powers of $(1/mc)$ for an arbitrary external color field. Therefore, the problem of a relativistic generalization of the nonrelativistic classical equations for color particles obtained within the described approach is of great importance.

In this paper we would like to present a solution to this problem. The solution is nontrivial because it requires a subtle analysis of operators whose expectation values define classical variables such as the position, spin, and color. In general, we can say that there are two particularly interesting types of operators in Dirac quantum mechanics—canonical and covariant ones. The canonical operators are not suitable from the point of view of the relativistic generalization problem but appropriate for studying the canonical structure of the Dirac theory. The covariant operators which we shall construct give the solution to the problem of the relativistic generalization of the classical equations for a particle with color and spin. The expectation values of the covariant operators have good transformation properties with respect to the Poincaré group. This is why we believe that the problem of the relativistic generalization is in fact interesting. It simply concerns the formalism of Dirac quantum mechanics with an external color field.

In our opinion particularly interesting is the fact that there exist two types of color operators—canonical and covariant ones. Only the expectation values of the covariant color operators are scalars with respect to the Poincaré group. The expectation values of apparently naturally defined canonical color operators do not have good geometrical properties. It is an open question whether this fact has some physical implications.

The paper is organized as follows. In Sec. II we recall the nonrelativistic classical equations of motion for a color spinning particle obtained in Ref. 4 and we present the problem of their relativistic generalization. In Sec. III we analyze the covariant operators in the free-particle case in order to facilitate the construction of the covariant operators for a particle in an external color field. This construction is performed in Sec. IV; the new scalar color operator is constructed in this section. We also emphasize the role of the gauge transformation in the method we use. In Sec. V we find the relativistic generalization of the equations from Sec. II by means of the covariant operators defined in Sec. IV.

II. NONRELATIVISTIC CLASSICAL EQUATIONS FOR A COLOR SPINNING PARTICLE

Let us recapitulate the main facts from papers[4] concerning the derivation of the nonrelativistic equations for a color particle based on the classical limit of the Dirac equation. We use the FW representation of the Dirac equation. Thus, the Dirac Hamiltonian calculated up to order $(1/mc)^2$ takes the form

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$$H_{\rm FW}=\beta m+g\hat{A}_0+\frac{\beta}{2m}\hat{\pi}^2-\frac{g\hbar}{2m}\beta\hat{\sigma}\cdot\hat{\mathbf{B}}-\frac{g\hbar}{4m^2}\hat{\sigma}\cdot(\hat{\pi}\times\hat{\mathbf{E}}). \tag{1}$$

We put $c=1$ for simplicity and neglect terms of the order $\hbar^2$. Here $\hat{\pi}$ is the kinetic momentum

$$\hat{\pi}=\hat{\mathbf{p}}-g\hat{\mathbf{A}} \quad\text{and}\quad \hat{\mathbf{p}}=-i\hbar\frac{\partial}{\partial\mathbf{R}} .$$

$\hat{A}_\mu=A_\mu^a\hat{T}^a$ are color potentials, $\hat{T}^a$ are the Hermitian generators of SU(N). $\hat{\sigma}=\mathrm{diag}(\sigma,\sigma)$ where σ are Pauli matrices and $\beta=\mathrm{diag}(I,-I)$. $\hat{\mathbf{E}}$ and $\hat{\mathbf{B}}$ are color-electric and -magnetic fields, respectively, defined as

$$\hat{\mathbf{E}}^i=E^{ia}\hat{T}^a=\hat{F}_{0i},\quad \hat{\mathbf{B}}^i=B^{ia}\hat{T}^a=-\tfrac{1}{2}\epsilon^{ikl}\hat{F}_{kl} ,$$

and

$$\hat{F}_{\mu\nu}=\partial_\mu\hat{A}_\nu-\partial_\nu\hat{A}_\mu+\frac{ig}{\hbar}[\hat{A}_\mu,\hat{A}_\nu] ,$$

where g is a color coupling constant (it has the dimension of an electric charge). The terms of order $(1/mc)^3$ and $(1/mc)^4$ in the Dirac Hamiltonian may be found in Ref. 7.

In the papers in Ref. 4, the classical limit of the Dirac equation with the Hamiltonian, (1) was studied with the help of the equation

$$\frac{d}{dt}\langle\psi|\hat{A}|\psi\rangle=\left\langle\psi\left|\frac{1}{i\hbar}[\hat{A},H_{\rm FW}]+\frac{\partial\hat{A}}{\partial t}\right|\psi\right\rangle .$$

Assuming that the state ψ has the form of a wave packet (for more details, see Ref. 4) the following closed set of classical equations has been obtained:

$$m\frac{d^2\mathbf{R}}{dt^2}=g\left[\mathbf{E}^a+\frac{d\mathbf{R}}{dt}\times\mathbf{B}^a\right]I^a , \tag{2a}$$

$$\frac{d\mathbf{s}}{dt}=\frac{g}{m}\mathbf{s}\times\left[\mathbf{B}^a-\frac{1}{2}\frac{d\mathbf{R}}{dt}\times\mathbf{E}^a\right]I^a , \tag{2b}$$

$$\frac{dI^a}{dt}=\frac{g}{\hbar}f^{abc}\left[A_0^b-\frac{d\mathbf{R}}{dt}\cdot\mathbf{A}^b\right]I^c - \frac{g}{m}f^{abc}\mathbf{s}\cdot\left[\mathbf{B}^b-\frac{1}{2}\frac{d\mathbf{R}}{dt}\times\mathbf{E}^b\right]I^c , \tag{2c}$$

where the classical quantities—position $\mathbf{R}$, spin $\mathbf{s}$, and color I^a—are defined as the expectation values

$$\mathbf{R}=\langle\psi|\hat{\mathbf{R}}|\psi\rangle ,\quad \mathbf{s}=\langle\psi|\tfrac{1}{2}\hat{\sigma}|\psi\rangle ,\quad I^a=\langle\psi|\hat{T}^a|\psi\rangle . \tag{3}$$

