# Aspects of the chiral quark model

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We point out intriguing connections between the chiral quark model in which the nucleon is treated as a soliton and the ordinary potential-binding quark model. The axial-vector coupling comes out near the value $\frac{5}{3}$ rather than around 0.7 as in the Skyrme description and can be easily "fine-tuned" to its experimental value. A "linear potential" in some sense is generated as a collective effect of the "meson cloud." Contact with QCD is made by introducing a gluonium field so as to satisfy the trace-anomaly equation. In the simplest treatment of this field one predicts a scalar-isoscalar state at 900 MeV in agreement with a recent analysis of experiments. The model is used to discuss how a "nontrivial winding number" emerges for the meson fields. A number of additional features of the model are treated.

## I. INTRODUCTION

In the last few years there has been a great deal of work by many different groups with somewhat different points of view which nevertheless points to a similar picture for the structure of the nucleon. Everyone agrees that, at some level, the nucleon contains three valence quarks. These are traditionally confined in an *ad hoc* though reasonable way by imposing baglike boundary conditions.[1] An important realization[2] has been that chiral symmetry requires the existence of a "meson cloud" (as believed long ago) surrounding this bag. In fact, the successful revival of the Skyrme model[3] which pictures the nucleon purely as a solitonic excitation of the effective chiral Lagrangian describing the meson cloud suggests that the cloud plays a major role in the low-energy description of the nucleon. The precise delineation of the role of the quarks versus that of the cloud, is, of course, model dependent but an interesting conceptual argument, the "Cheshire cat" picture,[4] suggests that one may get the same physics with an *arbitrary* division between the core and the cloud. A possible way in which this might work in practice is that as one introduces more different kinds of mesons in the chiral Lagrangian describing the cloud, the role of the quark core diminishes. For example, if the core is neglected completely the chiral Lagrangian should contain in addition to the pions, vector mesons and perhaps other still heavier particles. This is reasonable since it is now well known that the Skyrme model of pions gives only a crude description of the nucleon which is improved (but not perfected) by the addition of vector mesons.[5]

In this paper we shall be concerned with a model in which the pions but not the vector mesons contribute to the cloud. There are several options for treating the meson coupling to the core. The most conceptually straightforward involve the explicit introduction of a bag.[2] This has the advantage that confinement is built in but the disadvantage that the inclusion of the bag degree of freedom complicates the calculation. Here, instead, we shall investigate some aspects of the chiral quark model,[6–15] in which no mechanism for confinement is explicitly assumed. We are not trying to demonstrate that this model is superior to others for fitting nucleon properties. (The uncertainties associated with the precise formulation and method of handling each model would probably preclude a fair comparison anyway.) Rather we are attracted by the fact that the chiral quark model is about the simplest one available. Furthermore, by not imposing confinement it leaves open the relation between nucleon structure and the QCD gauge fields and so may enable one to test various pictures of this relationship. In particular we will be interested in investigating the effects of a gluonium-type order-parameter field $H$ which will be introduced into the effective Lagrangian in such a way as to mock up the trace anomaly[16] of QCD. It was previously shown[17] that such an approach could produce, in the Skyrme and related pure meson models, a "nonperturbative vacuum" outside the matter-field region, thereby simulating an important aspect of bag confinement. The same general picture will be seen to hold in the chiral quark model. It will also be seen that the process of confinement is most directly related to the matter, rather than glue, fields in this model. We will see how this may be reconciled with the fact that asymptotic freedom (which essentially amounts to confinement) *requires* the glue fields to play a dominant role.

In addition, we shall also study the way in which the nucleon mass and axial-vector coupling constant depend on the parameters and the way in which the topological aspect of the Skyrme model emerges. We will demonstrate that another possible matter term, which cannot be denied on *a priori* grounds, enables one to improve the agreement of the model with experiment.

Section II contains a review of the chiral quark model, a discussion of the behavior of the soliton mass and axial-vector coupling $g_A$ in both the linear and nonlinear cases, and a discussion of the effects of an extra derivatively coupled quark-meson term. It furthermore contains a discussion of two conceptual aspects of the model: how a nontrivial "winding number" for the meson fields emerges (in the linear case) and how a "linear potential" arises as a collective effect of the meson cloud. In Sec. III we treat a model for adding gluon effects based on the QCD trace anomaly. Assuming a parallel structure to the effective Lagrangian for the U(1) anomaly in QCD one is led to an interesting relation between the masses of the $0^+$ isovector and (nonstrange) isoscalar mesons. Section IV is devoted to alternative models for adding gluonium-type fields to the chiral quark model. Finally in the Appendix we investigate, for a speculative application of the chiral quark model to heavy "fourth"-generation quarks in the standard model, the nontrivial effects of a noninfinite Higgs-meson mass.

## II. THE CHIRAL QUARK MODEL

Here, for orientation and for establishing notation, we will briefly review the chiral quark model (CQM). We will also make some new remarks. In our present context the chiral quark model[6-15] is essentially the SU(2) Gell-Mann Levy[18] $\sigma$ model (with the $u$- and $d$-quark fields replacing the proton and neutron) treated with the mean-field approach suggested by Kahana, Ripka, and Soni[6] and by Birse and Banerjee.[7] The Lagrangian density is

$$\mathcal{L}=-\tfrac{1}{2}\mathrm{Tr}(\partial_\mu M\partial_\mu M^\dagger)-\bar{q}\gamma_\mu\partial_\mu q -\sqrt{2}g(\bar{q}_L M q_R+\bar{q}_R M^\dagger q_L) -c[\mathrm{Tr}(MM^\dagger)-v^2]^2 , \tag{2.1}$$

where the left- and right-handed quark-field projections are defined by $q_{L,R}=[(1\pm\gamma_5)/2]q$ and the meson matrix $M$ is related to the pion and a scalar isosinglet $\sigma$ by

$$M\equiv\frac{1}{\sqrt{2}}(\sigma+i\boldsymbol{\tau}\cdot\boldsymbol{\pi})\equiv\frac{1}{\sqrt{2}}\rho U . \tag{2.2}$$

Here we have also indicated an alternative decomposition of $M$ with $\rho$ a chiral singlet and $U$ a $2\times2$ unitary unimodular matrix. The pion decay constant is $F_\pi=\sqrt{2}v=\sqrt{2}\langle\sigma\rangle=\sqrt{2}\langle\rho\rangle\approx132$ MeV. $g$ is the Yukawa coupling constant and $c$ is a dimensionless parameter related to the mass of the $\sigma$. In the limit when $c$ gets very large $\rho$ gets frozen to its vacuum value and we have the nonlinear version of the model.

Although the chiral quark model has been around for quite a while[19,20] it is only recently that the mean-field treatment of the baryon state in it has been pursued. A number of authors have made advances in computing corrections of the "cranking" type to the collective quantization[11-15] necessary for the baryonic states. Furthermore a justification for the nonlinear model from QCD based on the "instanton liquid" model has been given[21] as has a connection[22] with the Nambu–Jona-Lasinio model in the large-$N$ limit.

In this model one solves for the quark wave functions (using the Dirac equation) in the self-consistently determined (classical) meson field. An ansatz for the meson fields which reflects their intrinsic properties is the static hedgehog form

$$\pi_{ic}=\hat{x}_i\pi(r),\quad \sigma_c=\sigma(r) , \tag{2.3}$$

with $\pi(0)=0$ to keep $\pi_{ic}$ well defined at the origin. In the nonlinear limit this is restricted to the Skyrme form:

$$\pi(r)=v\sin\theta(r),\quad \sigma(r)=v\cos\theta(r) , \tag{2.4}$$

where $\theta(0)=-n\pi$, $n$(winding number)$=0,\pm1,\ldots$ . With the hedgehog ansatz for the meson fields the Lagrangian (2.1) has an additional symmetry under simultaneous rotations in space and isospin space. The resulting solutions for the quark fields are not eigenstates of $\mathbf{J}$ and $\mathbf{I}$ but of a grand spin $\mathbf{K}\equiv\mathbf{I}+\mathbf{J}$. This is rectified by a collective quantization as in the Skyrme model. The appropriate ansatz for a quark spinor with positive parity and $K=0$ has the form

$$q_c=\begin{bmatrix}\dfrac{iF(r)}{r}\chi\\ \boldsymbol{\sigma}\cdot\hat{\mathbf{x}}\dfrac{G(r)}{r}\chi\end{bmatrix},\quad \chi=\frac{1}{\sqrt{2}}(\alpha_1\beta_2-\alpha_2\beta_1) , \tag{2.5}$$

where $\alpha_i$ and $\beta_i$ are two-component spin and isospin wave functions, respectively. The Dirac equation with energy eigenvalue $\epsilon$ obtained from (2.1) is

$$\begin{aligned}&-G'-\frac{G}{r}+gF\sigma-gG\pi=\epsilon F ,\\ &F'-\frac{F}{r}-gG\sigma-gF\pi=\epsilon G ,\end{aligned} \tag{2.6}$$

with the normalization $4\pi\int_0^\infty dr(F^2+G^2)=1$. $\epsilon$ is a functional of $\pi(r)$ and $\sigma(r)$. The total energy contains also a piece $E_m[\pi,\sigma]$ due to the meson fields

$$E_{\mathrm{tot}}[\pi,\sigma]=N\epsilon[\pi,\sigma]+E_m[\pi,\sigma] , \tag{2.7}$$

where

$$E_m[\pi,\sigma]=4\pi\int_0^\infty r^2dr\left[\frac{\pi'^2}{2}+\frac{\sigma'^2}{2}+\frac{2\pi^2}{r^2}+c(\sigma^2+\pi^2-v^2)^2\right] \tag{2.8}$$

and $\epsilon$ may be expressed as

$$\epsilon[\pi,\sigma]=4\pi\int_0^\infty dr[g\sigma(F^2-G^2)-2FG(g\pi+1/r)+GF'-FG'] . \tag{2.9}$$

