Substructure of the strongly interacting Higgs sector

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If the electroweak symmetry-breaking sector becomes strong at energies above 1 TeV as a result of the absence of a low-mass Higgs boson, and if only the minimal complement of Higgs particles is present, one expects an isovector spin-1 meson of mass 2 TeV/c^2, and width 400 GeV, decaying mostly to $W^{\pm}Z$ or W^+W^-, on very general grounds. Thus, if longitudinal W's and Z's and Higgs bosons are actually fermion-antifermion composites, one must study systems other than this heavy vector meson to learn the nature of the constituents. The role in such studies played by the corresponding spin-1 *isoscalar* meson, also expected to have a mass of ≈ 2 TeV/c^2, is examined, and compared with the corresponding role played by the ω in hadron physics.

I. INTRODUCTION

The price for the successful unification[1] of weak and electromagnetic interactions has been the introduction of spinless Nambu-Goldstone particles[2] associated with electroweak symmetry breaking. These particles may be regarded as the longitudinal components of massive gauge bosons.[3] The physics of these pseudoscalar particles and their scalar partner(s), the Higgs boson(s), can be investigated in the limit of vanishing gauge coupling with nontrivial consequences.[4–9] The low-energy interactions of the theory are then described by an effective σ model,[10] with a close analogy to low-energy pion-pion scattering except for an energy scale some 2650 times greater.[11] We shall refer to such a set of fields as a *strongly interacting Higgs sector.*

One wishes to know whether the spinless bosons in the electroweak theory are elementary or composite. If they are elementary, their masses must be protected from large quadratic divergences by some mechanism such as supersymmetry.[12] If, on the other hand, they are composite,[13–15] an immediate question is the nature of their constituents. In the present paper we assume for purposes of discussion that the second scenario is the correct one. We find under rather general circumstances that the substructure of the spinless bosons remains well hidden, demanding considerable ingenuity on the part of theorists and experimentalists to uncover this substructure.

We thus consider the possibility that no Higgs bosons or superpartners of observed particles are found with masses below about 1 TeV/c^2. In that case, what experiments are most likely to shed light on the underlying structure of spinless particles in the electroweak theory? We find the following.

(1) A scalar Higgs boson in the 1–2 TeV/c^2 mass range is unavoidable. However, it is likely to be extremely broad and unimpressive in its experimental signatures.[16]

(2) An isovector spin-1 meson, which we shall call ρ_T (the subscript is a mnemonic for TeV) is a nearly universal feature of the theory.[17] Its mass should be about 2 TeV/c^2 and its width about 400 GeV. Its major decay modes will be $\rho_T^{\pm} \to W_L^{\pm} Z_L^0, \rho_T^0 \to W_L^+ W_L^-$, where the subscript L denotes a longitudinally polarized vector meson. Such a resonance has been discussed recently in the literature.[6,7,9,18–20] It may be produced via mixing with $W^{\pm}$ and Z^0 in Drell-Yan processes at multi-TeV energies,[9] and via gauge-boson fusion in high-energy collisions.[6]

(3) The underlying properties of the theory are much more sensitively probed by experiments involving the *isoscalar* partner of ρ_T, a vector meson which we shall call ω_T. The ω meson of the strong interactions provides correspondingly fruitful information.[21] The mass of ω_T is also expected to be about 2 TeV/c^2. While we shall give suggestions for its observation, we find it to be difficult to produce even at supercollider energies of $\sqrt{s}$ up to 40 TeV.

This paper is thus intended as an appeal for help, both theoretical and experimental. If the Higgs sector is strongly interacting, its underlying substructure may be very difficult to learn directly via any but the highest-energy experiments which can be contemplated at present. We are thus hoping to stimulate further suggestions, whether for clever studies at supercolliders or for less direct tests of substructure in the Higgs sector.

This paper is organized as follows. In Sec. II we discuss the Higgs sector (including Nambu-Goldstone bosons) of the electroweak theory, recall its analogy with low-energy pion-pion scattering, and predict an $I=J=1$ meson of mass 2 TeV/c^2. In Sec. III we discuss the stability of this prediction with regard to underlying substructure, and note that while the quark substructure of hadrons is not probed by the strong interactions of pions and ρ mesons, experiments involving anomalies (in $\pi^0 \to \gamma\gamma$, for example) and ω mesons allow one to learn about colors and quark charges. The corresponding lessons for physics at 2 TeV are then applied to the properties of the isoscalar spin-1 meson ω_T. Strategies for producing ρ_T and ω_T are mentioned in Sec. IV. A possibility that the ω_T does not mix at all with electroweak gauge bosons is mentioned in Sec. V, where we suggest one specific substructure for ρ_T and ω_T. Section VI summarizes.

II. STRONG INTERACTIONS OF SPINLESS BOSONS

The Higgs sector in the electroweak theory can be characterized by one unknown parameter which we can choose to be the Higgs-scalar-boson mass M_H, or the self-coupling constant λ. At the tree level,

$$M_H^2=\sqrt{2}\lambda/G_F\simeq(350\text{ GeV})^2\lambda\ , \quad (2.1)$$

where G_F is the Fermi coupling constant. In a model with a single complex Higgs doublet,

$$\Phi(x)=\begin{bmatrix}\phi^+(x)\\ \phi^0(x)\end{bmatrix}\ , \quad (2.2)$$

the Lagrangian is given by

$$L_H=[\partial_\mu\Phi(x)]^\dagger[\partial^\mu\Phi(x)]-\lambda[\Phi^\dagger(x)\Phi(x)-\mu^2/2\lambda]^2\ . \quad (2.3)$$

We can write the complex fields $\phi^+(x)$ and $\phi^0(x)$ in terms of four real fields:

$$\phi^+(x)=[\phi_1(x)+i\phi_2(x)]/\sqrt{2}\ , \quad (2.4a)$$

$$\phi^0(x)=[\sigma(x)-i\chi(x)]/\sqrt{2}\ , \quad (2.4b)$$

and the Lagrangian L_H can be recast into the form[11]

$$L_H=\tfrac{1}{4}\text{Tr}(\partial_\mu M^\dagger\partial^\mu M)-\tfrac{1}{4}\lambda[\tfrac{1}{2}\text{Tr}(M^\dagger M)-\mu^2/\lambda]^2\ , \quad (2.5)$$

where $M(x)$ is a 2×2 matrix:

$$M(x)=\sigma(x)+i\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)\ , \quad (2.6)$$

with

$$\boldsymbol{\pi}(x)=(\phi_2(x),\phi_1(x),\chi(x))\ . \quad (2.7)$$

Here $\boldsymbol{\tau}$ denote the Pauli matrices.

The Lagrangian (2.5) is invariant under a global $SU(2)_L\times SU(2)_R$ transformation:

$$M(x)\to g_L M(x) g_R^\dagger\ . \quad (2.8)$$

This symmetry is spontaneously broken down to $SU(2)_{\text{diagonal}}$ by the nonzero vacuum expectation value of $M(x)$:

$$\langle M^\dagger M\rangle\equiv v^2=\mu^2/\lambda\ . \quad (2.9)$$

The fields $\boldsymbol{\pi}(x)$ become identified with the Goldstone bosons generated in the symmetry breaking. In the presence of gauged $SU(2)_L\times U(1)$ interactions, the fields $\boldsymbol{\pi}(x)$ become the longitudinal components of the gauge field $\mathbf{W}(x)$, which then develops a mass term (for charged W's)

$$M_W=gv/2\ . \quad (2.10)$$

Here g is the $SU(2)_L$ coupling constant. At low momentum transfers, the identification of massive W exchange with the four-fermion interaction entails the relation $G_F/\sqrt{2}=g^2/(8M_W^2)=1/(2v^2)$, and so

$$v=2^{-1/4}G_F^{-1/2}=246\text{ GeV}\ . \quad (2.11)$$

The neutral member of the triplet (2.7) couples not only to W^3 but also to the U(1) gauge field B. It thus mixes W^3 and B, providing the longitudinal component of the Z^0.

It can be shown to all orders in perturbation theory that at high energies ($s\gg M_W^2$) the scattering of longitudinally polarized gauge bosons is given by the scattering amplitude for the fields $\boldsymbol{\pi}(x)$ (governed by L_H) plus corrections of order $M_W/\sqrt{s}$ (Ref. 22).

Since we have no experimental information about M_H or λ, we may ask what happens to the theory when these parameters get large. If $\lambda\sim1$, the Higgs sector becomes strongly self-interacting and we expect perturbation theory to break down. In fact, careful analyses have shown that for $M_H\gtrsim1$ TeV the S-wave scattering amplitude for longitudinal gauge bosons exceeds the unitarity limit. (See Lee, Quigg, and Thacker, Ref. 4.)

It is useful to work in the limit $M_H\to\infty$, in which case the value of the Higgs potential is frozen at its minimum, given by Eq. (2.9) with $v=246$ GeV. In terms of the unitary matrix

$$U\equiv M/v,\quad U^\dagger U=1\ , \quad (2.12)$$

the Lagrangian L_H in this limit is given by

$$L_H \underset{M_H\to\infty}{=} \frac{v^2}{4}\text{Tr}(\partial_\mu U^\dagger\partial^\mu U)\ . \quad (2.13)$$

At this point we realize that in this limit the Higgs sector with doublet structure becomes identical to the pion system described by a nonlinear σ model if we replace the vacuum expectation value v of the Higgs field by the pion decay constant $f_\pi=93$ MeV (Ref. 11). The analogue of the isospin symmetry is the so-called custodial or residual SU(2) (Ref. 23) which guarantees the equality $\rho\equiv M_W^2/M_Z^2\cos^2\theta_W=1$ at the tree level.

