$\langle q\bar{q} \rangle$ and higher-dimensional-condensate contributions to the nonperturbative quark mass

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Dimension-3, -4, and -5 condensate contributions to the quark propagator are evaluated in the presence of a nonzero current-quark mass (m_L) explicitly breaking Lagrangian chiral symmetry through the use of an operator-product expansion (OPE) of appropriate nonperturbative vacuum expectation values in the fixed-point gauge. A gauge-parameter-independent shift away from the purely perturbative propagator pole at m_L is seen to occur, provided the corrected pole is identified self-consistently with the OPE mass parameter, a mass which differs from m_L by incorporating contributions from nonperturbative condensates. The results obtained are argued to be valid to all orders in the OPE, and are shown to encompass the entire $O(g_s^2)$ contribution of nonperturbative condensates to the quark propagator. Phenomenological consequences at (and near) the $m_L=0$ chiral limit are also discussed.

I. INTRODUCTION

One of the outstanding problems within standard-model physics is the mass spectrum of fundamental fermions. Within the context of standard electroweak physics,[1] quarks and leptons are assumed to acquire mass entirely through their Yukawa couplings to the electroweak vacuum expectation value (VEV) $\langle \phi \rangle$, a dimension-1 order parameter characterizing spontaneous breakdown of $SU(2)\times U(1)$ gauge symmetry. Within any given fermion family, the fact that quarks are heavier than leptons would seem to imply that Yukawa coupling strengths of color-nonsinglet fermions are larger than those of color-singlet fermions. This "QCD awareness" on the part of electroweak Yukawa couplings must be regarded as unnatural.[2] If the electroweak interactions are responsible for lepton masses, it is more reasonable to suppose that the heavier masses of quarks are a dynamical manifestation of the additional QCD forces which quarks experience.[3]

Indeed, there is strong phenomenological support for the existence of both Yukawa (or current) and dynamical components to the u- and d-quark masses. The current mass denotes the small effective u- and d-quark masses appropriate for current-algebra applications and deep-inelastic phenomenology.[4] These small masses are themselves one (or more[5,6]) orders of magnitude larger than the electron mass, a discrepancy that, within grand-unification contexts, perhaps may be a consequence of additional $SU(3)_{\text{color}}$ contributions to the perturbatively generated renormalization-group equations for running quark masses.[7–9] We know, however, that u- and d-quark masses on a much larger scale ($\sim$300 MeV) have been quite successful in explaining hadron spectroscopy and nucleon magnetic moments.[10] Such large quark masses cannot themselves devolve from the renormalization-group behavior of the small masses generated through the VEV $\langle \phi \rangle$; since $\langle \phi \rangle$ has dimensions of mass, masses generated through $\langle \phi \rangle$ can depend at most logarithmically on external momentum scales.[7,8] In order to develop the large constituent-quark masses appropriate for bound-state physics, the strong interactions must somehow generate additional chiral-symmetry-breaking order parameters of higher dimension than $\langle \phi \rangle$ (Ref. 11), yielding "dynamical" quark masses even in the explicit chiral-symmetry limit of vanishing Yukawa couplings.[12,13]

In particular, the strong interactions are expected to be sufficiently strong to condense local, Lorentz-scalar, SU(3)-color-invariant but chiral-symmetry-violating products of fields. The lowest-dimension operator of this type is the quark condensate $\langle \bar{q}q \rangle$, an operator of mass dimension 3. From dimensional considerations alone, tree-level $\langle \bar{q}q \rangle$ insertions on massless quark lines (analogous to tree-level Yukawa-coupling-dependent $\langle \phi \rangle$ insertions from spontaneous symmetry breaking) necessarily correspond to effective masses that have inverse-square dependence on off-shell fermion momenta: $M_{\text{eff}}(p^2) \sim |\langle \bar{q}q \rangle / p^2|$. Moreover, renormalization-group effects can at most modify this inverse-square behavior logarithmically.[12–14] Dynamical mass contributions are therefore expected to be significant at low $|p^2|$, where constituent-quark masses dominate low-energy hadron physics, and unimportant at high $|p^2|$, the regime of current-quark-mass phenomenology.[13]

From the perspective of perturbative quantum field theory, a propagator pole is an "observable." Indeed, the pole positions of propagators do not depend on the choice of gauge; for example, the gauge independence of quark propagator poles in purely perturbative QCD has been verified to two-loop order.[15] In a truly perturbative theory (such as QED), fermion propagator poles are identified with observed fermion masses (e.g., electron mass). For QCD, such identification becomes obscured at sufficiently large distances by quark confinement, which must be understood to arise wholly from nonperturbative effects. Nevertheless, one would like to continue identifying "constituent"-quark masses (as in observable static hadron properties) with any propagator poles that may occur in the absence of explicit nonperturbative confinement-generating mechanisms.

It is of interest for such a program that confinement (as evidenced by the $q\bar{q}$ potential's string tension) and dynamical mass generation are widely held to be manifestations of distinct nonperturbative order parameters, the former devolving[16] from the chirally invariant gluon condensate of mass dimension 4, and the latter arising[12] from the chiral-symmetry-breaking quark condensate of mass dimension 3. Thus, there is reason to hope for a decoupling between confinement mechanisms and the mechanism dynamically generating the scale of the constituent-quark mass.

One approach to incorporating nonperturbative physics, pioneered in the development of QCD sum rules,[11] is to allow nonperturbative order parameters to enter the Feynman-Dyson perturbation series. Such condensates characterize operator-product expansions of VEV's of normal-ordered products of uncontracted fields arising residually from the Wick expansion,[17] VEV's which are taken to be zero in purely perturbative contexts. The short-distance character of the operator-product expansion, corresponding to the perturbative domain of QCD with $\alpha_s \leq 1$, suggests the appropriateness of such an approach for studying nonperturbative contributions at subconfining distance scales.

The approach described above has already been shown to continue to support the same gauge-independent quark propagator poles that characterize purely perturbative QCD (Refs. 18 and 19). Indeed, earlier work has demonstrated how the dynamical contribution to the quark self-energy arising from the dimension-3 quark-condensate still yields a gauge-parameter-independent quark propagator pole, provided the quark mass appearing in subleading operator-product expansion (OPE) terms is self-consistently identified with that pole.[14] Subsequent work has pointed toward the gauge independence of further contributions to the quark propagator pole from the dimension-5 chiral-symmetry-breaking mixed quark-gluon condensate.[20] These results suggest a correspondence between the constituent-quark masses of static hadron properties and the quark propagator poles obtained from the above-described "OPE-augmented" QCD, a correspondence which is the central focus of this paper.

In this paper we examine in detail the gauge dependence of tree-level contributions to the quark self-energy arising from nonperturbative vacuum expectation values sensitive to the dimension-3, -4, and -5 order parameters of nonperturbative QCD. We find gauge independence to be a signature of the quark propagator pole, provided the pole is required to coincide with the quark mass characterizing the OPE of the particular nonperturbative VEV under consideration. This self-consistency property enables the construction of a relationship between the current-quark mass and the quark mass one obtains after contributions from nonperturbative condensates have been included.

In Sec. II we consider $O(g_s^2)$ contributions to the quark propagator arising from the chiral-symmetry-violating dimension-3 quark condensate ($\langle \bar{q}q \rangle$). Prior to such corrections, the quark propagator is assumed to have a nonzero spontaneous-symmetry-breaking component corresponding to the current- (or electroweak-Lagrangian-) quark mass. We find the new propagator pole remains gauge-parameter independent, provided the pole coincides with the nonperturbative mass characterizing the OPE of the nonperturbative VEV contributing to the Wick expansion of the fermion two-point amplitude (through a pair of uncontracted normal-ordered quark fields). This self-consistency condition is then utilized to obtain a gauge-independent relation between the current-quark mass and the quark mass one obtains after nonperturbative $\langle \bar{q}q \rangle$ corrections are included.

In Sec. III we consider further contributions to the quark propagator arising from the chiral-symmetric dimension-4 gluon condensate ($\langle GG \rangle$). We first demonstrate that any contributions to the quark propagator from condensates of dimension >4 are necessarily accompanied by additional factors of the QCD coupling constant g_s. Consequently, only $\langle \bar{q}q \rangle$ and $\langle GG \rangle$ contribute to $O(g_s^2)$ nonperturbative corrections to the quark propagator. We then calculate the full $O(g_s^2)$ contribution to the propagator pole. The expression we obtain, utilizing OPE's of nonperturbative VEV's obtained in the Appendix, is shown to be valid to all orders in the OPE mass parameter m (Ref. 18). In the chiral limit of vanishing current- (or Lagrangian) quark mass, the $\langle GG \rangle$ contribution to the quark propagator is seen to vanish,[19] perhaps indicative of a decoupling of nonperturbative order parameters phenomenologically characterizing chiral-symmetry breakdown ($\langle \bar{q}q \rangle$) and confinement ($\langle GG \rangle$).

In Sec. IV we discuss the $O(g_s^3)$ contributions to the quark propagator generated through the chirally noninvariant quark-gluon mixed condensate ($\langle \bar{q}G\cdot\sigma q \rangle$). The contributions are evaluated using expressions obtained in the Appendix for $\langle \bar{q}G\cdot\sigma q \rangle$ projections of nonperturbative VEV's valid to $O(m^3)$ in the OPE mass parameter m. The contribution of such VEV's to amplitudes corresponding to "Abelian graphs"—graphs insensitive to non-Abelian (multiple) gluon couplings—appears to truncate after $O(m^2)$ contributions are considered. The $O(m^3)$ contributions arising from the only contributing "non-Abelian graph" (involving a triple-gluon coupling) appear to generate a geometric series in $m\not{p}/p^2$. If the Abelian-graph contributions from the $\langle \bar{q}G\cdot\sigma q \rangle$ projection of the OPE indeed truncate after $O(m^2)$, and if all $O(m^n; n\geq 4)$ non-Abelian-graph contributions correspond to higher-order terms in the geometric series already obtained explicitly to $O(m^3)$, then an $O(g_s^3)$ expression for the mixed-condensate contribution to the quark propagator can be obtained that is valid *to all orders* of the OPE mass parameter m. Upon analysis, this expression leaves the quark propagator pole unshifted from the value obtained from $O(g_s^2)$ nonperturbative corrections. This failure of $\langle \bar{q}G\cdot\sigma q \rangle$ to contribute to the propagator pole is independent of the choice of gauge or the magnitude of the current-quark mass.

In Sec. V we discuss the phenomenological implications of our results as the limit of explicit Lagrangian chiral symmetry is approached, a limit appropriate for the discussion of u- and d-quark masses. In particular, we discuss the magnitude and gauge independence of dynamical contributions to u- and d-quark masses, and we predict an enhancement of the current u-d mass difference relative to the constituent u-d mass difference. We also obtain a chiral-limiting relationship between the quark condensate and the dynamical quark mass through an "inverse approach," involving identification of the condensate with a nonperturbative quark loop. This approach, which has already been discussed elsewhere,[14] is reconsidered here for arbitrary color number N_c in order to avoid spurious equality at $N_c=3$ between the anomalous mass-dimension exponent $d\equiv g_s\gamma_m(g_s)/2\beta(g_s)$ and the asymptotic-freedom parameter $d'\equiv[\alpha_s(p^2)/\pi]\ln(p^2/\Lambda^2)$. The result we obtain is shown to be consistent with the $\langle \bar{q}q \rangle$ contribution to the quark condensate obtained for arbitrary N_c (Ref. 21), providing a useful crosscheck of our results.

Section VI discusses and summarizes the field-theoretical results of our paper. A speculation on the link between unbroken non-Abelian symmetry and confinement, within the context of OPE-augmented QCD, is also presented.

The Appendix to our paper provides a detailed exposition as to how OPE coefficients proportional to a given condensate are extracted from residual normal-ordered terms within the Wick expansion (i.e., the nonperturbative VEV's of Figs. 1–3). In particular, a closed-form expression [Eq. (A30)] for the quark-condensate component of the VEV $\langle :\bar{\Psi}_i^\alpha(x)\Psi_j^\beta(y):\rangle$ is obtained to *all* orders of the operator-product expansion. The gluon-condensate projection of the nonlocal two-gluon VEV is shown to involve a single OPE term, which is explicitly obtained. Finally, the mixed (dimension-5) condensate components of $\langle :\bar{\Psi}_i^\alpha(x)\Psi_j^\beta(y):\rangle$ and $\langle :\bar{\Psi}_i^\alpha(x)B_\mu^\alpha(y)\Psi_j^\beta(z):\rangle$ are also extracted to $O(m^3)$ in the OPE.

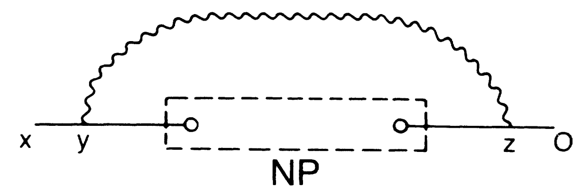

FIG. 1. Lowest-order graph generating dimension-3 quark-condensate contributions to the fermion propagator. A dimension-5 mixed-condensate contribution also arises through this graph, as discussed in Sec. IV.

II. THE QUARK CONDENSATE COMPONENT OF THE QUARK MASS

We begin by considering the second-order contribution to the fermion propagator $S_F(p)$ arising from a nonperturbative (NP) vacuum expectation value of normal-ordered quark fields (Fig. 1). In the Appendix we show that the quark-condensate component of this VEV is given by

$$-\langle 0|:\Psi_n^\alpha(y)\bar{\Psi}_r^\beta(z):|0\rangle_{\rm NP}=\delta^{\alpha\beta}\langle\bar{q}q\rangle\left[\sum_{j=0}^{\infty}C_j(-im)^j[\gamma\cdot(y-z)]_{nr}^j\right]$$
$$+(\text{contributions from higher-dimensional condensates})$$
$$=\delta^{\alpha\beta}\langle\bar{q}q\rangle\{\delta_{nr}/12-im[\gamma\cdot(y-z)]_{nr}/48-m^2(y-z)^2\delta_{nr}/96+\cdots\}+O(g_s\langle\bar{q}G\cdot\sigma q\rangle)\,, \tag{2.1}$$

where fermion field subscripts are Dirac-spinorial indices, fermion field superscripts are color indices, and where, from Eq. (A29) of the Appendix, the coefficients C_j are given by

$$C_j=\begin{cases}[3(j/2)!(j/2+1)!4^{(j/2+1)}]^{-1}, & j\text{ even}\,,\\ [6(j/2-\frac{1}{2})!(j/2+\frac{3}{2})!4^{(j+1)/2}]^{-1}, & j\text{ odd}\,.\end{cases} \tag{2.2}$$

As discussed in the Appendix, the fermion mass parameter m appearing in the operator-product expansion (2.1) arises from incorporating the (QCD) equation of motion

$$\not{D}\Psi(x)=-im\Psi(x) \tag{2.3}$$

in the covariantized Taylor-series expansion of the nonperturbative vacuum expectation value on the left-hand side of (2.1). Since m characterizes the expansion of this nonperturbative quantity, we necessarily distinguish between m and the purely perturbative (current-) quark mass m_L characterizing the renormalized electroweak Lagrangian. Thus for any particular choice of quark flavor, the current-quark mass m_L is generated entirely through the dimension-1 order parameter $\langle\phi\rangle$ of spontaneous symmetry breaking, whereas the quark mass m characterizing the OPE (2.1) also includes the effects of higher-dimensional order parameters, the condensates of nonperturbative QCD.

Of particular interest for up- and down-quark physics is the *dynamical* quark mass—the value of m in the limit of vanishing m_L—corresponding to a dynamical breakdown of explicit chiral symmetry. We are also interested in studying migration of the quark propagator pole away from its perturbative position m_L once nonperturbative condensate contributions are incorporated into the quark propagator. Although we expect m_L to continue to correspond to the apparent location of the quark propagator pole in the large-p^2 limit (corresponding to our intuitive understanding of current-quark masses), we also expect the operator-product mass m to correspond to the quark propagator pole upon inclusion of contributions from condensates of nonperturbative origin.

The $O(g_s^2)$ correction to the fermion propagator generated by Fig. 1 is given by[19]

$$i[S_F^{(2)}(p)]_{kl}^{\rho\sigma} = -\frac{g_s^2}{4}\int d^4x\, e^{ip\cdot x}\int d^4y\int d^4z[\langle 0|T\Psi_k^\rho(x)\bar\Psi_i^\tau(y)|0\rangle_{\rm pert}\,\gamma_{in}^\mu\lambda_{\tau\alpha}^b\langle 0|:\Psi_n^\alpha(y)\bar\Psi_r^\beta(z):|0\rangle_{\rm NP}$$
$$\times\gamma_{rj}^\nu\lambda_{\beta\omega}^c\langle 0|T\Psi_j^\omega(z)\bar\Psi_l^\sigma(0)|0\rangle_{\rm pert}\langle 0|TB_\mu^b(y)B_\nu^c(z)|0\rangle_{\rm pert}]\ . \tag{2.4}$$

In generating (2.4) the normal-ordered piece arises from the Wick expansion for the time-ordered product of several fields: for example,

$$T\phi(x)\phi(y) = \phi(x)\phi(y) + :\phi(x)\phi(y): = \langle 0|T\phi(x)\phi(y)|0\rangle_{\rm pert} + :\phi(x)\phi(y):\ . \tag{2.5}$$

The vacuum expectation value of the normal-ordered fields no longer vanishes in QCD [cf. Eq. (2.1)] (Ref. 17) as in "condensate-free" perturbation theory, as is evidenced by the role of condensates in sum-rule physics.[11,17] The "perturbative" time-ordered products represent the usual fermion and gluon propagators generated explicitly from inversion of the Lagrangian's bilinear terms:

$$\langle 0|T\Psi_k^\rho(x)\bar\Psi_i^\tau(y)|0\rangle_{\rm pert} = i\delta^{\rho\tau}\int\frac{d^4q}{(2\pi)^4}e^{-iq\cdot(x-y)}\left[\frac{\not q_{ki}+m_L\delta_{ki}}{q^2-m_L^2}\right], \tag{2.6}$$

$$\langle 0|TB_\mu^b(y)B_\nu^c(z)|0\rangle_{\rm pert} = i\delta^{bc}\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot(y-z)}\left[\frac{-g_{\mu\nu}}{k^2}+\frac{(1-a)k_\mu k_\nu}{(k^2)^2}\right]. \tag{2.7}$$

Note the presence of an arbitrary gauge parameter a in (2.7); believable results for any condensate-corrected quark mass m must be independent of this quantity.