Note that $N$ in (2.7) is the occupancy number for the orbital of energy $\epsilon$. Finally the total energy must be minimized with respect to $\pi$ and $\sigma$:

$$\frac{\delta}{\delta\pi}E_{\mathrm{tot}}(\pi,\sigma)=\frac{\delta}{\delta\sigma}E_{\mathrm{tot}}(\pi,\sigma)=0 . \tag{2.10}$$

For the meson energy in Eq. (2.8) to be finite we must impose the boundary conditions

$$\pi(\infty)=0, \quad \sigma(\infty)=v \ . \tag{2.11}$$

The Lagrangian (2.1) with the linear realization of the meson fields represents, of course, a renormalizable theory. However there is no need for an effective chiral Lagrangian to be renormalizable. We should mention here that Soni[23] and others[24] have shown that including all the "Dirac sea" levels in the energy estimate (2.7) (the "Casimir effect") leads to an unstable vacuum state for the Lagrangian (2.1). This arises from meson-cloud configurations of very small size and may thus be conceivably related to the lack of asymptotic freedom of (2.1). In any event, it is not clear that one should push an effective Lagrangian to such high energies and the problem may be cured with a reasonable large momentum cutoff.

To get a feeling for the dynamics of the mean-field approximation let us first consider the nonlinear model. Following Ref. 6 we make a rough calculation using (2.4) with a meson cloud having a trial size $R$: $\theta(r)=-\pi(1-r/R)$, $r<R$ and $\theta(r)=0$, $r>R$. The energy of the cloud is then, from a numerical integration, $E_m(R)\approx 9\pi v^2 R$. If only the cloud were present, this would correspond to the situation in the Skyrme model without the stabilizing "Skyrme term." Then minimization of $E_m(R)$ would cause the meson cloud to collapse. In the chiral quark model the baryon is stabilized because the $K=0$ energy eigenvalue $\epsilon$ descends rapidly from positive to negative energy (becoming asymptotic to the "Dirac sea" orbital eigenvalues) as the trial meson cloud size $R$ increases. In the range of interest this dependence is shown in Fig. 1 along with other $K^P$ levels. Very roughly the energy of the $0^+$ level is fitted by $\epsilon \simeq \pi/R - gv$ so that the total energy in (2.7) then reads

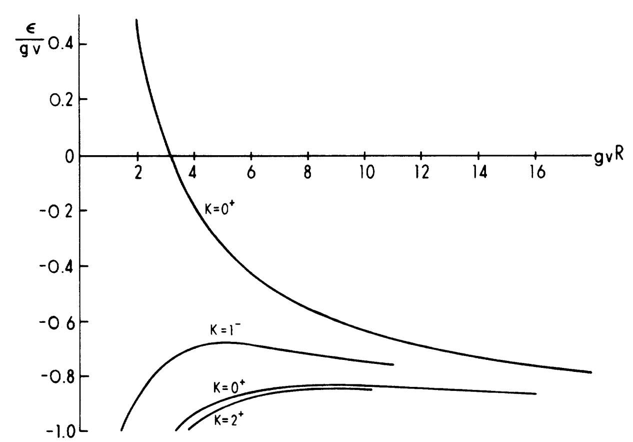


FIG. 1. Plot of the normalized valence energy level $\epsilon/gv$ (rapidly falling) as a function of the normalized size $gvR$ of the meson cloud with a linear ansatz. Some other levels are also shown. These were obtained by direct numerical integration of the differential equation and agree well qualitatively with those shown in Fig. 2 of Ref. 8 found by a different method.

$$E_{\rm tot} \approx N\left[\frac{\pi}{R}-gv\right]+9\pi R v^2 \ . \tag{2.12}$$

Minimizing this with respect to $R$ gives the cloud size $\bar{R}\simeq\sqrt{N}/3v$ and the total energy

$$\bar{E}_{\rm tot}(R)\simeq 6\pi v\sqrt{N}-Ngv \ . \tag{2.13}$$

The nucleon corresponds to three quarks ($N=3$) in the $K=0$ orbital so the Yukawa coupling constant $g$ is around 7.5 (a more accurate variational calculation reduces this to around 5). We remark here that the solution for the minimum requires the quarks to be relativistic. However it will be interesting to draw parallels between this model and the old nonrelativistic quark model so let us consider the case when $\epsilon \simeq gv = m$, the effective mass of the quarks. A nonrelativistic reduction of the Dirac equation yields an effective potential

$$V(r)=g[\sigma(r)-v] \tag{2.14}$$

which for the approximation above is $V=-gv(\cos\pi r/R+1)$ for $r\le R$. Using the standard result for the appearance of a bound state in a square well of depth $V_0$ and width $R$, $V_0R^2>\pi^2/2m$ one expects a threshold around $gvR\simeq\pi/2$ which is roughly the case in Fig. 1. Increasing the strength of the mesonic interaction drives the quark valence level down sharply and for $gvR$ large enough the level joins the "Dirac sea." This extreme case corresponds to the adiabatic limit in the Skyrme model in which the topological winding number of the meson cloud $[\theta(\infty)-\theta(0)/\pi=1]$ is equal to the baryon number of the Dirac sea. We also point out that the existence of other valence levels shown in Fig. 1 suggests that it may be possible to find bound states of quarks and antiquarks with an appropriate pion cloud. (Presumably such a cloud should have winding number zero.)

The above picture is quite appealing but there is the apparent conceptual difficulty that a quark is not confined. Note that Eq. (2.13) with $N=1$ for a single quark and $g=7.5$ from the fit for the nucleon, predicts a mass for a quark in a meson cloud larger than its free mass ($gv$) so we do not expect bound systems containing one quark. The objection to having free quarks in the model is reasonably answered by noting that we are interested in computing the nucleon properties and confinement may not play a big role unless we consider very large orbital excitations (so the binding "string" gets stretched). A hint of confinement and possible connection with glue fields will be discussed later in this section.

Let us now discuss the full self-consistent treatment of the nonlinear model allowing $\theta(r)$ to go to its true minimizing profile. This problem was first solved numerically by Kahana, Ripka, and Soni[6] and by Birse and Banerjee.[7] We will be interested in the general dependence of nucleon properties on the parameters in the model rather than doing a detailed fit with collective quantization. Two quantities which characterize the nucleon and do not to lowest order sensitively depend on the method of collective quantization are its mass and its axial-vector coupling $g_A$. As previously mentioned, the static energy given by Eq. (2.7) is not the mass of an eigenstate of $I$ or $J$. To get the physical states, one must choose a method of collective quantization which adds a rotational energy to the system. In the Skyrme model the static energy is fitted to $E_{\rm tot}=\frac{5}{4}M_N-\frac{1}{4}M_\Delta=0.867$ GeV. However, in the chiral quark model, some authors[11] have given a reasonable argument that the energy in (2.7) already contains some part of the quarks' rotational energy. They suggest fitting $E_{\rm tot}=\frac{1}{2}M_N+\frac{1}{2}M_\Delta=1.08$ GeV to avoid double counting. We will consider solutions with $E_{\rm tot}$ in this range. We can simply approximate $g_A$ by the strength of the pion tail:

$$g_A \simeq \frac{4\pi}{3}F_\pi^2 B \ , \tag{2.15}$$

where $B$ is given by the asymptotic behavior:

$$\pi(r) \underset{r\to\infty}{\sim} vB\left[m_\pi+\frac{1}{r}\right]\frac{e^{-m_\pi r}}{r} \ . \tag{2.16}$$

We will include in our calculations the standard pion mass term:

$$\mathcal{L}_m=\frac{m_\pi^2 v}{2\sqrt{2}}\mathrm{Tr}(M+M^\dagger-\sqrt{2}v) \ .$$

In Fig. 2 we plot the quantity $E_{\rm tot}$ vs $g$ (solid line) for the case $N=3$ valence quarks and winding number $n=1$. Note that $E_{\rm tot}$ decreases as $g$ increases. The straight line rising linearly from the origin in Fig. 2 is the mass of three free quarks, $3gv$. As pointed out by Soni[10] there is a maximum mass for a system of three quarks where these two curves intersect at $g \simeq 4.4$. For $g$ below this, the lowest-energy state would be three free quarks and for $g$ above this critical value, the quarks with a meson cloud would have lower energy. This does not have profound effects on the picture for the nucleon since the nucleon mass is safely below the upper bound of 1.25 GeV. However this same analysis can be applied to quite a different problem. The model in (2.1) can also describe the Yukawa sector for a heavy fourth-quark generation in the minimal standard model with (necessarily, because of the "$\rho$-parameter" bound) similar up- and down-quark masses. This was shown[25,26] to lead to an upper bound on the quark mass of about 2 TeV as well as an interesting phenomenology. In this paper, as a by-product, we shall discuss the effect of the value of the Higgs-particle mass on the above bound.