Now, it has been shown that the low-energy limit of pion-pion scattering, combined with unitarity in such a way as to preserve approximate crossing symmetry, is sufficient to reproduce certain aspects of the low-lying resonance spectrum.[24] The key predictions include an $I=J=0$ resonance (sometimes known as the ϵ) around 700 MeV, an $I=J=1$ resonance (the ρ) with similar mass, and the current-algebra prediction[25] for the ρ width, which is related to the $\rho\pi\pi$ coupling by

$$\Gamma(\rho\to\pi\pi)=\frac{g_{\rho\pi\pi}^2}{48\pi}\frac{(m_\rho^2-4m_\pi^2)^{3/2}}{m_\rho^2}\ . \quad (2.14)$$

In the limit of zero pion mass, the ϵ and ρ masses are predicted to be a number of order 1 times $2\pi f_\pi$. The $\rho\pi\pi$ coupling is related to m_ρ and f_π by[25]

$$g_{\rho\pi\pi}=m_\rho/\sqrt{2}f_\pi\ . \quad (2.15)$$

The ϵ of pion-pion scattering may have to be interpreted in terms of the large (but not necessarily resonant) $I=J=0$ phase shift observed over a considerable energy range. However, recent data[26,27] on $\gamma\gamma\to\pi\pi$ appear strikingly similar to predictions[28,29] based on the Brown-Goble approach,[24] which has an explicit ϵ around 700 MeV. [An explicit ϵ has been called for[30] on the basis of the shape of the mass spectrum[31] in $\Upsilon(3S)\to\Upsilon(1S)\pi\pi$, but values of the ϵ mass below 700 MeV (suggested in Ref. 30) are disfavored by elastic $\pi\pi$ scattering and by the new $\gamma\gamma\to\pi\pi$ results.[32]]

If the low-energy limit of the Higgs sector is really identical to that for pion-pion scattering except for a change of energy scale, that scale factor is roughly

$$v/f_\pi\approx2650\ . \quad (2.16)$$

Hence we naively expect a scaled-up version of the resonance spectrum of the pion system to be reproduced by the strongly interacting Higgs sector. Notice that the predictions[24] of the ϵ and ρ did not depend on any underlying quark dynamics. In similar fashion, we expect that predictions of $I=J=0$ and $I=J=1$ resonances decaying to longitudinal W's and Z's will be insensitive to details of the underlying structure leading to dynamical electroweak symmetry breaking.

The $I=J=0$ resonance is the so-called Higgs "particle." By scaling via v/f_π from QCD, we expect this "particle" in a strongly coupled theory to lie somewhere between 1 and 2 TeV in mass. Because of its large width it will be very difficult to detect. Analyses of future experiments at supercolliders[6,33] are not optimistic about prospects for observing the Higgs particle if M_H lies much above 1 TeV/c^2, even if $\sqrt{s}=40$ TeV. Nonetheless this resonance can show up as an enhancement in the scattering cross section of longitudinally polarized gauge bosons.

The $I=J=1$ resonance (ρ_T) analogous to the ρ meson in QCD is expected to have a mass

$$m(\rho_T)=(v/f_\pi)m_\rho \simeq 2 \text{ TeV} \tag{2.17}$$

and width

$$\Gamma(\rho_T)=(v/f_\pi)\Gamma_\rho \simeq 400 \text{ GeV} . \tag{2.18}$$

Several authors have stressed the independence of these predictions of any underlying substructure.[6,7,9,18–20] In the next section we discuss this independence, and show that experiments involving the $I=0$, $J=1$ meson (ω for QCD, ω_T for the strongly interacting Higgs sector) are much more sensitive to this substructure.

III. PROBES OF SUBSTRUCTURE

Low-energy pion-pion scattering leads to ϵ and ρ mesons, independently of colors and charges. Observables associated with anomalies and the ω, on the other hand, are suitable probes of these details.[21,34,35]

A. $\pi^0\to\gamma\gamma$

Consider, for example, the decay $\pi^0\to\gamma\gamma$. Let the decay amplitude $A(\pi^0\to\gamma\gamma)$ be defined in such a way that

$$\Gamma(\pi^0\to\gamma\gamma)=\frac{1}{16\pi m_\pi}\,|A(\pi^0\to\gamma\gamma)|^2 . \tag{3.1}$$

At the quark level, the amplitude A is described by the graph of Fig. 1(a). One finds[36]

$$A(\pi^0\to\gamma\gamma)=\frac{\alpha m_\pi^2 N_c(Q_u^2-Q_d^2)}{2\pi f_\pi} . \tag{3.2}$$

The observed rate[37] of $\Gamma(\pi^0\to\gamma\gamma)=7.29\pm0.19$ eV is compatible with the prediction of Eqs. (3.1) and (3.2),

$$\Gamma(\pi^0\to\gamma\gamma)=7.6[N_c(Q_u^2-Q_d^2)]^2 \text{ eV} \tag{3.3}$$

as long as $N_c(Q_u^2-Q_d^2)=1$. This is the case for the conventional quark model, $N_c=3$, $Q_u=\frac{2}{3}$, $Q_d=-\frac{1}{3}$.

Vector-meson dominance provides an alternate description of $\pi^0\to\gamma\gamma$, according to the graph of Fig. 1(b). The effective Lagrangians for the $\rho\pi\omega$, $\rho^0\gamma$, and $\omega\gamma$ couplings are

$$L_{\text{eff}}^{\rho\pi\omega}=g_{\rho\pi\omega}\epsilon_{\alpha\beta\gamma\delta}(\partial^\alpha\boldsymbol{\rho}^\beta\partial^\gamma\omega^\delta)\cdot\boldsymbol{\pi} , \tag{3.4}$$

where the scalar product acts in isospin space,

$$L_{\text{eff}}^{\rho^0\gamma}=(e/g_{\rho\pi\pi})m_\rho^2(Q_u-Q_d)\rho^\mu A_\mu \tag{3.5}$$

and

$$L_{\text{eff}}^{\omega\gamma}=(e/g_{\rho\pi\pi})m_\omega^2(Q_u+Q_d)\omega^\mu A_\mu . \tag{3.6}$$

Here ρ^μ,ω^μ, A^μ stand for the ρ, ω, and photon fields. (In fact, it is always true that $Q_u-Q_d=1$, but we retain this factor for comparison purposes.) Combining the contributions (3.4)–(3.6) to the graph of Fig. 1(b), we find

$$A(\pi^0\to\gamma\gamma)=\frac{e^2m_\pi^2(Q_u^2-Q_d^2)g_{\rho\pi\omega}}{g_{\rho\pi\pi}^2} . \tag{3.7}$$

Comparing (3.7) with (3.2), we find (cf. Ref. 21)

$$g_{\rho\pi\omega}=\frac{g_{\rho\pi\pi}^2}{4\pi}\frac{N_c}{2\pi f_\pi} . \tag{3.8}$$

Notice that while the scale of $g_{\rho\pi\omega}$ is set by $2\pi f_\pi$, the ratio N_c/f_π appears in $g_{\rho\pi\omega}$. Thus, in a theory where $g_{\rho\pi\pi}^2/4\pi$ (≈ 2.73; see Ref. 38) and $f_\pi=93$ MeV are taken from experiment, the $\rho\pi\omega$ coupling is a probe of the number of colors. (Recall, that through the use of unitarity, crossing, and current algebra, $g_{\rho\pi\pi}$ may be regarded as fixed.)

B. $\omega\to\pi^0\gamma$

The partial width for radiative ω decay to $\pi^0\gamma$ may be expressed in terms of an invariant amplitude as

$$\Gamma(\omega\to\pi^0\gamma)=\frac{2}{3}\frac{k_\gamma}{8\pi m_\omega^2}\,|A(\omega\to\pi^0\gamma)|^2 . \tag{3.9}$$

Here the factor of $\frac{2}{3}$ comes from a polarization average. The photon energy in the ω rest frame is k_γ. The amplitude A describes the decay of an ω polarized transversely to the photon direction in the ω rest frame. It is given by

$$A(\omega\to\pi^0\gamma)=(e/g_{\rho\pi\pi})m_\omega k_\gamma g_{\rho\pi\omega} \tag{3.10}$$

if we use the vector-dominance picture illustrated in Fig.

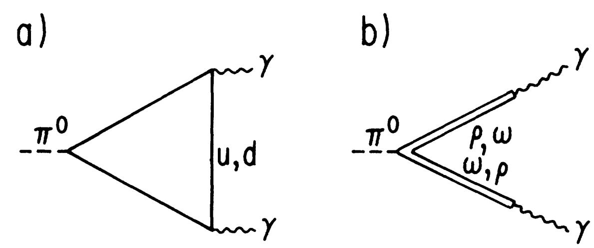

FIG. 1. Descriptions of $\pi^0\to\gamma\gamma$. (a) Quark loop; (b) vector dominance.