External fields $A_\mu^b, F_{\mu\nu}^b$ are taken at the position $\mathbf{R}$. The operators in formulas (3) are the canonical operators of the position, spin, and color which obey the following canonical commutation relations:

$$[\hat{R}^i,\hat{P}^j]=i\hbar\delta^{ij} ,\quad [\tfrac{1}{2}\hat{\sigma}^i,\tfrac{1}{2}\hat{\sigma}^k]=i\epsilon^{ikl}\tfrac{1}{2}\hat{\sigma}^l ,\quad [\hat{T}^a,\hat{T}^b]=if^{abc}\hat{T}^c . \tag{4}$$

$\hat{P}^i$ is the canonical momentum and f^{abc} are the SU(N) group structure constants. The rest of possible commutators vanish. In Eq. (2) terms of the order $\hbar$ with covariant gradients of color fields $\mathbf{E}^a$ and $\mathbf{B}^a$ were neglected as smaller then the terms of the order $\hbar^0$. Equations (2) are the nonrelativistic classical equations for a color spinning particle following from the classical limit of the Dirac equation with an external color field. Note that the classical spin is necessary in order to find the closed set of classical equations.

The problem that arises here quite naturally is to find a relativistic generalization of Eq. (2). This may seem an easy task. If we assume that $\mathbf{R}$ is the spatial part of the four-vector of position defined in the lab frame as $R^\mu=(t,\mathbf{R})$ and that I^a are scalars with respect to the Lorentz group, we shall find that Eq. (2a) is a nonrelativistic limit of the non-Abelian Lorentz equation

$$m\frac{d^2R^\mu}{d\tau^2}=gF^{\mu\nu a}\frac{dR_\nu}{d\tau}I^a . \tag{5}$$

Namely, Eq. (2a) can be regarded as an approximation to Eq. (5) (which is Lorentz covariant) linear in the velocity. In (5) τ is the proper time of a particle.

Similarly, if we additionally assume that s is the rest-frame spin of a particle (see, e.g., Ref. 8 for a detailed description of a classical relativistic spin) we shall recognize Eq. (2b) as the Thomas equation[8,9] for the spin in a non-Abelian field. It has the following manifestly covariant form:

$$\frac{dW^\mu}{dt}=\frac{g}{m}F^{\mu\nu a}W_\nu I^a , \tag{6}$$

where W^μ is the Pauli-Lubański four-vector which describes the classical relativistic spin. The relation between s and W^μ may be found in Ref. 8.

Thus, we have identified the classical variables $\mathbf{R}$, s, and I^a from a geometrical point of view. If it was done properly we should find the relativistic generalization of the last equation (2c) without difficulty. The only relativistic equation which is possible in this case has the form

$$\frac{dI^a}{d\tau}=\frac{g}{h}f^{abc}A_\mu^b\frac{dR^\mu}{d\tau}I^c - \frac{g}{m}f^{abc}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}^bW_\alpha\frac{dR_\beta}{d\tau}I^c . \tag{7}$$

Unfortunately Eq. (7) written in the lab frame up to the terms linear in the velocity is different from Eq. (2c). The only difference is in the spin term. There is no $\frac{1}{2}$ factor in Eq. (7) when written in the lab frame. Therefore, the naive method of constructing the relativistic generalization, based on *ad hoc* assumptions about the geometrical properties of the classical variables with respect to the Lorentz group fails. The question arises whether Eqs. (2) have a relativistic generalization at all.

In the following part of this paper we show that the answer to this question is in the affirmative. Analysis of operators which we want to take as the position, spin, and color operators will show that expectation values of the canonical operators defined by relations (4) are not good geometrical objects. We shall construct new operators—the covariant ones, the expectation values of which have good transformation properties with respect to the Poincaré group. The method of construction of the covariant operators comes from the works of de Groot and Suttorp.[10] We refine and generalize this method in order to apply it to the non-Abelian case.

III. COVARIANT OPERATORS FOR A FREE PARTICLE

Before constructing the covariant operators for a Dirac particle in an external color field it is worthwhile to analyze this problem for a free particle described by the Dirac equation

$$i\hbar\frac{\partial\psi}{\partial t}=H_D\psi,\quad H_D=\beta E_P=\beta(m^2+\mathbf{P}^2)^{1/2}\ . \tag{8}$$

We use the FW representation of the Dirac equation. The analysis is based on paper in Ref. 10. We would like to describe it briefly.

We want to find operators $\hat{\omega}=\hat{\omega}(\hat{\mathbf{R}},\hat{\mathbf{P}},\gamma)$ which are built out of the canonical operators of the position and momentum and Dirac γ matrices in such a manner that their expectation values defined in the lab frame as

$$\begin{aligned}\omega(t)&=\langle\psi(t)\mid\hat{\omega}\mid\psi(t)\rangle_{\text{lab}}\\&=\int d^3\mathbf{R}\,\psi^\dagger(\mathbf{R},t)\hat{\omega}(\hat{\mathbf{R}},\hat{\mathbf{P}},\gamma)\psi(\mathbf{R},t)\end{aligned}$$

are components of tensors; i.e., they have good transformation properties with respect to the Poincaré group. Using the results described in Appendix A we can compute the difference between the expectation values $\omega'(t')$ of the operators $\hat{\omega}$ computed in the new coordinate frame S' and $\omega(t)$ from the lab frame,

$$\begin{aligned}\omega'(x_0'=t)-\omega(x_0=t)&\\&=\int d^3\mathbf{R}'\psi'^\dagger(\mathbf{R}',t)\hat{\omega}(\hat{\mathbf{R}}',\hat{\mathbf{P}}',\gamma)\psi'(\mathbf{R}',t)\\&\quad-\int d^3\mathbf{R}\,\psi^\dagger(\mathbf{R},t)\hat{\omega}(\hat{\mathbf{R}},\hat{\mathbf{P}},\gamma)\psi(\mathbf{R},t)\\&=\langle\psi(t)\mid\frac{i}{\hbar}\boldsymbol{\epsilon}\cdot[\mathbf{G},\hat{\omega}]\mid\psi(t)\rangle\end{aligned}\tag{9}$$

when the two coordinate frames are connected by an infinitesimal translation or rotation or pure Lorentz boost. $\mathbf{G}$ are generators of these transformations in the spinorial representation. Their manifest form may be found in Appendix A. Note that the expectation values in formula (9) are computed at numerically the same instant of time but for different spacelike hypersurfaces.