In Fig. 3 we display the variation of the axial-vector coupling $g_A$ with the parameter $g$. Note that the values of $g_A$ are a little too large, $1.55<g_A<1.60$, over the range of interest while $g_A=1.23$ experimentally. This is reminiscent of the nonrelativistic quark model's prediction of $g_A=\frac{5}{3}$ which originated from the fact that the quarks in (2.1) have an axial-vector coupling of one. It has been known for a long time in the old-fashioned treatment of baryons in the chiral Lagrangian that the axial-vector coupling could be adjusted by adding suitable derivative terms.[27] In the present context we may add the term

$$\mathcal{L}'=\frac{-\tilde{g}}{2}(\bar{q}_L \not\partial U U^\dagger q_L-\bar{q}_R U^\dagger \not\partial U q_R) \ . \tag{2.17}$$

One sees, using Noether's theorem, that this modifies the effective quark axial-vector coupling to $1+\tilde{g}$ so a value of $\tilde{g}\simeq -0.25$ would be needed to account for the measured nucleon-axial-vector coupling. With a change of variables $q_R'=U^{1/2}q_R, q_L'=U^{-1/2}q_L$ the Lagrangian (2.1) (in the nonlinear limit) and (2.17) may be seen to be equivalent to that discussed by Manohar and Georgi.[20]

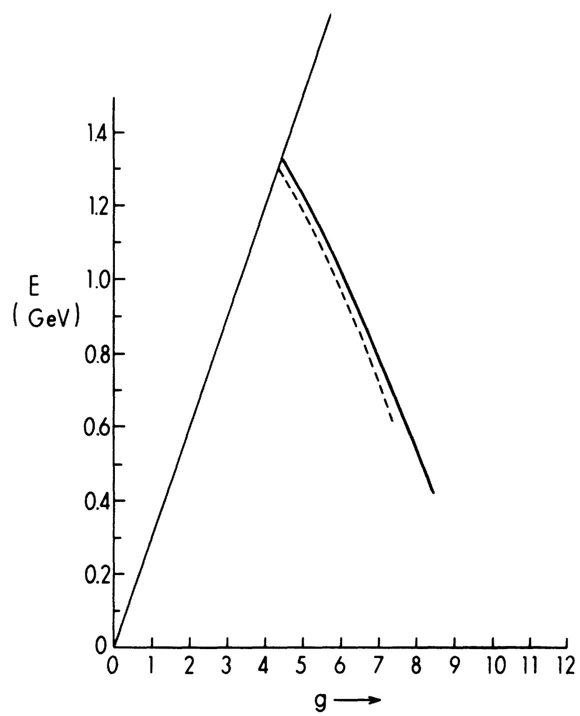


FIG. 2. The self-consistently determined nucleon energy $E$ plotted against $g$ for both the nonlinear (solid line) and linear (dashed line) models. The pion mass term is included for both cases and for the linear case the parameter $c=20$. The straight line rising from the origin represents the mass of three free quarks.

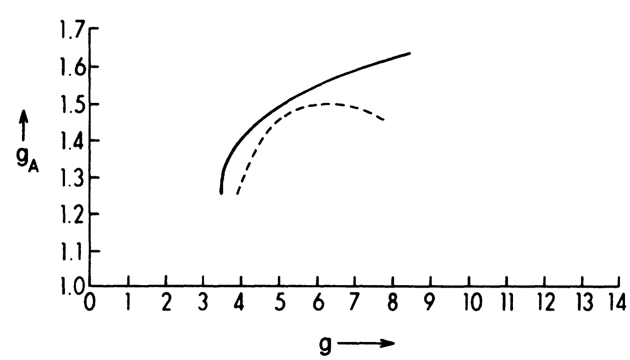


FIG. 3. The axial-vector coupling constant $g_A$ plotted against the Yukawa strength $g$ for both the nonlinear (solid line) and linear (dotted line) models. As for Fig. 2 the pion mass term is included and $c=20$.

Those authors had the same motivation, but did not employ the present mean-field approach to the nucleon in which the meson cloud plays a role. $\mathcal{L}'$ was neglected in earlier papers treating the chiral quark model by the mean-field approach although naive power-counting arguments[20] seem to suggest that it is on an equal footing with the quark-pion interaction term in (2.1). In the presence of $\mathcal{L}'$ the following extra term should be added to the energy in (2.9):

$$-4\pi\tilde{g}\int_0^\infty dr\left[-\frac{F^2}{2}\left[\theta'+\frac{\sin 2\theta}{r}\right]+\frac{G^2}{2}\left[-\theta'+\frac{\sin 2\theta}{r}\right]+\frac{2FG\sin^2\theta}{r}\right], \quad (2.18)$$

and the equations of motion read

$$G'=\left[-\frac{1}{r}-gv\sin\theta\right]G+(gv\cos\theta-\epsilon)F+\tilde{g}\left[\left[\frac{\theta'}{2}+\frac{\sin 2\theta}{2r}\right]F-\frac{\sin^2\theta}{r}G\right],$$

$$F'=\left[\frac{1}{r}+gv\sin\theta\right]F+(\epsilon+gv\cos\theta)G-\tilde{g}\left[\left[\frac{\theta'}{2}-\frac{\sin 2\theta}{2r}\right]G-\frac{\sin^2\theta}{r}F\right], \quad (2.19)$$

$$\tfrac{1}{2}F_\pi^2(\theta' r^2)'=\tfrac{1}{2}F_\pi^2\sin 2\theta+Ngv[-(F^2-G^2)\sin\theta-2FG\cos\theta]+\tilde{g}N\left[(F^2-G^2)\frac{\cos 2\theta}{r}-2FG\frac{\sin 2\theta}{r}-FF'-GG'\right].$$

The effect of $\mathcal{L}'$ on the full solution in the chiral quark model is to raise $E_{\rm tot}$ and $g_A$ when $\tilde{g}$ is positive and to lower them when $\tilde{g}$ is negative. Interestingly, if we choose $gv \simeq 370$ MeV and $\tilde{g}\simeq -0.25$ (as in the nonrelativistic quark model) we find that $E_{\rm tot}\simeq 966$ MeV and $g_A\simeq 1.3$. Of course, it is possible to fine-tune these parameters. For example, if one now considers $m_\pi = 0.14$ GeV, the choices $g=4.3$ and $\tilde{g}=-0.16$ fit the nucleon values $E_{\rm tot}=1.1$ GeV and $g_A=1.2$. It seems that the $\mathcal{L}'$ term is quite useful for improving the experimental agreement of the nonlinear model with quarks and pions. Some of the effects of adding vector mesons and still heavier particles might be simply modeled by adjusting $\tilde{g}$.