2(a). If we substitute Eq. (3.8) for $g_{\rho\pi\omega}$, we find

$$\Gamma(\omega\to\pi^0\gamma)=\frac{\alpha}{12\pi}\left[\frac{N_c}{2\pi f_\pi}\right]^2\frac{g_{\rho\pi\pi}^2}{4\pi}k_\gamma^3 . \tag{3.11}$$

With $N_c=3$, $f_\pi=93$ MeV, $g_{\rho\pi\pi}^2/4\pi=2.73$, and $k_\gamma=380$ MeV, one obtains[21] $\Gamma(\omega\to\pi^0\gamma)=764$ keV, to be compared with the experimental values

$$\Gamma(\omega\to\pi^0\gamma)=\begin{cases}853\pm56 \text{ keV (Ref. 39)} , & (3.12a)\\ 731\pm43 \text{ keV (Ref. 40)} . & (3.12b)\end{cases}$$

The important point regarding the prediction (3.11) is not its agreement with experiment, but the fact that it probes the number of quark colors once we regard f_π, $g_{\rho\pi\pi}$, and m_ω as known.[21]

The quark model[41] also predicts the rate for $\omega\to\pi^0\gamma$, in terms of the magnetic moments of u and d quarks [see Fig. 2(b)]:

$$\Gamma(\omega\to\pi^0\gamma)=\frac{k_\gamma^3}{3\pi}(\mu_u-\mu_d)^2\,|I|^2 , \tag{3.13}$$

where

$$\mu_{u,d}=(e/2m_{u,d})Q_{u,d} , \tag{3.14}$$

and I represents the overlap of spatial wave functions between the ${}^3S_1$ (ω) and ${}^1S_0$ (π) states. With $Q_u-Q_d=1$ and $m_u\approx m_d$, we find

$$\Gamma(\omega\to\pi^0\gamma)=\frac{\alpha}{3}\frac{k_\gamma^3}{m_u^2}\,|I|^2 . \tag{3.15}$$

In Eq. (3.14), $m_u\approx m_d$ is the mass of a *constituent* quark, $m_u\approx310$ MeV (Ref. 42). With this value, the right-hand side of (3.15) is 1.39 MeV$(|I|^2)$ so $|I|^2\simeq0.5$–0.6. Comparing (3.15) with (3.11), we find

$$\frac{m_u}{I}=\frac{2\pi f_\pi}{N_c}\frac{4\pi}{g_{\rho\pi\pi}} . \tag{3.16}$$

Again, N_c dependence enters when a description at the quark level is required to be consistent with one based on vector dominance.

C. Scaling with number of colors

We have regarded $f_\pi=93$ MeV as an input into the chiral low-energy dynamics of pions. One then finds[24] that $m_\rho/(2\pi f_\pi)$ is a fixed number of order 1; $g_{\rho\pi\pi}$ also is fixed and independent of substructure. On the other hand, $g_{\rho\pi\omega}$ scales as N_c, while the constituent-quark mass scales as $1/N_c$.

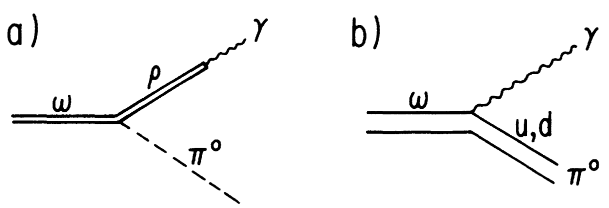

FIG. 2. Descriptions of $\omega\to\pi^0\gamma$. (a) Vector dominance; (b) quark model.

In many discussions of dynamical symmetry-breaking schemes,[43] the "techni-ρ" (the particle corresponding to our ρ_T) is found to have a mass which scales as $N^{-1/2}$, if SU(N) is the group associated with the superstrong interactions (sometimes called "technicolor"). It is important to understand why we find different behavior. The difference appears to lie in the incompatibility of the large-N limit with the Brown-Goble[24] prescription for satisfying the constraints of current algebra, crossing, and unitarity with smoothly varying functions.

In an asymptotically free theory, the strong coupling constant g_s increases logarithmically as the momentum transfer Q at which it is probed decreases:

$$\frac{g_s^2}{4\pi}=\left[\text{const}\times\ln\frac{Q^2}{\Lambda^2}\right]^{-1} . \tag{3.17}$$

Here Λ defines the scale at which the coupling constant becomes strong. This scale is usually adopted as the definition of the fundamental energy scale in the theory.

If Λ is regarded as fixed, one finds[44]

$$f_\pi\sim\Lambda\sqrt{N} , \tag{3.18}$$

$$g_{\rho\pi\pi}\sim1/\sqrt{N} , \tag{3.19}$$

and so, via Eq. (2.14),

$$m_\rho\sim\Lambda \tag{3.20}$$

without any N dependence. Thus

$$\frac{m_\rho}{f_\pi}\sim N^{-1/2} , \tag{3.21}$$

as is usually noted in discussions of the superstrong interactions.

In the large-N limit, meson-meson elastic scattering amplitudes scale as $f_\pi^{-2}\sim(N\Lambda^2)^{-1}$. The P-wave $\pi\pi$ scattering amplitude remains small until the ρ meson is reached, whereupon it suddenly attains the unitary limit. Smooth parametrizations such as that assumed in Ref. 24 cannot anticipate such behavior. Thus, one could argue that the successful prediction of the ρ mass in Ref. 24 is merely an accident.

The structure of the superstrong interactions is as yet unknown. However, if N is large, Eq. (3.21) suggests that the onset of new physics will occur at a *lower* energy than we have estimated; for example, we would expect

$$m(\rho_T)<2 \text{ TeV}/c^2 . \tag{3.22}$$

Thus, our estimates should be regarded as the *most conservative* ones for new physics associated with a strongly interacting Higgs sector. One also expects new physics to set in at lower energies than 2 TeV$/c^2$ if there is more than one complex doublet of Higgs bosons.[7]

To continue the discussion of the large-N limit, we note that Eqs. (3.18)–(3.21), when combined with Eq. (3.8), imply that

$$g_{\rho\pi\omega}\sim f_\pi^{-1}\sim\Lambda^{-1}N^{-1/2} , \tag{3.23}$$

so that the $\rho\pi\omega$ coupling has the expected $N^{-1/2}$ behavior of a three-meson coupling.[44] Moreover, one finds from (3.16) that

$$m_u/I \sim \Lambda \; , \tag{3.24}$$

so that m_u and m_ρ are both independent of N. Thus, a constituent-quark description makes sense in the large-N limit. The limit we are discussing is, rather, one which is expected to be valid for modest N, such that current-algebra amplitudes join smoothly onto resonant behavior.

D. Dependence on quark charges

The ρ-γ coupling in Eq. (3.5) is proportional to $Q_u - Q_d = 1$. On the other hand, the ω-γ coupling (3.6) is proportional to $Q_u + Q_d$, since ω is an isoscalar, so ω-γ couplings are sensitive to details of quark charges. Both the quark-loop and vector-dominance amplitudes (3.2) and (3.7) for $\pi^0 \to \gamma\gamma$ contain a factor of $Q_u + Q_d$. Thus, one learns not only about quark colors but also about quark charges from processes involving anomalies and ω's.

E. The isoscalar spin-1 meson ω_T

In analogy with the ρ-ω degeneracy, understood from the standpoint of constituent quarks,[45] one expects the isovector spin-1 meson ρ_T to be accompanied by an isoscalar partner ω_T of nearly the same mass:

$$m(\omega_T) \approx (v/f_\pi) m_\omega \approx 2 \text{ TeV}/c^2 \; . \tag{3.25}$$

In the large-N limit, Eq. (3.25) is to be regarded as an upper limit. By further analogy with the strong interactions, we expect the properties of ω_T to be much more intimately related to underlying substructure than those of ρ_T.

Let π now stand for (W_L^1, W_L^2, Z_L), the longitudinal components of the charged W's and Z^0. By analogy with Eq. (3.8), we expect

$$g_{\rho_T \pi \omega_T} = \frac{g_{\rho\pi\pi}^2}{4\pi} \frac{N_s}{2\pi v} \; , \tag{3.26}$$

for a superstrong interaction group SU(N_s).

Let the subconstituents ("techniquarks") of Higgs bosons and longitudinally polarized W's and Z's consist of a single weak isospin doublet (U, D), and denote the corresponding charges by $(A, A-1)$:

$$Q \begin{bmatrix} U \\ D \end{bmatrix} = \begin{bmatrix} A \\ A-1 \end{bmatrix} . \tag{3.27}$$

The coupling of transverse gauge bosons to ρ_T is proportional to $Q_U - Q_D = A - (A-1)$ and hence is independent of A, while the corresponding coupling to ω_T is proportional to $Q_U + Q_D = A + (A-1) = 2A - 1$. Thus, the ω_T is a much more useful probe than ρ_T of underlying substructure. As we shall see in the next section, however, the ω_T-gauge-boson coupling has a good chance of being very small or zero, so that this substructure is likely to remain hidden unless ingenious means are found to uncover it.