We will use (2.4) to relate m_L and m by incorporating the "m-sensitive" expression (2.1) for the nonperturbative vacuum expectation value in (2.4). We first note, however, that two configuration-space integrals can be easily evaluated by writing the factor $e^{ip\cdot x}$ in (2.4) as $e^{ip\cdot(x-y)}e^{ip\cdot(y-z)}e^{ip\cdot z}$ and by changing integration variables from $(d^4x\, d^4y\, d^4z)$ to $[d^4(x-y)d^4(y-z)d^4z]$. The integrals over $(x$-$y)$ and over z yield momentum-space δ functions centered at the external momentum p; integrals over these δ functions then yield fermion propagators of momentum p:

$$i[S_F^{(2)}(p)]_{kl}^{\rho\sigma} = \left[\frac{\not p+m_L}{p^2-m_L^2}\right]_{ki}\left[\frac{ig_s^2\lambda_{\rho\alpha}^b\lambda_{\beta\sigma}^b}{4(2\pi)^4}\gamma_{in}^\mu\int d^4(y-z)\int d^4k\langle 0|:\Psi_n^\alpha(y)\bar\Psi_r^\beta(z):|0\rangle_{\rm NP}\,e^{i(p-k)\cdot(y-z)}\right.$$
$$\left.\times\left[\frac{-g_{\mu\nu}}{k^2}+\frac{(1-a)k_\mu k_\nu}{(k^2)^2}\right]\gamma_{rj}^\nu\right]\left[\frac{\not p+m_L}{p^2-m_L^2}\right]_{jl}. \tag{2.8}$$

The full propagator $S_F(p)$ is related to its self-energy via the defining relationship

$$S_F(p)\equiv[\not p-m_L-\Sigma(p)]^{-1} = (\not p-m_L)^{-1}+(\not p-m_L)^{-1}\Sigma(p)(\not p-m_L)^{-1}+\cdots$$
$$=(\not p-m_L)^{-1}+S_F^{(2)}(p)+\cdots\ . \tag{2.9}$$

We see from (2.8) that the self-energy does not depend on the Lagrangian mass m_L:

$$\Sigma_{tu}^{\rho\sigma}(p)\equiv(\not p-m_L)_{tk}[S_F^{(2)}(p)]_{kl}^{\rho\sigma}(\not p-m_L)_{lu} \tag{2.10a}$$

$$=\frac{g_s^2}{4(2\pi)^4}\lambda_{\rho\alpha}^b\lambda_{\beta\sigma}^b\gamma_{tn}^\mu\int d^4(y-z)\int\frac{d^4k}{(k^2)^2}\langle 0|:\Psi_n^\alpha(y)\bar\Psi_r^\beta(z):|0\rangle_{\rm NP}\,e^{i(p-k)\cdot(y-z)}[-g_{\mu\nu}k^2+(1-a)k_\mu k_\nu]\gamma_{ru}^\nu\ . \tag{2.10b}$$

Equation (2.10b) may now be evaluated using the expansion (2.1) for the nonperturbative vacuum expectation value. We use the identity

$$\int d^4(y-z)\int d^4k[\gamma\cdot(y-z)]^j e^{i(p-k)\cdot(y-z)}f(k)$$
$$=\left[-i\gamma\cdot\frac{\partial}{\partial p}\right]^j\int d^4(y-z)\int d^4k\, e^{i(p-k)\cdot(y-z)}f(k) = (2\pi)^4\left[-i\gamma\cdot\frac{\partial}{\partial p}\right]^j f(p) \tag{2.11}$$

in order to obtain[14,19,22]

$$\Sigma_{tu}(p) = -\frac{4g_s^2}{3}\,|\langle\bar{q}q\rangle|\,\gamma^\mu\sum_{j=0}^{\infty}C_j(-im)^j\left[-i\gamma\cdot\frac{\partial}{\partial p}\right]^j\left[\frac{-g_{\mu\nu}}{p^2}+\frac{(1-a)p_\mu p_\nu}{(p^2)^2}\right]\gamma^\nu$$
$$= \frac{g_s^2\,|\langle\bar{q}q\rangle|}{9p^2}[(3+a)-am\not{p}/p^2]\ . \tag{2.12}$$

In obtaining (2.12) we note that all $j\geq 2$ terms explicitly vanish when contracted into the bracketing factors of γ^μ and γ^ν so as to truncate the infinite summation within the operator-product expansion of the nonperturbative vacuum expectation value.[18] This truncation is *essential* to the development of sensible physics on the $\not{p}=m$ mass shell. In principle, the ratio of successive terms for successive values of j is scaled by the operator-product-expansion parameter $m\not{p}/p^2$, which is unity on the $\not{p}=m$ mass shell. Convergence is not assured unless the series truncates.

The salient feature of the last line of (2.12) is its gauge-parameter independence when $\not{p}$ equals the OPE quark mass m of Eq. (2.3) (Ref. 14). This gauge independence is a signal of self-consistent physics, as we shall demonstrate shortly. We first utilize (2.9) to write the $\langle\bar{q}q\rangle$-corrected inverse quark propagator as

$$S_F^{-1}(p)=\not{p}-m_L-\frac{g_s^2\,|\langle\bar{q}q\rangle|}{9p^2}[(3+a)-am\not{p}/p^2] = \left[1+\frac{g_s^2am\,|\langle\bar{q}q\rangle|}{9p^4}\right]\left[\not{p}-\frac{m_L+g_s^2(3+a)\,|\langle\bar{q}q\rangle|\,/9p^2}{1+g_s^2am\,|\langle\bar{q}q\rangle|\,/9p^4}\right]\ . \tag{2.13}$$

The *apparent* pole of the quark propagator mass in (2.13) continues to be at the current-quark mass in the large-p^2 limit:

$$\lim_{p^2\to\infty}\frac{m_L+g_s^2(3+a)\,|\langle\bar{q}q\rangle|\,/9p^2}{1+g_s^2am\,|\langle\bar{q}q\rangle|\,/9p^4}=m_L\ . \tag{2.14}$$

This gauge-independent result is hardly surprising; the contributions of the dimension-1 order parameter $\langle\phi\rangle$ necessarily dominate those of the dimension-3 order parameter $\langle\bar{q}q\rangle$ at large p^2.

However, to retain consistency with the equation of motion (2.3), we now require the pole of the $\langle\bar{q}q\rangle$-corrected propagator to be at the OPE mass m, as discussed earlier. Consequently, we see from (2.13) that the effective mass

$$M_{\rm eff}(p^2)=\frac{m_L+g_s^2(3+a)\,|\langle\bar{q}q\rangle|\,/9p^2}{1+g_s^2am\,|\langle\bar{q}q\rangle|\,/9p^4} \tag{2.15a}$$

must satisfy the constraint $M_{\rm eff}(m^2)=m$, in which case (2.15a) yields the algebraic relation

$$m(1+g_s^2a\,|\langle\bar{q}q\rangle|\,/9m^3) = m_L+g_s^2(3+a)\,|\langle\bar{q}q\rangle|\,/9m^2\ . \tag{2.15b}$$

The factor of $g_s^2a\,|\langle\bar{q}q\rangle|\,/9m^2$ cancels algebraically from both sides of (2.15b), leading to a gauge-parameter independent relationship between quark masses before (m_L) and after (m) the inclusion of $\langle\bar{q}q\rangle$ effects:

$$m=m_L+g_s^2\,|\langle\bar{q}q\rangle|\,/3m^2\ . \tag{2.16}$$

It is worth noting that (2.16) could also have been derived without explicitly constraining m, the operator-product-expansion-parameter mass, to be the pole of the $\langle\bar{q}q\rangle$-corrected quark propagator. If we denote the pole position of the quark propagator by a new parameter μ, such that

$$\mu\equiv\lim_{p^2\to\mu^2}M_{\rm eff}(p^2)\ , \tag{2.17}$$

we obtain an algebraic constraint of the form

$$0=G(\mu,m,m_L,a)\ . \tag{2.18}$$

G is seen from (2.15a) to be given by

$$G(\mu,m,m_L,a)\equiv(m_L-\mu)+\frac{g_s^2\,|\langle\bar{q}q\rangle|}{9\mu^2}[3+a(1-m/\mu)]\ . \tag{2.19}$$

The constraint (2.18) serves formally to define μ implicitly as a function of the independent variables m, m_L, and a. However, if the pole μ is to be phenomenologically meaningful, it must be independent of the gauge parameter a. Upon differentiating both sides of (2.18) with respect to a, we find that

$$0=\frac{\partial G}{\partial\mu}\frac{\partial\mu}{\partial a}+\frac{\partial G}{\partial a}\ . \tag{2.20}$$

We see from (2.20) that gauge-parameter independence of the quark propagator pole μ ($\partial\mu/\partial a=0$) is impossible unless $\partial G/\partial a$ vanishes. Such is the case provided $\mu=m$ (2.19), thereby demonstrating the intimate connection between gauge independence and the requirement that the mass in (2.3) correspond self-consistently to the propagator pole μ. Indeed, we see from (2.19) that the solution to the simultaneous equations $G=0$, $\partial G/\partial a=0$, is given by equating μ to *both* sides of (2.16).

If we use the chiral limit of (2.16) to define a dynamical quark mass,

$$\lim_{m_L\to 0}(m)\equiv m_{\rm dyn}\ , \tag{2.21}$$

we then find that m is related to the current mass m_L via the constraint[6]

$$m = m_L + m_{\text{dyn}}^3/m^2 \ . \tag{2.22}$$

Moreover, the product $g_s^2|\langle\bar{q}q\rangle|/3$ occurring in (2.16) corresponds to m_{dyn}^3 (Ref. 14). Some phenomenological consequences of these relations are discussed in Sec. V.

III. THE FULL $O(g_s^2)$ NONPERTURBATIVE COMPONENT OF THE QUARK MASS

In order to determine to $O(g_s^2)$ the *full* contribution of nonperturbative condensates to the quark propagator, we must now consider the second-order contribution arising from a nonperturbative vacuum expectation value (VEV) of two normal-ordered gluon fields (Fig. 2). In the Appendix [Eq. (A35)] we show that the component of this VEV proportional to the dimension-4 gluon condensate is given by

$$\langle 0|:B_\mu^b(y)B_\nu^c(z):|0\rangle_{\text{NP}} = \delta^{bc}(g_{\tau\eta}g_{\mu\nu} - g_{\tau\nu}g_{\eta\mu})y^\tau z^\eta\langle GG\rangle/394 + \text{terms proportional to higher-dimensional condensates} \ , \tag{3.1a}$$

where

$$\langle GG\rangle \equiv \langle 0|:G^{\mu\nu a}(0)G_{\mu\nu}^a(0):|0\rangle \ . \tag{3.1b}$$

The terms proportional to higher-dimensional condensates in both (2.1) *and* (3.1a) necessarily involve at least one additional power of the coupling constant g_s. For the case of (2.1), this property is demonstrated explicitly in the Appendix for the dimension-5 mixed-condensate component of the two-fermion VEV (A37). All the covariant derivatives (D) appearing in (A37) are eliminated, either through systematic use of (2.3), which eliminates a single D *without* injecting into the VEV any additional "higher-dimensional-condensate generating" fields, or through use of the relation

$$[D^\mu(0),D^\nu(0)]_{\alpha\beta} = -g_s G_{\alpha\beta}^{\mu\nu}(0) \quad \{\equiv g_s[(-i\lambda_{\alpha\beta}^a/2)G_a^{\mu\nu}(0)]\} \ , \tag{3.2a}$$

which replaces the product of two D's within the VEV with a field strength $[G_{\mu\nu}(0)]$ accompanied by an additional factor of g_s. This latter relation leads to the generation of a higher-dimensional condensate; e.g., $\langle:\bar{\Psi}_i(x)\Psi_j(y):\rangle$ contains

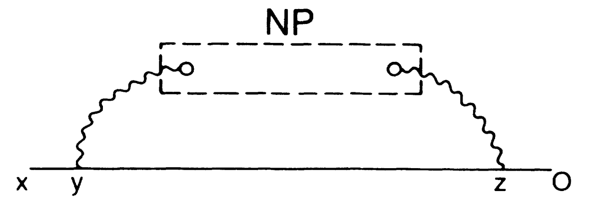

FIG. 2. Lowest-order graph generating dimension-4 gluon condensate contributions to the fermion propagator.

$$x_\mu y_\nu\langle:\bar{\Psi}(0)D^\mu D^\nu\Psi(0):\rangle \rightarrow g_s x_\mu y_\nu\langle:\bar{\Psi}(0)G^{\mu\nu}(0)\Psi(0):\rangle \sim g_s x_\mu y_\nu \sigma_{ji}^{\mu\nu}\langle\bar{q}G\cdot\sigma q\rangle \ .$$

For the case of (3.1a), it is argued in the discussion following (A35) in the Appendix that evaluation of *all* nonleading terms in (3.1a) necessarily entails use of either (3.2a) or the field equation

$$[D^\mu(x),G_{\mu\nu}(x)]_{\alpha\beta} = g_s\left[-i\lambda_{\alpha\beta}^a\sum_{\text{flavor}}\bar{\Psi}^\tau(x)\lambda_{\tau\sigma}^a\gamma_\nu\Psi^\sigma(x)\right] \equiv g_s j_\nu(x) \tag{3.2b}$$

for $x=0$. Consequently, an additional factor of g_s necessarily accompanies the collapse of two D's in (A31) into an additional field strength (3.2a), or alternatively, the reduction via (3.2b) of one factor of D and a field strength into a fermion current $j_\nu(0)$ [or, through combination of (3.2a) and (3.2b), the replacing of three D's with a fermion current]. Thus D's can be eliminated from VEV's of Taylor-series coefficients only at the price of either generating additional fields within the VEV [through (3.2a) and (3.2b)] or additional powers of m [through (2.3)]. Generation of additional fields, a prerequisite to obtaining higher-dimensional condensates, necessarily entails additional factors of the QCD coupling g_s.

We see, therefore, that the $O(g_s^2)$ contribution to the quark propagator from Fig. 2 arises entirely from the gluon-condensate projection of (3.1a), just as the $O(g_s^2)$ contribution from Fig. 1 arises entirely from the quark condensate projection of (2.1). The $O(g_s^2)$ contribution to the quark propagator from Fig. 2 is then obtained in precisely the same way the $O(g_s^2)$ contribution (2.4) was obtained, from Fig. 1 (Ref. 19),

$$i[S_F^{(2)}(p)]_{kl}^{\rho\sigma} = \left[\frac{-g_s^2}{4}\right]\int d^4x\, e^{ip\cdot x}\int d^4y\int d^4z[\langle 0|T\Psi_k^\rho(x)\bar{\Psi}_i^\tau(y)|0\rangle_{\text{pert}}\gamma_{in}^\mu\lambda_{\tau\alpha}^b\langle 0|T\Psi_n^\alpha(y)\bar{\Psi}_r^\beta(z)|0\rangle_{\text{pert}} \times\gamma_{rj}^\nu\lambda_{\beta\omega}^c\langle 0|T\Psi_j^\omega(z)\bar{\Psi}_l^\sigma(0)|0\rangle_{\text{pert}}\langle 0|:B_\mu^b(y)B_\nu^c(z):|0\rangle_{\text{NP}}] \ , \tag{3.3}$$

where we now consider only the $\langle GG\rangle$ projection of the nonperturbative VEV in (3.3). Using (2.6) and (3.1), we can evaluate (3.3) from precisely the same procedures as are delineated immediately after (2.7). We then obtain

$$[S_F^{(2)}(p)]_{kl}^{\rho\sigma} = \frac{g_s^2\delta^{\rho\sigma}}{288(2\pi)^{12}}(g_{\tau\eta}g_{\mu\nu}-g_{\tau\nu}g_{\eta\mu})\langle GG\rangle$$
$$\times\int d^4(x-y)\int d^4(y-z)\int d^4z\, e^{ip\cdot(x-y)}e^{ip\cdot(y-z)}e^{ip\cdot z}[(y-z)^\tau+z^\tau]z^\eta$$
$$\times\int d^4k\, e^{-ik\cdot(x-y)}\int d^4t\, e^{-it\cdot(y-z)}\int d^4q\, e^{-iq\cdot z}\left[\frac{\not k+m_L}{k^2-m_L^2}\gamma^\mu\frac{\not t+m_L}{t^2-m_L^2}\gamma^\nu\frac{\not q+m_L}{q^2-m_L^2}\right]_{kl}$$
$$=g_s^2\delta^{\rho\sigma}(g_{\tau\eta}g_{\mu\nu}-g_{\tau\nu}g_{\eta\mu})\frac{\langle GG\rangle}{288}\left[\frac{\not p+m_L}{p^2-m_L^2}\gamma^\mu\left\{\left[-i\frac{\partial}{\partial p_\tau}\left[\frac{\not p+m_L}{p^2-m_L^2}\right]\right]\gamma^\nu\left[-i\frac{\partial}{\partial p_\eta}\left[\frac{\not p+m_L}{p^2-m_L^2}\right]\right]\right.\right.$$
$$\left.\left.+\left[\frac{\not p+m_L}{p^2-m_L^2}\right]\gamma^\nu\left[-\frac{\partial}{\partial p_\tau}\frac{\partial}{\partial p_\eta}\left[\frac{\not p+m_L}{p^2-m_L^2}\right]\right]\right\}\right]_{kl}$$
$$=\left[\frac{\not p+m_L}{p^2-m_L^2}\right]_{kj}\left[\frac{g_s^2\langle GG\rangle\delta^{\rho\sigma}m_L\not p(\not p-m_L)}{12(p^2-m_L^2)^3}\right]_{jn}\left[\frac{\not p+m_L}{p^2-m_L^2}\right]_{nl}. \tag{3.4}$$