One of the most interesting of the heavier mesons to include is the scalar singlet already present in the linear version of the model (2.1). We will discuss in the next section how this particle may have a natural connection with a gluonium-type field. Although the scalar singlets' mass is around 1 GeV its broad width suggests that it might be important even if the effective Lagrangian is truncated to include only the very light particles. In this model it is appropriate to use the ansatz (2.3) in the calculation. Apart from $g$ the only parameter is $c$, related to the scalar-meson mass $m_\sigma$ by

$$m_\sigma^2 = 8cv^2 . \quad (2.20)$$

A scalar mass around 1 GeV corresponds to $c\approx 14.4$; the results are not very sensitive to small variations of $c$. We show in Fig. 4 a curve for the Dirac wave functions $F(r)$ and $G(r)$ and the meson "profiles" $\sigma(r)$ and $\pi(r)$ for the typical case $g=6.41$ and $c=20.6$ (corresponding to a $\sigma$ mass about 1.2 GeV). The soliton mass is 0.88 GeV which is about right for the nucleon mass when the effects of collective quantization are included in the simplest way suggested by the Skyrme model. Notice that the axial-vector renormalization constant is $g_A = 1.50$, which is a small improvement over the nonlinear model without the $\tilde{g}$ term. The energy eigenvalue of the Dirac equation is $-0.15gv$ which has already "crossed over" but is still quite far from the region near $-gv$ where the valence level may be considered part of the Dirac sea. We show in Fig. 3 (dotted lines) the effect on $g_A$ of varying the soliton mass by varying $g$ in a relevent range. We see that $g_A$ is not very sensitive to the mass in this range. We also show in Fig. 2 (dotted lines) how the soliton mass varies with the coupling constant $g$. It is somewhat lower than the nonlinear case (solid line). The application of the soliton mass versus $g$ curve to finding the maximum allowed heavy fourth-generation quark mass is discussed in the Appendix.

Now we would like to emphasize some interesting conceptual aspects of the linear model containing the scalar field $\sigma$. The first thing to note is that the use of the ansatz (2.3) rather than (2.4) no longer guarantees a "topo-

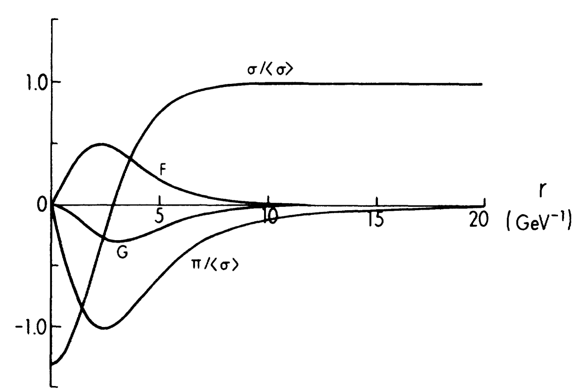


FIG. 4. For the linear model of (2.1) plus pion mass term, the Dirac wave functions $F(r)$, $G(r)$, and the scalar- and pseudoscalar-meson profiles $\sigma(r)/\langle\sigma\rangle$ and $\pi(r)/\langle\sigma\rangle$ are plotted against $r$ measured in units of GeV$^{-1}$. Here $g=6.41$ and $c=20.6$.

logical" number one for the meson cloud. The winding number will be determined dynamically. From (2.4) we see that a winding-number-one solution is characterized by $\sigma(r)$ starting out negative at the origin and rising to its positive vacuum value while $\pi(r)$ starts from zero at the origin, goes through negative values, and then goes to zero again at $r=\infty$. A winding-number-zero solution is characterized by a $\sigma(r)$ which remains positive. Which winding number emerges depends on the strength of the coupling constant $g$. This is illustrated in Figs. 5(a) and 5(b) where $\sigma(r)$ and $\pi(r)$ are plotted against $r$ for various values of $g$. We have chosen the parameter $c$ to be 0.01. As $g$ decreases below about 4, we notice that the $\sigma(r)$ curve goes from crossing to not crossing, corresponding to an adiabatic evolution of winding number from 1 to 0. At around $g=2.3$, the uniform phase $\sigma(r)=gv=\text{const}$, $\pi(r)=0$ is reached. It should be remarked that since $\pi^2+\sigma^2$ is not being constrained to be constant the winding number is not required to be conserved. The dynamics of the evolution of the $\sigma(r)$ curve may be understood by referring to the expression for the quark energy in (2.9). We see that for large $g$, negative values of $\sigma(r)$ in the region where the quark fields are nonzero can lower the energy through the term[28] $g\sigma(r)(F^2-G^2)$. ($F$ is larger than $G$ in magnitude for a positive-energy quark.) For smaller values of $g$, the extra energy required by the $\frac{1}{2}\sigma'^2$ term for $\sigma(r)$ to depart from its asymptotic value makes this possibility uncompetitive.

Note that the evolution of winding number was illustrated for a value of $c$ which is rather low compared to the expected value for strong interactions (although not necessarily low for the application to the standard model discussed in the Appendix). If $c$ is larger, the transition from winding number one to winding number zero as $g$ decreases no longer takes place smoothly in the bound system. This may be understood from Fig. 6. The solid line shows the critical $g$ (for given $c$) where the winding-number transition occurs [i.e., where $\sigma(0)$ becomes positive]. However, the dotted line shows the lowest $g$ (for given $c$) for which the bound-state energy is less than the energy of a free system, $3gv$. Clearly if $c>c_0\simeq 1.4$, as $g$ is decreased the winding-number-one solution becomes energetically unstable and jumps to the unbound solution (with a trivial $\sigma=v$, $\pi=0$, winding-number-zero background) before $\sigma(0)$ can pass smoothly to positive values.

We see that the present model is reminiscent of the old-fashioned quark model in a number of respects. In the first place the quark wave functions are being determined as solutions of a Dirac equation. Furthermore the axial-vector renormalization constant $g_A$ tends to come out roughly around the quark-model value $\frac{5}{3}$. Now we would like to point out an amusing similarity between the usual quark-model confining potential and the effective potential here, which is dynamically generated by the meson cloud. In the nonrelativistic approximation our effective potential is given by (2.14). This approximation is clearly valid for small $g$ where [see Fig. 5(a)] $\sigma(r)$ does not vary too much. For larger and more realistic $g$ where

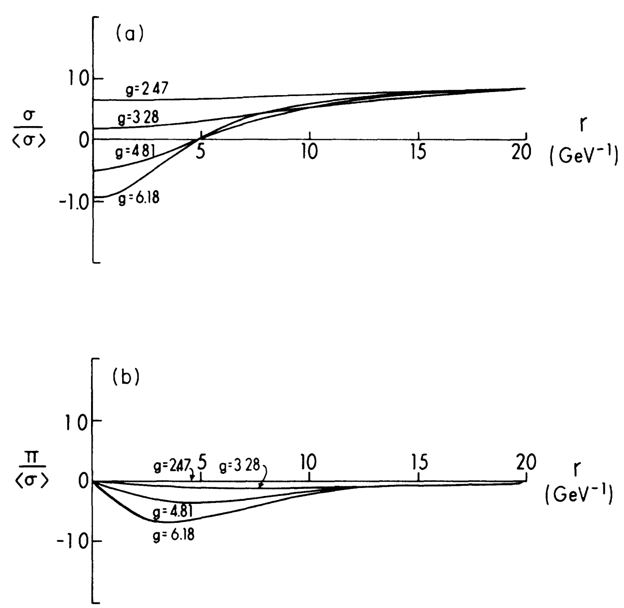


FIG. 5. Evolution of winding number for the linear model with $c=0.01$. (a) shows $\sigma(r)/\langle\sigma\rangle$ for various values of $g$ while (b) shows $\pi(r)/\langle\sigma\rangle$.

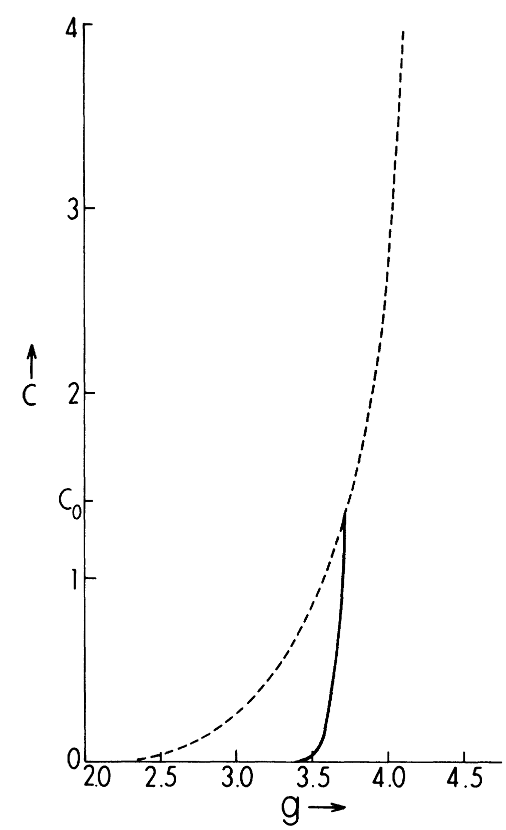


FIG. 6. Phase diagram for the linear model in the $c$-$g$ plane. The solid line separates phases of zero winding number on the left from winding number one on the right. The dotted line separates the bound state region on the right from the unbound (free quark) region on the left.