We have computed the difference (9) based on the covariance properties of the Dirac quantum mechanics. On the other hand, this difference is known from the classical considerations. Namely, if we postulate that $\omega(t)$ are components of some tensor we shall know the expressions $\omega'(t)-\omega(t)$ from the transformation laws of the tensor under the Poincaré group. These expressions may be written in the classical limit as expectation values of certain operators. Comparing these operators with the commutators from formula (9) we obtain operatorial relations which enable us to find the covariant operators $\hat{\omega}$.

Let us illustrate the method described above taking as an example the covariant position operator $\hat{\omega}=\hat{\mathbf{x}}$. We would like to find such a position operator that its expectation value $\mathbf{x}(t)=\langle\psi(t)\mid\mathbf{x}\mid\psi(t)\rangle$ is the spatial part of the four-vector of position defined in the lab frame as $x^\mu=(t,\mathbf{x}(t))$. Assuming the four-position defined in such a manner is a four-vector for every time t one can find the following expressions for the difference:

$$\delta\mathbf{x}(t)=\mathbf{x}'(t)-\mathbf{x}(t)\ .$$

We obtain

$$\delta\mathbf{x}(t)=\begin{cases}\boldsymbol{\epsilon}=\langle\psi(t)\mid\boldsymbol{\epsilon}\cdot\hat{1}\mid\psi(t)\rangle\ \text{for translation}\ , & \text{(10a)}\\ \boldsymbol{\epsilon}\times\mathbf{x}(t)=\langle\psi(t)\mid\boldsymbol{\epsilon}\times\mathbf{x}(t)\mid\psi(t)\rangle\ \ \text{for rotation}\ , & \text{(10b)}\\ -\boldsymbol{\epsilon}t+\boldsymbol{\epsilon}\cdot\mathbf{x}(t)\dfrac{d\mathbf{x}}{dt}(t)=\langle\psi\mid-\boldsymbol{\epsilon}t\cdot 1+\dfrac{i}{2\hbar}(\boldsymbol{\epsilon}\cdot\hat{\mathbf{x}},[H_D,\hat{\mathbf{x}}])\mid\psi\rangle\ \text{for boost}\ . & \text{(10c)}\end{cases}$$

Equation (10c) is valid in the classical limit. Comparing the right-hand sides of formulas (9) and (10) we obtain the following commutation relations for the operator $\hat{\mathbf{x}}$:

$$\begin{aligned}&[\hat{P}^i,\hat{x}^k]=-i\hbar\delta^{ik}\ ,\\&[\hat{J}^i,\hat{x}^k]=i\hbar\epsilon^{ikl}\hat{x}^l\ ,\\&[\hat{N}^i,\hat{x}^k]=\tfrac{1}{2}(\hat{x}^i,[H_D,\hat{x}^k])\ ,\end{aligned}\tag{11}$$

where generators $\hat{\mathbf{P}}$, $\hat{\mathbf{J}}$, and $\hat{N}$ are listed in Appendix A, and (,) means an anticommutator. Assuming that the operator $\hat{\mathbf{x}}$ is even, the following unique form of the covariant position operator in the FW representation may be found with the help of relations (11):[10]

$$\hat{\mathbf{x}}=\hat{\mathbf{R}}+\frac{\hbar\hat{\sigma}\times\mathbf{p}}{2m(m+E_P)}\ . \tag{12}$$

The expectation values of this operator together with the lab time t form in the classical limit a four-vector. Note that the components of the operator (12) do not commute.

In the same spirit the covariant operators of the spin and color may be found. The covariant operator of spin $\hat{\mathbf{S}}$ is constructed in such a manner that the expectation values

$$\mathbf{S}=\langle \psi | \hat{\mathbf{S}} | \psi \rangle$$

are spatial components of the classical spin tensor $S_{\mu\nu}$: i.e.,

$$S^i=\tfrac{1}{2}\epsilon^{ijk}S_{jk} \ .$$

The antisymmetric tensor $S_{\mu\nu}$ gives an alternative description of the classical relativistic spin. It may be defined by means of the Pauli-Lubański four-vector W^μ

$$S_{\mu\nu}=\epsilon_{\mu\nu\alpha\beta}u^\alpha W^\beta \ ,$$

where u^μ is a covariant velocity. Note that $u^\mu S_{\mu\nu}=0$, therefore the tensor $S^{\mu\nu}$ has only three independent components as has the four-vector W^μ.

As a result we have obtained the following form of the even covariant spin operator in the FW representation:[10]

$$\hat{\mathbf{S}}=\frac{E_P}{2m}\hat{\sigma}-\frac{\mathbf{p}(\mathbf{p}\cdot\hat{\sigma})}{2m(m+E_P)} \ . \tag{13}$$

For a free particle the generators of the SU(N) group $\hat{T}^a$ are scalar operators. Their expectation values are scalars with respect to the Poincaré group. This statement will be no longer valid with a color field.