Note that this contribution vanishes, as expected from chiral invariance of $\langle GG\rangle$, if $m_L=0$ (Ref. 23). This contribution is also trivially gauge independent: there are no gauge-parameter-dependent gluon propagators in the Fig. 2 self-energy. Using (2.10a), we see that the $\langle GG\rangle$ contribution to the self-energy $\Sigma(p)$ is just the middle term in large parentheses on the final line of (3.4):[24]

$$\Delta\Sigma(p)_{\langle GG\rangle}=\frac{g_s^2\langle GG\rangle m_L(p^2-m_L\not p)}{12(p^2-m_L^2)^3}. \tag{3.5}$$

The total $O(g_s^2)$ nonperturbative-condensate contribution to the quark propagator is obtained by combining the self-energies (2.12) and (3.5), so as to obtain the inverse propagator

$$S_F^{-1}(p)=\not p-m_L-\Sigma(p)$$
$$=(1+g_s^2\{am\,|\,\langle\bar qq\rangle\,|\,/9p^4+m_L^2\langle GG\rangle/[12(p^2-m_L^2)^3]\})[\not p-M_{\text{eff}}(p^2)], \tag{3.6a}$$

where

$$M_{\text{eff}}(p^2)=\frac{36p^4(p^2-m_L^2)^3m_L+g_s^2[4\,|\,\langle\bar qq\rangle\,|\,p^2(p^2-m_L^2)^3(3+a)+3\langle GG\rangle m_Lp^6]}{36p^4(p^2-m_L^2)^3+g_s^2[4\,|\,\langle\bar qq\rangle\,|\,(p^2-m_L^2)^3am+3\langle GG\rangle m_L^2p^4]}. \tag{3.6b}$$

As in the previous section, the pole-mass μ is defined implicitly by the relationship

$$\mu=\lim_{p^2\to\mu^2}M_{\text{eff}}(p^2). \tag{3.7}$$

After a little algebra, this constraint may be expressed in the form

$$0=\mathcal{G}(\mu,m_L,m,a), \tag{3.8a}$$

where

$$\mathcal{G}(\mu,m_L,m,a)=36\mu^4(\mu^2-m_L^2)^3(m_L-\mu)+g_s^2[12\,|\,\langle\bar qq\rangle\,|\,(\mu^2-m_L^2)^3\mu^2+3\langle GG\rangle m_L\mu^5(\mu-m_L)$$
$$+4a\,|\,\langle\bar qq\rangle\,|\,(\mu^2-m_L^2)^3\mu(\mu-m)]. \tag{3.8b}$$

The solutions to (3.8), obtained by choosing $\mu=m_L$ or $\mu=0$, are trivial in that they are completely insensitive to the condensates $\langle GG\rangle$ and $\langle\bar qq\rangle$. These solutions correspond to singularities of the self-energies (3.5) and (2.12). A nontrivial solution, however, may be obtained as in the previous section by equating μ with m to ensure gauge independence ($\partial\mathcal{G}/\partial a=0$ implies $\partial\mu/\partial a=0$), and by then having $\mathcal{G}$ vanish nontrivially by choosing

$$m=\frac{m_L\left[1+\dfrac{g_s^2\langle GG\rangle m^2}{12(m^2-m_L^2)^3}\right]+\dfrac{g_s^2\,|\,\langle\bar qq\rangle\,|}{3m^2}}{1+\dfrac{g_s^2\langle GG\rangle m_L^2}{12(m^2-m_L^2)^3}}+O(g_s^4)=m_L\left[1+\frac{g_s^2\langle GG\rangle m(m-m_L)}{12(m^2-m_L^2)^3}\right]+\frac{g_s^2\,|\,\langle\bar qq\rangle\,|}{3m^2}+O(g_s^4). \tag{3.9}$$

The quark condensate's contribution to the propagator pole clearly dominates that of the gluon condensate for the phenomenologically motivated $m_L \ll m$ region.[25] Moreover, the gluon-condensate correction to the quark mass is of order $g_s^2 m_L \langle GG\rangle / m^4$, which falls far more rapidly with m than the dynamical term $(\sim g_s^2 \mid \langle \bar{q}q\rangle \mid /m^2)$ generated by the quark condensate in (2.22).

IV. THE MIXED CONDENSATE COMPONENT OF THE QUARK MASS

In this section we examine whether the chiral-symmetry-breaking dimension-5 "mixed condensate" order parameter

$$\langle \bar{q}G\cdot\sigma q\rangle \equiv \langle 0 \mid :\bar{\Psi}_r^{\alpha}(0)\left[\frac{i}{2}G_{\mu\nu}^{a}(0)\lambda_{\alpha\beta}^{a}\sigma_{rn}^{\mu\nu}\right]\Psi_n^{\beta}(0): \mid 0\rangle \tag{4.1}$$

shifts the position of the quark propagator pole (μ) from that already obtained in (3.9) through insertions of lower-dimensional order parameters. To determine whether such a further shift in the pole position occurs, we consider the lowest-order contribution to the quark propagator arising from $\langle \bar{q}G\cdot\sigma q\rangle$ insertions. Tree graphs leading to such insertions through the nonperturbative vacuum expectation value $\langle 0 \mid :\bar{\Psi}_r^{\alpha}(z)\Psi_n^{\beta}(y)B_{\mu}^{b}(w): \mid 0\rangle$ (B is a gluon field) are given in Figs. 3(a), 3(b), and 3(c). In addition, $\langle \bar{q}\sigma\cdot Gq\rangle$ coefficients also arise from the expansion of the $\langle 0 \mid :\Psi_n^{\alpha}(y)\bar{\Psi}_r^{\beta}(z): \mid 0\rangle$ nonperturbative vacuum expectation value of Fig. 1. We will first examine this latter set of coefficients.

The mixed condensate projection of the nonperturbative VEV in Fig. 1 is calculated in the Appendix [Eq. (A69)]. Recall from Sec. II that contributions to Fig. 1 from terms in $\langle :\Psi(y)\bar{\Psi}(z):\rangle$ proportional to $[\gamma\cdot(y-z)]^j$ *vanish* for $j \geq 2$. A similar argument shows that contributions to Fig. 1 from terms proportional to $\sigma_{\tau\eta}y^{\tau}z^{\eta}[\gamma\cdot(y-z)]^j$ also vanish for $j\geq 2$. Consequently, we see from the general form (A70) given in the Appendix that the *only* nonvanishing $\langle \bar{q}G\cdot\sigma q\rangle$ contribution to the Fig. 1 amplitude (2.4) is obtained through use of the $\sigma^{\mu\nu}y_{\mu}z_{\nu}$ portion of the lead term in (A69). The resulting expression may be obtained as in Sec. II by changing variables of configuration-space integration to $[d^4(x-y)d^4(y-z)d^4z]$. Replacement of $e^{ip\cdot x}$ with $e^{ip\cdot(x-y)}e^{ip\cdot(y-z)}e^{ip\cdot z}$ and y_{τ} with $[(y-z)_{\tau}+z_{\tau}]$ facilitates a factorization of $(x-y)$, $(y-z)$, and z integrals, which are then evaluated using (2.11). We then find the mixed condensate projection of the Fig. 1 ampltiude to be given by

$$iS_1^{(2)}(p) = \frac{-g_s^3\langle \bar{q}G\cdot\sigma q\rangle(\not{p}+m_L)\not{p}(1-a)}{36p^4(p^2-m_L^2)^2}\ . \tag{4.2}$$

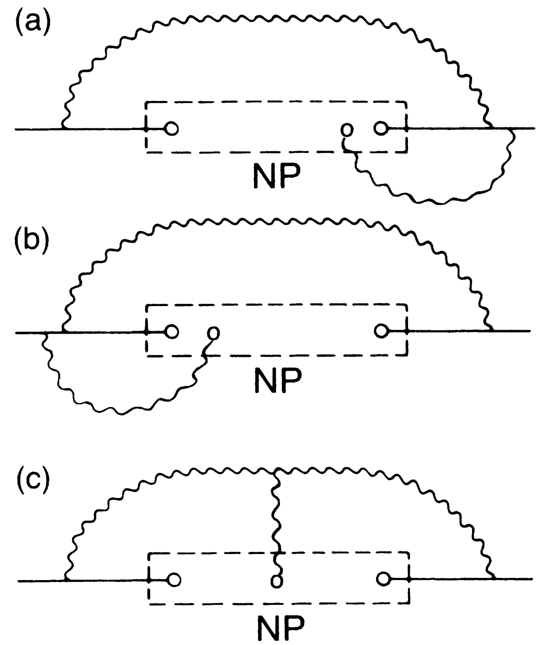

FIG. 3. Other graphs generating $O(g_s^3)$ dimension-5 mixed condensate contributions to the fermion propagator.

We now consider the Fig. 3 contributions obtained from expanding the nonperturbative VEV of two quark fields and a gluon field. The mixed condensate projection of this VEV is calculated to $O(m^3)$ in Eq. (A88) of the Appendix. The contribution of Fig. 3(a) to the fermion propagator is obtained through utilization of (A88), (2.6), and (2.7) and (2.11):

$$\begin{aligned} i[S_{3a}^{(2)}(p)]_{ij}^{\alpha\beta} = {} & \frac{-g_s^3}{4}\int d^4(x-y)e^{ip\cdot(x-y)}\int d^4(y-z)e^{ip\cdot(y-z)} \\ & \times \int d^4(z-w)e^{ip\cdot(z-w)}\int d^4w\, e^{ip\cdot w}\langle 0 \mid T\Psi_i^{\alpha}(x)\bar{\Psi}_l^{\epsilon}(y)\mid 0\rangle_{\text{pert}}\gamma_{ln}^{\rho}\lambda_{\epsilon\gamma}^{a}\langle 0 \mid TB_{\rho}^{a}(y)B_{\nu}^{b}(z)\mid 0\rangle_{\text{pert}}\gamma_{rt}^{\nu}\lambda_{\delta\tau}^{b} \\ & \times \langle 0 \mid T\Psi_t^{\tau}(z)\bar{\Psi}_u^{\omega}(w)\mid 0\rangle_{\text{pert}}\gamma_{uv}^{\mu}\lambda_{\omega\phi}^{c}\langle 0 \mid T\Psi_v^{\phi}(w)\bar{\Psi}_j^{\beta}(0)\mid 0\rangle_{\text{pert}}\frac{i}{2}\langle 0 \mid :\Psi_n^{\gamma}(y)\bar{\Psi}_r^{\delta}(z)B_{\mu}^{c}(w): \mid 0\rangle_{\text{NP}} \\ = {} & \frac{g_s^3\delta^{\alpha\beta}\langle \bar{q}G\cdot\sigma q\rangle(\not{p}+m_L)_{il}}{288p^4(p^2-m_L^2)^3}\{[-\not{p}p^2(1-a)]+m[2\not{p}(\not{p}+m_L)]+m^2[\not{p}(1-a)]+m^3[0]+\cdots\}_{lj}\ . \end{aligned} \tag{4.3}$$

As indicated in (4.3), the coefficient of m^3 in (A88) does not contribute to the Fig. 3(a) amplitude. This result is suggestive of a truncation of operator-product contributions beyond $O(m^2)$ analogous to the $O(m)$ truncation already obtained within the Fig. 1 amplitude, as is discussed in Sec. II and Refs. 18. The coefficient of m^3 in (A88) also fails to contribute to the amplitude of Fig. 3(b), which yields the following correction to the quark two-point function:

$$iS_{3b}^{(2)}(p)=\frac{g_s^3\langle\bar{q}G\cdot\sigma q\rangle(\not p+m_L)}{288p^4(p^2-m_L^2)^3}\{[-\not p(2p^2+m_L^2)(1-a)]+m[4m_L\not p]+m^2[3\not p(1-a)]+m^3[0]+\cdots\}\ . \tag{4.4}$$

However, the m^3 term does *not* vanish when substituted in the "non-Abelian" two-point amplitude of Fig. 3(c):

$$\begin{aligned}iS_{3c}^{(2)}(p)=\frac{g_s^3\langle\bar{q}G\cdot\sigma q\rangle(\not p+m_L)}{288p^8(p^2-m_L^2)^2}\Bigg[&(-54-18a)\not p p^4+(-45-9a)m_L p^4\\&+(m-m_L)\left[\frac{-18a(p^6+m_L p^4\not p)}{(p^2-m_L^2)}\right]+m[(45+9a)(p^4+m_L\not p p^2)]\\&+m^2[-(45+9a)(\not p p^2+m_L p^2)]+m^3[(45+9a)(p^2+m_L\not p)]+\cdots\Bigg]\ .\end{aligned} \tag{4.5}$$

We now employ (2.9) to relate (4.2)–(4.5) to the quark self-energy, so as to obtain the following $O(g_s^3)$ mixed-condensate projections for Figs. 1, 3(a), 3(b), and 3(c):[26]

$$\Delta\Sigma_1(p)\equiv(\not p-m_L)S_1^{(2)}(p)(\not p-m_L)=\frac{ig_s^3\langle\bar{q}G\cdot\sigma q\rangle}{288p^4}\left[\frac{(1-a)8(p^2-m_L\not p)}{p^2-m_L^2}\right], \tag{4.6}$$

$$\begin{aligned}\Delta\Sigma_{3a}(p)=\frac{-ig_s^3\langle\bar{q}G\cdot\sigma q\rangle}{288p^4}\Bigg[&\left[\frac{-(1-a)p^2(p^2-m_L\not p)}{(p^2-m_L^2)^2}\right]+m\left[\frac{2\not p}{p^2-m_L^2}\right]\\&+m^2\left[\frac{(1-a)(p^2-m_L\not p)}{(p^2-m_L^2)^2}\right][+O(m^4)]\Bigg]\ ,\end{aligned} \tag{4.7}$$

$$\begin{aligned}\Delta\Sigma_{3b}(p)=\frac{-ig_s^3\langle\bar{q}G\cdot\sigma q\rangle}{288p^4}\Bigg[&\left[\frac{-(1-a)(2p^2+m_L^2)(p^2-m_L\not p)}{(p^2-m_L^2)^2}\right]+m\left[\frac{4m_L(p^2-m_L\not p)}{(p^2-m_L^2)^2}\right]\\&+m^2\left[\frac{3(1-a)(p^2-m_L\not p)}{(p^2-m_L^2)^2}\right][+O(m^4)]\Bigg]\ ,\end{aligned} \tag{4.8}$$

$$\begin{aligned}\Delta\Sigma_{3c}(p)=\frac{ig_s^3\langle\bar{q}G\cdot\sigma q\rangle}{288p^4}\Bigg[&-9(1+a)\frac{p^2-m_L\not p}{p^2-m_L^2}-\frac{18a\not p(m-m_L)}{p^2-m_L^2}\\&-9(5+a)[1-m\not p/p^2+m^2/p^2-m^3\not p/p^4+O(m^4)]\Bigg]\ .\end{aligned} \tag{4.9}$$

Additional contributions involving zero-momentum propagators have been ignored, as discussed in Ref. 20. Note that the results of (4.6), (4.7), (4.8), and (4.9) coincide with those of Ref. 20 provided we keep terms only linear in m *and* m_L [i.e., $(\not p-m_L)^{-1}\to(\not p+m_L)/p^2$], and provided no distinction is made between the propagator mass m_L and the mass m characterizing the operator-product expansion of nonperturbative VEV's.

To proceed further, we shall assume that the series in the final line of (4.9) continues to alternate in powers of $m\not p/p^2$, corresponding to a series representation of $\not p/(\not p+m)$. We shall also assume that $\Delta\Sigma_{3a}(p)$ and $\Delta\Sigma_{3b}(p)$ truncate after the $O(m^2)$ terms listed in (4.7) and (4.8). This assumption is motivated by the explicit vanishing of the $O(m^3)$ contribution to the nonperturbative VEV (A88) upon insertion of that contribution into the amplitudes of Figs. 3(a) and 3(b), as noted in (4.4) and (4.5). An analogous truncation of higher-order contributions in m for the Fig. 1 amplitude leads to closed-form expressions [(4.6) and (2.12)] for the appropriate projections of the Fig. 1 self-energy. As remarked earlier, general proof[18] of such truncation was possible for Fig. 1's two-fermion VEV through use of the all-orders expressions (2.1) and (A70). Unfortunately, no analogous all-orders expression has so far been derived for Fig. 3's higher-dimensional VEV. Consequently, truncation beyond $O(m^2)$ of $\Delta\Sigma_{3a}$ and $\Delta\Sigma_{3b}$ must be regarded only as plausible, as evidenced by the vanishing of $O(m^3)$ contributions.