$\sigma(r)$ crosses the axis the model is intrinsically relativistic but it seems reasonable to still consider (2.14) as our effective potential for the purpose of comparing with the quark-model approach. With a realistic fit corresponding to the case shown in Fig. 4 we notice that the $\sigma(r)$ curve is quite linear in the region where the quark wave functions are peaking. The slope of the effective potential curve $dV/dr = g\, d\sigma/dr$, depending on how much of the region of nontrivial quark wave function one wishes to cover, varies from about 0.2 to 0.3 GeV$^2$. In the string model this should be equated to the string tension $1/2\pi\alpha'$. We thus find $\alpha'$ somewhere between 0.80 GeV$^{-2}$ and 0.53 GeV$^{-2}$. Qualitatively, this is quite comparable to (though a little smaller than) the values $\alpha' = 0.9-1.0$ found from experimental Regge trajectories. We have found that fitting to a soliton mass 1.06 GeV improves the agreement. Using the argument discussed at the beginning of this section which [see (2.13)] gives the rough size of the soliton to be $\bar{R} \approx 1/(\sqrt{3}\, v)$ we may estimate the slope $dV/dr$ as $2\sqrt{3}\, gv^2$. Hence the predicted $\alpha'$ depends on the Yukawa coupling and the pion decay constant as $\alpha' \approx (2\sqrt{3}\, \pi g F_\pi^2)^{-1}$.

## III. THE CHIRAL QUARK MODEL WITH GLUONIUM

We have seen that the chiral quark model has the intriguing aspect of generating something like a linear confining potential for the quarks bound in a baryon. Of course this is not complete confinement since the effective potential does not continue rising after a certain value of radius. Nevertheless it does represent what one would normally takes as the confining potential in a nonrelativistic quark-model treatment. This raises the puzzle that since the potential is due to the meson cloud it would appear to have no relation to gluon fields and to QCD. However it is known that the breaking of chiral symmetry in QCD is closely related[29] to confinement so the gluon fields must be playing an indirect role in the establishment of the meson cloud. It would seem desirable to construct a simple model in which this indirect role could be illustrated. We shall present such a model now, based on a simple generalization of our earlier work[17] in which a "zeroth-order" treatment of glue confinement effects in the chiral Lagrangian framework was given using the trace-anomaly equation of QCD. We shall also note an interesting consistency of this model with a recent analysis of the isoscalar scalar mesons.

We would like the energy-momentum tensor of our effective Lagrangian to obey the QCD scaling law (with massless quarks)

$$\theta_{\mu\mu} = -\frac{\beta(g)}{g}\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu}) \equiv H \; . \tag{3.1}$$

We will regard $H$ as an order parameter field for describing gluon effects. As discussed in detail elsewhere[30] we may satisfy (1) in a theory of "pure" QCD without matter by using the field $H$ in the unique (up to two derivatives) Lagrangian

$$\mathcal{L}_H = -\tfrac{1}{2}aH^{-3/2}(\partial_\mu H)^2 - \tfrac{1}{4}H\ln\frac{H}{\Lambda^4} \; , \tag{3.2}$$

where $a$ is a dimensionless parameter and $\Lambda$, with mass dimension 1 is a scale for QCD. This Lagrangian already displays an aspect of confinement in that it leads to a negative vacuum energy density $-\Lambda^4/4e$. If matter is added, as in a modified Skyrme model, a bubble with higher $H$-field energy density in the vicinity of the matter tends to form,[17] mocking up the bag model.

Notice that the first term of (3.2) is scale invariant and does not contribute to the scale anomaly in (3.1). The second term is the crucial one for this purpose. Actually, in the presence of matter one may satisfy the scale-anomaly equation with a more general set of terms. All that is really needed is for the potential $V$ (here constructed out of the fields $H, M, M^\dagger$) to satisfy[30]

$$H = -4V + 4H\frac{\partial V}{\partial H} + d\,\mathrm{Tr}\left[M\frac{\partial V}{\partial M} + M^\dagger\frac{\partial V}{\partial M^\dagger}\right] , \tag{3.3}$$

where $d$ is an effective scaling dimension[31] for the field $M$. Furthermore the kinetic term for $H$ is not necessarily needed. Whether it appears is the dynamical question of whether the trace of the energy-momentum tensor is dominated at low energies mainly by a glueball field. If the kinetic term does not appear $H$ would get eliminated by its equation of motion $\partial V/\partial H = 0$ in terms of the quarkonium scalar singlet $\rho$. Now, we have seen that the quarkonium scalar field $\sigma$ (equal to $\rho\cos\theta$ with the "hedgehog" ansatz) is responsible for the "linear potential." The field $\rho$ controls the overall strength so, by the above, we have an explicit relation between the binding and the glue. It should be remarked that an effective Lagrangian for the U(1) problem can be formulated[32] in a similar way. There the relevant glue "order parameter" is

$$G = \frac{-3ig^2}{16\pi^2}\epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}(F_{\mu\nu}F_{\alpha\beta})$$

and it is necessary that this gluonic field get eliminated in terms of the quarkonium field $\eta'(960)$. Thus if we formulate the model without an $H$ kinetic term there is an interesting parallel structure between the trace and U(1) anomaly effective Lagrangians. We shall first discuss this approach in the chiral model with quarks.

The simplest model is obtained just by replacing the potential term (last term) in (2.1) by

$$V = c[\mathrm{Tr}(MM^\dagger) - RH^{1/2}]^2 + \tfrac{1}{4}H\ln\frac{H}{\Lambda^4} \; , \tag{3.4}$$

which satisfies (3.3). $R$ is a dimensionless parameter which is obtained from the stability condition $\langle \partial V/\partial\rho\rangle = 0$ to be $R = \langle\rho\rangle^2/\langle H\rangle^{1/2}$, a ratio of quarkonium and gluonium condensates. With the value[33]

$$\langle H\rangle = \frac{\Lambda^4}{e} = 0.0135\ \mathrm{GeV}^4 \tag{3.5}$$

found by analysis of QCD sum rules, we have $R = 0.075$. The equation of motion for the field $H$ results in

$$H = \frac{\Lambda^4}{e}\exp\left[4cR^2\left[-1 + \frac{\rho^2}{RH^{1/2}}\right]\right] , \tag{3.6}$$

which we solve numerically to eliminate $H$ in terms of $\rho$. For simplicity the pion mass term is being neglected in the present discussion. Using as input (3.5), $c=21$ (corresponding to a mass for the field $\rho$ of 1.2 GeV) and $g=6.45$ we find results which are, all in all, not very different from the model (2.1) without the $H$ field. What may be interesting however, is to estimate the energy density (above the nonperturbative background[34]) associated with the scalar fields, namely,

$$\frac{\rho'^2}{2}+c(\rho^2-RH^{1/2})^2+\tfrac{1}{4}H\ln\left[\frac{H}{\Lambda^4}\right]+\frac{\langle H\rangle}{4}\ .$$

This is shown in Fig. 7. We interpret this as a measure of the bag energy density. It is confined within a region of about 0.4 fm, and rises to a maximum of about $3.4\times10^{-4}$ GeV$^4$. This may be compared with in increase of energy density in the matter region of about $4.2\times10^{-4}$ GeV$^4$ in the bag model.[1] However the energy density is sensitive to the exact choice of $\langle H\rangle$, which is not precisely determined. We point out that in the matter region, the $H$ field rises above its asymptotic value rather than, as for the case[17] in a generalized Skyrme model without explicit quarks, dropping below it. This feature may be understood because the scalar field $\rho$ [roughly $R^{1/2}H^{1/4}$ by (3.6)] governs the strength of the attractive "linear potential" and so, for sufficiently large $g$, it will be energetically favorable to increase $\rho(r)$ [and hence $H(r)$] at small $r$.

What can be said about the scalar meson, described by the field $\rho$, in this model? The physics of eliminating $H$ in terms of $\rho$ by (3.6) is the statement that the matrix elements of the operator $\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu})$ are dominated at low energy by the singlet quarkonium state with the structure $(1/\sqrt{2})(u\bar{u}+d\bar{d})$ (in the two-flavor case) or $(1/\sqrt{3})(u\bar{u}+d\bar{d}+s\bar{s})$ (in the three-flavor case). For simplicity we will restrict our attention to the two-flavor case. It is useful to compare this situation with the analogous effective Lagrangian for the U(1) problem.[32] There, in order for the $\eta$ (in the two-flavor case) to acquire a mass, it is necessary that the pseudoscalar gluonium operator $G\propto\mathrm{Tr}(F_{\mu\nu}F^d_{\mu\nu})$ be eliminated in terms of $\eta$.