The covariant operators of the position and spin do not obey the canonical commutation relations. Below we present the nonvanishing relations:

$$[\hat{x}^i,\hat{x}^k]=\frac{i\hbar}{E_P^2}\epsilon^{ikl}\left[\hat{S}^l+\frac{p^l(\mathbf{p}\cdot\hat{\mathbf{S}})}{m^2}\right] ,$$

$$[\hat{x}^i,\hat{S}^k]=i\hbar\left[\frac{\mathbf{p}\cdot\hat{\mathbf{S}}}{m^2}\left[\frac{p^ip^k}{m^2}-\delta^{ik}\right]+\frac{p^i\hat{S}^k}{E^2}\right] ,$$

$$[\hat{S}^i,\hat{S}^k]=i\hbar\epsilon^{ikl}\left[\hat{S}^l+\frac{p^l(\mathbf{p}\cdot\hat{\mathbf{S}})}{m^2}\right] ,$$

$$[\hat{x}^i,p^k]=i\hbar\delta^{ik} \ .$$

IV. COVARIANT OPERATORS FOR THE DIRAC PARTICLE IN AN EXTERNAL COLOR FIELD

We will generalize the method described in the previous section to the case of the Dirac particle interacting with an external color field. The particle is described by the Dirac equation in the FW representation with the Hamiltonian (1) [up to the terms of order $(1/m)^2$]. In this case the covariant operators $\hat{\omega}$ which we want to construct depend moreover on an external color field $\hat{A}_\mu$. We indicate it as

$$\hat{\omega}=\hat{\omega}(\hat{\mathbf{R}},\hat{\mathbf{P}},\gamma,\hat{\mathbf{A}})\equiv\hat{\omega}(A) \ .$$

The rest of the notation is the same as in Sec. III. We have to calculate the difference

$$\delta\omega(t)=\omega'(t)-\omega(t)$$
$$=\langle \psi'(t) | \hat{\omega}(A') | \psi'(t) \rangle_{s'}-\langle \psi(t) | \hat{\omega}(A) | \psi(t) \rangle_s$$

for the three infinitesimal transformations defined in Sec. III. Note that the expectation value in the S' frame is computed for an external field A'_μ as seen in this frame. The relations between A_μ from the frame S and A'_μ may be found in Appendix B. Using the results from Appendixes A and B one can obtain the following formula up to the terms linear in ϵ:

$$\delta\omega(t)=\langle \psi(t) | \delta\hat{\omega} | \psi(t) \rangle \ , \tag{14}$$

where

$$\delta\hat{\omega}=\frac{i}{\hbar}\epsilon\cdot[\mathbf{G},\hat{\omega}]+\hat{\omega}(A'(x))-\hat{\omega}(A(x))$$
$$=\frac{i}{\hbar}\epsilon\cdot[\mathbf{G},\hat{\omega}]+\left[\frac{\partial\hat{\omega}}{\partial A^a_\mu}\mathcal{L}A^a_\mu+\cdots\right] .$$

Here the operators $\mathbf{G}$ are operators of the infinitesimal transformations (see Appendix A) and $\mathcal{L}A_\mu\cdots$ are Lie derivatives of fields A_μ listed in Appendix B.

Similarly to a free-particle case the form of the classical quantity $\delta\omega(t)$ results from the assumptions about the geometrical properties of the quantity $\omega(t)$. The expressions for $\delta\omega(t)$ may be written in the classical limit as expectation values of certain operators. Comparing these operators with the operators $\delta\hat{\omega}$ from the formula (14) we obtain the relations which the covariant operators $\hat{\omega}$ must obey. Let us illustrate the method taking as an example the covariant color operators.

A. Covariant color operators

We would like to find the operators $\tilde{T}^a(A)$ such that the classical color

$$\tilde{I}^a(t)=\langle \psi(t) | \tilde{T}^a(A) | \psi(t) \rangle$$

is a scalar with respect to the Poincaré group. It means that the classical quantity $\delta\tilde{I}^a(t)=\tilde{I}'^a(t)-\tilde{I}^a(t)$ must be equal to

$$\delta\tilde{I}^a(t)=\begin{cases}0 \text{ for translation} \ , \\ 0 \text{ for rotation} \ , \\ \epsilon\cdot\mathbf{x}(t)\dfrac{d\tilde{I}^a}{dt}=\langle \psi | \tfrac{1}{2}\left[\epsilon\cdot\hat{\mathbf{x}},\dfrac{i}{\hbar}[H_{\mathrm{FW}},\tilde{T}^a]+\dfrac{\partial\tilde{T}^a}{\partial t}\right] | \psi \rangle \text{ for boost} \ .\end{cases} \tag{15}$$

Comparing the right-hand sides of the formulas (14) (for $\hat{\omega}=\tilde{T}^a$) and (15) one can obtain the following relations for the operators $\tilde{T}^a$:

$$\frac{i}{\hbar}[\hat{P}^k,\tilde{T}^a]-\frac{1}{2}\left[\frac{\partial \tilde{T}^a}{\partial A^{mb}},\frac{\partial A^{mb}}{\partial R^k}\right]=0\ , \tag{16a}$$

$$\frac{i}{\hbar}[\hat{J}^k,\tilde{T}^a]+\frac{1}{2}\epsilon^{klm}\left[\frac{\partial \tilde{T}^a}{\partial A^{mb}},A^{lb}\right]-\frac{1}{2}\epsilon^{klm}\left[\frac{\partial \tilde{T}^a}{\partial A^{rb}},R^l(\partial_m A^{rb})\right]=0\ , \tag{16b}$$

$$\frac{i}{\hbar}[\hat{N}^k,\tilde{T}^a]-\frac{1}{2}\left[\frac{\partial \tilde{T}^a}{\partial A^{kb}},A^{0b}\right]+\frac{1}{2}\left[\frac{\partial \tilde{T}^a}{\partial A^{mb}},R^k(\partial_0 A^{mb})\right]=\frac{1}{2}\left[\hat{x}^k,\frac{i}{\hbar}[H_{\rm FW},\tilde{T}^a]+\frac{\partial \tilde{T}^a}{\partial t}\right]\ , \tag{16c}$$

where the operators $\hat{P}^k$, $\hat{J}^k$, and $\hat{N}^k$ are listed in Appendix A (the case with a color field).

