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Brief Reports

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Do we understand $D_s^+ \to \eta\pi^+$ and $D_s^+ \to \eta'\pi^+$?

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We have analyzed a recent observation of $D_s^+ \to \eta\pi^+$ and $D_s^+ \to \eta'\pi^+$ in two theoretical schemes for charm→two-body hadron decays.

Recently[1,2] the decays $D_s^+ \to \eta\pi^+$ and $D_s^+ \to \eta'\pi^+$ have been observed with quite large branching ratios. The Mark III Collaboration observes[1] $B(D_s^+ \to \eta\pi^+)/B(D_s^+ \to \phi\pi^+) = (2.5 \pm 0.8 \pm 0.8)$, whereas the Mark II Collaboration quotes[2] a preliminary number of (3.0 ± 1.3) for this ratio. With the world average[3] $B(D_s^+ \to \phi\pi^+)$ around 3.5%, this implies $B(D_s^+ \to \eta\pi^+)$ in the region of 10%. The Mark III Collaboration also quotes[1] $B(D_s^+ \to \eta\pi^+)/B(D_s^+ \to \bar{K}^0 K^+) = (2.3 \pm 0.7 \pm 0.8)$. With[3] $B(D_s^+ \to \bar{K}^0 K^+) = (3.7 \pm 2.0)$, one again finds $B(D_s^+ \to \eta\pi^+)$ in the vicinity of 10%.

The Mark II Collaboration reports[2] $B(D_s^+ \to \eta'\pi^+)$ "at least as large as $B(D_s^+ \to \eta\pi^+)$." The second work by Wormser in Ref. 2 quotes $B(D_s^+ \to \eta'\pi^+)$ "around 18%." The observed large branching ratios for $D_s^+ \to \eta\pi^+$ and $\eta'\pi^+$ go a long way in accounting for the missing rates in the hadronic decays of D_s^+. The question is the following: Are these branching ratios understandable in a theoretical framework?

In the following we have looked at the predictions of two models, one due to Kamal and Sinha[4] (KS) and the second due to Bauer, Stech, and Wirbel[5] (BSW), and explored the predictions with various η-η' mixing schemes.

The weak Hamiltonian for Cabibbo-angle-favored decays of charm is

$$H_W = \frac{G_F \cos^2\theta_C}{\sqrt{2}} \left[\tfrac{1}{2}(C_+ + C_-)(\bar{u}d)(\bar{s}c) + \tfrac{1}{2}(C_+ - C_-)(\bar{u}c)(\bar{s}d) \right] . \tag{1}$$

Here C_+ and C_- are the well-known QCD coefficients which are unity in the absence of QCD corrections, $(\bar{u}d)$, etc., are left-handed currents. θ_C is the Cabibbo angle and G_F the weak Fermi coupling constant.

Both Refs. 4 and 5 use the factorization approximation. Following the method of KS (Ref. 4) we obtain the following decay amplitude:

$$\begin{aligned} A(D_s^+ \to \eta\pi^+) &= \frac{2}{\sqrt{3}} C_1 f_\pi (m_{D_s}^2 - m_\eta^2) \\ &\quad \times \left(\cos\theta_P + \frac{1}{\sqrt{2}} \sin\theta_P\right) , \\ A(D_s^+ \to \eta'\pi^+) &= \sqrt{2/3}\, C_1 f_\pi (m_{D_s}^2 - m_{\eta'}^2) \\ &\quad \times (\cos\theta_P - \sqrt{2}\sin\theta_P) , \\ A(D_s^+ \to K^+\bar{K}^0) &= \sqrt{2}\, C_2 f_K (m_{D_s}^2 - m_K^2) , \\ A(D_s^+ \to \phi\pi^+) &= 2\sqrt{2}(\epsilon \cdot k) C_1 f_\pi f_{D_s} g_{VPP} . \end{aligned} \tag{2}$$