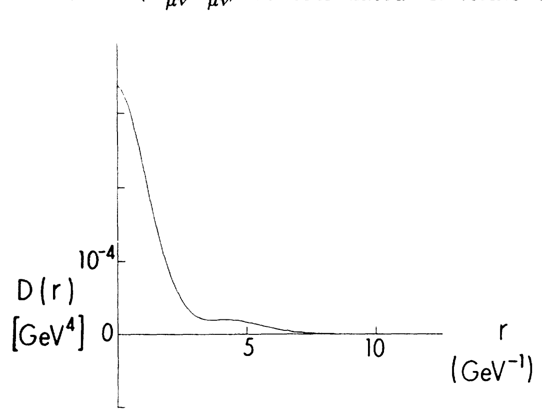


FIG. 7. For the model given by (3.4) we show the quantity

$$D(r)=\frac{\rho'^2}{2}+c(\rho^2-RH^{1/2})^2+\tfrac{1}{2}H\ln\left[\frac{H}{\Lambda^4}\right]+\frac{\langle H\rangle}{4}\ ,$$

a measure of the bag energy density. Here $c=20$, $\Lambda=0.44$ GeV, and $E=0.89$ GeV. The pion mass has been set to zero for simplicity.

This yields a neutral pseudoscalar-squared mass matrix proportional to

$$m_\pi^2\begin{bmatrix}1&0\\0&1\end{bmatrix}+B\begin{bmatrix}1&1\\1&1\end{bmatrix} \tag{3.7}$$

in the $u\bar{u},d\bar{d}$ basis. Here $B$ is a positive quantity, which is related to the U(1) anomaly, and $m_\pi^2=0$ in the current-algebra limit. The U(1) problem is that $B$ vanishes by a current-algebra theorem. The existence of the anomaly is a nonperturbative (e.g., instanton) effect which solves the problem. In the quark-model approach $B\neq0$ is often considered to be related to the two-gluon annihilation diagram shown in Fig. 8. In the effective Lagrangian model the same effect is expressed by the elimination of the gluonium field $G$ in terms of $\eta$. Note that in the $(u\bar{u}-d\bar{d})/\sqrt{2},(u\bar{u}+d\bar{d})/\sqrt{2}$ basis (3.7) becomes

$$\begin{bmatrix}m_\pi^2&0\\0&m_\pi^2+2B\end{bmatrix} \tag{3.8}$$

which shows how the $\eta$ mass is boosted above that of the pion. Now let us return to the present problem. We denote the scalar isovector meson by $\delta$ and the scalar isoscalar by $\epsilon$. The two-flavor mass squared matrix in the basis analogous to (3.8) [i.e., $(\delta,\epsilon)$] is

$$\begin{bmatrix}m_\delta^2&0\\0&m_\delta^2+2B'\end{bmatrix}\ . \tag{3.9}$$

Here $m_\delta^2$ is not close to zero and $B'$ should represent the effect of the matter-glue "duality." Specifically, we interpret $m_\delta^2$ as $\langle\partial^2V/\partial\rho^2\rangle$ while $m_\epsilon^2=m_\delta^2+2B'$ should be obtained by computing $\langle d^2V[\rho,H(\rho)]/d\rho^2\rangle$ which includes a contribution due to the elimination of the gluonium field $H$. This calculation was carried out at the end of Sec. II of Ref. 30; adapting that result to the two-flavor case and allowing an arbitrary scaling dimension $d$ for the field $\rho$ yields the formula

$$\left\langle\frac{d^2V}{d\rho^2}\right\rangle=\frac{\left\langle\dfrac{\partial^2V}{\partial\rho^2}\right\rangle}{1+\dfrac{d^2F_\pi^2}{8\langle H\rangle}\left\langle\dfrac{\partial^2V}{\partial\rho^2}\right\rangle}\ . \tag{3.10}$$

Notice that the inverse coefficient $8\langle H\rangle/d^2F_\pi^2$ in (3.10) represents the maximum possible mass for the lightest particle in the scalar channel. This holds[30] even when mixing with an additional scalar particle is included. Now if we identify $\langle\partial^2V/\partial\rho^2\rangle$ with the squared mass of the $\delta(980)$ particle, use $\langle H\rangle$ from (3.5), and adopt the canonical value $d=1$ we predict the $\epsilon$ particle's mass to be

$$m(\epsilon)=900\ \text{MeV}\ . \tag{3.11}$$

It is interesting that, unlike the case in the pseudoscalar

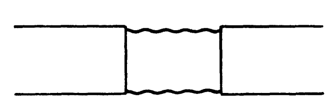

FIG. 8. Quark "annihilation" graph.

TABLE I. Comparison of the isovector and "nonstrange" isoscalar masses for the low-lying $q\bar{q}$ multiplets.

| Orbital state, $L$ | $J^{PC}$ | Isoscalar mass—isovector mass (MeV) |
|---|---|---|
| 0 | $0^{-+}$ | $\eta-\pi=411$ |
| 0 | $1^{--}$ | $\omega-\rho=14\pm3$ |
| 1 | $0^{++}$ | $\epsilon(900)-\delta(975)=-75$ |
| 1 | $1^{+-}$ | $H(1190)-B(1234)=-44\pm70$ |
| 1 | $1^{++}$ | $D(1283)-A_1(1275)=8\pm35$ |
| 1 | $2^{++}$ | $f(1274)-A_2(1318)=-44\pm10$ |
| 2 | $3^{--}$ | $\omega(1668)-g(1691)=-23\pm10$ |

neutral channel, the $(u\bar{u}+d\bar{d})$-type state is *reduced* in mass from the $(u\bar{u}-d\bar{d})$-type state; $B'$ in (3.9) is predicted to be negative, about $-0.065$ GeV$^2$. The isoscalar particle $\epsilon$ should have an extremely large width into two pions;[30] it is just like the old, relatively low-mass, and wide scalar singlet greatly desired by nuclear theorists.

Does the prediction (3.11) for the mass of an isoscalar-scalar meson with flavor content $(u\bar{u}+d\bar{d})/\sqrt{2}$ agree with experiment? As is well known the experimental analysis of the scalar mesons is fraught with uncertainty. A brief summary from the present point of view is presented in Sec. III of Ref. 30. The standard candidate for the $(u\bar{u}+d\bar{d})/\sqrt{2}$ scalar state has been the $\epsilon(1300)$ which, though rather broad, clearly differs substantially from (3.11). More recently, however, a reanalysis of the low-energy scalar isoscalars has been carried out by Au, Morgan, and Pennington.[35] These authors use new constraints coming from an experiment by the AFS Collaboration[36] on the double-Pomeron-exchange reaction and find the lowest-lying scalar to be consistent with a $(u\bar{u}+d\bar{d})/\sqrt{2}$ structure at the pole location (roughly $M-i\Gamma/2$) in the complex energy plane $(0.91-0.35i)$ GeV. This is in nice agreement with (3.11) and qualitatively with the extremely large width predicted. They also find a state $\epsilon'(1430)$ at $(1.43-0.20i)$ GeV which may be a radial recurrence of $\epsilon(900)$ and, most spectacularly, a narrow SU(3)-singlet glueball candidate $s_1(991)$ at $(0.991+0.021i)$ GeV. They confirm an $s\bar{s}$-type state $s_2(998)$ at 0.988 GeV. One way to formulate the present model to include the glueball candidate $s_1(991)$ is to replace $H$ by $H_1+H_2$ where $\langle H\rangle=\langle H_1\rangle$ and to include[37] a kinetic term only for the $H_2$ field (which gets identified with $s_1$). A similar decomposition $G=G_1+G_2$ was made in the neutral pseudoscalar channel to study[38] both the $\eta'(960)$ and the $\iota(1440)$. Clearly a very detailed analysis, including also SU(3)-symmetry breaking and the $s\bar{s}$ type state, is beyond the scope of this paper. We just mention that if one neglects mixing with the glueball candidate state $s_1(991)$ the inclusion of the $s\bar{s}$ state generalizes (3.9) in the standard way to

$$\begin{bmatrix} m_\delta^2 & 0 & 0 \\ 0 & m_\delta^2+2B' & \sqrt{2}B' \\ 0 & \sqrt{2}B' & 2m_\kappa^2-m_\delta^2+B' \end{bmatrix} . \tag{3.12}$$

This is roughly consistent with our prediction for $B'$ if $m_\kappa$ is taken to be around 1 GeV rather than the usually quoted 1.35 GeV. Of course, the unitarity[39] and SU(3)-violating corrections may play an important role. Let us see how the "inverted spectrum" for the scalars predicted by (3.10) fits into the empirical systematics of the quark-model spectroscopy.[40] In the limit of exact Okubo-Zweig-Iizuka (OZI) rule (or leading $1/N_c$ behavior) the $(u\bar{u}+d\bar{d})$ and $(u\bar{u}-d\bar{d})$ states are degenerate. Empirically we have Table I, neglecting strange-quark contamination of the isoscalar states. With the interpretation of $\epsilon(900)$ as the correct scalar isoscalar it is seen that only the $1^{++}$ mass splitting is likely to be not inverted among the $p$-wave multiplets. The axial-vector mesons are clearly a special case, however, since in the chiral-symmetric framework their masses are related to those of the vectors. Furthermore the precise mass to be assigned to the $A_1$ is still not completely settled.