With the assumptions delineated above for (4.7), (4.8), and (4.9), we obtain the following expression for the mixed-condensate contribution to the quark self-energy:[27]

$$\Delta\Sigma_{\langle\bar{q}G\cdot\sigma q\rangle}=\frac{-ig_s^3\langle\bar{q}G\cdot\sigma q\rangle E(p^2)}{288p^4(p^2-m^2)(p^2-m_L^2)^2}\left[\not p+\frac{F(p^2)}{E(p^2)}\right], \tag{4.10}$$

where

$$E(p^2)\equiv p^4[(47m+20m_L)+a(16m_L-9m)]+p^2[(-16m_L^3-96m_L^2m-24m_Lm^2-2m^3)+a(-20m_L^3-12m_Lm^2+18m^3)]+[(45m_L^4m+16m_L^3m^2+6m_L^2m^3+4m_Lm^4)+a(9m_L^4m+20m_L^3m^2-18m_L^2m^3-4m_Lm^4)]\ , \tag{4.11a}$$

$$F(p^2)\equiv p^6(-65-7a)+p^4[(106m_L^2+4m_Lm+24m^2)+a(20m_L^2-6m^2)]+p^2[(-45m_L^4-16m_L^2m^2-4m_Lm^3-4m^4)+a(-9m_L^4-2m_L^2m^2+4m^4)]\ . \tag{4.11b}$$

At the end of the previous section, the propagator pole was found to occur at $\mu=m$, where [to $O(g_s^2)$] m is related to dimension-3 and -4 condensates by (3.9). Equation (4.10) is higher order in g_s than the corresponding self-energies [(2.12) and (3.5)] used to derive (3.9). Consequently, Eq. (4.10) generates an $O(g_s^3)$ correction to (3.9) unless

$$\lim_{p^2\to m^2}\frac{F(p^2)}{E(p^2)}=-m\ , \tag{4.12}$$

in which case $\Delta\Sigma_{\langle\bar{q}G\cdot\sigma q\rangle}$ can be absorbed entirely into an on-shell renormalization of the quark wave function. Indeed, Eq. (4.12) is seen to be true for *arbitrary* "current mass" m_L and for *arbitrary* gauge-parameter a, as Eqs. (4.11a) and (4.11b) simplify to the following expressions when $p^2=m^2$:

$$E(m^2)=9(5+a)m(m^2-m_L^2)^2\ , \tag{4.13a}$$

$$F(m^2)=-9(5+a)m^2(m^2-m_L^2)^2\ . \tag{4.13b}$$

We therefore conclude that the mass relation (3.9) remains upheld after $O(g_s^3)$ mixed-condensate corrections to the fermion propagator are included. This result, however, is contingent on the validity of (4.12) to all orders in m, which rests in turn on the truncation and geometric-series assumptions delineated in the paragraph preceding (4.10).

V. THE CHIRAL LIMIT

We have seen so far that the dimension-3 and -4 condensates generate a nonperturbative component to the quark mass, which is given to $O(g_s^2)$ by (3.9). The $O(g_s^3)$ contributions generated through the dimension-5 mixed condensate appear not to alter this relationship. The only other possible nonperturbative $O(g_s^3)$ contributions arise from the dimension-6 gluon condensate $\langle G^3\rangle$. Since this condensate, like $\langle GG\rangle$, is trivially chiral-symmetry invariant, it cannot contribute to the quark mass in the $m_L\to 0$ limit of Lagrangian chiral symmetry. We therefore expect that (2.16) and (2.22) are appropriate equations for the chiral limit up to $O(g_s^4)$ corrections.

In Sec. II, this limit has already been seen to imply that the dynamical component of the quark mass is given by

$$m_{\rm dyn}=(4\pi\alpha_s\,|\langle\bar{q}q\rangle|/3)^{1/3}\ . \tag{5.1}$$

The standard estimate of the quark condensate's magnitude obtained from QCD sum rules is $|\langle\bar{q}q\rangle_M|\approx(250\ \text{MeV})^3$ at $M=1$ GeV (Refs. 11, 28, and 29). If we substitute into (5.1) this value for the condensate and the corresponding ($M=1$ GeV) value for the QCD coupling [$\alpha_s(1\ \text{GeV}^2)\approx 0.5$], we find that[14,19]

$$m_{\rm dyn}\approx 320\ \text{MeV}\simeq m_{\rm nucleon}/3\ . \tag{5.2}$$

Although this approximate quark mass scale has been anticipated for quite some time, it is nonetheless satisfying to note that (5.1) is derived here from a *relativistic* quantum field theory in a *gauge independent* manner. The original guess that $m_{\rm quark}=m_{\rm nucleon}/3$, prevalent even in physics of the late 1960s, was regarded to be a nonrelativistic loose-binding quark mass accompanied by a large (and relativistic) Fermi momentum $p\sim R_c^{-1}\sim 400$ MeV. The derivation of (5.2) presented in Sec. II puts the many successes of the original nonrelativistic quark model on a firmer relativistic and gauge-independent footing.

The dynamical contribution to light (u- and d-) quark masses clearly dominates any current-mass contributions, which vanish entirely in the limit of explicit Lagrangian chiral symmetry. A curious consequence of (2.22), however, is that current-quark mass *differences* are magnified relative to constituent quark-mass differences. (We will henceforth denote "constituent"-quark masses to be quark masses which include contributions from nonperturbative QCD condensates.) Consider, for example, a naive application of (2.22) to the u-d mass difference,

$$m^{(d)}-m^{(u)}=m_L^{(d)}-m_L^{(u)}+m_{\rm dyn}^3[(m^{(d)})^{-2}-(m^{(u)})^{-2}]=(m_L^{(d)}-m_L^{(u)})-(m^{(d)}-m^{(u)})[m_{\rm dyn}^3(m^{(d)}+m^{(u)})/(m^{(d)}m^{(u)})^2]\ , \tag{5.3}$$

where $m_{\rm dyn}$ in (5.1) is a flavor-blind quantity. We see from (2.22) that as $m_L^{(u,d)}/m^{(u,d)}\to 0$ (the chiral-symmetry limit), then the constituent-quark masses $m^{(u,d)}$ approach $m_{\rm dyn}$. From (5.3), we find in the chiral limit that the ratio of the current to the constituent d-u mass difference is

$$\frac{m_L^{(d)}-m_L^{(u)}}{m^{(d)}-m^{(u)}}=\left[1+\frac{m_{\rm dyn}^3(m^{(d)}+m^{(u)})}{(m^{(d)}m^{(u)})^2}\right]\xrightarrow[m_L/m\to 0]{}3\ . \tag{5.4}$$

This numerical enhancement factor of 3 cannot be taken too seriously; if $\langle GG \rangle$ effects are also included via (3.9), we find that the ratio reduces to $3/[1+g_s^2\langle GG \rangle / 12m_{\rm dyn}^4]$. We note that enhancement of $(m^{(d)}-m^{(u)})_{\rm curr}$ relative to $(m^{(d)}-m^{(u)})_{\rm const}$ is also expected from bag-model considerations,[30] and that some enhancement (by a factor of ~ 1.5) may be motivated by phenomenology as well.[6,31]

We have seen so far that the chiral limit appears to correspond to quark masses generated *entirely* through the dimension-3 $\langle \bar{q}q \rangle$ condensate: the dimension-1 order parameter $\langle \phi \rangle$ is decoupled from quark masses in the chiral limit (through vanishing Yukawa couplings), and the chirally noninvariant order parameter of dimension 5 appears not to destabilize the position of the quark propagator pole, as evidenced by (4.12). A graphical depiction of this overall result, in which the quark condensate is represented by the nonperturbative quark-loop graph of Fig. 4, has already been suggested for color number $N_c=3$ (Ref. 14). In this "inverse approach" to relating $\langle \bar{q}q \rangle$ to $m_{\rm dyn}$, the dynamically generated quark mass in the Fig. 4 fermion loop runs according to the renormalization-group expression

$$m_{\rm dyn}(p^2)=\left[\frac{M^2}{p^2}\right] m_{\rm dyn}(M^2)\left[\frac{\ln(M^2/\Lambda^2)}{\ln(p^2/\Lambda^2)}\right]^{1-d} \tag{5.5}$$

in the deep Euclidean region, where $d=12(33-2N_f)^{-1}$ for color number $N_c=3$ (Ref. 32). Since Fig. 4 does not explicitly depend on a perturbative gluon, it is not surprising that the relationship of the quark condensate to this running quark mass is *independent* of any perturbative gauge parameter.

But to emphasize that this inverse graphical connection between Figs. 1 and 4 corresponds to the chiral limit of (2.16), we first restate the result of Sec. II for arbitrary color number N_c. Equation (2.16), which is obtained from Fig. 1, then generalizes to[33]

$$m_{\rm dyn}=\frac{N_c^2-1}{2N_c^2}\frac{3\pi\alpha_s(M^2)\,|\langle \bar{q}q \rangle_M|}{m_{\rm dyn}^2} . \tag{5.6}$$

In (5.6), the coupling constant runs according to the renormalization-group for $SU(N_c)$ symmetry[34]

$$\alpha_s(p^2)=-\frac{(4\pi b)^{-1}}{\ln(p^2/\Lambda^2)}=\frac{\pi d'}{\ln(p^2/\Lambda^2)} , \tag{5.7}$$

such that

$$d'=-(4\pi^2 b)^{-1}=12(11N_c-2N_f)^{-1} . \tag{5.8}$$

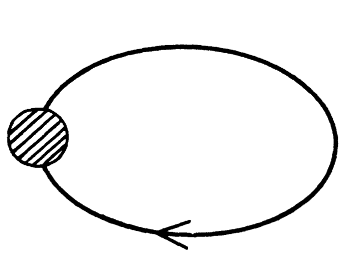

FIG. 4. Momentum-space quark-loop representation of the dimension-3 quark condensate. The shaded circle represents a quark propagator with mass $m_{\rm dyn}(p^2)$.

We shall now obtain the result (5.6) directly through use of (5.5) within the nonperturbative quark loop of Fig. 4. Although d' is equal to d when $N_c=3$, the value for the anomalous mass-dimension exponent d obtained for $N_c\neq 3$ differs from that of d' (Refs. 7 and 8):

$$\begin{aligned} d&=\gamma_m/2bg_s^2\\ &=(9/2N_c)(N_c^2-1)(11N_c-2N_f)^{-1} . \end{aligned} \tag{5.9}$$

For the correspondence between Fig. 1 and Fig. 4 to be meaningful, (5.6) should be obtainable from the Fig. 4 quark loop for *arbitrary* N_c. At renormalization cutoff M, Fig. 4 generates a quark condensate in terms of the running dynamical mass of (5.5):[14]

$$\langle \bar{q}q \rangle_M=\frac{-i4N_c}{(2\pi)^4}\int_\Lambda^M\frac{d^4p\, m_{\rm dyn}(p^2)}{p^2-m_{\rm dyn}(p^2)} \tag{5.10a}$$

$$\simeq -\frac{N_c}{4\pi d}m_{\rm dyn}(M^2)M^2\ln(M^2/\Lambda^2) . \tag{5.10b}$$

In obtaining (5.10b) we have dropped $m_{\rm dyn}(p^2)$ in the denominator of (5.10a) for large M [implying $p^2 \gg m_{\rm dyn}(p^2)$], thereby permitting a Wick rotation of the p_0 contour with $y=-p^2>0$ and $d^4p=i\pi^2 y\,dy$. If we use (5.7) to replace the logarithm in (5.10b) with a factor proportional to $\alpha_s^{-1}(M^2)$, and if we assume from the running mass structure (5.5) that $m_{\rm dyn}(M^2)=m_{\rm dyn}^3/M^2$ for values of M above the coupling-constant freeze-out point but beneath the deep Euclidean range, we then find from (5.10b) that

$$-\langle \bar{q}q \rangle_M=\frac{N_c(d'/d)}{4\pi\alpha_s(M^2)}m_{\rm dyn}^3 . \tag{5.11}$$

Finally, we substitute (5.8) and (5.9) into (5.11), an equation generated from Fig. 4 for arbitrary color number N_c, to again obtain (5.6), an equation previously generated for arbitrary N_c from Fig. 1. Note that the "inverse approach" through Fig. 4 successfully accounts for both the scale and sign of $\langle \bar{q}q \rangle$, the former ($\sim |250$ MeV$|^3$) linked to $m_{\rm dyn}\sim m_{\rm nucleon}/3$, and the latter (negative) following from the minus sign of the Fig. 4 fermion loop.

VI. FIELD-THEORETICAL RESULTS

In the Appendix of this paper we have used fixed-point techniques to determine the $\langle \bar{q}q \rangle$, $\langle GG \rangle$, and $\langle \bar{q}G\cdot\sigma q \rangle$ projections of nonperturbative VEV's occurring in the Wick expansion of the fermion two-point function in QCD. We regard the following as central results of our paper: (1) the $\langle \bar{q}q \rangle$ projections given by (A29) and (A30); (2) the form of the $\langle \bar{q}G\cdot\sigma q \rangle$ projection given by (A70); (3) the $\langle GG \rangle$ projection given by (A35). These projections are valid *to all orders* of the OPE of the VEV's under consideration. The specific $O(m^3)$ expressions for $\langle \bar{q}G\cdot\sigma q \rangle$ projections given by (A69) and (A88) should also prove useful, particularly for future QCD sum-rule applications.

Upon insertion of appropriate condensate projections of nonperturbative VEV's into those terms in the 2-

point-function Wick-expansion corresponding to Figs. 1–3, we have been able to determine the contribution of nonperturbative condensates (of dimension ≤ 5) to the location of the quark propagator pole. We have proved that only the first two terms [leading and $O(m)$] of the infinite OPE series can contribute to the $\langle \bar{q}q \rangle$ and $\langle \bar{q}G\cdot\sigma q \rangle$ projections of the Fig. 1 amplitude. We have also proved that the OPE series for the $\langle GG \rangle$ component of the nonperturbative VEV in Fig. 2 has only a single term (A35). Moreover, we have provided plausible evidence that only the leading $O(m)$ and $O(m^2)$ terms of the OPE for the $\langle \bar{q}G\cdot\sigma q \rangle$ projection of the VEV in Figs. 3 can contribute to the Fig. 3(a) and 3(b) amplitudes. Although no such truncation of the infinite OPE series seems to occur for the "non-Abelian" Fig. 3(c) amplitude, we have found that higher-order OPE contributions [listed explicitly to $O(m^3)$] appear to generate the geometric series for $\not{p}/(\not{p}+m)$ $[=(p^2-m\not{p})/(p^2-m^2)]$.

We therefore conclude that the results obtained for $\langle \bar{q}q \rangle$ and $\langle GG \rangle$ components of the quark propagator are valid to all orders of the OPE. The $\langle \bar{q}G\cdot\sigma q \rangle$ component (4.10) and (4.11) may also be valid to all orders of the OPE, provided the apparent truncation of higher-order OPE contributions to Figs. 3(a) and 3(b) and the assumed series summation of such contributions to Fig. 3(c) are indeed correct.

In dealing with condensates of successively higher dimension, we have demonstrated (in Sec. III) that the $O(g_s^2)$ contribution of nonperturbative condensates to the fermion 2-point function arises *entirely* from condensates of dimension ≤ 4; condensates of dimension ≥ 5 necessarily appear in conjunction with additional powers of the coupling constant g_s. This result suggests the possibility of a perturbative ordering of contributions from higher-dimensional condensates.[35]

It is interesting to speculate of the effect of higher-dimensional condensates in light of our salient field-theoretical result: the fermion propagator pole must be self-consistently identified with the condensate-sensitive mass parameter m characterizing the OPE of nonperturbative VEV's in order for that pole to be gauge-parameter independent. We first note that the self-energies generated through $\langle GG \rangle$ and $\langle \bar{q}G\cdot\sigma q \rangle$ exhibit singular behavior at the old perturbative propagator pole ($\not{p}=m_L$) but *not* at $\not{p}=m$ (Ref. 36). If condensates of dimension >5 exhibit singular contributions to the self-energy at $\not{p}=m$, so as to render the propagator $(\not{p}-m-\Sigma)^{-1}$ *nonsingular* at $\not{p}=m$, then quarks may be prevented from propagating entirely. Such conjectured singular behavior would be highly suggestive of confinement through nonperturbative order-parameter physics.

It is worth recalling that all "Abelian" contributions [Figs. 1, 3(a) and 3(b)] to the $\langle \bar{q}G\cdot\sigma q \rangle$ component of the quark 2-point function were argued, through OPE truncation, to be polynomials of finite order in m (or, strictly speaking, $m\not{p}/p^2$). If OPE truncation is indeed a general characteristic of Abelian graphs (i.e., those graphs insensitive to multigluon couplings), then such graphs can never yield self-energies that are singular at $\not{p}=m$. Conversely, we have seen that the *non-Abelian* graph Fig. 3(c) appears to yield an infinite series in m, as discussed in Sec. IV. For condensates of dimension >5, any singular behavior at $\not{p}=m$ would similarly have to manifest itself in an untruncated infinite series in the OPE mass parameter m, as opposed to the finite polynomials anticipated from Abelian graphs. Such results suggest, within the context of contributions of nonperturbative condensates, the possibility of an intimate connection between confinement and the non-Abelian character of QCD.