Now, we describe briefly the construction of the even color operators $\tilde{T}^a$ which obey the relations (16). Closer inspection shows that in order to satisfy the relations (16a) and (16b) the operator $\tilde{T}^a$ must not contain the canonical position operator **R** and all spacelike indices $i,k,l,\ldots$ should be contracted. In order to satisfy relation (16c) the covariant position operator in an external color field $\hat{x}^k$ should be known. It is defined by the formula (24) (cf. part B of this section). Now, we are fully prepared for construction of the scalar color operators. We shall construct the even color operators. The only independent and dimensionless quantities out of which the operator $\tilde{T}^a$ can be built are

$$1,\ \sigma^k,\ \beta,\ \epsilon^{klm},\ f^{abc},\ \hat{T}^a,\ \frac{\hat{p}^k}{m},\ \frac{g}{m}A^{ka},\ \frac{g}{m}A^{0a}\ .$$

The general expression for $\tilde{T}^a$ which is built according to the rules described above contains unknown coefficients. Inserting that expression into relation (16c) we can determine these coefficients. Therefore, after very long calculations we can obtain the following form of the scalar color operators up to order $(1/m)^2$:

$$\tilde{T}^a=\hat{T}^a+\frac{i}{8m^2}\epsilon^{klm}\hat{\sigma}^k(\hat{\pi}^l,[\hat{\pi}^m,\hat{T}^a])\ . \tag{17}$$

The expectation values $\tilde{I}^a=\langle\psi\,|\,\tilde{T}^a\,|\,\psi\rangle$ are scalars with respect to the Poincaré group in the classical limit.

B. Covariant operators of position and spin

The covariant operators of the position and spin $\hat{\omega}(A)$ should also be covariant under the transformations from the gauge group, i.e.,

$$\hat{\omega}(A^{\omega})=\Omega(x)\hat{\omega}(A)\Omega^{\dagger}(x)\ , \tag{18}$$

where A^{ω} denotes the gauge-transformed color potential

$$\hat{A}^{\omega}_{\mu}(x)=\Omega(x)\hat{A}_{\mu}(x)\Omega^{\dagger}(x)+\frac{i\hbar}{g}[\partial_\mu\Omega(x)]\Omega^{\dagger}(x)$$

and $\Omega(x)$ is a local matrix from the SU(N) group. Then the classical position and spin defined as the expectation values $\omega(t)=\langle\psi(t)\,|\,\hat{\omega}(A)\,|\,\psi(t)\rangle$ are gauge invariant as should be expected. [Remember that the gauge transformation changes also the bispinor ψ: $\psi^{\omega}(x)=\Omega(x)\psi(x)$.]

In the case of the gauge-covariant operators (18) we can perform the gauge transformation in addition to the coordinate one without changing the difference:

$$\delta\omega(t)=\langle\psi'(t)\,|\,\hat{\omega}(A')\,|\,\psi'(t)\rangle_{s'}-\langle\psi(t)\,|\,\hat{\omega}(A)\,|\,\psi(t)\rangle_s\ .$$

Hence, we can write

$$\delta\omega(t)=\langle\psi'^{\omega}(t)\,|\,\hat{\omega}(A'^{\omega})\,|\,\psi'^{\omega}(t)\rangle_{s'}-\langle\psi(t)\,|\,\hat{\omega}(A)\,|\,\psi(t)\rangle_s\ . \tag{19}$$

Let us compute the difference (19) for an infinitesimal translation, rotation, boost, and for additionally performed infinitesimal gauge transformation $\Omega(x)=1-(i/\hbar)\epsilon\cdot\mathbf{W}(x)$, where

$$\mathbf{W}(x)=\begin{cases}-g\,\hat{\mathbf{A}}(x)\ \text{for translation}\ ,\\ -g\mathbf{R}\times\hat{\mathbf{A}}(x)\ \text{for rotation}\ ,\\ -g\hat{A}_0(x)\mathbf{R}+gt\,\hat{\mathbf{A}}(x)\ \text{for boost}\ ,\end{cases} \tag{20}$$

and ϵ are just the same as in the coordinate transformations. In this case we obtain up to terms linear in ϵ:

$$\delta\omega(t)=\langle\psi\,|\,\delta\hat{\omega}\,|\,\psi\rangle\ , \tag{21}$$

where

$$\delta\hat{\omega}=\epsilon^k\left[\frac{i}{\hbar}[\hat{\pi}^k,\hat{\omega}]+\frac{1}{2}\epsilon^{klm}\left[\frac{\partial\hat{\omega}}{\partial A^{ma}},B^{la}\right]\right]\ ,$$

$$\delta\hat{\omega}=\epsilon^k\left\{\frac{i}{\hbar}\left[\epsilon^{klm}R^l\hat{\pi}^m+\frac{\hbar}{2}\hat{\sigma}^k,\hat{\omega}\right]+\frac{1}{2}\epsilon^{klm}\left[\frac{\partial\hat{\omega}}{\partial B^{ma}},B^{la}\right]+\frac{1}{2}\epsilon^{klm}\left[\frac{\partial\hat{\omega}}{\partial E^{ma}},E^{la}\right]-\frac{1}{2}\epsilon^{klm}\left[\frac{\partial\hat{\omega}}{\partial A^{pa}},\epsilon^{mpq}R^lB^{qa}\right]\right\}\ ,$$

$$\delta\hat{\omega}=\epsilon^k\left[\frac{i}{\hbar}[(\hat{N}^k-g\hat{A}_0R^k)-t\hat{\pi}^k,\hat{\omega}]-\frac{1}{2}\left[\frac{\partial\hat{\omega}}{\partial A^{ma}},R^kE^{ma}\right]-\frac{1}{2}t\left[\frac{\partial\hat{\omega}}{\partial A^{ma}},\epsilon^{klm}B^{la}\right]\right]\ .$$

Here $\hat{\pi}=\hat{\mathbf{p}}-g\hat{\mathbf{A}}$ and the operator $\hat{\mathbf{N}}$ may be found in Appendix A (the case with a color field). The gauge transformations (20) were defined in such a manner that the operators in the commutators in the relations (21) the gauge covariant. This form of relation (21) is more convenient for the construction of the covariant position and spin operators.

