Thermal expansion and critical temperature in a geometric representation of quark deconfinement

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The critical temperature of the quark deconfinement transition is calculated combining a geometrical model of thermal expansion with a phenomenological description of hadronization. The value thus obtained is perfectly compatible with lattice QCD and bag-model estimates.

Considering a single hadronic bag in thermal equilibrium in the framework of the MIT bag model, it was recently argued[1] that at finite internal temperature the bag radius acquires a thermal dependence: in particular the radius diverges as T increases towards a limiting temperature T_c; at this critical temperature quark deconfinement is assumed to occur.

The aim of this paper is to show that an increase of the hadronic volume at finite temperature can also be deduced using completely different arguments, in the context of a geometric representation of the confinement process based on the anti–de Sitter geometry. One obtains in this way a thermal behavior of the effective confining radius in qualitative agreement with Ref. 1. What is remarkable is that, in this context, it is possible to compute the value of the critical temperature and, even more remarkable, this is very close to what is predicted within lattice QCD.

It is known[2] that the confining aspects of strong-interaction dynamics can be represented formally by embedding the quark in an anti–de Sitter manifold, with negative cosmological constant ($\Lambda < 0$). By using in particular such geometry[3] in order to reproduce the effective potential responsible for the exponential damping of the quark wave function postulated in a phenomenological hadronization model,[4] one obtains a relation between the characteristic length of confinement, x_0, and the effective cosmological constant,[5] i.e., $\Lambda = -3/x_0^2$. A possible variation of Λ describes therefore, in this model, a variation of the effective radius of the corresponding hadronic bag.

At zero temperature, the experimental data on charmed-meson decays give the estimate[4] $x_0 \simeq 1\ \text{GeV}^{-1}$ so that $(-3/\Lambda)^{1/2} \simeq 1\ \text{GeV}^{-1}$. At finite temperature the geometry is modified because of direct thermal contributions, and is also indirectly modified because of the thermal corrections to the matter sources. The contribution of the latter corrections to the field equations, however, is weighted by the Newton coupling constant, so that it may become relevant only for temperatures sufficiently near the Planck mass, for example, in a cosmological context, unless one makes rather wild assumptions on the effective coupling constant of gravity on the level of hadronic matter.

Consequently, we shall consider only the direct thermal contributions to the effective microscopic geometry. As discussed in Ref. 6, they can be classically evaluated by using the projective invariance of a metric affine geometric structure, on the grounds of a generalized principle of equivalence. An average of the thermal corrections over a scale of lengths and times larger than the Planck one gives then, as the only correction to the geometry, the contribution $12\pi^2T^2$ to the cosmological constant.[6]

By including this thermal contribution, the cosmological constant of the anti–de Sitter vacuum representing quark confinement is then modified as

$$\Lambda(T) = -3/x_0^2 + 12\pi^2T^2 , \tag{1}$$

where T denotes the intrinsic temperature of the geometric background.

Supposing that the hadronic density is high enough to keep in thermal contact matter and geometry,[6,7] we can thus immediately predict, with this model, that the deconfining temperature [corresponding to the divergence of the effective radius, i.e., $\Lambda(T)=0$] is

$$T_c = 1/2\pi x_0 \simeq 160\ \text{MeV} \tag{2}$$

in very good agreement with QCD lattice calculations and estimates based on the bag model (see, for example, Ref. 8), which suggest $T_c \simeq 150-200$ MeV.

According to Eq. (1), the geometric model moreover predicts, for the effective confining radius $x_0(T) = [-3/\Lambda(T)]^{1/2}$, the thermal behavior

$$x_0(T) = x_0(1 - T^2/T_c^2)^{-1/2} . \tag{3}$$

This expression describes an expansion of the hadronic bag in qualitative agreement with Ref. 1, since $x_0 \to \infty$ as $T \to T_c$, but with a different power-law dependence on T. Following Ref. 1, the critical baryon-number density ρ_c at which quark deconfinement occurs may behave, in a suitable approximation, as $1/x_0^3(T)$. As a consequence of Eq. (3) the geometrical model gives then for ρ_c the thermal dependence $(1-T^2/T_c^2)^{3/2}$, instead of $(1-T^4/T_c^4)^{3/4}$ obtained in Ref. 1.

In conclusion it should be noted that a thermal variation of x_0 may have testable phenomenological consequences in the context of this geometric scheme, as may induce an energy dependence in the effective quark potential describing hadronization,[4] like the energy dependence empirically introduced and discussed in Ref. 9. From the experimental data relative to the hadronization process at high energy one could obtain then indirect indications on a possible variation of x_0, and on the type of thermal behavior, to be compared with the one obtained geometrically here, and the one suggested in Ref. 1.

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