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APPENDIX: CONDENSATE PROJECTIONS OF NONPERTURBATIVE VEV'S

A. $\langle \bar{q}q \rangle$ projection of the two-fermion VEV

Consider the nonperturbative VEV

$$\langle :\bar{\Psi}_i^{\rho}(y)\Psi_j^{\sigma}(z): \rangle_{\rm NP} .$$

Expanding in a Taylor series about $y=z=0$ we have

$$\langle :\bar{\Psi}_i^{\rho}(y)\Psi_j^{\sigma}(z): \rangle = \langle :\bar{\Psi}_i^{\rho}(0)\Psi_j^{\sigma}(0): \rangle + y^{\alpha}\langle :[\partial_\alpha\bar{\Psi}_i^{\rho}(0)]\Psi_j^{\sigma}(0): \rangle + z^{\alpha}\langle :\bar{\Psi}_i^{\rho}(0)[\partial_\alpha\Psi_j^{\sigma}(0)]: \rangle + \text{higher-order terms} . \tag{A1}$$

(Fermion superscript and subscript indices are over color and Dirac-spinorial space, respectively.) We assume this quantity leads to a gauge-independent nonperturbative contribution to the Fig. 1 self-energy. Consequently, we are free to utilize the fixed-point gauge condition[38]

$$x^{\mu}B_{\mu}(x)=0 \tag{A2}$$

to reexpress (A1) in terms of gauge-covariant quantities. Repeated differentiations of (A2) evaluated at $x=0$ lead to the relations[39]

$$B_\mu(0)=0, \partial_{(\alpha_1\alpha_2\cdots\alpha_n}B_{\mu)}(0)=0 \ . \tag{A3}$$

(Subscript parentheses indicate symmetrization.) This result implies that all derivatives (∂) in the Taylor series (A1) may be replaced by covariant derivatives (D). We then integrate by parts with respect to covariant derivatives, so as to obtain[17]

$$\langle :\bar{\Psi}_i^\rho(y)\Psi_j^\sigma(z): \rangle = \langle :\bar{\Psi}_i^\rho(0)\Psi_j^\sigma(0): \rangle - (y^\alpha - z^\alpha)\langle :\bar{\Psi}_i^\rho[D_\alpha(0)\Psi_j(0)]^\sigma: \rangle + \left[\frac{y^\alpha y^\beta}{2}+\frac{z^\alpha z^\beta}{2}-y^\alpha z^\beta\right]\langle :\bar{\Psi}_i^\rho(0)[D_\alpha(0)D_\beta(0)\Psi_j(0)]^\sigma: \rangle + \cdots \ . \tag{A4}$$

The first term on the right-hand side (RHS) of (A4) is proportional to the $\langle \bar{q}q \rangle$ condensate, as is evident from Lorentz invariance:[17]

$$\langle :\bar{\Psi}_i^\rho(0)\Psi_j^\sigma(0): \rangle = (\delta^{\rho\sigma}/3)(\delta_{ij}/4)\langle :\bar{\Psi}_k^\phi(0)\Psi_k^\phi(0): \rangle \equiv (\delta^{\rho\sigma}/3)\delta_{ij}\langle \bar{q}q \rangle/4 \ . \tag{A5}$$

Color superscript indices (ρ,σ) in (A4) will always lead to a trivial $(\delta^{\rho\sigma}/3)$ Kronecker-δ factor; we shall suppress such indices henceforth. Consider a general term in (A4), containing the object

$$\langle :\bar{\Psi}_i(0)D_{\alpha_1}(0)\cdots D_{\alpha_n}(0)\Psi_j(0): \rangle \ .$$

Only the completely symmetric part of this object has a $\langle \bar{q}q \rangle$ component. Terms with any antisymmetry in the indices $\alpha_1\cdots\alpha_n$ will end up contributing terms proportional to field strengths $G_{\mu\nu}(\equiv G_{\mu\nu}^{\rho\sigma}(x)=i\lambda_{\rho\sigma}^a G_{\mu\nu}^a(x)/2;$ $[D_\alpha(0),D_\beta(0)]=g_s G_{\beta\alpha}(0))$ and covariant derivatives of field strengths. Such objects cannot be eliminated through equations of motion to project out a term linear in $\langle \bar{q}q \rangle$. If only the symmetric part of (A4) is retained, one then finds that

$$\langle :\bar{\Psi}_i(y)\Psi_j(z): \rangle = \sum_{n=0}^{\infty}\frac{(y-z)^{\alpha_1}\cdots(y-z)^{\alpha_{2n}}}{(2n)!}\langle :\bar{\Psi}_i(0)D_{(\alpha_1}(0)\cdots D_{\alpha_{2n})}(0)\Psi_j(0): \rangle - \sum_{n=0}^{\infty}\frac{(y-z)^{\alpha_1}\cdots(y-z)^{\alpha_{2n+1}}}{(2n+1)!}\langle :\bar{\Psi}_i(0)D_{(\alpha_1}(0)\cdots D_{\alpha_{2n+1})}(0)\Psi_j(0): \rangle + (\text{terms proportional to higher-dimensional condensates}) \ . \tag{A6}$$

The "even" and "odd" terms in (A6) have been separated for convenience. Let us first consider the term in (A6) in which an even number of indices are contracted. Consider the quantity

$$\langle :\bar{\Psi}_i(0)D_{\alpha_1}(0)\cdots D_{\alpha_{2n}}(0)\Psi_j(0): \rangle_{(\text{sym})} \equiv A_{2n}\delta_{ij}S_{\alpha_1\cdots\alpha_{2n}} \ , \tag{A7}$$

where $S_{\alpha_1\cdots\alpha_{2n}}$ is a Lorentz-invariant and completely symmetric tensor. For example,

$$S_{\alpha_1\alpha_2\alpha_3\alpha_4}=g_{\alpha_1\alpha_2}g_{\alpha_3\alpha_4}+g_{\alpha_1\alpha_3}g_{\alpha_2\alpha_4}+g_{\alpha_1\alpha_4}g_{\alpha_2\alpha_3} \ . \tag{A8}$$

In general, the tensor $S_{\alpha_1\cdots\alpha_{2n}}$ will contain the sum of N_{2n} products of n metric tensors; e.g., from (A8) we see that $N_4=3$. These factors of N_{2n} are important because they occur when (A7) is substituted into the "even" term of (A6):

$$\sum_{n=0}^{\infty}\frac{(y-z)^{\alpha_1}\cdots(y-z)^{\alpha_{2n}}}{(2n)!}\langle :\bar{\Psi}_i(0)D_{\alpha_1}(0)\cdots D_{\alpha_{2n}}(0)\Psi_j(0): \rangle = \sum_{n=0}^{\infty}N_{2n}(y-z)^{2n}A_{2n}\delta_{ij}/(2n)! \ . \tag{A9}$$

To find N_{2n} explicitly, we note that

$$S_{\alpha_1\cdots\alpha_{2n+2}}=g_{\alpha_{2n+1}\alpha_{2n+2}}S_{\alpha_1\cdots\alpha_{2n}}+g_{\alpha_{2n+1}\alpha_1}S_{\alpha_{2n+2}\alpha_2\cdots\alpha_{2n}}+\cdots+g_{\alpha_{2n+1}\alpha_{2n}}S_{\alpha_1\cdots\alpha_{2n+2}} \ , \tag{A10}$$

in which case

$$N_{2n+2}=(2n+1)N_{2n} \ . \tag{A11}$$

We see from (A11) and our knowledge of N_0, $N_2(=1)$, and $N_4(=3)$ that

$$N_{2n}=\begin{cases}1, & n=0 \ ,\\ (2n-1)!!=(2n-1)!2^{1-n}/(n-1)!, & n\neq 0 \ .\end{cases} \tag{A12}$$

The constant A_{2n} appearing in (A7) and (A9) can be determined by contracting the products of n metric tensors and a spinor-index Kronecker δ

$$\delta_{ji}g^{\alpha_1\alpha_2}g^{\alpha_3\alpha_4}\cdots g^{\alpha_{2n-1}\alpha_{2n}}$$

into both sides of (A7). When one uses the equation of motion (2.3), one finds upon contraction that

$$(-im)^{2n}\langle\bar{q}q\rangle = 4A_{2n}(g^{\alpha_1\alpha_2}\cdots g^{\alpha_{2n-1}\alpha_{2n}})S_{\alpha_1\cdots\alpha_{2n}} \equiv 4A_{2n}S_{2n} , \tag{A13}$$

in which case

$$A_{2n} = (-im)^{2n}\langle\bar{q}q\rangle/4S_{2n} . \tag{A14}$$

To find A_{2n}, we first note that $S_2 = g^{\alpha_1\alpha_2}g_{\alpha_1\alpha_2} = 4$, and that $S_4 = g^{\alpha_1\alpha_2}g^{\alpha_3\alpha_4}(g_{\alpha_1\alpha_2}g_{\alpha_3\alpha_4} + g_{\alpha_1\alpha_3}g_{\alpha_2\alpha_4} + g_{\alpha_1\alpha_4}g_{\alpha_2\alpha_3}) = 24$. By contracting $g^{\alpha_1\alpha_2}\cdots g^{\alpha_{2n+1}\alpha_{2n2}}$ into both sides of (A10), we find that

$$S_{2n+2} = (2n+4)S_{2n} . \tag{A15}$$

We then see from (A15) and our known values for S_2 and S_4 that

$$S_{2n} = \tfrac{1}{2}(2n+2)!! = 2^n(n+1)! . \tag{A16}$$

Upon substitution of (A16) into (A14), we see that

$$A_{2n} = \frac{(-im)^{2n}\langle\bar{q}q\rangle}{2^{n+2}(n+1)!} . \tag{A17}$$

Further substitution of (A17) and (A12) into (A9) completely determines the "even" term of (A6):

$$\sum_{n=0}^{\infty}\frac{(y-z)^{\alpha_1}\cdots(y-z)^{\alpha_{2n}}}{(2n)!}\langle :\bar{\Psi}_i(0)D_{\alpha_1}(0)\cdots D_{\alpha_{2n}}(0)\Psi_j(0):\rangle = \sum_{n=0}^{\infty}\frac{(-im)^{2n}(y-z)^{2n}\delta_{ij}\langle\bar{q}q\rangle}{4^{n+1}n!(n+1)!} . \tag{A18}$$

We now consider the "odd" term in (A6), involving contraction of an odd number of indices. As before, we consider the quantity

$$\langle :\bar{\Psi}_i(0)D_{\alpha_1}(0)\cdots D_{\alpha_{2n+1}}(0)\Psi_j(0):\rangle_{\text{sym}} \equiv A_{2n+1}(S_{\alpha_1\cdots\alpha_{2n+1}})_{ji} , \tag{A19}$$

where the tensor $(S_{\alpha_1\cdots\alpha_{2n+1}})_{ji}$ is Lorentz invariant and symmetric; e.g.,

$$(S_{\alpha_1\alpha_2\alpha_3})_{ji} = (\gamma_{\alpha_1})_{ji}g_{\alpha_2\alpha_3} + (\gamma_{\alpha_2})_{ji}g_{\alpha_1\alpha_3} + (\gamma_{\alpha_3})_{ji}g_{\alpha_1\alpha_2} . \tag{A20}$$

The tensor $(S_{\alpha_1\cdots\alpha_{2n+1}})_{ji}$ will be the sum of N_{2n+1} terms, each of which being the product of a γ matrix and n metric tensors; e.g., from (A20), $N_3 = 3$. Such factors of N_{2n+1} occur when (A19) is substituted into the "odd" term of (A6):

$$\sum_{n=0}^{\infty}\frac{(y-z)^{\alpha_1}\cdots(y-z)^{\alpha_{2n+1}}}{(2n+1)!}\langle :\bar{\Psi}_i(0)D_{\alpha_1}(0)\cdots D_{\alpha_{2n+1}}(0)\Psi_j(0):\rangle = \sum_{n=0}^{\infty}\frac{N_{2n+1}[\gamma\cdot(y-z)]_{ji}(y-z)^{2n}A_{2n+1}}{(2n+1)!} . \tag{A21}$$

Using the relation

$$S_{\alpha_1\cdots\alpha_{2n+1}} = \gamma_{\alpha_1}S_{\alpha_2\cdots\alpha_{2n+1}} + \gamma_{\alpha_2}S_{\alpha_1\alpha_3\cdots\alpha_{2n+1}} + \cdots + \gamma_{\alpha_{2n+1}}S_{\alpha_1\cdots\alpha_{2n}} , \tag{A22}$$

it is easy to verify that $N_{2n+1} = (2n+1)N_{2n}$ [$=N_{2n+2}$ via (A11)], in which case we see from (A12) that

$$N_{2n+1} = (2n+1)!2^{-n}/n! . \tag{A23}$$

The quantity A_{2n+1} may be obtained by contracting

$$(\gamma^{\alpha_1})_{ij}g^{\alpha_2\alpha_3}\cdots g^{\alpha_{2n}\alpha_{2n+1}}$$

into both sides of (A19). We perform this contraction using the equation of motion (2.3), so as to obtain

$$A_{2n+1} = (-im)^{2n+1}\langle\bar{q}q\rangle/S_{2n+1} , \tag{A24}$$

where

$$S_{2n+1} = [(\gamma^{\alpha_1})_{ij}g^{\alpha_2\alpha_3}\cdots g^{\alpha_{2n}\alpha_{2n+1}}]_{ij}(S_{\alpha_1\cdots\alpha_{2n+1}})_{ji} . \tag{A25}$$

For example, we see from (A20) that $S_3 = 96$. In general, we see from (A22) that

$$S_{2n+1} = (16+8n)S_{2n} , \tag{A26}$$

in which case we find from (A16) that

$$S_{2n+1}=2^{n+3}(n+2)! \ . \tag{A27}$$

Upon substitution of (A27), (A24), and (A23) into (A21), we find that the "odd" term of (A6) is given by

$$\sum_{n=0}^{\infty}\frac{(y-z)^{\alpha_1}\cdots(y-z)^{\alpha_{2n+1}}}{(2n+1)!}\langle :\bar{\Psi}_i(0)D_{\alpha_1}(0)\cdots D_{\alpha_{2n+1}}(0)\Psi_j(0):\rangle=\sum_{n=0}^{\infty}\frac{[\gamma\cdot(y-z)]_{ji}(y-z)^{2n}(-im)^{2n+1}\langle\bar{q}q\rangle}{2^{2n+3}n!(n+2)!} \ . \tag{A28}$$

Equations (2.1) and (2.2) of the text correspond to substitution of (A28) and (A18) into (A6):

$$\langle :\bar{\Psi}_i^{\rho}(y)\Psi_j^{\sigma}(z):\rangle=(\delta^{\rho\sigma}/3)\langle\bar{q}q\rangle\sum_{n=0}^{\infty}\frac{(-im)^{2n}(y-z)^{2n}}{4^{n+1}n!(n+1)!}\left[\delta_{ji}+\frac{im[\gamma\cdot(y-z)]_{ji}}{2(n+2)}\right]$$
$$+(\text{contributions of higher-dimensional condensates}) \ . \tag{A29}$$

Thus, we have obtained the full coefficient of the $\langle\bar{q}q\rangle$ component of the nonperturbative two-fermion vacuum expectation value.

It is interesting to note that this coefficient (A29) can be expressed in closed form. If $x\equiv y-z$, $x^2<0$, the series in (A29) are summable[40] and

$$\langle :\bar{\Psi}_i^{\rho}(y)\Psi_j^{\sigma}(z):\rangle=\delta^{\rho\sigma}\frac{\langle\bar{q}q\rangle}{6m}\left[\frac{\delta_{ji}}{\sqrt{-x^2}}I_1(m\sqrt{-x^2})+\frac{i(\gamma\cdot x)_{ji}}{-x^2}I_2(m\sqrt{-x^2})\right]$$
$$+(\text{contributions of higher-dimensional condensates}) \ , \tag{A30}$$

where I_1,I_2 are modified Bessel's functions. A similar expression exists for timelike x^2, containing ordinary Bessel functions.