Exactly as in the free-particle case the expectation values of the position operator $\mathbf{x}(t)=\langle\psi|\hat{\mathbf{x}}(A)|\psi\rangle$ together with the lab time t form a four-vector. It means that [compare (10)]

$$\delta\mathbf{x}(t)=\mathbf{x}'(t)-\mathbf{x}(t)=\langle\psi|\delta\hat{\mathbf{x}}|\psi\rangle\ , \tag{22}$$

where

$$\delta\hat{\mathbf{x}}=\begin{cases}\epsilon\cdot 1 \text{ for translation },\\ \epsilon\times\mathbf{x} \text{ for rotation },\\ -t\epsilon\cdot 1+\frac{1}{2}\left\{\epsilon\cdot\hat{\mathbf{x}},\frac{i}{\hbar}[H_{\mathrm{FW}},\hat{\mathbf{x}}]+\frac{\partial\hat{\mathbf{x}}}{\partial t}\right\} \text{ for boost }.\end{cases}$$

Comparing the operator $\delta\hat{\mathbf{x}}$ defined above with the right-hand side of formulas (21) for $\hat{\omega}(A)=\hat{\mathbf{x}}(A)$ we obtain the relations which the covariant operator of the position must obey:

$$\begin{aligned}&\frac{i}{\hbar}[\hat{\pi}^k,\hat{x}^l]+\frac{1}{2}\epsilon^{kmn}\left\{\frac{\partial\hat{x}^l}{\partial A^{na}},B^{ma}\right\}=\delta^{kl}\ ,\\ &\frac{i}{\hbar}\left[\epsilon^{kpq}R^p\hat{\pi}^q+\frac{\hbar}{2}\hat{\sigma}^k,\hat{x}^l\right]-\frac{1}{2}\epsilon^{kmn}\left\{\frac{\partial\hat{x}^l}{\partial A^{ra}},\epsilon^{nrt}R^mB^{ta}\right\}=-\epsilon^{klm}\hat{x}^m\ ,\\ &\frac{i}{\hbar}[(\hat{N}^k-g\hat{A}_0R^k),\hat{x}^l]-\frac{1}{2}\left\{\frac{\partial\hat{x}^l}{\partial A^{ma}},R^kE^{ma}\right\}=\frac{1}{2}\left\{\hat{x}^k,\frac{i}{\hbar}[H_{\mathrm{FW}},\hat{x}^l]+\frac{\partial\hat{x}^l}{\partial t}\right\}\ .\end{aligned} \tag{23}$$

We have assumed that the operator $\hat{\mathbf{x}}$ does not depend on fields E^{ma} and B^{ma}. Using dimensional analysis we can prove that this is really the case when we neglect terms of the order $\hbar^2$ in the operator of the position.

It is not an easy task to construct the general form of the even operator $\hat{\mathbf{x}}$ which obeys relations (23). After long calculations which are similar to those leading to the scalar color operators and described in Sec. A, the following form of the even operator $\hat{\mathbf{x}}$ emerges from relations (23):

$$\hat{\mathbf{x}}=\mathbf{R}+\frac{\hbar}{4m^2}\hat{\sigma}\times\hat{\pi}\ , \tag{24}$$

up to order $(1/m)^2$ and neglecting terms proportional to $\hbar^2$. Note that this covariant operator is closely related to the operator in the free-particle case (12). Namely, if we expand formula (12) in powers of $(1/m)$ up to the second order and substitute $\hat{\pi}$ for $\mathbf{p}$ we obtain the covariant operator of the position (24).

Considerations similar to those above allow us to construct the covariant operator of the spin $\mathbf{S}(A)$. This operator is defined in the same way as for a free particle. Its expectation value $\mathbf{S}=\langle\psi|\hat{\mathbf{S}}(A)|\psi\rangle$ is the spatial component of the classical spin tensor $S_{\mu\nu}$. We do not present this construction here because of its length. We shall write only the result.

The even covariant operator of the spin $\hat{\mathbf{S}}(A)$ has the following form [up to order $(1/m)^2$ and neglecting terms of order $\hbar^2$]:

$$\hat{\mathbf{S}}(A)=\left[1+\frac{\hat{\pi}^2}{2m^2}\right]\frac{1}{2}\hat{\sigma}-\frac{1}{8m^2}(\hat{\pi}\cdot\hat{\sigma},\hat{\pi})\ . \tag{25}$$

This form of the spin operator may also be obtained from the spin operator in the free particle case (13) according to the formal rules described for the operator of position (24).

To sum up the considerations of this section we compare in Table I the even covariant operators of the position, spin, and color for a free particle with those for the particle in an external color field.

TABLE I. Comparison of even covariant operators of the position, spin, and color for a free particle with those for the particle in an external color field.

Covariant operators for a free particle	Covariant operators for a particle in an external color field
$\hat{\mathbf{x}}=\hat{\mathbf{R}}+\frac{\hbar\hat{\sigma}\times\mathbf{p}}{2m(m+E_P)}$	$\hat{\mathbf{x}}(A)=\hat{\mathbf{R}}+\frac{\hbar}{4m^2}\hat{\sigma}\times\hat{\pi}$
$\hat{\mathbf{S}}=\frac{E_P}{2m}\hat{\sigma}-\frac{\mathbf{p}(\mathbf{p}\cdot\hat{\sigma})}{2m(m+E_P)}$	$\hat{\mathbf{S}}(A)=\left[1+\frac{\hat{\pi}^2}{2m^2}\right]\frac{1}{2}\hat{\sigma}-\frac{1}{8m^2}(\hat{\pi}\cdot\hat{\sigma},\hat{\pi})$
$\hat{T}^a$	$\hat{T}^a(A)=\hat{T}^a+\frac{i}{8m^2}\epsilon^{klm}\hat{\sigma}^k(\hat{\pi}^l,[\hat{\pi}^m,\hat{T}^a])$

particle in an external color field. The canonical operators for the particle in external color fields do not obey the canonical commutation relations (4). Therefore, the scalar color operators $\tilde{T}^a$ are not canonical ones.

V. RELATIVISTIC GENERALIZATION OF NONRELATIVISTIC EQUATIONS FOR COLORED PARTICLE

Having the covariant operators of the position, spin, and color we can easily find the relativistic generalization of Eqs. (2). We remember that the naive method of Sec. II failed because of the trouble with the $\frac{1}{2}$ factor in Eq. (2c). It is very easy to understand that problem in the light of the results from the previous section. Simply, the classical color I^a defined as the expectation value of the canonical color operator $\hat{T}^a$ (3) is not a scalar in the presence of an external color field. Similarly, the canonical position **R** does not form a four-vector together with the time t. To sum up, the canonical variables defined by relations (3) are not good geometrical objects.