All the above amplitudes are to be multiplied by $G_F/\sqrt{2}\cos^2\theta_C$. k is the π^+ momentum in the D_s rest frame and θ_P is the singlet-octet mixing angle. f_π and f_K are normalized to 93 and 120 MeV, respectively. g_{VPP}, assuming SU(4) symmetry, is related to the $\rho\pi\pi$ coupling constant, $g_{VPP}^2/4\pi = 3$. C_1 and C_2 are defined by

$$\begin{aligned} C_1 &= \tfrac{1}{2}[(C_+ + C_-) + \xi(C_+ - C_-)] , \\ C_2 &= \tfrac{1}{2}[(C_+ - C_-) + \xi(C_+ + C_-)] , \end{aligned} \tag{3}$$

ξ is the color factor, $\frac{1}{3}$ for $\mathrm{SU}(3)_c$. However, phenomenology suggests[5] $\xi = 0$ fits charm decay data very well.

It must be pointed out that annihilation plays no role in the four decay amplitudes exhibited in (2). This is a consequence of the conserved-vector-current (CVC) hypothesis in the $(\bar{u}d)$ sector for the vector current and the absence of second-class axial-vector current (G-parity even) in the $(\bar{u}d)$ sector. The work of Bauer, Stech, and Wirbel[5] (BSW) uses a pseudoscalar mixing angle $\theta_P = -11°$. We have redone their calculation with

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different mixing angles. The difference between their amplitudes and those in (2) lies in the hadronic form factors. BSW calculate these from $(q\bar{q})$ wave functions[6] for the mesons, while in the KS model[4] SU(4) symmetry is used with

$$f_+(m_\pi^2) \approx f_+(m_K^2) \approx f_+(0) = 1 ,$$

to generate the form factors. In (2) the contribution of the form factors $f_-(m_\pi^2)$ and $f_-(m_K^2)$ is neglected since they come multiplied by m_π^2 and m_K^2, respectively.

In Table I we have summarized the results of our calculations for different relevant rates. These rates depend on the QCD coefficients C_1 and C_2 and the η-η' mixing angle. We have used $\theta_P = -11°$ and[7] $\theta_P = -19°$ in an orthogonal mixing scheme involving η and η' only.

As the magnitude of the mixing angle is increased, $\Gamma(D_s^+ \to \eta\pi^+)$ decreases and $\Gamma(D_s^+ \to \eta'\pi^+)$ increases. For a mixing angle of $\approx -19°$ we find, as is seen from Table I, that both KS (Ref. 4) and BSW (Ref. 5) models give $B(D_s^+ \to \eta'\pi^+)/B(D_s^+ \to \eta\pi^+) \approx 1$. However, since the effect of a larger (and negative) mixing angle is to lower the rate for $D_s^+ \to \eta\pi^+$, $B(D_s^+ \to \eta\pi^+)/B(D_s^+ \to \phi\pi^+)$ for $\theta_P = -19°$ is lower than that for $\theta_P = -11°$. In the KS model[4] this ratio still stays well within the experimental error; however, in the BSW model[5] this ratio drops well below the minimum value allowed by data.[1]

In Table II we have converted the rate formulas of Table I into branching ratios. We have used[3] $\tau_D = 4.33 \times 10^{-13}$ sec and calculated C_1 and C_2 from next-to-leading-log (NLL) perturbation theory[8] with $m_t = 50$ GeV and $\xi = 0$. We have shown three numbers in Table II for each rate. They correspond to NLL-calculated C_1 and C_2, with $\xi = 0$, for QCD parameters (μ_c, Λ) in GeV $= (1.5, 0.1)$, $(1.5, 0.2)$, and $(1.5, 0.3)$, as indicated in the table caption. It is interesting to note that both KS and BSW models are consistent with data for $B(D_s^+ \to \phi\pi^+)$ and $B(D_s^+ \to \eta\pi^+)/B(D_s^+ \to \bar{K}^0K^+)$ for both $\theta_P = -11°$ and $-19°$. However, the BSW model gives low values for both $B(D_s^+ \to \eta\pi^+)$ and $B(D_s^+ \to \bar{K}^0K^+)$ individually. In the KS model $B(D_s^+ \to \bar{K}^0K^+)$ is consistent with the data, while for $\theta_P = -19°$ $B(D_s^+ \to \eta\pi^+)$ is barely consistent with the data.[1]

Summarizing our results so far, in the standard orthogonal η-η' mixing formalism we find the following.