IV. PRODUCTION OF ρ_T AND ω_T

A. Couplings to gauge bosons

1. Isovector bosons

The coupling of ρ_T to a transverse gauge boson is evaluated in exactly the same manner as the ρ-γ coupling. Let us recapitulate the argument[46] which leads to Eq. (3.5), in order to adapt it to present purposes. The coupling of a photon to $\pi^+\pi^-$ is proportional to a kinematic factor times e [Fig. 3(a)]. This same kinematic factor characterizes the $\rho\pi^+\pi^-$ coupling [Fig. 3(b)], and vector-meson dominance of the low-q^2 behavior of Fig. 3 requires

$$e = g_{\rho\gamma} \frac{1}{m_\rho^2} g_{\rho\pi\pi} \tag{4.1}$$

or

$$g_{\rho\gamma} = \frac{e m_\rho^2}{g_{\rho\pi\pi}} \; , \tag{4.2}$$

the result of Eq. (3.5).

Similarly, one can assume that the neutral isovector electroweak current is dominated by ρ_T^0, as illustrated in Fig. 4. A short calculation, which we now perform, indicates that the W^3-W_L^+-W_L^- coupling is $g/2$, where g is the SU(2) gauge coupling.

The Yang-Mills Lagrangian for SU(2) is

$$L^{\text{YM}} = -\tfrac{1}{4} F_{\mu\nu}^i F^{\mu\nu i} \; , \tag{4.3}$$

where

$$F_{\mu\nu}^i = \partial_\mu A_\nu^i - \partial_\nu A_\mu^i + g \epsilon_{ijk} A_\mu^j A_\nu^k \; . \tag{4.4}$$

The term in (4.3) trilinear in gauge fields may be written

$$L^{(3)} = -g \epsilon_{ijk} \partial_\mu A_\nu^i A^{\mu j} A^{\nu k} \; , \tag{4.5}$$

making use of the antisymmetry of $F_{\mu\nu}^i$ in μ and ν and of ϵ_{ijk} in pairs of indices. We wish to evaluate

$$\langle W_L^+(p_1) W_L^-(p_2) | L | W^3(P,\lambda) \rangle \; ,$$

where λ denotes the polarization state of the W^3 and P its momentum.[47] One needs the value of the normalization constant c in

$$\langle W_L^i(p) | A^{\mu j} | 0 \rangle = \delta^{ij} p^\mu c \; . \tag{4.6}$$

The W_L propagator in momentum space is $(p^2 - m_W^2)^{-1}$,

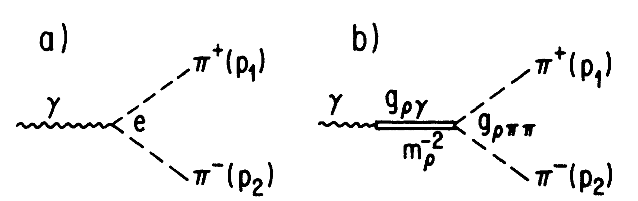

FIG. 3. Illustration of vector-meson dominance. (a) Direct $\gamma\pi^+\pi^-$ coupling; (b) $\gamma\pi^+\pi^-$ coupling dominated by ρ^0.

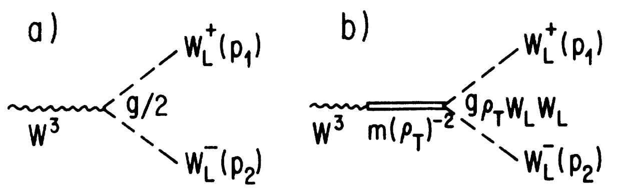

FIG. 4. Dominance of W^3 current by ρ_T^0. (a) W^3 coupling to $W_L^+ W_L^-$; (b) dominance by ρ_T^0.

if we treat W_L as a spinless boson, while that of (the longitudinal) A^μ is $(p^\mu p^\nu/m_W^2)(p^2-m_W^2)^{-1}$. Comparing, we see that

$$c=m_W^{-1} \ . \tag{4.7}$$

We then find that

$$\langle W_L^+(p_1)W_L^-(p_2)\,|\,L\,|\,W^3(P,\lambda)\rangle =gm_W^{-2}(\epsilon^{(\lambda)}\cdot p_1 P\cdot p_2-\epsilon^{(\lambda)}\cdot p_2 P\cdot p_1) \ . \tag{4.8}$$

With $P^2=(p_1+p_2)^2=p_1^2=p_2^2=m_W^2$, we find

$$P\cdot p_1=P\cdot p_2=m_W^2/2 \ , \tag{4.9}$$

and so

$$\langle W_L^+(p_1)W_L^-(p_2)\,|\,L\,|\,W^3(P,\lambda)\rangle=(g/2)\epsilon^{(\lambda)}\cdot(p_1-p_2) \ . \tag{4.10}$$

This is the appropriate coupling for Fig. 4(a).

If the W^3 current is dominated by a ρ_T^0 meson, the amplitude (4.10) is also expressed via the graph of Fig. 4(b):

$$\langle W_L^+(p_1)W_L^-(p_2)\,|\,L\,|\,W^3(P,\lambda)\rangle =g_{\rho_T W^3}\frac{1}{m(\rho_T)^2}g_{\rho_T W_L W_L}\epsilon^{(\lambda)}\cdot(p_1-p_2) \ . \tag{4.11}$$

The coupling constant $g_{\rho_T W_L W_L}$ is the analogue of the coupling $g_{\rho\pi\pi}$ in the strong-interaction case. A scaled-up version of the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation[25] implies

$$g_{\rho_T W_L W_L}=m(\rho_T)/\sqrt{2}v \ . \tag{4.12}$$

With $m(\rho_T)/v=m_\rho/f_\pi$, we then find

$$g_{\rho_T W_L W_L}=g_{\rho\pi\pi} \ , \tag{4.13}$$

and $\Gamma(\rho_T^0\to W_L^+W_L^-)=(v/f_\pi)\Gamma(\rho\to\pi\pi)\approx 400$ GeV, as mentioned in Sec. II.

Identifying (4.10) with (4.11), we find

$$g_{\rho_T W^3}=gm(\rho_T)^2/(2g_{\rho\pi\pi}) \ . \tag{4.14}$$

2. *Isoscalar bosons*

The ω_T will mix with the isoscalar gauge boson B associated with the U(1) of the electroweak theory. Now, W^3 couples to gI_3 and I_3 is well defined for the constituents of the isovector ρ_T, but B couples to $g'Y/2$, where Y is the weak hypercharge. The value of Y for the ω_T's constituents depends on their charges.

Since $Q=I_3+Y/2$, the isodoublet $(U,D)_L$ has assignments

$$\begin{bmatrix}U\\D\end{bmatrix}_L:\ Q=\begin{bmatrix}A\\A-1\end{bmatrix},\ \ I_3=\begin{bmatrix}\frac{1}{2}\\-\frac{1}{2}\end{bmatrix},\ \ \frac{Y}{2}=\begin{bmatrix}A-\frac{1}{2}\\A-\frac{1}{2}\end{bmatrix}; \tag{4.15}$$

$$\begin{bmatrix}U\\D\end{bmatrix}_R:\ Q=\begin{bmatrix}A\\A-1\end{bmatrix},\ \ I_3=0,\ \ \frac{Y}{2}=\begin{bmatrix}A\\A-1\end{bmatrix} \ . \tag{4.16}$$

The (vector) couplings of B to ω_T and W^3 to ρ_T^0 are in the ratio

$$\frac{g_{\omega_T B}}{g_{\rho_T W^3}}=\frac{g'}{2g}\frac{Y(U_L)+Y(D_L)+Y(U_R)+Y(D_R)}{I_3(U_L)-I_3(D_L)+I_3(U_R)-I_3(D_R)} =(2g'/g)(2A-1) \ . \tag{4.17}$$

Thus we expect

$$g_{\omega_T B}=g'm(\omega_T)^2(2A-1)/g_{\rho\pi\pi} \ . \tag{4.18}$$

B. Drell-Yan processes

A major contribution to production of ρ_T is expected to be the Drell-Yan[48] process illustrated in Fig. 5. As we shall see, the corresponding cross section for production of ω_T is expected to be much smaller.

The ρ_T is produced in Fig. 5 via mixing with a virtual W, while the ω_T is produced via mixing with a virtual B. The cross section per unit rapidity for production of a vector boson of mass M by colliding hadrons A and B is[49]

$$\frac{d\sigma}{dy}=\frac{4\pi^2 x_1 x_2}{3M^3}\sum_{i,j}f_i^{(A)}(x_1)f_j^{(B)}(x_2)\Gamma_{ij} \ , \tag{4.19}$$

where $f_i^{(A,B)}$ are the structure functions of quark i in hadrons A and B, and Γ_{ij} is the partial width of the vector boson into the quark pair $q_i q_j$. The momentum fractions x_i and x_2 are related to the rapidity y by

$$\left.\begin{matrix}x_1\\x_2\end{matrix}\right\}=(M/\sqrt{s}\,)e^{\pm y} \ . \tag{4.20}$$

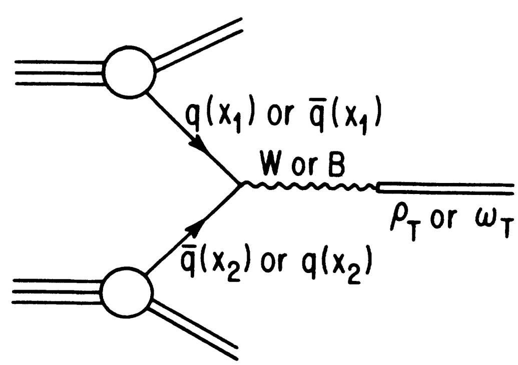

FIG. 5. Drell-Yan production of ρ_T,ω_T.