The classical quantities defined as the expectation values of the covariant color operators transform properly under the Poincaré group. Therefore, we should write the classical equations of motion of the Ehrehfest type in terms of these quantities. Let us define them first:

$$\mathbf{x}(t)=\langle\psi|\hat{\mathbf{x}}(A)|\psi\rangle=\mathbf{R}+\frac{\hbar}{2m^2}\mathbf{s}\times\boldsymbol{\pi}\ , \tag{26a}$$

$$\mathbf{S}(t)=\langle\psi|\hat{\mathbf{S}}(A)|\psi\rangle=\left[1+\frac{\pi^2}{2m^2}\right]\mathbf{s}-\frac{1}{2m^2}(\mathbf{s}\cdot\boldsymbol{\pi})\boldsymbol{\pi}\ , \tag{26b}$$

$$\tilde{I}^a(t)=\langle\psi|\tilde{T}^a(A)|\psi\rangle=I^a-\frac{g}{2m^2}f^{abc}(\mathbf{s}\times\boldsymbol{\pi})\cdot\mathbf{A}^bI^c\ , \tag{26c}$$

and $\boldsymbol{\pi}=\langle\psi|\hat{\boldsymbol{\pi}}|\psi\rangle=m\dot{\mathbf{R}}$. The wave packet ψ is the same one which was used in Sec. II to derive the classical equations (2) (for a detailed description of this packet, see Ref. 4). The relations (26) follow from the definition of the covariant operators (17), (24), and (25) and the form of the wave packet ψ.

Now we write the dynamical equations for the covariant variables with the help of relations (26) and Eqs. (2). We must neglect for consistency terms proportional to $\hbar^1$ in the equations we obtained because those terms were also neglected in Eqs. (2). We obtain the following set of equations:

$$m\frac{d^2\mathbf{x}}{dt^2}=g\left[\mathbf{E}^a(\mathbf{x},t)+\frac{d\mathbf{x}}{dt}\times\mathbf{B}^a(\mathbf{x},t)\right]\tilde{I}^a,\quad \frac{d\mathbf{S}}{dt}=\frac{g}{m}\left[\mathbf{S}\times\mathbf{B}^a(\mathbf{x},t)-\left[\mathbf{S}\times\frac{d\mathbf{x}}{dt}\right]\times\mathbf{E}^a(\mathbf{x},t)\right]\tilde{I}^a\ ,$$
$$\frac{d\tilde{I}^a}{dt}=\frac{g}{\hbar}f^{abc}\left[A_0^b(\mathbf{x},t)-\frac{d\mathbf{x}}{dt}\cdot\mathbf{A}^b(\mathbf{x},t)\right]\tilde{I}^c-\frac{g}{m}f^{abc}\mathbf{S}\cdot\left[\mathbf{B}^b(\mathbf{x},t)-\frac{d\mathbf{x}}{dt}\times\mathbf{E}^b(\mathbf{x},t)\right]\tilde{I}^c\ . \tag{27}$$

Note that there is no $\frac{1}{2}$ coefficient in the spin term in the last equation.

The external fields in Eqs. (27) are taken at the new position **x** whereas in Eqs. (2) the fields were taken at the canonical position **R**. This change of the position is allowed. Deriving the classical equations of the Ehrenfest type (2) or (27) we have assumed that the wave packet ψ is built out of the positive-energy states. Therefore, the width of the wave packet δr must be much greater than the Compton wavelength of the Dirac particle (see Ref. 5):

$$\delta r\gg\frac{\hbar}{mc}\ .$$

It is easy to check [by inspection of formula (26a)] that the difference between trajectories is of the order of the Compton length:

$$|\mathbf{x}(t)-\mathbf{R}(t)|\propto\frac{\hbar}{mc}\ .$$

Both trajectories are inside the wave packet. So, both may be used to describe the motion of the wave packet. The covariant trajectory $\mathbf{x}(t)$ is the only one which gives covariant equations.

The relativistic generalization of Eqs. (27) is straightforward. We obtain the following manifestly covariant equations:

$$m\frac{d^2x^\mu}{d\tau}=gF^{\mu\nu a}(x)\frac{dx_\nu}{d\tau}\tilde{I}^a\ ,$$
$$\frac{dS^{\mu\nu}}{d\tau}=\frac{g}{m}[F^{\mu\beta a}(x)S_\beta^\nu-F^{\nu\beta a}(x)S_\beta^\mu]\tilde{I}^a\ , \tag{28}$$
$$\frac{d\tilde{I}^a}{d\tau}=\frac{g}{m}f^{abc}A_\mu^b(x)\frac{dx^\mu}{d\tau}\tilde{I}^c-\frac{g}{2m}f^{abc}S_{\mu\nu}F^{\mu\nu b}(x)\tilde{I}^c\ .$$

In this case we describe the classical spin by means of the tensor $S_{\mu\nu}$. The equation for $S_{\mu\nu}$ is equivalent to the equation for W^μ,

$$\frac{dW^\mu}{d\tau}=\frac{g}{m}F^{\mu\nu a}W_\nu\tilde{I}^a\ ,$$

provided the field $F_{\mu\nu}^a$ is homogeneous. Equations (28) written in the lab frame up to the terms linear in the velocity, are identical to Eqs. (27). Hence they are the equations we were looking for.

VI. REMARKS

The main goal of this paper was to solve the problem of the relativistic generalization of the equations for a color particle (2). This problem revealed the fact that in Dirac quantum mechanics the canonical color operator is different from the covariant one. At this point we would like to emphasize that the covariance of operators is strongly connected with the assumption about the existence of the classical limit of the Dirac quantum mechanics. The classical limit may exist only when we impose very severe limitations on physical conditions. The case with an external color field is especially difficult owing to the problem of deriving the classical trajectory from quantum mechanics.[11] Therefore, the problem we presented here requires further study.