(1) Both KS and BSW models predict $B(D_s^+ \to \eta'\pi^+)/B(D_s^+ \to \eta\pi^+) \approx 1$ for $\theta_P = -19°$. For $\theta_P = -11°$ this ratio is predicted to be ≈ 0.6. We note that this ratio is independent of the QCD coefficients C_1 and C_2.

(2) The KS model predicts $B(D_s^+ \to \eta\pi^+)/B(D_s^+ \to \phi\pi^+) = 1.89$ for $\theta_P = -11°$ and 1.35 for $\theta_P = -19°$. Both these values are consistent with data[1] $(2.5 \pm 0.8 \pm 0.8)$. BSW model predicts this ratio to be 1.05 $(\theta_P = -11°)$ and 0.75 $(\theta_P = -19°)$. While the prediction for $\theta_P = -11°$ is barely consistent with data, the prediction for $\theta_P = -19°$ is too low. Note again that this ratio is independent of the QCD coefficients C_1 and C_2.

Once we determine C_1 and C_2 with $\xi = 0$ from a NLL perturbation calculation with $m_t = 50$ GeV, we can make the following statements.

(3) $B(D_s^+ \to \phi\pi^+)$ is predicted to be consistent with data[3] in both the KS and BSW models.

(4) $B(D_s^+ \to \bar{K}^0K^+)$ in the KS model is consistent with data[3] with QCD coefficients calculated with $\mu_c = 1.5$ GeV and Λ in the range 0.1–0.3 GeV. In the BSW model this branching ratio is predicted to be too low for $\Lambda = 0.1$ and 0.2 GeV. The prediction is consistent with data for $\Lambda = 0.3$ GeV.

(5) $B(D_s^+ \to \eta\pi^+)/B(D_s^+ \to \bar{K}^0K^+)$ is consistent with data[1] in both the KS and BSW models for both mixing angles. However, in the BSW model this agreement is secured at the expense of lowering both rates below their experimental values.

TABLE I. Rates (in 10^{10} sec^{-1}) and ratios of rates for various decays in the KS and BSW models.

Mode	KS model (Ref. 4)		BSW model (Ref. 5)		Experiment
	$\theta_P = -11°$	$\theta_P = -19°$	$\theta_P = -11°$	$\theta_P = -19°$	
$\Gamma(D_s^+ \to \eta'\pi^+)$	$5.64C_1^2$	$7.11C_1^2$	$2.87C_1^2$	$3.63C_1^2$	
$\Gamma(D_s^+ \to \eta\pi^+)$	$9.11C_1^2$	$6.50C_1^2$	$4.89C_1^2$	$3.50C_1^2$	
$\Gamma(D_s^+ \to \phi\pi^+)$	$4.83C_1^2$		$4.67C_1^2$		
$\Gamma(D_s \to \bar{K}^0K^+)$	$30.91C_2^2$		$12.76C_2^2$		
$\dfrac{\Gamma(D_s^+ \to \eta'\pi^+)}{\Gamma(D_s^+ \to \eta\pi^+)}$	0.62	1.09	0.59	1.04	
$\dfrac{\Gamma(D_s^+ \to \eta\pi^+)}{\Gamma(D_s^+ \to \phi\pi^+)}$	1.89	1.35	1.05	0.75	$2.5 \pm 0.8 \pm 0.8$ (Ref. 1)
$\dfrac{\Gamma(D_s^+ \to \eta\pi^+)}{\Gamma(D_s^+ \to \bar{K}^0K^+)}$	$0.29\left[\dfrac{C_1}{C_2}\right]^2$	$0.21\left[\dfrac{C_1}{C_2}\right]^2$	$0.38\left[\dfrac{C_1}{C_2}\right]^2$	$0.27\left[\dfrac{C_1}{C_2}\right]^2$	$2.3 \pm 0.7 \pm 0.8$ (Ref. 1)

TABLE II. Branching ratios (in %) (with $\tau_{D_s^+}=4.33\times10^{-13}$ sec) for various decays on the KS and BSW models. The three entries (a,b,c) are obtained with $(\mu_c,\Lambda)=(1.5,0.1)$, $(1.5,0.2)$, $(1.5,0.3)$ GeV, respectively.