The partial width Γ_{ij} contains the factor of 3 appropriate for color. For example, the partial width of an ordinary W^+ into $u\bar{d}$ is

$$\Gamma(W^+ \to u\bar{d}) = \frac{G_F M_W^2}{\sqrt{2}} \frac{M_W}{2\pi} \approx 700 \text{ MeV} , \quad (4.21)$$

while that into $e^+\nu_e$ is one-third this amount. In what follows we shall neglect small quark mixing angles, and shall ignore contributions of quarks other than u, d, $\bar{u}$, and $\bar{d}$.

Integrating Eq. (4.19) with respect to y, we find

$$\sigma = \frac{4\pi^2}{3M^3} \sum_{i,j} \tau \frac{dL_{ij}}{d\tau} \Gamma_{ij} , \quad (4.22)$$

where

$$\tau \equiv M^2/s = x_1 x_2 \quad (4.23)$$

and

$$\frac{dL_{ij}}{d\tau} \equiv \int_\tau^1 \frac{dx}{x} f_i^{(A)}(x) f_j^{(B)} \left[\frac{\tau}{x} \right] . \quad (4.24)$$

The partial width of ρ_T or ω_T into a quark-antiquark pair is related by vector dominance to that for a W or B with the same mass as ρ_T or ω_T, which we shall take to be $M_0 \equiv 2$ TeV/c^2. (See Fig. 6.) A W^3 or B, which dominates ρ_T^0 or ω_T decay, is to be thought of as the appropriate linear combination of γ and Z. In our calculations we will neglect the mass of a W or Z in comparison with $m(\rho_T)$ or $m(\omega_T)$. We then find

$$\Gamma(\rho_T \to q_i \bar{q}_j) = \left[\frac{g_{\rho_T W^3}}{M_0^2} \right]^2 \Gamma(W_{2\,\text{TeV}} \to q_i \bar{q}_j) , \quad (4.25)$$

$$\Gamma(\omega_T \to q_i \bar{q}_j) = \left[\frac{g_{\omega_T B}}{M_0^2} \right]^2 \Gamma(B_{2\,\text{TeV}} \to q_i \bar{q}_j) , \quad (4.26)$$

where $g_{\rho_T W^3}$ and $g_{\omega_T B}$ are given by (4.14) and (4.18).

We neglect small logarithmic variations of g and g' between $M_W \equiv 81$ GeV/c^2 or $M_Z \equiv 92$ GeV/c^2 and $M_0 \equiv 2$ TeV/c^2. The partial widths on the right-hand side of (4.25) are then

$$\Gamma(W^+_{2\,\text{TeV}} \to u\bar{d}) = \left[\frac{M_0}{M_W} \right] \Gamma(W^+ \to u\bar{d}) \approx 17.2 \text{ GeV} , \quad (4.27)$$

and, similarly for $W^-_{2\,\text{TeV}} \to d\bar{u}$,

$$\Gamma(W^3_{2\,\text{TeV}} \to u\bar{u}) = \Gamma(W^3_{2\,\text{TeV}} \to d\bar{d}) = \tfrac{1}{2}\Gamma(W^+_{2\,\text{TeV}} \to u\bar{d}) \simeq 8.61 \text{ GeV} . \quad (4.28)$$

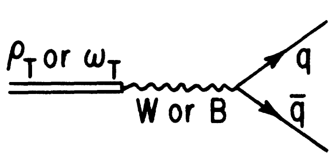

FIG. 6. Vector-dominance contribution to quark-antiquark partial width of ρ_T or ω_T.

To calculate $\Gamma(B \to q\bar{q})$ we note that

$$\frac{\Gamma(B \to q\bar{q})}{\Gamma(W^3 \to q\bar{q})} = \left[\frac{g'}{2g} \right]^2 \frac{Y(q_L)^2 + Y(q_R)^2}{I_3(q_L)^2 + I_3(q_R)^2} . \quad (4.29)$$

Since $Y(u_L) = Y(d_L) = \frac{1}{3}$, $Y(u_R) = \frac{4}{3}$, $Y(d_R) = -\frac{2}{3}$, $I_3(u_L) = -I_3(d_L) = \frac{1}{2}$, and $I_3(u_R) = I_3(d_R) = 0$, we find

$$\Gamma(B_{2\,\text{TeV}} \to u\bar{u}) = \tfrac{17}{9}\tan^2\theta_W \Gamma(W^3_{2\,\text{TeV}} \to u\bar{u}) \simeq 4.86 \text{ GeV} , \quad (4.30)$$

$$\Gamma(B_{2\,\text{TeV}} \to d\bar{d}) = \tfrac{5}{9}\tan^2\theta_W \Gamma(W^3_{2\,\text{TeV}} \to d\bar{d}) \simeq 1.43 \text{ GeV} . \quad (4.31)$$

Applying (4.14) and (4.25) to (4.27) and (4.28), we find

$$\Gamma(\rho_T^+ \to u\bar{d}) = \Gamma(\rho_T^- \to d\bar{u}) = \frac{1}{4} \frac{g^2}{4\pi} \frac{4\pi}{g^2_{\rho\pi\pi}} \Gamma(W^+_{2\,\text{TeV}} \to u\bar{d}) . \quad (4.32)$$

We use $g^2/4\pi = \alpha/\sin^2\theta_W$, $\alpha = \frac{1}{128}$ (the value at M_W; in fact a slightly larger value would be appropriate for 2 TeV); $\sin^2\theta_W = 0.23$, and $g^2_{\rho\pi\pi}/4\pi = M_0^2/(8\pi v^2)$, as indicated. We then find

$$\Gamma(\rho_T^+ \to u\bar{d}) = \Gamma(\rho_T^- \to d\bar{u}) = \frac{\alpha M_W^2}{2M_0 \sin^2\theta_W} = 55.7 \text{ MeV} , \quad (4.33a)$$

$$\Gamma(\rho_T^0 \to u\bar{u}) = \Gamma(\rho_T^0 \to d\bar{d}) = \tfrac{1}{2}\Gamma(\rho_T^+ \to u\bar{d}) = 27.9 \text{ MeV} , \quad (4.33b)$$

where the values are quoted for $M_0 = 2$ TeV/c^2.

A similar calculation for $\omega_T \to q\bar{q}$ yields

$$\Gamma(\omega_T \to q\bar{q}) = \frac{\alpha M_W^2 \sin^2\theta_W}{M_0 \cos^4\theta_W} (2A-1)^2 [Y(q_L)^2 + Y(q_R)^2] , \quad (4.34)$$

or, for $M_0 = 2$ TeV/c^2,

$$\Gamma(\omega_T \to u\bar{u}) = 18.8 \text{ MeV}(2A-1)^2 , \quad (4.35a)$$

$$\Gamma(\omega_T \to d\bar{d}) = 5.52 \text{ MeV}(2A-1)^2 . \quad (4.35b)$$

The partial widths for ρ_T and ω_T decays into quark pairs are summarized in Table I.

TABLE I. Partial widths for $q\bar{q}$ decays of ρ_T and ω_T of mass 2 TeV/c^2.

Decaying particle	Mode	Partial width (MeV)
ρ_T^+	$u\bar{d}$	55.7
ρ_T^-	$d\bar{u}$	55.7
ρ_T^0	$u\bar{u}$	27.9
	$d\bar{d}$	27.9
ω_T	$u\bar{u}$	$18.8(2A-1)^2$
	$d\bar{d}$	$5.5(2A-1)^2$

With $M=2$ TeV/c^2 in Eq. (4.22), and with the partial widths just calculated, we find

$$\sigma(pp\to\rho_T^{\pm}+\cdots)=3.57\times10^{-38}\ \mathrm{cm}^2\left[\tau\frac{dL}{d\tau}\right]_{u\bar d,\ d\bar u}, \quad (4.36)$$

$$\sigma(pp\to\rho_T^{0}+\cdots)=[\sigma(pp\to\rho_T^{+}+\cdots)+\sigma(pp\to\rho_T^{-}+\cdots)]/2\ , \quad (4.37)$$

$$\sigma(pp\to\omega_T+\cdots)=\left[1.20\times10^{-38}\ \mathrm{cm}^2\left[\tau\frac{dL}{d\tau}\right]_{u\bar u}+3.53\times10^{-39}\ \mathrm{cm}^2\left[\tau\frac{dL}{d\tau}\right]_{d\bar d}\right]\times(2A-1)^2\ . \quad (4.38)$$

The appropriate luminosities for $\sqrt{s}=17$ and 40 TeV, based on structure functions from Ref. 33 (set 1, with $\Lambda=200$ MeV), and the corresponding cross sections, are summarized in Table II.