B. $\langle GG\rangle$ projection of the two-gluon VEV

Consider the evaluation of the nonperturbative VEV of two gluon fields at different space-time points

$$\langle :B_{\lambda}^{a}(y)B_{\rho}^{b}(z):\rangle \ .$$

If we demand that the external gluon fields satisfy the fixed-point gauge condition (A2), then the fields may be expanded in a gauge-covariant fashion:[17]

$$B_{\mu}^{a}(x)=\tfrac{1}{2}x^{\nu}G_{\nu\mu}^{a}(0)+\tfrac{1}{3}x^{\nu}x^{\omega}[D_{\omega}(0),G_{\nu\mu}^{a}(0)]$$
$$+\cdots+\frac{1}{(n+1)(n-1)!}x^{\alpha_1}x^{\alpha_2}\cdots x^{\alpha_n}[D_{\alpha_1}(0),[D_{\alpha_2}(0),[\cdots[D_{\alpha_{n-1}}(0),G_{\alpha_n\mu}^{a}(0)]\cdots]]]+\cdots \ . \tag{A31}$$

The lowest-order contribution to the nonperturbative VEV of two-gluon fields is generated through the lead term of (A31):

$$\langle :B_{\lambda}^{a}(y)B_{\rho}^{b}(z):\rangle=\tfrac{1}{4}y^{\eta}z^{\tau}\langle :G_{\eta\lambda}^{a}(0)G_{\tau\rho}^{b}(0):\rangle+\cdots \ . \tag{A32}$$

The RHS of (A32) can be expressed in terms of the gluon-gluon condensate, defined by

$$\langle :G_{\alpha\beta}^{a}(0)G_{\alpha\beta}^{a}(0):\rangle\equiv\langle GG\rangle \ , \tag{A33}$$

by noting from symmetry considerations that

$$\langle :G_{\eta\lambda}^{a}(0)G_{\tau\rho}^{b}(0):\rangle=C\delta^{ab}(g_{\eta\tau}g_{\lambda\rho}-g_{\eta\rho}g_{\lambda\tau}) \ . \tag{A34a}$$

Contracting $\delta^{ab}g_{\eta\tau}g_{\lambda\rho}$ into both sides of (A34a) we find that

$$C=\langle GG\rangle/96 \ . \tag{A34b}$$

The gluon-condensate component of the two-gluon nonperturbative VEV is then obtained from substitution of (A34) into (A32):

$$\langle :B_{\lambda}^{a}(y)B_{\rho}^{b}(z):\rangle=\frac{y^{\eta}z^{\tau}}{384}(g_{\eta\tau}g_{\lambda\rho}-g_{\eta\rho}g_{\lambda\tau})\langle GG\rangle+\cdots \ . \tag{A35}$$

The ellipsis on the RHS of (A35) refers to contributions arising from nonleading terms in (A31). These contributions are not proportional to $\langle GG\rangle$, but only to higher-than-4-dimensional condensates. To see this, consider the general form of the VEV coefficient of $y^{\alpha_1}\cdots y^{\alpha_n}z^{\beta_1}\cdots z^{\beta_m}$ within (A35) arising from nonleading terms of (A31):

$$\langle :[D_{\alpha_1}(0),[\cdots[D_{\alpha_{n-1}}(0),G_{\alpha_n\lambda}(0)]\cdots]][D_{\beta_1}(0),[\cdots[D_{\beta_{m-1}}(0),G_{\beta_m\rho}(0)]\cdots]]:\rangle \ .$$

Lorentz invariance implies proportionality of this VEV coefficient to products of metric tensors, as this VEV coefficient does not contain any Dirac-spinorial indices.[41] Hence, contractions analogous to the one generating (A34b) necessarily involve contracting products of metric tensors into the above VEV coefficient. Such contractions necessarily remove covariant-derivative factors of D by generating fermion fields through the equation of motion (3.2b) or through increasing the number of field strengths G through the equation of motion (3.2a). Consequently, we see that QCD equations of motion can be used to eliminate covariant derivatives $D_\alpha(0)$ from VEV coefficients generated by nonleading terms in (A31) *only* by increasing the number of fermions $(\Psi,\bar{\Psi})$ or field strengths (G) appearing in these coefficients after contraction is performed. In particular, multiple-metric-tensor contractions of such higher derivative VEV coefficients can never yield a quantity involving only two field strengths (GG) once all covariant derivatives are eliminated via field equations of motion. The $\langle GG\rangle$ projection of the two-gluon VEV is therefore given *entirely* by (A35).

C. $\langle \bar{q}G\cdot\sigma q\rangle$ projection of the two-fermion VEV

We now consider the $\langle \bar{q}G\cdot\sigma q\rangle$ "mixed-condensate" projection of the two-fermion VEV on the left-hand side of (A4). The mixed condensate is a dimension-5 object containing both fermion fields and a field strength (G):[26]

$$\langle \bar{q}G\cdot\sigma q\rangle \equiv \langle :\bar{\Psi}_i^{\alpha}(0)G_{\mu\nu}^{\alpha\beta}(0)\sigma_{ij}^{\mu\nu}\Psi_j^{\beta}(0):\rangle \ , \quad \text{(A36a)}$$

$$G_{\mu\nu}^{\alpha\beta}(x)\equiv i\lambda^a_{\alpha\beta}G^a_{\mu\nu}(x)/2 \ . \quad \text{(A36b)}$$

In (A4), the covariantized Taylor-series expansion of the dimension-3 VEV, only second- and higher-order terms can have Taylor-series coefficients of dimension-5. The first four terms relevant for obtaining the components of $\langle :\bar{\Psi}_i(z)\Psi_j(y):\rangle$ proportional to the mixed condensate are just[17]

$$\langle :\bar{\Psi}_i(z)\Psi_j(y):\rangle_{\rm II}\equiv \frac{z^\mu z^\nu}{2}\langle :\bar{\Psi}_i D_{(\mu}D_{\nu)}\Psi_j:\rangle+\frac{y^\mu y^\nu}{2}\langle :\bar{\Psi}_i D_{(\mu}D_{\nu)}\Psi_j:\rangle - y^\mu z^\nu\langle :\bar{\Psi}_i D_\nu D_\mu\Psi_j:\rangle \ , \quad \text{(A37a)}$$

$$\langle :\bar{\Psi}_i(z)\Psi_j(y):\rangle_{\rm III}\equiv \tfrac{1}{6}(y^\mu y^\nu y^\omega - z^\mu z^\nu z^\omega)\langle :\bar{\Psi}_i D_{(\mu}D_\nu D_{\omega)}\Psi_j:\rangle+\tfrac{1}{2}z^\mu z^\nu y^\omega\langle :\bar{\Psi}_i D_{(\mu}D_{\nu)}D_\omega\Psi_j:\rangle$$
$$-\tfrac{1}{2}(z^\omega y^\mu y^\nu)\langle :\bar{\Psi}_i D_\omega D_{(\mu}D_{\nu)}\Psi_j:\rangle \ , \quad \text{(A37b)}$$

$$\langle :\bar{\Psi}_i(z)\Psi_j(y):\rangle_{\rm IV}\equiv \tfrac{1}{24}(y^\alpha y^\beta y^\mu y^\nu+z^\alpha z^\beta z^\mu z^\nu)\langle :\bar{\Psi}_i D_{(\alpha}D_\beta D_\mu D_{\nu)}\Psi_j:\rangle-\tfrac{1}{6}(y^\alpha z^\beta z^\mu z^\nu)\langle :\bar{\Psi}_i D_{(\mu}D_\nu D_{\beta)}D_\alpha\Psi_j:\rangle$$
$$-\tfrac{1}{6}(y^\mu y^\nu y^\beta z^\alpha)\langle :\bar{\Psi}_i D_\alpha D_{(\mu}D_\nu D_{\beta)}\Psi_j:\rangle+\tfrac{1}{4}(y^\mu y^\nu z^\alpha z^\beta)\langle :\bar{\Psi}_i D_{(\alpha}D_{\beta)}D_{(\mu}D_{\nu)}\Psi_j:\rangle \ , \quad \text{(A37c)}$$

$$\langle :\bar{\Psi}_i(z)\Psi_j(y):\rangle_{\rm V}\equiv \tfrac{1}{120}(y^\alpha y^\beta y^\mu y^\nu y^\lambda - z^\alpha z^\beta z^\mu z^\nu z^\lambda)\langle :\bar{\Psi}_i D_{(\alpha}D_\beta D_\mu D_\nu D_{\lambda)}\Psi_j:\rangle$$
$$+\tfrac{1}{24}(y^\alpha z^\beta z^\mu z^\nu z^\lambda)\langle :\bar{\Psi}_i D_{(\beta}D_\mu D_\nu D_{\lambda)}D_\alpha\Psi_j:\rangle-\tfrac{1}{24}(y^\beta y^\mu y^\nu y^\lambda z^\alpha)\langle :\bar{\Psi}_i D_\alpha D_{(\beta}D_\mu D_\nu D_{\lambda)}\Psi_j:\rangle$$
$$-\tfrac{1}{12}(y^\alpha y^\beta z^\mu z^\nu z^\lambda)\langle :\bar{\Psi}_i D_{(\mu}D_\nu D_{\lambda)}D_{(\alpha}D_{\beta)}\Psi_j:\rangle+\tfrac{1}{12}(y^\mu y^\nu y^\lambda z^\alpha z^\beta)\langle :\bar{\Psi}_i D_{(\alpha}D_{\beta)}D_{(\mu}D_\nu D_{\lambda)}\Psi_j:\rangle \ . \quad \text{(A37d)}$$

On the RHS of (A37), all covariant derivatives and all fields are evaluated at $x=0$. Subscript parentheses indicate symmetrization of indices. Note also the color indices have been ignored, since they lead trivially to a Kronecker-$\delta/3$ factor, as discussed earlier in this appendix.

To evaluate the mixed condensate contributions in (A37) two identities [(A39) and (A40)] are required. The first follows from an obvious Dirac-matrix relation

$$\gamma_\mu\gamma_\nu=g_{\mu\nu}-i\sigma_{\mu\nu} \ .$$

This implies [upon application of (32a)] that

$$\not{D}\not{D}=D^2-i\sigma_{\mu\nu}D_\mu D_\nu=D^2+g_s(i/2)\sigma^{\mu\nu}G_{\mu\nu} \ , \quad \text{(A38)}$$

in which case

$$D^2=\not{D}\not{D}-g_s(i/2)G\cdot\sigma \ , \quad \text{(A39)}$$

where $G\cdot\sigma=\sigma^{\mu\nu}G_{\mu\nu}$ and $G_{\mu\nu}$ is given by (3.2a). The second identity (A40) is obtained by setting $[D^\mu,G_{\mu\nu}]=0$. This simplification of the QCD equation (3.2b), in which the fermion current generated by the commutator is dropped, is justified since such currents will not contribute to the $\langle \bar{q}G\cdot\sigma q\rangle$ projection of the two-fermion VEV, but only to condensate components of dimension ≥ 6. With $[D^\mu,G_{\mu\nu}]=0$, it is easy to obtain the simplified relation

$$D^\mu D_\nu D_\mu=\tfrac{1}{2}(D^2D_\nu+D_\nu D^2) \ . \quad \text{(A40)}$$

Consider now those terms in (A37a) requiring evaluation of $\langle \bar{\Psi}D_\mu D_\nu\Psi\rangle$. The expectation value of a gauge covariant is Lorentz invariant; therefore,

$$\langle \bar{\Psi}_i D_\mu D_\nu \Psi_j\rangle = A\delta_{ij}g_{\mu\nu}+B(\sigma_{\mu\nu})_{ji} \ . \quad \text{(A41)}$$

Contracting both sides with $\delta_{ij}g^{\mu\nu}$ and $\sigma_{ij}^{\mu\nu}$ we obtain

$$\langle \bar{\Psi}D^2\Psi\rangle=16A \ , \quad \text{(A42)}$$

$$-\frac{g_s}{2}\langle \bar{\Psi}\sigma^{\mu\nu}G_{\mu\nu}\Psi\rangle\equiv\frac{-g_s}{2}\langle \bar{q}G\cdot\sigma q\rangle=48B \ . \quad \text{(A43)}$$

Using the identity (A39) to find the mixed condensate component of (A42), we obtain $A=-ig_s\langle\bar{q}G\cdot\sigma q\rangle/32$, in which case (A41) becomes

$$\langle\bar{\Psi}_i D_\mu D_\nu \Psi_j\rangle = g_s[(-i\delta_{ij}g_{\mu\nu})/32 - (\sigma_{\mu\nu})_{ji}/96]\langle\bar{q}G\cdot\sigma q\rangle \ . \quad \text{(A44)}$$

Substituting (A44) into (A37a) gives the lowest-order mixed-condensate contribution to $\langle:\bar{\Psi}(z)\Psi(y):\rangle$:

$$\langle:\bar{\Psi}_i(z)\Psi_j(y):\rangle_{\text{II}} = g_s\langle\bar{q}G\cdot\sigma q\rangle\left[-\frac{1}{96}y_\mu z_\nu \sigma_{ji}^{\mu\nu} - \frac{i}{64}(y-z)^2\delta_{ij}\right] . \quad \text{(A45)}$$

Now consider the third order terms (A37b). From covariance, we find that

$$\langle:\bar{\Psi}_i D_{(\mu}D_\nu D_{\omega)}\Psi_j:\rangle = A(\gamma_{\mu_{ji}}g_{\nu\omega}+\gamma_{\nu_{ji}}g_{\mu\omega}+\gamma_{\omega_{ji}}g_{\mu\nu}) \ . \quad \text{(A46)}$$

Contracting both sides of (A46) with $\gamma_{ij}^{\mu}g_{\nu\omega}$ gives

$$\tfrac{1}{3}\langle:\bar{\Psi}(\not{D}D^2+D^2\not{D}+D^\mu\not{D}D_\mu)\Psi:\rangle = A\,16(4+1+1) \ . \quad \text{(A47)}$$

Identities (A39) and (A40), as well as the equation of motion (2.3) permit simplification of (A47):

$$g_s(-im)(-i/2)\langle\bar{q}G\cdot\sigma q\rangle = 96A \quad \text{(A48)}$$

in which case

$$\langle:\bar{\Psi}_i D_{(\mu}D_\nu D_{\omega)}\Psi_j:\rangle = -g_s\frac{m}{2(96)}\langle\bar{q}G\cdot\sigma q\rangle(\gamma_{\mu_{ji}}g_{\nu\omega}+\gamma_{\nu_{ji}}q_{\mu\omega}+\gamma_{\omega_{ji}}g_{\mu\nu}) \ . \quad \text{(A49)}$$

Now consider

$$\langle:\bar{\Psi}_i D_{(\mu}D_{\nu)}D_\omega\Psi_j:\rangle = Ag_{\mu\nu}\gamma_{\omega_{ji}}+B(g_{\nu\omega}\gamma_{\mu_{ji}}+g_{\mu\omega}\gamma_{\nu_{ji}}) \ . \quad \text{(A50)}$$

Contracting with $g^{\mu\nu}\gamma_{ij}^{\omega}, g^{\mu\omega}\gamma_{ij}^{\nu}$ yields two equations:

$$\langle:\bar{\Psi}D^2\not{D}\Psi:\rangle = 16(4)A+16(2)B \ , \quad \text{(A51)}$$

$$\tfrac{1}{2}\langle:\bar{\Psi}(\not{D}D^2+D^\mu\not{D}D_\mu)\Psi:\rangle = 16A+16(5)B \ . \quad \text{(A52)}$$

Using (A39), (A40), and (2.3) we find that

$$-g_s\frac{m}{2(16)}\langle\bar{q}G\cdot\sigma q\rangle = 4A+2B \ ,$$

$$-g_s\frac{m}{2(16)}\langle\bar{q}G\cdot\sigma q\rangle = A+5B \ .$$

Solving the above equations for A, B, and substituting back into (A50), we obtain the same expression as on the right-hand side of (A49):

$$\langle:\bar{\Psi}_i D_{(\mu}D_{\nu)}D_\omega\Psi_j:\rangle = -g_s\frac{m}{2(96)}\langle\bar{q}G\cdot\sigma q\rangle(\gamma_{\mu_{ji}}g_{\nu\omega}+\gamma_{\nu_{ji}}g_{\mu\omega}+\gamma_{\omega_{ji}}g_{\mu\nu}) \ . \quad \text{(A53)}$$

This result is also applicable for $\langle:\bar{\Psi}_i D_\omega D_{(\mu}D_{\nu)}\Psi_j:\rangle$. Substitution of (A49) and (A53) into (A37b) yields

$$\begin{aligned}\langle:\bar{\Psi}_i(z)\Psi_j(y):\rangle_{\text{III}} &\equiv g_s\frac{-m\langle\bar{q}G\cdot\sigma q\rangle}{1152}(y^\mu y^\nu y^\omega-3y^\mu y^\nu z^\omega+3z^\mu z^\nu y^\omega-z^\mu z^\nu z^\omega)(\gamma_\mu g_{\nu\omega}+\gamma_\nu g_{\mu\omega}+\gamma_\omega g_{\mu\nu})_{ji}\\ &= g_s\frac{-m\langle qG\cdot\sigma q\rangle}{384}[\gamma\cdot(y-z)]_{ji}(y-z)^2 \ . \end{aligned} \quad \text{(A54)}$$

One can similarly evaluate the mixed condensate component of the fourth- and fifth-order terms in $\langle:\bar{\Psi}(z)\Psi(y):\rangle$. For example, consider within (A37c) the term

$$\langle:\bar{\Psi}_i D_{(\alpha}D_{\beta)}D_{(\mu}D_{\nu)}\Psi_j:\rangle \equiv Ag_{\mu\nu}g_{\alpha\beta}\delta_{ij}+B(g_{\nu\beta}g_{\mu\alpha}+g_{\mu\beta}g_{\nu\alpha})\delta_{ij}+C(g_{\nu\beta}\sigma_{\alpha\mu}+g_{\mu\beta}\sigma_{\alpha\nu}+g_{\nu\alpha}\sigma_{\beta\mu}+g_{\mu\alpha}\sigma_{\beta\nu})_{ji} \ . \quad \text{(A55)}$$

Contracting both sides of (A55) as indicated and extracting only the mixed condensate component yields

$$g^{\mu\nu}g^{\alpha\beta}\delta_{ij}\times\text{Eq. (A55)}\rightarrow 64A+32B=\langle:\bar{\Psi}D^2D^2\Psi:\rangle \ , \quad \text{(A56a)}$$

$$g^{\alpha\mu}g^{\beta\nu}\delta_{ij}\times\text{Eq. (A55)}\rightarrow 16A+80B=\tfrac{1}{2}\langle:\bar{\Psi}(D^\alpha D^\mu D_\alpha D_\mu+D^\alpha D^2 D_\alpha)\Psi:\rangle \ , \quad \text{(A56b)}$$

$$g^{\alpha\mu}\sigma_{ij}^{\beta\nu}\times\text{Eq. (A55)}\rightarrow 288C=\tfrac{1}{4}\langle:\bar{\Psi}(\sigma^{\beta\nu}D^\alpha D_\beta D_\alpha D_\nu+D^\alpha\sigma^{\beta\nu}D_\beta D_\nu D_\alpha+\sigma^{\beta\nu}D_\beta D^2 D_\nu+\sigma^{\beta\nu}D_\beta D_\alpha D_\nu D^\alpha)\Psi:\rangle \ . \quad \text{(A56c)}$$