An earlier version of the solution to the relativistic generalization problem was published in a paper by Arodź and Golec.[12] The idea was similar to the one presented here. The change of operators was performed in *ad hoc* manner in order to obtain the equations which were easy to generalize to the covariant form. In this paper the change of operators is independent of equations we expect to obtain. It simply follows from considerations about the covariance properties of the expectation values. We feel that this method is the most natural one. Up to now we have not found any connection between the operators from Ref. 12 and those constructed here.

Nonrelativistic equations of motion for the color particle (2) are simpler versions of equations from papers in Ref. 4. Namely, deriving Eq. (2) it was assumed that

$$\langle \psi | \hat{T}^a \tfrac{1}{2}\hat{\sigma}^k | \psi \rangle = \langle \psi | \hat{T}^a | \psi \rangle \langle \psi | \tfrac{1}{2}\hat{\sigma}^k | \psi \rangle , \quad (29)$$

i.e., the expectation values of the operator $\hat{J}^{ak} = \hat{T}^a \frac{1}{2}\hat{\sigma}^k = \hat{T}^a \otimes \frac{1}{2}\hat{\sigma}^k$ factorize. This is a so-called case without mixing of spin and color. The mixing of spin and color case [assumption about factorization (29) is no longer valid] leads to a more complicated set of nonrelativistic equations.[4] There appear new dynamical variables $J^{ak} = \langle \psi | \hat{J}^{ak} | \psi \rangle$ and new equations of motion for these. However, the problem of relativistic generalization of those equations remains. The solution of the problem seems to be similar to the one presented in this paper. We should additionally construct new covariant operators $\tilde{J}^{ak}$ in the same spirit as in this paper. The details of the construction will be presented in a forthcoming paper.

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APPENDIX A

Let us consider the three infinitesimal Poincaré transformations:

(a) translation $t' = t, \quad \mathbf{x}' = \mathbf{x} + \boldsymbol{\epsilon}$,
(b) rotation $t' = t, \quad \mathbf{x}' = \mathbf{x} + \boldsymbol{\epsilon} \times \mathbf{x}$,
(c) boost $t' = t - \boldsymbol{\epsilon}\cdot\mathbf{x}, \quad \mathbf{x}' = \mathbf{x} - \boldsymbol{\epsilon} t$.

The bispinor ψ which obeys the Dirac equation in the FW representation

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi ,$$

where the Hamiltonian H is defined by (8) for a free particle or (2) for a particle in external color fields, undergoes the following transformation under the transformations (a), (b), and (c):

$$\psi'(\mathbf{x},t) = \left[1 - \frac{i}{\hbar} \boldsymbol{\epsilon}\cdot\mathbf{G} \right] \psi(\mathbf{x},t) .$$

The operators $\mathbf{G}$ are equal to

$$\mathbf{G} = \begin{cases} \hat{\mathbf{P}} \text{ for translation} , \\ \hat{\mathbf{J}} = \hat{\mathbf{R}} \times \hat{\mathbf{P}} + \dfrac{\hbar}{2}\hat{\boldsymbol{\sigma}} \text{ for rotation} , \\ \hat{\mathbf{K}} = \hat{\mathbf{N}} - t\hat{\mathbf{P}} \text{ for boost} . \end{cases}$$

$\hat{\mathbf{R}}$ and $\hat{\mathbf{P}}$ are canonical operators of the position and momentum and $\hat{\boldsymbol{\sigma}}$ are Pauli matrices. The operator $\hat{\mathbf{N}}$ is defined

$$\hat{\mathbf{N}} = \begin{cases} \frac{1}{2}\{\hat{\mathbf{R}}, \beta E_P\} + \dfrac{\hbar\beta \mathbf{p} \times \hat{\boldsymbol{\sigma}}}{2(m + E_P)} \quad \text{for a free particle} , \\ \frac{1}{2}\{\hat{\mathbf{R}}, H_{\mathrm{FW}}\} + \dfrac{\beta\hbar}{4m}(\hat{\boldsymbol{\pi}} \times \hat{\boldsymbol{\sigma}}) + O(\hbar^2/m^2) \quad \text{for a particle interacting with a field} . \end{cases}$$

APPENDIX B

The four-potential $\hat{A}_\mu = (A_0, \mathbf{A})$ transforms in the following way under the infinitesimal Poincaré transformations from Appendix A:

(a) translation

$$A_0'(x) = A_0(x) - \epsilon^k \partial_k A_0(x) ,$$

$$\mathbf{A}'(x) = \mathbf{A}(x) - \epsilon^k \partial_k \mathbf{A}(x) ,$$

(b) rotation

$$A'_0(x) = A_0(x) - (\boldsymbol{\epsilon}\times\mathbf{x})^k \partial_k A_0(x) ,$$

$$\mathbf{A}'(x) = \mathbf{A}(x) + \boldsymbol{\epsilon}\times \mathbf{A}(x) - (\boldsymbol{\epsilon}\times\mathbf{x})^k \partial_k \mathbf{A}(x) ,$$

(c) boost

$$A'_0(x) = A_0(x) - \boldsymbol{\epsilon}\cdot \mathbf{A}(x) + t[\epsilon^k \partial_k A_0(x)] + (\boldsymbol{\epsilon}\cdot\mathbf{x})\partial_0 A_0(x) ,$$

$$\mathbf{A}'(x) = \mathbf{A}(\mathbf{x}) - \boldsymbol{\epsilon}\cdot A_0(x) + t[\epsilon^k \partial_k \mathbf{A}(x)] + (\boldsymbol{\epsilon}\cdot\mathbf{x})\partial_0 \mathbf{A}(x) .$$

The Lie derivative of the potential $A_\mu(x)$ is defined by the equation

$$A'_\mu(x) = A_\mu(x) + \epsilon^k \cdot \mathcal{L}_k A_\mu(x) .$$

It is easy to find the Lie derivatives for our infinitesimal transformations.

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