The ρ_T and ω_T are to be detected via their (predominant) decays to longitudinal gauge bosons:

$$\rho_T^{\pm}\to W_L^{\pm}Z_L\ , \quad (4.39a)$$

$$\rho_T^{0}\to W_L^{+}W_L^{-}\ , \quad (4.39b)$$

$$\omega_T\to W_L^{+}W_L^{-}Z_L\ , \quad (4.39c)$$

in analogy with $\rho\to2\pi$ and $\omega\to\pi^+\pi^-\pi^0$ decays. The cross sections in Table II for production of ω_T should be compared with those which would be obtained by convoluting the values of $\tau dL/d\tau$ quoted there (essentially, effective $q\bar q$ luminosities[33]) with the standard-model values[50]

$$\sigma\left[\begin{Bmatrix}u\bar u\\ d\bar d\end{Bmatrix}\to W_L^{+}W_L^{-}Z_L\right]=\begin{Bmatrix}4\\5\end{Bmatrix}\times10^{-40}\ \mathrm{cm}^2 \quad (4.40)$$

arising from diagrams involving Higgs-boson–gauge-boson couplings. Unfortunately, the ω_T signal in the Drell-Yan process may be buried by the production of transverse gauge bosons in the standard model.[51]

The cross sections for production of the ρ_T states (see Ref. 52 for comparable estimates) are small but just barely measurable, requiring an integrated luminosity of 10^{40} $\mathrm{cm}^{-2}=10\ \mathrm{fb}^{-1}$ at $\sqrt{s}=40$ TeV or some five times that figure at $\sqrt{s}=17$ TeV. However, the cross sections for ω_T production contain the unknown factor $(2A-1)^2$. As we shall note presently, this factor could easily be very small or even zero. Moreover, the detection of ω_T may be rather challenging. It is this secretive nature of the ω_T that has led to our appeal for clever suggestions for its detection.

C. Effective-gauge-boson processes

Another process which has received much attention for production of resonances by pairs of gauge bosons is illustrated in Figs. 7 and 8 for ρ_T and ω_T production.

In Fig. 7, the $\rho_T^{\pm}$-$W^{\pm}$-Z and ρ_T^0-W^+-W^- couplings are the analogue of the $\rho\pi\pi$ coupling, for which $\Gamma(\rho\pi\pi)\simeq150$ MeV. We estimated in Sec. II that $\Gamma(\rho_T^{\pm}\to W_L^{\pm}Z)=\Gamma(\rho_T^0\to W_L^+W_L^-)\simeq400$ GeV. It has been estimated[52] that the process of Fig. 7 and the Drell-Yan process of the previous subsection give comparable contributions to ρ_T production. Our calculations, to be presented below, confirm these estimates.

The ω_T-Z_L-W_T^3 coupling of Fig. 8, on the other hand, is the analogue of the ω-π^0-γ coupling. The partial width $\Gamma(\omega\to\pi^0\gamma)$ is less than 1 MeV. The corresponding partial decay width for ω_T is

$$\Gamma(\omega_T\to Z_L W_T^3)=\left[\frac{g}{2g_{\rho\pi\pi}}\right]^2\frac{g^2_{\rho_T\pi\omega_T}m(\omega_T)^3}{96\pi}\ , \quad (4.41)$$

in analogy with Eqs. (3.9) and (3.10), if the effective ω_T-ρ_T-W_L coupling is

$$L_{\mathrm{eff}}=g_{\rho_T\pi\omega_T}\epsilon_{\alpha\beta\gamma\delta}\partial^\alpha\rho_T^\beta\partial^\gamma\omega_T^\delta\cdot\mathbf{W}_L \quad (4.42)$$

in analogy with Eq. (3.4). Pursuing the analogy with hadron physics further, we may use Eq. (3.26) and the KSRF relation to write

$$\Gamma(\omega_T\to Z_L W_T^3)=\frac{g^2}{4\pi}\left[\frac{g^2_{\rho\pi\pi}}{4\pi}\right]^2\frac{N_s^2}{192\pi^2}M_0 \quad (4.43a)$$

$$\approx280\ \mathrm{MeV}(N_s^2)\ , \quad (4.43b)$$

where we recall $M_0\equiv2$ TeV/c^2 and N_s is the number of members of the fundamental representation of the superstrong-interaction group. Equation (4.43b) implies that $\Gamma(\omega_T\to Z_L W_T^3)/\Gamma(\rho_T)\approx10^{-2}(N_s/4)^2$, and we would expect the cross sections described by Figs. 8 and 7 to be roughly in the same ratio.

A further contribution to ω_T production, of the same order of magnitude as that shown in Fig. 8, comes from $W_L^+W_T^-$ and $W_L^-W_T^+$ fusion. Whereas ω can decay only to a neutral pion and a gauge boson (the photon), ω_T can decay to $W_L^+W_T^-$ and $W_L^-W_T^+$ as well as to $Z_LW_T^3$. Each

TABLE II. Parton-parton luminosities in pp collisions and corresponding cross sections for production of ρ_T and ω_T with mass 2 TeV/c^2 via the Drell-Yan process.

$\sqrt{s}$ (TeV)	$\left[\tau\frac{dL}{d\tau}\right]_{u\bar u}=\left[\tau\frac{dL}{d\tau}\right]_{u\bar d}$	$\left[\tau\frac{dL}{d\tau}\right]_{d\bar d}=\left[\tau\frac{dL}{d\tau}\right]_{d\bar u}$	Cross sections (units of 1 fb$=10^{-39}$ cm^2) $pp\to\rho_T^+$	$pp\to\rho_T^-$	$pp\to\rho_T^0$	$pp\to\omega_T$
17	0.174	6.77×10^{-2}	6.2	2.4	4.3	$2.3\times(2A-1)^2$
40	0.932	0.452	33	16	25	$12.8\times(2A-1)^2$

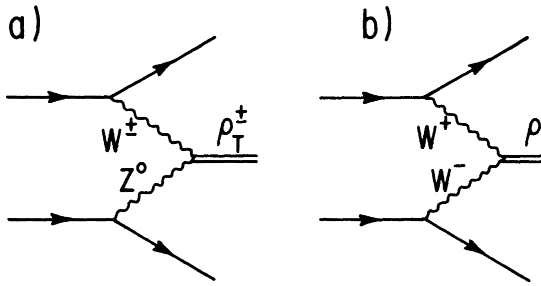

FIG. 7. Effective-gauge-boson processes contributing to ρ_T production. (a) $\rho_T^{\pm}$ production; (b) ρ_T^0 production.

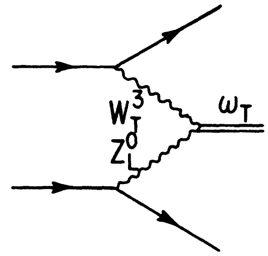

FIG. 8. Example of graph for effective-gauge-boson production of ω_T. Other contributions come from $W_L^+ W_T^-$ and $W_L^- W_T^+$ fusion.

decay amplitude involves the effective Lagrangian (4.42) and the ρ_T-W_T coupling, and so

$$\Gamma(\omega_T \to W_L^+ W_T^-) = \Gamma(\omega_T \to W_L^- W_T^+) \approx 280\ \mathrm{MeV}(N_s^2)\ , \tag{4.44}$$

as in Eq. (4.43).

To calculate the cross sections corresponding to Figs. 7 and 8, we need the effective luminosity of a gauge-boson pair $V_1 V_2$ in a system composed of hadrons A and B:

$$\left.\frac{dL}{d\tau}\right|_{AB/V_1V_2} = \sum_{i,j} \int_\tau^1 \frac{d\tau'}{\tau'} \int_{\tau'}^1 \frac{dx}{x} f_i^{(A)}(x) f_j^{(B)}(\tau'/x) \times \left.\frac{dL}{d\xi}\right|_{q_i q_j/V_1 V_2}\ , \tag{4.45}$$

where

$$\xi \equiv \tau/\tau'\ , \tag{4.46}$$

and $\tau = M^2/s$ as in Sec. IV B. In order to estimate $(dL/d\xi)_{q_i q_j/V_1 V_2}$ we use the effective gauge-boson approximation,[53] in which the gauge bosons are considered as partons inside the quarks. One computes $f_{q/V}$, which represents the number of gauge bosons V in a quark carrying a fraction x of the quark's momentum. In terms of these distribution functions, we have

$$\left.\frac{dL}{d\xi}\right|_{q_i q_j/V_1 V_2} = \int_\xi^1 \frac{dx}{x} \left[f_{q_i/V_1}(x) f_{q_j/V_2}\left(\frac{\xi}{x}\right) + f_{q_i/V_2}(x) f_{q_j/V_1}\left(\frac{\xi}{x}\right) \right]\ . \tag{4.47}$$

At high energies $(E \gg M_W)$ the distribution functions which we shall need are given by

$$f_{u/W_T^+}(x) = f_{d/W_T^-}(x) = f_{\bar u/W_T^-}(x) = f_{\bar d/W_T^+}(x) = 2 f_{u/W_T^3}(x) = 2 f_{d/W_T^3}(x) = \frac{g^2}{32\pi^2 x}[x^2 + 2(1-x)] \ln(4E^2/M_W^2)\ , \tag{4.48a}$$

$$f_{u/W_L^+}(x) = f_{d/W_L^-}(x) = f_{\bar u/W_L^-}(x) = f_{\bar d/W_L^+}(x) = \frac{g^2}{16\pi^2 x}(1-x)\ , \tag{4.48b}$$

$$f_{u/Z_L}(x) = f_{\bar u/Z_L}(x) = \frac{g^2}{16\pi^2 x \cos^2\theta_W}[(\tfrac{1}{2} - \tfrac{4}{3}\sin^2\theta_W)^2 + (\tfrac{1}{2})^2](1-x)\ , \tag{4.49a}$$

$$f_{d/Z_L}(x) = f_{\bar d/Z_L}(x) = \frac{g^2}{16\pi^2 x \cos^2\theta_W}[(-\tfrac{1}{2} + \tfrac{2}{3}\sin^2\theta_W)^2 + (\tfrac{1}{2})^2](1-x)\ . \tag{4.49b}$$

In Eq. (4.48a) we take $4E^2 = M^2$, where M is the effective mass of the two-gauge-boson system.