We extract the mixed condensate components of the right-hand sides of (A56) first through use of the relations ($j\equiv$fermion current; $G\equiv$field strength)

$$D^\alpha D^\mu D_\alpha D_\mu+D^\alpha D^2 D_\alpha = 2D^2D^2+O(g_s^2Dj)+O(g_s^2G^2) \ , \quad \text{(A57a)}$$

$$\sigma^{\beta\nu}D_\beta D^2 D_\nu = i\not{D}D^2\not{D}-iD^2D^2+O(g_s^2G^2)+O(g_s^2Dj) \ , \quad \text{(A57b)}$$

$$\sigma^{\beta\nu}D^\alpha D_\beta D_\alpha D_\nu = \tfrac{1}{2}\sigma^{\beta\nu}D_\beta D^2 D_\nu-\tfrac{1}{4}g_s D^2\sigma\cdot G+O(g_s^2Dj) \ , \quad \text{(A57c)}$$

$$\sigma^{\beta\nu}D_\beta D_\alpha D_\nu D^\alpha = \tfrac{1}{2}\sigma^{\beta\nu}D_\beta D^2 D_\nu-\tfrac{1}{4}g_s\sigma\cdot GD^2+O(g_s^2Dj)+O(g_s^2G^2) \ , \quad \text{(A57d)}$$

$$\sigma^{\beta\nu}D_\alpha D_\beta D_\nu D^\alpha = -\tfrac{1}{4}g_s D^2\sigma\cdot G - \tfrac{1}{4}g_s\sigma\cdot G D^2 + O(g_s^2G^2) + O(g_s^2Dj) \;, \tag{A57e}$$

and then through use of (2.3) and (A39) in order to find the following mixed condensate projections:

$$64A + 32B = g_s i m^2\langle \bar{q}G\cdot\sigma q\rangle \;, \tag{A58a}$$

$$16A + 80B = g_s i m^2\langle \bar{q}G\cdot\sigma q\rangle \;, \tag{A58b}$$

$$288C = g_s(m^2/2)\langle \bar{q}G\cdot\sigma q\rangle \;. \tag{A58c}$$

Solving (A58) for A, B, and C and substituting back into (A55), one finds that

$$\langle :\bar{\Psi}_i D_{(\alpha}D_{\beta)}D_{(\mu}D_{\nu)}\Psi_j:\rangle = g_s\langle \bar{q}G\cdot\sigma q\rangle\frac{im^2}{576}[6\delta_{ij}(g_{\alpha\beta}g_{\mu\nu}+g_{\alpha\mu}g_{\beta\nu}+g_{\beta\mu}g_{\alpha\nu}) - i(g_{\alpha\mu}\sigma_{\beta\nu}+g_{\alpha\nu}\sigma_{\beta\mu}+g_{\beta\mu}\sigma_{\alpha\nu}+g_{\beta\nu}\sigma_{\alpha\mu})_{ji}] \;. \tag{A59}$$

Similarly, one can show that

$$\langle :\bar{\Psi}_i D_{(\alpha}D_\beta D_\mu D_{\nu)}\Psi_j:\rangle = g_s\langle \bar{q}G\cdot\sigma q\rangle\frac{im^2}{96}\delta_{ij}(g_{\alpha\beta}g_{\mu\nu}+g_{\alpha\mu}g_{\beta\nu}+g_{\alpha\nu}g_{\beta\mu}) \;, \tag{A60}$$

$$\langle :\bar{\Psi}_i D_\alpha D_{(\mu}D_\nu D_{\beta)}\Psi_j:\rangle = g_s\langle \bar{q}G\cdot\sigma q\rangle\frac{im^2}{576}[36\delta_{ij}(g_{\alpha\mu}g_{\beta\nu}+g_{\alpha\nu}g_{\beta\mu}+g_{\alpha\beta}g_{\mu\nu}) - i(\sigma_{\alpha\mu}g_{\beta\nu}+\sigma_{\alpha\beta}g_{\mu\nu}+\sigma_{\alpha\nu}g_{\beta\mu})_{ji}] \;, \tag{A61}$$

$$\langle :\bar{\Psi}_i D_{(\mu}D_\nu D_{\beta)}D_\alpha\Psi_j:\rangle = g_s\langle \bar{q}G\cdot\sigma q\rangle\frac{im^2}{576}[36\delta_{ij}(g_{\alpha\mu}g_{\beta\nu}+g_{\alpha\nu}g_{\beta\mu}+g_{\alpha\beta}g_{\mu\nu}) + i(\sigma_{\alpha\mu}g_{\beta\nu}+\sigma_{\alpha\beta}g_{\mu\nu}+\sigma_{\alpha\nu}g_{\beta\mu})_{ji}] \;. \tag{A62}$$

Upon substituting (A59)–(A62) into (A37c) one finds that the fourth-order term of the nonperturbative vacuum expectation value is

$$\langle :\bar{\Psi}_i(z)\Psi_j(y):\rangle_{\rm IV} = g_s\langle \bar{q}G\cdot\sigma q\rangle\frac{im^2}{24\cdot 96}[3(y-z)^4\delta_{ij} + 2iz_\alpha y_\mu\sigma^{\alpha\mu}_{ji}(y-z)^2] \;. \tag{A63}$$

Evaluation of the mixed condensate projection of (A37d) requires determination of the coefficients A–F in the expressions

$$\begin{aligned}\langle :\bar{\Psi}_i D_{(\alpha}D_{\beta)}D_{(\mu}D_\nu D_{\lambda)}\Psi_j:\rangle = {} & A g_{\alpha\beta}(\gamma_\lambda g_{\mu\nu}+\gamma_\mu g_{\nu\lambda}+\gamma_\nu g_{\mu\lambda})_{ji}\\ & + B[\gamma_\lambda(g_{\alpha\mu}g_{\beta\nu}+g_{\beta\mu}g_{\alpha\nu})+\gamma_\nu(g_{\alpha\mu}g_{\beta\lambda}+g_{\beta\mu}g_{\alpha\lambda})+\gamma_\mu(g_{\alpha\nu}g_{\beta\lambda}+g_{\alpha\lambda}g_{\beta\nu})]_{ji}\\ & + C[\gamma_\alpha(g_{\beta\mu}g_{\nu\lambda}+g_{\beta\nu}g_{\mu\lambda}+g_{\beta\lambda}g_{\mu\nu})+\gamma_\beta(g_{\alpha\mu}g_{\nu\lambda}+g_{\alpha\nu}g_{\mu\lambda}+g_{\alpha\lambda}g_{\mu\nu})]_{ji} \;,\end{aligned} \tag{A64}$$

$$\begin{aligned}\langle :\bar{\Psi}_i D_\alpha D_{(\beta}D_\mu D_\nu D_{\lambda)}\Psi_j:\rangle = {} & D\gamma_{\alpha_{ji}}(g_{\beta\mu}g_{\nu\lambda}+g_{\beta\nu}g_{\mu\lambda}+g_{\beta\lambda}g_{\mu\nu})\\ & + E[\gamma_\beta(g_{\alpha\mu}g_{\nu\lambda}+g_{\alpha\nu}g_{\mu\lambda}+g_{\alpha\lambda}g_{\mu\nu})+\gamma_\mu(g_{\alpha\beta}g_{\nu\lambda}+g_{\alpha\nu}g_{\beta\lambda}+g_{\alpha\lambda}g_{\beta\nu})\\ & \quad + \gamma_\nu(g_{\alpha\beta}g_{\mu\lambda}+g_{\alpha\mu}g_{\beta\lambda}+g_{\alpha\lambda}g_{\beta\mu})+\gamma_\lambda(g_{\alpha\beta}g_{\mu\nu}+g_{\alpha\mu}g_{\beta\nu}+g_{\alpha\nu}g_{\beta\mu})]_{ji} \;,\end{aligned} \tag{A65}$$

$$\begin{aligned}\langle :\bar{\Psi}_i D_{(\alpha}D_\beta D_\mu D_\nu D_{\lambda)}\Psi_j:\rangle = {} & F[\gamma_\alpha(g_{\beta\mu}g_{\nu\lambda}+g_{\beta\nu}g_{\mu\lambda}+g_{\beta\lambda}g_{\mu\nu})+\gamma_\beta(g_{\alpha\mu}g_{\nu\lambda}+g_{\alpha\nu}g_{\mu\lambda}+g_{\alpha\lambda}g_{\mu\nu})\\ & + \gamma_\mu(g_{\alpha\beta}g_{\nu\lambda}+g_{\alpha\nu}g_{\beta\lambda}+g_{\alpha\lambda}g_{\beta\nu})+\gamma_\nu(g_{\alpha\beta}g_{\mu\lambda}+g_{\alpha\mu}g_{\beta\lambda}+g_{\alpha\lambda}g_{\beta\mu})\\ & + \gamma_\lambda(g_{\alpha\beta}g_{\mu\nu}+g_{\alpha\mu}g_{\beta\nu}+g_{\alpha\nu}g_{\beta\mu})]_{ji} \;.\end{aligned} \tag{A66}$$

We contract the indices of (A64)–(A66) into the indicated tensor structures in order to obtain mixed condensate projections of the coefficients A–F:

$$g^{\alpha\beta}g^{\mu\nu}\gamma^\lambda_{ij}\times \text{Eq. (A64)} \rightarrow 4A + 2B + 2C = g_s m^3\langle \bar{q}G\cdot\sigma q\rangle/96 \;, \tag{A67a}$$

$$g^{\alpha\mu}g^{\beta\nu}\gamma^\lambda_{ij}\times \text{Eq. (A64)} \rightarrow A + 5B + 2C = g_s m^3\langle \bar{q}G\cdot\sigma q\rangle/96 \;, \tag{A67b}$$

$$g^{\beta\lambda}g^{\mu\nu}\gamma^\alpha_{ij}\times \text{Eq. (A64)} \rightarrow A + 2B + 5C = g_s m^3\langle \bar{q}G\cdot\sigma q\rangle/96 \;, \tag{A67c}$$

$$g^{\beta\mu}g^{\nu\lambda}\gamma^\alpha_{ij}\times \text{Eq. (A65)} \rightarrow 4D + 4E = g_s m^3\langle \bar{q}G\cdot\sigma q\rangle/96 \;, \tag{A67d}$$

$$g^{\alpha\mu}g^{\nu\lambda}\gamma^\beta_{ij}\times \text{Eq. (A65)} \rightarrow D + 7E = g_s m^3\langle \bar{q}G\cdot\sigma q\rangle/96 \;, \tag{A67e}$$

$$g^{\beta\mu}g^{\nu\lambda}\gamma^\alpha_{ij}\times \text{Eq. (A66)} \rightarrow F = g_s m^3\langle \bar{q}G\cdot\sigma q\rangle/(768) \;. \tag{A67f}$$

We see from (A67) that A–F all equal $g_s m^3\langle \bar{q}G\cdot\sigma q\rangle/(768)$, in which case we find from (A37d) that

$$\langle :\bar{\Psi}_i(z)\Psi_j(y):\rangle_V = -g_s \frac{m^3\langle \bar{q}G\cdot\sigma q\rangle}{(64)(96)}(y-z)^4\gamma^{\mu}_{ji}(y-z)_{\mu} . \tag{A68}$$

Consequently, the mixed condensate projection of the two-fermion vacuum expectation value is found to order m^3 to be

$$\begin{aligned}\langle 0|:\bar{\Psi}_i(z)\Psi_j(y):|0\rangle = g_s\langle \bar{q}G\cdot\sigma q\rangle\{&[-i\delta_{ij}(y-z)^2/64-\sigma^{\mu\nu}_{ji}y_{\mu}z_{\nu}/96]+m[-\gamma^{\mu}_{ji}(y-z)_{\mu}(y-z)^2/384]\\ &+m^2[i\delta_{ij}(y-z)^4/768-\sigma^{\mu\nu}_{ji}y_{\mu}z_{\nu}(y-z)^2/1152]\\ &+m^3[-\gamma^{\mu}_{ji}(y-z)_{\mu}(y-z)^4/6144]+O(m^4)\} .\end{aligned} \tag{A69}$$

Equation (A69) suggests that the mixed condensate projection of the two-fermion vacuum expectation value takes the general form

$$\langle 0|:\bar{\Psi}_i(z)\Psi_j(y):|0\rangle = g_s\langle \bar{q}G\cdot\sigma q\rangle \sum_{n=0}^{\infty}\{a_n[\gamma\cdot(y-z)]^n + b_n y_{\mu}z_{\nu}\sigma^{\mu\nu}[\gamma\cdot(y-z)]^{n-2}\}_{ji}m^n . \tag{A70}$$

This expression is justified by observing that any contributions to the RHS of (A70) involving more than one $\sigma^{\mu\nu}$ necessarily correspond to a further commutation $[D_{\mu},D_{\nu}]$ in the covariantized Taylor series (A37), thereby introducing via (3.2a) an additional field strength G. We note that there is no mixed condensate projection of any local vacuum expectation value coefficient unless that coefficient contains just one field strength G, as additional G's can be eliminated through (3.2b) only at the price of introducing into the coefficient VEV two more fermion fields, again eliminating the dimension-5 mixed-condensate projection.

D. $\langle \bar{q}G\cdot\sigma q\rangle$ projection of the quark-antiquark–gluon VEV

Consider the mixed condensate component of the nonperturbative vacuum expectation value $\langle 0|:\bar{\Psi}_i(z)B^a_{\mu}(w)\Psi_j(y):|0\rangle$. The gluon field may be expanded covariantly using the fixed-point gauge[17]

$$B^a_{\mu}(w)=\tfrac{1}{2}w^{\lambda}G^a_{\lambda\mu}(0)+\frac{w^{\lambda}w^{\sigma}}{3}[D_{\sigma}(0),G^a_{\lambda\mu}(0)] + \frac{w^{\lambda}w^{\sigma}w^{\tau}}{8}[D_{\sigma}(0),[D_{\tau}(0),G^a_{\lambda\mu}(0)]]+\cdots . \tag{A71}$$

Only the first term of (A71) will contribute to the mixed condensate projection of the nonperturbative vacuum expectation value under consideration. All subsequent terms, involving successively nested commutators of D's with field strengths G (if $G^{\omega\phi}_{\mu\nu}\equiv i\lambda^a_{\omega\phi}G^a_{\mu\nu}/2$, then $G^a_{\mu\nu}=-i\lambda^a_{\omega\phi}G^{\phi\omega}_{\mu\nu}/8$), eventually yield further field strengths ($[D^{\alpha},D^{\beta}]\to g_sG^{\beta\alpha}$) or additional fermion fields ($[D^{\alpha},G_{\alpha\beta}]\to g_sj_{\beta}$). As a result, we find the following covariantized Taylor series to be appropriate for the extraction of a mixed condensate projection of the VEV under consideration:

$$\begin{aligned}\frac{i}{2}\langle 0|:\bar{\Psi}_i(z)B^a_{\mu}(w)\Psi_j(y):|0\rangle = \frac{\lambda^a w^{\tau}}{16}\Bigg[&\left[\frac{1}{2}\langle :\bar{\Psi}_iG_{\tau\mu}\Psi_j:\rangle\right]+\left[\frac{y^{\alpha}}{2}\langle :\bar{\Psi}_iG_{\tau\mu}D_{\alpha}\Psi_j:\rangle-\frac{z^{\alpha}}{2}\langle :\bar{\Psi}_iD_{\alpha}G_{\tau\mu}\Psi_j:\rangle\right]\\ &+\left[\frac{z^{\alpha}z^{\beta}}{4}\langle :\bar{\Psi}_iD_{(\alpha}D_{\beta)}G_{\tau\mu}\Psi_j:\rangle+\frac{y^{\alpha}y^{\beta}}{4}\langle :\bar{\Psi}_iG_{\tau\mu}D_{(\alpha}D_{\beta)}\Psi_j:\rangle\right.\\ &\left.-\frac{z^{\alpha}y^{\beta}}{2}\langle :\bar{\Psi}_iD_{\alpha}G_{\tau\mu}D_{\beta}\Psi_j:\rangle\right]\\ &+\left[-\frac{z^{\alpha}z^{\beta}z^{\nu}}{12}\langle :\bar{\Psi}_iD_{(\alpha}D_{\beta}D_{\nu)}G_{\tau\mu}\Psi_j:\rangle+\frac{y^{\alpha}y^{\beta}y^{\nu}}{12}\langle :\bar{\Psi}_iG_{\tau\mu}D_{(\alpha}D_{\beta}D_{\nu)}\Psi_j:\rangle\right.\\ &\left.+\frac{z^{\alpha}z^{\beta}y^{\nu}}{4}\langle :\bar{\Psi}_iD_{(\alpha}D_{\beta)}G_{\tau\mu}D_{\nu}\Psi_j:\rangle-\frac{y^{\alpha}y^{\beta}z^{\nu}}{4}\langle :\bar{\Psi}_iD_{\nu}G_{\tau\mu}D_{(\alpha}D_{\beta)}\Psi_j:\rangle\right]+\cdots\Bigg] .\end{aligned} \tag{A72}$$

In (A72), all fields and covariantized derivatives on the RHS are evaluated at the origin of configuration space, and contracted color indices between λ^a and $G_{\tau\mu}$ have been suppressed ($\lambda^aG_{\tau\mu}\equiv\lambda^a_{\omega\phi}G^{\phi\omega}_{\tau\mu}$).