Mode	KS model (Ref. 4)		BSW model (Ref. 5)		Experiment
	$\theta_P=-11°$	$\theta_P=-19°$	$\theta_P=-11°$	$\theta_P=-19°$	
$B(D_s^+\to\eta'\pi^+)$	3.43[a]	4.33	1.74	2.20	
	3.70[b]	4.67	1.81	2.38	≈18% (Ref. 2)
	4.09[c]	5.16	2.07	2.62	
$B(D_s^+\to\eta\pi^+)$	5.55	3.96	2.97	2.12	
	5.98	4.25	3.20	2.29	≈11% (Ref. 2)
	6.61	4.72	3.54	2.53	
$B(D_s^+\to\phi\pi^+)$	2.94		2.84		3.3±1.6±0.4 (TASSO)
	3.17		3.06		3.2±0.7±0.5 (ARGUS)
	3.50		3.39		3.5±0.8 (E-691)
					4.4±1.1 (CLEO)
					3.3±1.0 (HRS)
					(Ref. 3)
$B(D_s^+\to\bar{K}^0K^+)$	1.96		0.81		3.7±2.0 (Mark III)
	2.79		1.15		(Ref. 3)
	4.01		1.69		
$\dfrac{B(D_s^+\to\eta\pi^+)}{B(D_s^+\to\bar{K}^0K^+)}$	2.83	2.02	3.67	2.62	2.3±0.7±0.8
	2.14	1.53	2.78	1.99	(Ref. 1)
	1.65	1.17	2.09	1.50	

In the standard η-η' mixing formalism the two ratios $B(D_s^+\to\eta\pi^+)/B(D_s^+\to\phi\pi^+)$ and $B(D_s^+\to\eta'\pi^+)/B(D_s^+\to\phi\pi^+)$ pull in opposite directions as function of the mixing angle θ_P, that is, raising the magnitude of θ_P has the effect of lowering the first ratio and raising the second. Thus, if

$$B(D_s^+\to\eta\pi^+)/B(D_s^+\to\phi\pi^+)\gtrsim1.5$$

and

$$B(D_s^+\to\eta'\pi^+)/B(D_s^+\to\eta\pi^+)\gtrsim1.0 \tag{4}$$

the standard orthogonal mixing model will have difficulty in explaining the data. The Mark II data[2] (see the second reference to Wormser in Ref. 2) indicate that the first ratio in (4) is about 3 (preliminary) and that the second ratio in (4) is about 1.6. Moreover,[2] $B(D_s^+\to\eta'\pi^+)$ is about 18%. As is evident from Table II such a large branching ratio poses a problem for standard orthogonal η-η' mixing model. The current prejudice favors[7] a mixing angle $\theta_P\simeq-19°$. The branching ratio for $D_s^+\to\eta'\pi^+$ could be enhanced by using a larger mixing angle; however, in that case $B(D_s^+\to\eta\pi^+)$ will drop to unacceptably low values.

The remainder of this paper is somewhat speculative in nature. If the standard orthogonal mixing for η and η' runs into trouble with $D_s^+\to\eta\pi^+$ and $D_s^+\to\eta'\pi^+$ data, one may seek to reconcile data with theory in one or more of the ways we discuss below.

The models we have discussed use factorization of the hadronic decay amplitude. One may suspect factorization, yet factorization appears to work[5] well in other hadronic decays of charm. One could also suspect nonperturbative contributions to the QCD coefficients. However, both the decay amplitudes for $D_s^+\to\eta\pi^+$ and $\eta'\pi^+$ involve the same combination of QCD coefficients. Hence, the relative normalization of the two rates should remain unaffected by nonperturbative effects.