We first calculate ρ_T^0 production, arising from $W_L^+ W_L^-$ fusion, and $\rho_T^{\pm}$ production, arising from $Z_L W_L^{\pm}$ fusion. These processes were also calculated in Ref. 22. The convolutions in Eq. (4.47) are

$$\left.\frac{dL}{d\xi}\right|_{q_i q_j/W_L^+ W_L^-} = \frac{1}{\xi}\left[\frac{g^2}{16\pi^2}\right]^2 \times \left[(1+\xi)\ln\frac{1}{\xi} - 2(1-\xi)\right]\ , \tag{4.50}$$

for those $q_i q_j$ pairs capable of emitting $W^+ W^-$, and

$$\left.\frac{dL}{d\xi}\right|_{q_i q_j/Z_L W_L^{\pm}} = \frac{1}{\xi \cos^2\theta_W}\left[\frac{g^2}{16\pi^2}\right]^2 \times \left[(1+\xi)\ln\frac{1}{\xi} - 2(1-\xi)\right] \times \{[I_3(q_{iL}) - 2Q_{qi}\sin^2\theta_W]^2 + [I_3(q_{iL})]^2\}\ , \tag{4.51}$$

for those $q_i q_j$ pairs capable of emitting $Z_L W_L^{\pm}$, where

$q_i \to Z_L$ and $q_j \to W_L^{\pm}$. A corresponding expression, with $i \to j$ on the right-hand side, applies for the case $q_i \to W_L^{\pm}, q_j \to Z_L$. Now we can compute the terms due to specific quarks i and j entering into the sum in Eq. (4.45):

$$\tau \left[\frac{dL}{d\tau} \right]_{ij} \Bigg|_{pp/V_1V_2} \equiv \tau \int_\tau^1 \frac{d\tau'}{\tau'} \int_{\tau'}^1 \frac{dx}{x} f_i^{(p)}(x) f_j^{(p)}(\tau'/x) \times \frac{dL}{d\xi} \Bigg|_{q_iq_j/V_1V_2} . \quad (4.52)$$

These terms and their total contribution to Eq. (4.45) are summarized in Table III.

The cross sections $\sigma_{\rm eff}$ for production of ρ_T^0 and $\rho_T^{\pm}$ with mass M via gauge-boson fusion are related to their partial decay widths as follows:

$$\sigma_{\rm eff}(pp \to \rho_T^0 + \cdots) = \frac{16\pi^2}{M^3} \Gamma(\rho_T^0 \to W_L^+ W_L^-) \times \tau \frac{dL}{d\tau} \Bigg|_{pp/W_L^+ W_L^-} , \quad (4.53)$$

$$\sigma_{\rm eff}(pp \to \rho_T^{\pm} + \cdots) = \frac{16\pi^2}{M^3} \Gamma(\rho_T^{\pm} \to Z_L W_L^{\pm}) \times \tau \frac{dL}{d\tau} \Bigg|_{pp/Z_L W_L^{\pm}} . \quad (4.54)$$

These are shown in Table III for $\sqrt{s} = 17$ and 40 TeV.

For ω_T production, one gauge boson must be transverse while the other is longitudinal. The convolutions in Eq. (4.47) are

$$\frac{dL}{d\xi} \Bigg|_{q_iq_j/W_L^{\pm} W_T^{\mp}} = \frac{1}{2\xi} \left[\frac{g^2}{16\pi^2} \right]^2 \times \left[2(1+\xi)\ln\frac{1}{\xi} - \frac{(1-\xi)(7+\xi)}{2} \right] \times \ln(4E^2/M_W^2) \quad (4.55)$$

for those q_iq_j pairs capable of emitting W^+W^-, and

$$\frac{dL}{d\xi} \Bigg|_{q_iq_j/Z_L W_T^3} = \frac{1}{\xi \cos^2\theta_W} \left[\frac{g^2}{32\pi^2} \right]^2 \times \left[2(1+\xi)\ln\frac{1}{\xi} - \frac{(1-\xi)(7+\xi)}{2} \right] \times \{ [I_3(q_{iL}) - 2Q_{q_i}\sin^2\theta_W]^2 + [I_3(q_{iL})]^2 + (i \to j) \} \ln(4E^2/M_W^2) \quad (4.56)$$

for any pair of quarks. These terms and their total contribution to Eq. (4.45) are shown in Table IV.

The cross sections $\sigma_{\rm eff}$ for production of ω_T with mass M via gauge-boson fusion are analogous to Eqs. (4.53) and (4.54) for ρ_T production, but the overall coefficient is half that in Eqs. (4.53) and (4.54) as a result of the polarization average over the transverse W:

$$\sigma_{\rm eff}(pp \to \omega_T + \cdots) = \frac{8\pi^2}{M^3} \left[\Gamma(\omega_T \to W_L^+ W_T^-) \tau \frac{dL}{d\tau} \Bigg|_{pp/W_L^+ W_T^-} + \Gamma(\omega_T \to W_L^- W_T^+) \tau \frac{dL}{d\tau} \Bigg|_{pp/W_L^- W_T^+} + \Gamma(\omega_T \to Z_L W_T^3) \tau \frac{dL}{d\tau} \Bigg|_{pp/Z_L W_T^3} \right] . \quad (4.57)$$

For $M = 2$ TeV/c^2, with the partial widths given in Eqs. (4.43b) and (4.44), we find the contributions to the cross section listed in Table IV.

The production of ρ_T via gauge-boson fusion occurs at a rate comparable to that due to the Drell-Yan process, as mentioned in Ref. 22. The cross section for produc-

TABLE III. Effective luminosities $\tau(dL/d\tau)$ and cross sections for gauge-boson collisions in pp interactions at $\sqrt{s} = 17$ and 40 TeV, producing ρ_T of mass 2 TeV/c^2. Partial contributions [Eq. (4.52)] and their sums [Eq. (4.45)] are shown.

Vector-meson pair / Quark pair	$W_L^+ W_L^-$		$Z_L W_L^+$		$Z_L W_L^-$	
	17 TeV	40 TeV	17 TeV	40 TeV	17 TeV	40 TeV
$u\bar{u} + \bar{u}u$	3.7×10^{-8}	9.6×10^{-7}	1.4×10^{-8}	3.6×10^{-7}	1.4×10^{-8}	3.6×10^{-7}
$ud + du$	2.8×10^{-7}	3.8×10^{-6}	1.3×10^{-7}	1.8×10^{-6}	1.0×10^{-7}	1.4×10^{-6}
$\bar{d}d + d\bar{d}$	1.1×10^{-8}	3.6×10^{-7}	5.4×10^{-9}	1.8×10^{-7}	5.4×10^{-9}	1.8×10^{-7}
$\bar{d}\bar{u} + \bar{u}\bar{d}$	9.8×10^{-10}	6.7×10^{-8}	3.7×10^{-10}	2.5×10^{-8}	4.7×10^{-10}	3.2×10^{-8}
uu			3.0×10^{-7}	3.3×10^{-6}	0	0
$\bar{u}\bar{u}$			0	0	3.7×10^{-10}	2.5×10^{-8}
dd			0	0	4.5×10^{-8}	7.4×10^{-7}
$\bar{d}\bar{d}$			4.7×10^{-10}	3.2×10^{-8}	0	0
$u\bar{d} + \bar{d}u$			3.1×10^{-8}	8.2×10^{-7}	0	0
$d\bar{u} + \bar{u}d$			0	0	9.6×10^{-9}	3.1×10^{-7}
Total	3.3×10^{-7}	5.2×10^{-6}	4.8×10^{-7}	6.5×10^{-6}	1.8×10^{-7}	3.0×10^{-6}
$\sigma_{\rm eff}(pp \to \rho_T)$	ρ_T^0		ρ_T^+		ρ_T^-	
(units of 10^{-39} cm^2=1 fb)	1.0	16	1.5	20	0.55	9.3

TABLE IV. Effective luminosities $\tau(dL/d\tau)$ and cross sections for gauge-boson collisions in pp interactions at $\sqrt{s}=17$ and 40 TeV, producing ω_T of mass 2 TeV/c^2. Partial contributions [Eq. (4.52)] and their sums [Eq. (4.45)] are shown.