Consider the lowest-order term on the RHS of (A72) with all color indices now inserted. By covariance, this term must have the following structure:

$$\lambda^a_{\omega\phi}\langle :\bar{\Psi}^{\alpha}_iG^{\phi\omega}_{\tau\mu}\Psi^{\beta}_j:\rangle = A\lambda^a_{\beta\alpha}(\sigma_{\tau\mu})_{ji} . \tag{A73}$$

Upon contraction of both sides of (A73) with $\lambda^a_{\alpha\beta}(\sigma^{\tau\mu})_{ij}$ we find that

$$16A[(\sigma^{\tau\mu}\sigma_{\tau\mu})_{ii}]=\langle :\bar{\Psi}_i^{\alpha}G_{\tau\mu}^{\phi\omega}\sigma_{ij}^{\tau\mu}\Psi_j^{\beta}:\rangle\lambda_{\alpha\beta}^{a}\lambda_{\omega\phi}^{a}$$
$$=8i\langle :\bar{\Psi}_i^{\alpha}G_{\tau\mu}^{a}\lambda_{\alpha\beta}^{a}\sigma_{ij}^{\tau\mu}\Psi_j^{\beta}:\rangle$$
$$=16\langle :\bar{\Psi}_i^{\alpha}G_{\tau\mu}^{\alpha\beta}\sigma_{ij}^{\tau\mu}\Psi_j^{\beta}:\rangle$$
$$\equiv 16\langle \bar{q}G\cdot\sigma q\rangle \ . \qquad \text{(A74)}$$

In evaluating A, all color indices in (A73) clearly could have been ignored $(A/\langle\bar{q}G\cdot\sigma q\rangle=[(\sigma^{\tau\mu}\sigma_{\tau\mu})_{ii}]^{-1}=\frac{1}{48})$ provided we note that the color indices $\omega\phi$ (contracted into G) are converted to the fermion field color indices $\beta\alpha$. Thus the lowest-order (LO) contribution to (A72) is given by

$$\frac{i}{2}\langle 0|:\bar{\Psi}_i^{\alpha}(z)B_{\mu}^{a}(w)\Psi_j^{\beta}(y):|0\rangle_{\mathrm{LO}}$$
$$=\frac{\lambda_{\beta\alpha}^{a}}{32\times 48}\langle\bar{q}G\cdot\sigma q\rangle w^{\tau}(\sigma_{\tau\mu})_{ji} \ . \qquad \text{(A75)}$$

We will henceforth ignore color indices in computing coefficients of covariant quantities, with the stipulation that the λ^a initiating the (RHS) of (A72) eventually acquires the uncontracted color indices of the left-hand side's fermion fields. Thus in the next-to-leading order term

$$\langle :\bar{\Psi}_i D_{\alpha}G_{\tau\mu}\Psi_j:\rangle=B(\sigma_{\tau\mu}\gamma_{\alpha})_{ji}+C(\gamma_{\tau}g_{\alpha\mu}-\gamma_{\mu}g_{\alpha\tau})_{ji} \ , \qquad \text{(A76)}$$

the coefficients B and C are obtained from the following contractions:

$$(\gamma^{\alpha}\sigma^{\tau\mu})_{ij}\times\text{Eq. (A76)}\rightarrow 2B-iC=-im\langle\bar{q}G\cdot\sigma q\rangle/96 \ , \qquad \text{(A77a)}$$

$$(\gamma^{\tau}g^{\alpha\mu})_{ij}\times\text{Eq. (A76)}\rightarrow iB+C=0 \ . \qquad \text{(A77b)}$$

The result thus obtained for (A76) (which is identical to that for $\langle :\bar{\Psi}_i G_{\tau\mu}D_{\alpha}\Psi_j:\rangle$) yields the following first-order (I) contribution to (A72):

$$\frac{i}{2}\langle 0|:\bar{\Psi}_i^{\delta}(z)B_{\mu}^{a}(w)\Psi_j^{\epsilon}(y):|0\rangle_{\mathrm{I}}\left[\equiv\frac{\lambda^a w^{\tau}}{16}\left[\frac{y^{\alpha}}{2}\langle :\bar{\Psi}_i^{\delta}G_{\tau\mu}D_{\alpha}\Psi_j^{\epsilon}:\rangle-\frac{z^{\alpha}}{2}\langle :\bar{\Psi}_i^{\delta}D_{\alpha}G_{\tau\mu}\Psi_j^{\epsilon}:\rangle\right]\right]$$
$$=\frac{\lambda_{\epsilon\delta}^{a}}{3072}m\langle\bar{q}G\cdot\sigma q\rangle w^{\tau}[i\sigma_{\mu\tau}\gamma\cdot(y-z)+(y-z)_{\tau}\gamma_{\mu}-(y-z)_{\mu}\gamma_{\tau}]_{ji} \ . \qquad \text{(A78)}$$

Consider now the second-order terms in (A72). The coefficients $\langle :\bar{\Psi}_i D_{(\alpha}D_{\beta)}G_{\tau\mu}\Psi_j:\rangle$ and $\langle :\bar{\Psi}_i G_{\tau\mu}D_{(\alpha}D_{\beta)}\Psi_j:\rangle$ are found to be identical; we sketch here the derivation of the first of these:

$$\langle :\bar{\Psi}_i D_{(\alpha}D_{\beta)}G_{\tau\mu}\Psi_j:\rangle=E(g_{\alpha\beta}\sigma_{\tau\mu})_{ji}+F(g_{\tau\beta}\sigma_{\alpha\mu}+g_{\tau\alpha}\sigma_{\beta\mu}+g_{\mu\beta}\sigma_{\tau\alpha}+g_{\mu\alpha}\sigma_{\tau\beta})_{ji} \ . \qquad \text{(A79)}$$

The coefficients E and F are obtained via the following contractions:

$$(g^{\alpha\beta}\sigma_{ij}^{\tau\mu})\times\text{Eq. (A79)}\rightarrow E+F=-m^2\langle\bar{q}G\cdot\sigma q\rangle/192 \ , \qquad \text{(A80a)}$$

$$(g^{\tau\beta}\sigma_{ij}^{\alpha\mu})\times\text{Eq. (A79)}\rightarrow E+4F=0 \ . \qquad \text{(A80b)}$$

Thus $E=-m^2\langle\bar{q}G\cdot\sigma q\rangle/144=-4F$. In deriving the RHS of (A80) liberal use is made of (A39), (A40), (2.3), and (A57). From covariance the remaining second-order coefficient has the following structure:

$$\langle :\bar{\Psi}_i D_{\alpha}G_{\tau\mu}D_{\beta}\Psi_j:\rangle=Hg_{\alpha\beta}(\sigma_{\tau\mu})_{ji}+K(\sigma_{\alpha\beta}\sigma_{\tau\mu})_{ji}+L(g_{\beta\tau}g_{\alpha\mu}-g_{\beta\mu}g_{\alpha\tau})\delta_{ji}$$
$$+N(g_{\alpha\mu}\sigma_{\tau\beta}-g_{\alpha\tau}\sigma_{\mu\beta})_{ji}+P(g_{\mu\beta}\sigma_{\alpha\tau}-g_{\tau\beta}\sigma_{\alpha\mu})_{ji} \ . \qquad \text{(A81)}$$

The coefficients in (A81) are obtained through the use of the following contractions:

$$(g^{\alpha\beta}\sigma_{ij}^{\tau\mu})\times\text{Eq. (A81)}\rightarrow 4H+2N-2P=-m^2\langle\bar{q}G\cdot\sigma q\rangle/48 \ , \qquad \text{(A82a)}$$

$$g^{\beta\tau}g^{\alpha\mu}\delta_{ij}\times\text{Eq. (A81)}\rightarrow -K+L=0 \ , \qquad \text{(A82b)}$$

$$(\sigma^{\tau\beta})_{ij}g^{\alpha\mu}\times\text{Eq. (A81)}\rightarrow H+2iK+3N-P=0 \ , \qquad \text{(A82c)}$$

$$(\sigma^{\alpha\tau})_{ij}g^{\beta\mu}\times\text{Eq. (A81)}\rightarrow -H+2iK-N+3P=0 \ , \qquad \text{(A82d)}$$

$$(\gamma^{\alpha}\sigma^{\tau\mu}\gamma^{\beta})_{ij}\times\text{Eq. (A81)}\rightarrow 2iL-6N+6P=-m^2\langle\bar{q}G\cdot\sigma q\rangle/48 \ . \qquad \text{(A82e)}$$

Substituting the solutions of (A82) and (A80) into (A81) and (A79), respectively, yields the following second-order contribution to the nonperturbative vacuum expectation value (A72):

$$\frac{i}{2}\langle :\bar{\Psi}_i^{\delta}(z)B_{\mu}^{a}(w)\Psi_j^{\epsilon}(y):\rangle_{\mathrm{II}}=\frac{-\lambda_{\epsilon\delta}^{a}m^2\langle\bar{q}G\cdot\sigma q\rangle}{12\cdot 1536}w^{\tau}[2(y-z)^2\sigma_{\tau\mu}-(y-z)_{\tau}(y-z)^{\alpha}\sigma_{\alpha\mu}-(y-z)^{\alpha}(y-z)_{\mu}\sigma_{\tau\alpha}]_{ji} \ . \qquad \text{(A83)}$$

[The color subscript indices of λ^a have been referenced to those of the fermion fields, as in (A75) and (A78).]

Consider now the following third-order terms in (A72):

$$\langle :\overline{\Psi}_i G_{\tau\mu} D_{(\alpha} D_\beta D_{\nu)} \Psi_j : \rangle = A[(g_{\beta\nu}\gamma_\alpha + g_{\alpha\nu}\gamma_\beta + g_{\alpha\beta}\gamma_\nu)\sigma_{\tau\mu}]_{ji} + B[(g_{\alpha\nu}g_{\beta\mu} + g_{\beta\nu}g_{\alpha\mu} + g_{\alpha\beta}g_{\mu\nu})\gamma_\tau - (g_{\alpha\nu}g_{\beta\tau} + g_{\beta\nu}g_{\alpha\tau} + g_{\alpha\beta}g_{\tau\nu})\gamma_\mu]_{ji} , \quad \text{(A84)}$$

$$\langle :\overline{\Psi}_i D_\nu G_{\tau\mu} D_{(\alpha} D_{\beta)} \Psi_j : \rangle = C g_{\alpha\beta}(\sigma_{\tau\mu}\gamma_\nu)_{ji} + E[\sigma_{\tau\mu}(\gamma_\alpha g_{\nu\beta} + \gamma_\beta g_{\nu\alpha})]_{ji} + F[g_{\nu\beta}(g_{\tau\alpha}\gamma_\mu - g_{\mu\alpha}\gamma_\tau)_{ji} + g_{\nu\alpha}(g_{\tau\beta}\gamma_\mu - g_{\mu\beta}\gamma_\tau)_{ji}] + H[(g_{\alpha\mu}\sigma_{\tau\beta} + g_{\beta\mu}\sigma_{\tau\alpha} - g_{\alpha\tau}\sigma_{\mu\beta} - g_{\beta\tau}\sigma_{\mu\alpha})\gamma_\nu]_{ji} + K[g_{\alpha\beta}(g_{\nu\mu}\gamma_\tau - g_{\nu\tau}\gamma_\mu)]_{ji} + L[g_{\nu\mu}(g_{\tau\beta}\gamma_\alpha + g_{\tau\alpha}\gamma_\beta)_{ji} - g_{\nu\tau}(g_{\mu\beta}\gamma_\alpha + g_{\mu\alpha}\gamma_\beta)_{ji}] . \quad \text{(A85)}$$

The coefficients A,B,C,E,F,H,K,L are obtained via the following contractions:

$$g^{\beta\nu}(\sigma^{\tau\mu}\gamma^\alpha)_{ij} \times \text{Eq. (A84)} \rightarrow 2A + iB = im^3\langle \overline{q}G\cdot\sigma q\rangle/576 , \quad \text{(A86a)}$$

$$g^{\alpha\nu}g^{\beta\mu}\gamma^\tau_{ij} \times \text{Eq. (A84)} \rightarrow -iA + B = 0 , \quad \text{(A86b)}$$

$$g^{\alpha\beta}(\gamma^\nu\sigma^{\tau\mu})_{ij} \times \text{Eq. (A85)} \rightarrow 4C + 2E + iF + 4H - 2iK - iL = im^3\langle \overline{q}G\cdot\sigma q\rangle/192 , \quad \text{(A87a)}$$

$$g^{\nu\beta}(\sigma^{\tau\mu}\gamma^\alpha)_{ij} \times \text{Eq. (A85)} \rightarrow -5iF - 6H + iK - iL = im^3\langle \overline{q}G\cdot\sigma q\rangle/96 , \quad \text{(A87b)}$$

$$g^{\nu\beta}g^{\tau\alpha}\gamma^\mu_{ij} \times \text{Eq. (A85)} \rightarrow C + 5E + 5iF + 4H - iK + iL = 0 , \quad \text{(A87c)}$$

$$g^{\alpha\mu}(\gamma^\nu\sigma^{\tau\beta})_{ij} \times \text{Eq. (A85)} \rightarrow 2C + E + 2iF + 8H - iK - 2iL = 0 , \quad \text{(A87d)}$$

$$g^{\alpha\beta}g^{\nu\mu}\gamma^\tau_{ij} \times \text{Eq. (A85)} \rightarrow -2C - E - iF - 2H + 2iK + iL = 0 , \quad \text{(A87e)}$$

$$g^{\nu\mu}g^{\tau\beta}\gamma^\alpha_{ij} \times \text{Eq. (A85)} \rightarrow -C + E + iF - 4H + iK + 5iL = 0 . \quad \text{(A87f)}$$

The solutions to the above are $A = im^3\langle \overline{q}G\cdot\sigma q\rangle/576 = -iB = C = E = -iF = +iK$, $H = L = 0$. Explicit calculation shows *for mixed condensate projections* that $\langle :\overline{\Psi}_i D_{(\alpha} D_\beta D_{\nu)} G_{\tau\mu} \Psi_j : \rangle = \langle :\overline{\Psi}_i G_{\tau\mu} D_{(\alpha} D_\beta D_{\nu)} \Psi_j : \rangle$, $\langle :\overline{\Psi}_i D_{(\alpha} D_{\beta)} G_{\tau\mu} D_\nu \Psi_j : \rangle = \langle :\overline{\Psi}_i D_\nu G_{\tau\mu} D_{(\alpha} D_{\beta)} \Psi_j : \rangle$, in which case the third-order term of (A72) can be completely determined. Combining this $O(m^3)$ contribution with the lead, first $(O(m))$, and second $(O(m^2))$ -order contributions of (A75), (A78), and (A83), respectively, we obtain the following expression for the vacuum expectation value in (A72):

$$\frac{i}{2}\langle 0 | :\overline{\Psi}{}^\delta_i(z) B^a_\mu(w) \Psi^\epsilon_j(y) : | 0 \rangle = \frac{\lambda^a_{\epsilon\delta}\langle \overline{q}G\cdot\sigma q\rangle w^\tau}{1536} \Bigg\{ (\sigma_{\tau\mu})_{ji} + \frac{m}{2}[-i\sigma_{\tau\mu}\gamma\cdot(y-z) + (y-z)_\tau\gamma_\mu - \gamma_\tau(y-z)_\mu]_{ji} - \frac{m^2}{12}[2\sigma_{\tau\mu}(y-z)^2 - (y-z)_\tau(y-z)^\eta\sigma_{\eta\mu} - (y-z)^\eta(y-z)_\mu\sigma_{\tau\eta}]_{ji} + \frac{m^3}{12}[i\sigma_{\tau\mu}\gamma\cdot(y-z)(y-z)^2 + \gamma_\tau(y-z)_\mu(y-z)^2 - (y-z)_\tau\gamma_\mu(y-z)^2]_{ji} + O(m^4) \Bigg\} . \quad \text{(A88)}$$

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[23] If m_L is zero, corresponding to full chiral symmetry of the initial Lagrangian, no $\langle GG\rangle$-generated shift from a pole at zero can occur, as $\langle GG\rangle$ is itself a chiral invariant. This is verified in Ref. 19.

[24] This expression for the self-energy is in agreement with that quoted by Reinders and Stam (Ref. 18) for fixed-point gauge. The conflicting expression obtained in Ref. 19 for the gluon-condensate contribution is erroneous.

[25] Contrary conclusions are drawn in the absence of the "reflection assumption" of note 22. The coefficient of $g_s^2|\langle\bar{q}q\rangle|$ in (3.9) is then *negative*, and the positive contribution through $\langle GG\rangle$ must clearly dominate that of $\langle\bar{q}q\rangle$ to realize a positive range of values for m, particularly if $m\gg m_L$.

[26] A factor of g_s was absorbed in the definition of the mixed condensate in Ref. 20, leading to corrections whose mixed-condensate projections appear to be $O(g_s^2)$.

[27] If $m_L=m$ and only terms linear in m and m_L are kept, Eqs. (4.10) and (4.11) correspond to the mixed condensate contribution derived in Ref. 20: $E(p^2)\to mp^4(67+7a)$ and $F(p^2)\to p^6(-65-7a)$.

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[30] We are grateful to R. Mendel (private communications) for pointing out that in the naive MIT bag model when one approaches the chiral limit of $m_{\text{curr}}\ll2/R$, where R is the ~1 fm bag radius,

$$(m^{(d)}-m^{(u)})_{\text{const}}\simeq0.48(m^{(d)}-m^{(u)})_{\text{curr}}+O(m_{\text{curr}}^2R)\ .$$

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[35] For example, only condensates of dimensions 5 and 6 will generate $O(g_s^3)$ contributions to the fermion 2-point function.

[36] The factor of (p^2-m^2) in the denominator of (4.10) arises from summing the series in (4.9), a series whose sum $[(p^2-m\not{p})/(p^2-m^2)]$ goes to $\frac{1}{2}$ as $\not{p}\to m$.

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[40] See Eqs. (5.18.5) and (5.22.7) of E. Hansen, *A Table of Series and Products* (Prentice-Hall, Englewood Cliffs, NJ, 1975).

[41] It is this absence of spinorial indices that ensures that the VEV (A32) does not have a dimension-5 mixed-condensate projection.