Earlier analyses[9,10] of the Mark III data on J/ψ decays, which had neglected double Okubo-Zweig-Iizuka-rule violating[11] (DOZI) amplitudes, in terms of a generalized mixing, had concluded that there was a significant gluonium content in η'. Defining[12]

$$\begin{aligned}|\eta\rangle&=X_\eta\,|N\rangle+Y_\eta\,|S\rangle+Z_\eta\,|G\rangle\ ,\\ |\eta'\rangle&=X_{\eta'}\,|N\rangle+Y_{\eta'}\,|S\rangle+Z_{\eta'}\,|G\rangle\ ,\end{aligned} \tag{5}$$

where

$$\begin{aligned}&|N\rangle=\frac{1}{\sqrt{2}}\,|u\bar{u}+d\bar{d}\rangle,\quad |S\rangle=|s\bar{s}\rangle\ ,\\ &|G\rangle=|\,\text{gluonium}\rangle\end{aligned} \tag{6}$$

with the normalization

$$X^2+Y^2+Z^2=1\ , \tag{7}$$

the 1985 analysis[9] by the Mark III Collaboration had yielded

$$\begin{aligned}&|X_\eta|=0.63\pm0.06,\quad |Y_\eta|=0.83\pm0.13\ ,\\ &|X_{\eta'}|=0.36\pm0.05,\quad |Y_{\eta'}|=0.72\pm0.12\ .\end{aligned} \tag{8}$$

Haber and Perrier[10] analysis gave similar results. The above set is consistent with $Z_\eta=0$ but allows $|Z_{\eta'}|=0.59\pm0.09$.

However, a recent analysis[13] of the Mark III J/ψ data, with the inclusion of DOZI amplitudes, concludes that if DOZI amplitudes are about 15% of OZI-violating amplitudes then there is no room for gluonium in η' and one can satisfactorily explain J/ψ decay data using the standard orthogonal η-η' mixing scheme with $\theta_P \approx -19°$. However, if the orthogonality condition $X_{\eta'} = -Y_\eta$ is dropped, unitarity condition (7) appears to be oversubscribed:[13]

$$X_{\eta'}^2 + Y_{\eta'}^2 = 1.44 \pm 0.26 \ . \tag{9}$$

The reader is also referred to the analysis in Ref. 14 and a summary in Ref. 15. Thus, if DOZI amplitudes are at the level of 15% of OZI-violating amplitudes, $Z_{\eta'} \approx 0$. However, if DOZI amplitudes are considerably smaller then $Z_{\eta'}$ is allowed to be nonzero and one could enhance the rate for $D_s^+ \to \eta'\pi^+$ while leaving the rate for $D_s^+ \to \eta\pi^+$ unaffected. In this connection it would be interesting to search for gluonium candidates, such as $\iota(1440)$, in D_s^+ decays into $(KK\pi)\pi^+$.

What about final-state interactions? In principle the three channels $\bar{K}^0K^+$, $\eta\pi^+$, and $\eta'\pi^+$ could mix through final-state interactions. However, we do not expect final-state interactions to play a significant role for the following reason. All these channels involve a single isospin state, $I=1$. The only resonance with $I=1$, $G=-1$, and $J=0$ appears[16] to be $a_0(980)$ well below the D_s^+ mass. Since the resonance activity occurs well below the D_s^+ mass we expect the effect of final-state interaction to be simply to rotate the amplitudes leaving the magnitude of the amplitudes largely unaffected. However, since the final states involve a single isospin amplitude, the phase of the amplitude is irrelevant. A coupled three-channel mode analysis carried out by us confirmed this speculation.

Lastly, penguin diagrams play no role in $D_s^+ \to \eta\pi^+$ and $\eta'\pi^+$ decays.

In conclusion, the large branching ratios observed for $D_s^+ \to \eta\pi^+$ and $\eta'\pi^+$ by the Mark III (Ref. 1) and Mark II (Ref. 2) Collaborations pose a problem for the standard orthogonal η-η' mixing model with a mixing angle in the region of $\theta_P \simeq -19°$.

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