Vector-meson pair / Quark pair	$W_T^+W_L^- + W_L^+W_T^-$ 17 TeV	$W_T^+W_L^- + W_L^+W_T^-$ 40 TeV	$Z_L W_T^3$ 17 TeV	$Z_L W_T^3$ 40 TeV
$u\bar{u}+\bar{u}u$	9.1×10^{-7}	1.9×10^{-5}	1.7×10^{-7}	3.6×10^{-6}
$ud+du$	6.1×10^{-6}	6.8×10^{-5}	1.3×10^{-6}	1.5×10^{-5}
$\bar{d}d+d\bar{d}$	2.9×10^{-7}	7.5×10^{-6}	6.9×10^{-8}	1.8×10^{-6}
$\bar{d}\bar{u}+\bar{u}\bar{d}$	2.8×10^{-8}	1.5×10^{-6}	6.0×10^{-9}	3.2×10^{-7}
uu			1.6×10^{-6}	1.5×10^{-5}
$\bar{u}\bar{u}$			2.6×10^{-9}	1.4×10^{-7}
dd			2.6×10^{-7}	3.5×10^{-6}
$\bar{d}\bar{d}$			3.4×10^{-9}	1.8×10^{-7}
$u\bar{d}+\bar{d}u$			1.9×10^{-7}	4.1×10^{-6}
$d\bar{u}+\bar{u}d$			6.1×10^{-8}	1.6×10^{-6}
Total	7.3×10^{-6}	9.7×10^{-5}	3.6×10^{-6}	4.4×10^{-5}
$\sigma_{\rm eff}(pp\to\omega_T)$ (units of 10^{-39} cm^2=1 fb)	$0.008N_s^2$	$0.105N_s^2$	$0.004N_s^2$	$0.048N_s^2$

tion of ω_T is sensitive to N_s. [Recall we have assumed the underlying substructure to be described by a group SU(N_s).] For example, with $N_s=4$, at $\sqrt{s}=40$ TeV, we find a total cross section of

$$\sigma_{\rm eff}(pp\to\omega_T+\cdots)\approx 2.4\ \text{fb}=2.4\times10^{-39}\ \text{cm}^2\ . \tag{4.58}$$

The cross section does not approach observable values unless N_s is quite large, in which case our assumption that hadron physics can simply be scaled by v/f_π probably breaks down. In that case, the cross section for ω_T production is likely to be larger than our estimates, simply because the ω_T is likely to be lighter than 2 TeV.

The ω_T thus remains an elusive particle, even when we know in principle that it can be produced via gauge-boson fusion.

V. A SPECIFIC SUBSTRUCTURE

The Drell-Yan production of ω_T was seen in the previous section to depend on the charges (4.15) and (4.16) of its constituents: $\sigma(\omega_T)\sim(2A-1)^2$. In this section we shall discuss one model in which $A=\frac{1}{2}$, and hence in which the ω_T does not mix at all with the isoscalar gauge boson B.

We assume that the longitudinal Z behaves very much like a π^0, in the spirit of the discussion in Sec. II. Now, a π^0 can decay to two photons, with an amplitude proportional to $Q_u^2-Q_d^2=Q_u+Q_d$ (since $Q_u-Q_d=1$). The Higgs sector's analogue of a π^0, composed of $(U\bar{U}-D\bar{D})/\sqrt{2}$, also should be able to decay to two photons, with an amplitude proportional to $Q_u+Q_d=A+(A-1)=2A-1$. But if the longitudinal Z is analogous to the pseudoscalar boson composed of $(U\bar{U}-D\bar{D})/\sqrt{2}$, an anomalous Z-γ-γ coupling will be generated unless $A=\frac{1}{2}$. The ω_T then cannot mix with B, and cannot be produced via the Drell-Yan ($q\bar{q}$ fusion) process.

The hypothesis $A=\frac{1}{2}$ for subunits of Higgs bosons, as well as of quarks and leptons, has appeared previously.[33,54,55] Indeed, a related argument based on anomaly cancellation appears in Ref. 54.

If ω_T is so hard to produce, how do we probe compositeness in the Higgs sector? A crucial question is whether this compositeness carries over in any way to the quarks and leptons. If it does so, and the quarks and leptons share some subunits with the Higgs bosons and their excitations, one is confronted with a host of theoretical difficulties.[56] The most notable of these is the likely appearance of flavor-changing processes. Present limits on strangeness-conserving amplitudes[57] indicate (from Bhabha scattering, for example) that

$$\frac{g_0^2}{\Lambda_c^2}\lesssim\left[\frac{1}{1\ \text{TeV}}\right]^2 , \tag{5.1}$$

where g_0 is a strangeness-conserving Yukawa coupling, and Λ_c is a compositeness scale. Limits from K-$\bar{K}$ mixing[58] imply

$$\frac{g_1^2}{\Lambda_c^2}\lesssim\left[\frac{1}{1000\ \text{TeV}}\right]^2 , \tag{5.2}$$

where g_1 describes a $\Delta S=1$ Yukawa coupling. Then $\Delta S=1$ processes should be characterized by amplitudes no larger than

$$\frac{g_0 g_1}{\Lambda_c^2}\lesssim\left[\frac{1}{30\ \text{TeV}}\right]^2 , \tag{5.3}$$

the geometric mean of Eqs. (5.1) and (5.2). Normally such amplitudes would be obscured by other contributions to the weak interactions, but the process $K_L\to\pi^0 e^+e^-$ is highly suppressed in the standard electroweak model by CP considerations.[59] An anomalous $\Delta S=1$ interaction which signals compositeness could well make its appearance in such a process.

The amplitude for $K_L \to \pi^{\pm} e^{\mp} \nu$ is characterized by a strength typical of $G_F \sin\theta_c / \sqrt{2} \approx (\frac{1}{5})/(300\ \text{GeV})^2$. Comparing this with Eq. (5.3), we see that

$$\frac{A(K_L \to \pi^0 e^+ e^-)}{A(K_L \to \pi^{\pm} e^{\mp} \nu)} \lesssim 5 \times 10^{-4} , \quad (5.4)$$

or

$$B(K_L \to \pi^0 e^+ e^-) \lesssim 10^{-7} . \quad (5.5)$$

This lies at the limits of present searches.[60] Higher sensitivity is expected soon, both in $K_L \to \pi^0 e^+ e^-$ (Refs. 60 and 61) and in the related process $K^+ \to \pi^+ e^+ e^-$ (Ref. 62).

VI. CONCLUSIONS

If the electroweak symmetry-breaking sector becomes strongly interacting above 1 TeV, there undoubtedly exists a deeper substructure to Higgs bosons and to longitudinally polarized W's and Z's. We have examined some ways of probing the most conventional of such substructures, in which a single complex doublet of Higgs fields is composed out of fundamental fermions of two different charges A and $A-1$.

We find that, as in the strong interactions, an isovector meson with $J=1$ (called ρ_T here) is expected to exist at a mass whose scale is set by the chiral-symmetry-breaking parameter $v = 2^{-1/4} G_F^{-1/2} = 246$ GeV. The corresponding parameter for the strong interactions is $f_\pi = 93$ MeV. Thus, one expects this vector meson to have a mass $m(\rho_T) = (v/f_\pi) m_\rho \approx 2\ \text{TeV}/c^2$ on rather general grounds, and experiments regarding such a meson are unlikely to probe its underlying structure, just as the decay $\rho \to \pi\pi$ tells us very little about quarks and gluons. [The key ingredient of the prediction of $m(\rho_T)$ is the analogy with the second of Ref. 24, in which crossing symmetry plays a crucial role. If one simply unitarizes the low-energy amplitude, as in Ref. 63, rather different results can be obtained.]

An isoscalar meson ω_T with $J=1$ is also expected around 2 TeV$/c^2$ in any subconstituent model which gives rise to ρ_T. Its coupling to the isoscalar gauge boson B is proportional to $2A-1$, and hence provides valuable information on subconstituents. We have noted, however, that one particular model of such subconstituents yields $A = \frac{1}{2}$ unless additional fermions are present to cancel anomalies. It appears difficult, but perhaps not impossible, to produce ω_T in Drell-Yan processes at multi-TeV pp colliders. The exception occurs for $A = \frac{1}{2}$, when alternate methods must be found.

The gauge boson-fusion method for producing ω_T has also been examined. Cross sections are found to be disappointingly small unless the gauge group for the interactions of subconstituents is very large. (The analogue of $\pi\pi \to \pi\omega$ may be useful for producing ω_T via the gauge-boson-fusion process. We are currently investigating this suggestion in more detail, and thank S. Sharpe for proposing it.)

(The production of ρ_T and ω_T at $\sqrt{s} = 40$ TeV has been considered recently in Ref. 64. As in Ref. 43, the masses of these particles are assumed to scale as $N^{-1/2}$.)

We are left with a dilemma. If the Higgs sector is strongly interacting, and a replay of QCD occurs at $v/f_\pi \approx 2650$ times the energy, what will shed light on the analogue of quarks and gluons for these new strong interactions? The ω_T could be the lightest particle in the spectrum carrying any characteristic information for the underlying theory. To use such a particle as an effective probe remains a challenge.

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