## IV. ALTERNATIVES IN GLUING THE CHIRAL QUARK MODEL

The model presented in Sec. III which mocks up the QCD scale anomaly and fits nicely with the phenomenological picture of the scalar mesons put forth by Au, Morgan, and Pennington[35] is by no means a unique prescription for incorporating both quarkonium and gluonium scalars into the chiral quark model. Considerations of scale symmetry alone do not supply enough constraints on a chiral-invariant model when there is more than one scalar present. This was discussed for the soliton model without quarks present.[41] In that work the authors tried to fit the $G(1590)$ scalar-glueball candidate to the field $H$ and a quarkonium singlet around 1 GeV to the field $\rho$. This option, in common with the picture discussed in Sec. III features large glueball-quarkonium mixing. However this mixing is manifested as a mixing angle rather than in the matter-glue "duality" sense.

Before discussing some general features of the two-scalar case it should be remarked that a model rather similar to the one discussed in Sec. III may be obtained without deleting the $H$ kinetic term but rather by freezing out the heavy scalar by taking the limit [in (3.4)] $c\rightarrow\infty$. In a similar manner to the discussion of Sec. III of Ref. 41, one will end up[42] with the nonlinear Lagrangian (here $\psi=H^{1/4}$)

$$\mathcal{L}=-\tfrac{1}{2}b^2(\partial_\mu\psi)^2-\frac{F_\pi^2}{8}\frac{\psi^2}{\langle\psi\rangle^2}\mathrm{Tr}(\partial_\mu U\partial_\mu U^\dagger)-\psi^4\ln\frac{\psi}{\Lambda}-\tfrac{1}{2}\bar{q}\gamma_\mu\overleftrightarrow{\partial}_\mu q-gv\frac{\psi}{\langle\psi\rangle}(\bar{q}_L Uq_R+\bar{q}_R U^\dagger q_L)+\mathcal{L}' , \quad (4.1)$$

where $b$ is a dimensionless constant and we have, for generality, included also the term $\mathcal{L}'$ given in (2.17) which is already scale invariant. It should be noted that the model in (4.1) can also be obtained by requiring the nonlinear model to satisfy the scale properties of QCD with one scalar field assumed to dominate the trace of the energy-momentum tensor. In (4.1) we have written $\mathcal{L}$ as if $q$ had the canonical scaling dimension of $\frac{3}{2}$. Note that if this were not the case (since $q$ is an "effective" quark field we do not know its scale dimension *a priori*), we could define a new field $q'$ as $q\psi^\gamma$ where $\gamma$ is an appropriate power to make the new field $q'$ have the canonical scaling dimension. The scale-invariant quark kinetic term $-\frac{1}{2}\bar{q}\overleftrightarrow{\partial\!\!\!/}q\psi^{2\gamma}$ can be seen to be equal to $-\frac{1}{2}\bar{q}'\overleftrightarrow{\partial\!\!\!/}q'$ giving again the same form as (4.1). A similar result is obviously true for the Yukawa term. The scale dimension of $\psi$ is fixed to be one by taking its fourth power to be equal to the scale anomaly in (3.1). The model in (4.1) describes an effective interaction for the light quarks, the pseudo-Goldstone bosons (pions) and the pseudodilaton (glueball). The mass of the glueball $(2\langle\psi\rangle/b)$ can be fitted with the parameter $b$ and the nucleon parameters $M$ and $g_A$ could be fitted with $g$ and $\tilde{g}$. The results of this model are, for the usual value of $\Lambda$ given in (3.5), rather similar to those for the nonlinear model in Sec. II. There is not then a large dependence on scalar mass. For example, taking $\mathcal{L}'=0$, we find that $g_A$ decreases from 1.8 to 1.6 as the scalar-particle mass[43] increases from 0.9 GeV to 1.7 GeV. The "bag" as measured by the percentage change in $\psi$ in the region of the quarks is a rather shallow one. As in the case without quarks,[41] decreasing $\Lambda$ can make the bag deeper, at the expense of the nucleon properties.

The situation is more complicated when two physical scalar particles appear in the spectrum. Let us take $\rho$ and $\psi$ to be the two scalars and to have mass dimension one. Then a general effective Lagrangian reflecting both chiral and scale symmetry can be written as

$$\mathcal{L}=-\bar{q}\partial\!\!\!/ q-gv\mathcal{S}_1(\bar{q}_L Uq_R+\bar{q}_R U^\dagger q_L)-\frac{F_\pi^2}{8}\mathcal{S}_2\mathrm{Tr}(\partial_\mu U\partial_\mu U^\dagger)-\tfrac{1}{2}(\partial_\mu\rho)^2-\tfrac{1}{2}(\partial_\mu\psi)^2-V(\rho,\psi,\Lambda)+\mathcal{L}' , \quad (4.2)$$

where $\mathcal{S}_1$ and $\mathcal{S}_2$ are functions of $\rho/\langle\rho\rangle$ and $\psi/\langle\psi\rangle$ having scale dimensions 1 and 2, respectively, and $V$ is a potential obeying the scale-anomaly condition

$$\langle\theta_\mu^\mu\rangle=-4\langle V\rangle+\langle\psi\rangle\left\langle\frac{\partial V}{\partial\psi}\right\rangle+\langle\rho\rangle\left\langle\frac{\partial V}{\partial\rho}\right\rangle . \quad (4.3)$$

The potential can be further constrained by fitting the scalar masses. We also have the constraint that $\langle\mathcal{S}_2\rangle=1$ but no such constraint on $\langle\mathcal{S}_1\rangle$. The energetics of binding the quarks in a background meson field would tend to enhance $\mathcal{S}_1$ in the region near $r=0$ where the quark wave function is large while the function $\mathcal{S}_2$ should be suppressed where the pion kinetic energy is large. How this manifests itself in the physical scalar states is clearly model dependent. Still, it is comforting to know that the qualitative nature of the nucleon is not too sensitive to these details as we shall illustrate in the following discussion.

Let us begin by first considering a model which is motivated from the linear $\sigma$ model. We take $\mathcal{S}_1=\rho/\langle\rho\rangle$ and $\mathcal{S}_2=(\rho/\langle\rho\rangle)^2$. Furthermore we assume that the scale anomaly is dominated by $\psi$ so that we can take, for our potential,[44]

$$V=c(\rho^2-R\psi^2)^2+\tfrac{1}{2}\psi^4\ln\frac{\psi^4}{\Lambda^4} . \quad (4.4)$$

This is essentially the model discussed in Ref. 30 to fit a glueball candidate at 1.59 GeV and a scalar-quarkonium state around 1 GeV. The parameter $R^{1/2}$ is fixed to be $\langle\rho\rangle/\langle\psi\rangle=F_\pi e^{1/4}/\sqrt{2}\Lambda$ and we take $\Lambda=0.44$ GeV from (3.5). Then to fit the scalar masses we choose $c=24.4$ and $b=0.66$. The resulting solution (taking $\tilde{g}=0$ for simplicity) for the nucleon is again very similar to the linear $\sigma$ model presented in Sec. II. The field $\psi$ deviates only slightly from its vacuum value. If one now tries the same model with $\mathcal{S}_1=\psi/\langle\psi\rangle$ and $\mathcal{S}_2=\rho^2/\langle\rho\rangle^2$ one finds quite different solutions for $\psi$ and $\rho$ but the same qualitative solution for the nucleon. Similarly if one tries $\mathcal{S}_1=\rho/\langle\rho\rangle$, $\mathcal{S}_2=\psi^2/\langle\psi\rangle^2$ one again finds different solutions for $\rho$ and $\psi$ with the same qualitative solution for the nucleon. These last two cases seem to lower the energy slightly due to the independence of $\mathcal{S}_1$ and $\mathcal{S}_2$ and the energetics of enhancing $\mathcal{S}_1$ and suppressing $\mathcal{S}_2$ in the region near $r=0$ as discussed earlier. Rough calculations indicate that one can get a 10% improvement in the fit for $g_A$ using two scalar fields in this way. The main point is that the scalar degrees of freedom do not, for $\Lambda$ given by (2.5), have a substantial effect on the nucleon properties. As we discussed in Sec. III, however, the general structure induced by the scalars seems to furnish a link with the underlying QCD model, which may lead to a deeper understanding of that theory.

Finally we remark on the possibility that $\mathcal{S}_1$ or $\mathcal{S}_2$ might contain a singularity. For example, if $\mathcal{S}_1=(\rho^2/\langle\rho\rangle^2)\langle\psi\rangle/\psi$ there is a singularity as $\psi$ tends to zero. This causes an instability in our numerical program due to indefinite negative-energy contribution near the origin. Another possibility is $\mathcal{S}_1=(\rho/\langle\rho\rangle)/(1-R^{1/2}\psi/\rho)^2$ which has a more controllable singularity at $r\to\infty$ when $R^{1/2}\psi=R^{1/2}\langle\psi\rangle=\langle\rho\rangle$. This model was proposed by a number of authors[45] in a different context to mimic QCD confinement. The effective quark mass in (4.2) is $gv\mathcal{S}_1$ and so would go to infinity as $r\to\infty$, thus confining the quarks. We refer to Ref. 45 for the phenomenology of this model. If $\langle\mathcal{S}_1\rangle$ is finite and there are no other singularities the qualitative picture of the nucleon is unchanged from the linear $\sigma$ model.

## V. SUMMARY AND DISCUSSION

It may be helpful to summarize the main results of this investigation. First, we have emphasized that the CQM provides a tantalizing link of the effective chiral Lagrangian approach to the ordinary quark-model approach. This is manifested in its prediction of the well-known axial-vector-current coupling $g_A$. Rather than yielding a value around 0.7 as in the Skyrme model, the CQM gives a value closer to the quark-model value of $\frac{5}{3}$. Specifically, as the soliton mass (which gets modified to the nucleon mass in a somewhat model-dependent way by collective quantization) varies from 880 to 1220 MeV, $g_A$ decreases from 1.57 to 1.49 in the nonlinear CQM and from 1.50 to 1.45 in the linear version. From the standpoint of considering the CQM as an *effective* chiral Lagrangian there is another derivative coupling quark-meson term which has as much justification as the original one; we have also investigated the model with this term included and shown that $g_A$ may be easily "fine-tuned" to its experimental value. In the ordinary quark model a binding potential (usually taken to be linear in $r$) is added in an *ad hoc* manner. We have pointed out that the CQM generates a rather linear potential with around the usual slope in the distance scale of interest. This is a collective effect of the meson cloud. We have also shown how in the linear version of the CQM the nontrivial winding number which is a characteristic feature of the Skyrme model, dynamically arises as one increases the Yukawa coupling $g$. Whether the change of winding number is a sudden or a smooth process depends on the value of the parameter $c$ in (2.1).

The addition of gluonium-type fields in such a way as to mock up the "trace anomaly" equation of QCD was also investigated. As in similar models without quarks present this procedure was seen to display an aspect of bag confinement in the sense of producing a depressed gluonic energy density (a "nonperturbative vacuum") outside of the matter region. In the present context the addition of gluonium-type fields shows how the "linear potential" generated as a collective effect of quarkonium-type mesons could be ultimately due to the glue fields, as one expects. This comes about because of the mixing of the gluonium and quarkonium fields required by the trace-anomaly constraint. What this means for the spectrum of the scalar-isoscalar mesons is model dependent. As discussed in detail elsewhere[30] the trace-anomaly constraint is not sufficiently strong to completely predict the masses and mixing angle if more than one scalar isoscalar is assumed to be present. Thus, guidance from experiment is required. Unfortunately the experimental situation in this channel is notoriously complicated. If one accepts the suggestion that the two lowest-lying scalar isoscalars are a glueball-type state in the 1.5-GeV region [e.g., $G(1590)$] and a broad quarkonium state slightly lower in mass, one can accommodate these mesons in the present framework and obtain a slightly improved fit for $g_A$ (see Sec. IV). On the other hand, a recent reanalysis[35] of the experimental data suggests that the lowest scalar isoscalar may in fact be a broad $(u\bar{u}+d\bar{d})$ state at 900 MeV. This may be naturally accommodated by postulating a "matter-glue duality" as previously seen to be required for the $\eta'$ meson in a very analogous model. This approach leads to the formula (3.10) which relates the quarkonium isoscalar mass to that of the isovector-scalar meson [$\delta(980)$] in good agreement with the new analysis of experiments. This corresponds to an "inverted spectrum" in the sense that the isoscalar lies lower than the isovector. We point out that the inverted spectrum seems to be more the rule than the exception for the $p$-wave $q\bar{q}$ multiplets.

In addition we have shown that, in the speculative application of this approach to the standard model, the finite mass (as opposed to infinite in the nonlinear version of the model) of the Higgs meson has an important effect in lowering the maximum possible quark mass.

We hope to have convinced the reader that the CQM possesses a number of fascinating features and can furthermore fit the nucleon mass and axial-vector coupling in a simple way. Thus it would seem to merit further investigation both of its phenomenological predictions and its relation to QCD. Here we have mainly taken an operational point of view in which the model is to be the tree level with neglect of quarks in the baryon-number zero sector and at the Dirac equation level in the nonzero baryon-number sector. This sidesteps a number of well-known conceptual problems (Casimir energy,[9] instability in the ultraviolet region,[23,24] double counting[20]) which would be present if one considered the Lagrangian to be a fundamental rather than an effective one. Nevertheless it is important to further clarify the justification for our operational point of view. The phenomenological success of this model suggests that that would not be an empty exercise.

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## APPENDIX

It was pointed out[25] that the Lagrangian (2.1) might also be relevant for describing a new very-heavy-quark generation in the standard model. While parity-violating effects are of course present, the "$\rho$ parameter bound"[46] suggests that the up and down members of this heavy generation should be similar in mass. This leads to a suppression of the parity-violating Yukawa terms. The only differences for this new application are (i) the "pion" mass term is not present, (ii) $v$ is about 246 GeV rather than 93 MeV. (iii) In addition to studying the color-singlet "nucleon" we are also interested in finding the quark mass. This corresponds to choosing $N=1$ in (2.7).

It was found[25] that the maximum allowed quark mass was around 2 TeV. If one uses the same collective quantization as suggested by the Skyrme model one finds that extra rotational energy boosts this value by about 20%. However if one uses the collective quantization scheme of the chiral quark model given in Ref. 11 one finds the mass bound not to be shifted at all. A different treatment[26] of the Skyrme collective quantization and inclusion of some Casimir energy effects gives about a 10% higher result. Both of these calculations assumed the $c \rightarrow \infty$, or essentially the nonlinear limit of (2.1). Here we would like to point out that the effect of a finite value of $c$ is nontrivial for this purpose. Note that the Higgs-boson mass in the standard model is given by (2.20) so, for example, $c = 4.13 \times 10^{-4}$, $10^{-2}$, and 1 correspond, respectively, to Higgs-boson masses of 14 GeV, 70 GeV, and 700 GeV. In Fig. 9 we show, for the $N=1$ situation suitable for discussing a heavy *quark,* the dependence of quark mass on the Yukawa coupling constant $g$ for various values of $c$. We also show the straight line $E_{\text{tot}} = gv$ corresponding to the uniform Higgs phase. We see that there is a rather non-negligible dependence of the maximum allowed quark mass on $c$. For example, decreasing the Higgs-boson mass from infinity to 70 GeV decreases the bound from 2 TeV to 1.3 TeV.

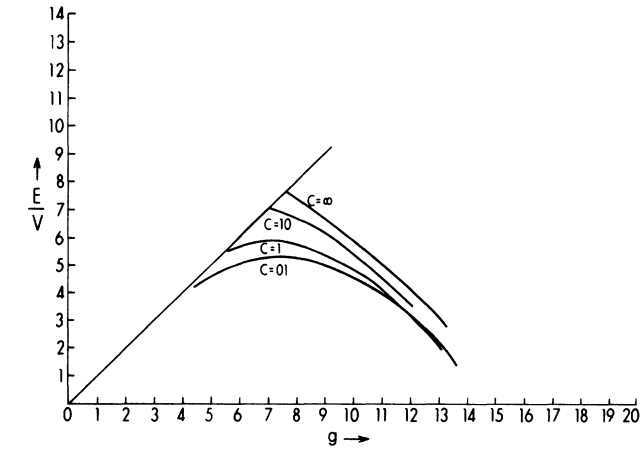


FIG. 9. Dependence of a hypothetical extremely large quark mass on the Yukawa coupling constant for various values of $c$.

The $N=3$ sector was also considered in Ref. 25 as a strongly bound color-singlet state of three fourth-generation quarks and it was found to have a mass upper bound of 3 TeV in the infinitely heavy-Higgs-boson limit. For a Higgs-boson mass of 50 GeV this bound is lowered to around 2.3 TeV. Our calculations indicate that for a finite Higgs-boson mass one should actually start seeing *some* strong Higgs binding effects in the fourth-generation quarks with Yukawa couplings as small as $g \simeq 3.5$. This would correspond to the situation where the quarks themselves are in the ordinary phase with Yukawa mass around 850 GeV.

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