Limits on photino and squark masses from proton lifetime in supergravity models

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It is shown that the current experimental lower limit on the proton lifetime via $p \to \bar{\nu} K^+$ puts severe constraints on the photino and squark masses in supersymmetric unification. For Higgs-triplet masses $\lesssim 10^{16}$ GeV (as required to maintain the gauge hierarchy in supergravity models) the Kamioka data imply that squarks would be so heavy as not to be observable at the Fermilab Tevatron (and probably not observable even at the Superconducting Super Collider) for photinos heavier than ~ 10 GeV. (W-ino and gluino signals would then still be possible signals of supersymmetry.) For very light photinos, a region of squark mass accessible to the Tevatron is still possible, due to an "accidental" cancellation of the low-lying and high-lying W-ino contribution to the decay amplitude.

I. INTRODUCTION

Lower bounds on the proton lifetime have been steadily increasing, and have for some time now eliminated the minimal SU(5) grand-unified-theory (GUT) model.[1] These enhanced lower bounds have begun also to significantly constrain supersymmetric (SUSY) grand unified models.[2,3] Thus SUSY models with two generations and low squark and slepton masses (i.e., ~ 100 GeV) can already be ruled out.[2] Models with three or more generations with constructive interference among generations make the disagreement with experiment even sharper. A possible resolution of this conflict was suggested some time ago if one allows for a distructive interference among generations.[4,5] Explicit models within the framework of $N=1$ supergravity unification[6] were exhibited,[4] consistent with the experimental bounds on nucleon decay modes then available.

Proton decay in supergravity models proceeds mainly through the exchange of superheavy Higgsino and Higgs color-triplet particles followed by gaugino dressing.[7] (See Fig. 1.) There are a number of contributions to the decay amplitude:[4] W-ino ($\tilde{W}$) dressing, gluino ($\tilde{g}$) dressing, Z-ino ($\tilde{Z}$), and photino ($\tilde{\gamma}$) dressing, as well as right-handed dimension-5 contributions. For $m_{\tilde{g}} \gtrsim 50$ GeV, the current experimental bound on the gluino mass,[8] the gluino contribution is larger when the up and down squarks ($\tilde{q}$) in the first two generations are not degenerate.[4] Since this would again produce disagreement with experiment, we will assume here that the squarks are nearly degenerate (a situation which arises naturally when either the squarks are much heavier than the Z boson or the Higgs mixing angle α_H is close to 45°). The Z-ino and photino dressing and right-handed contributions are generally small. We will assume in the following, therefore, that the dominant contribution to the decay amplitude comes from W-ino dressing (with or without higher-generation cancellations), and will compare the current data[9] with the theoretical predictions for the $p \to \bar{\nu} + K^+$ decay mode.

The theoretical formulas for proton decay in supersymmetry depend on the value of the Higgs-triplet mass M_H. In supergravity models, the known models[10] which maintain the gauge hierarchy require[2] $M_H \lesssim M_{\rm GUT}$ (and often M_H must be much less than the GUT mass). For the standard two-Higgs-doublet model one has

$$0.4\times 10^{16} \lesssim M_{\rm GUT} \lesssim 1.6\times 10^{16}\ \text{GeV}\ . \qquad (1.1)$$

We will assume here then, that

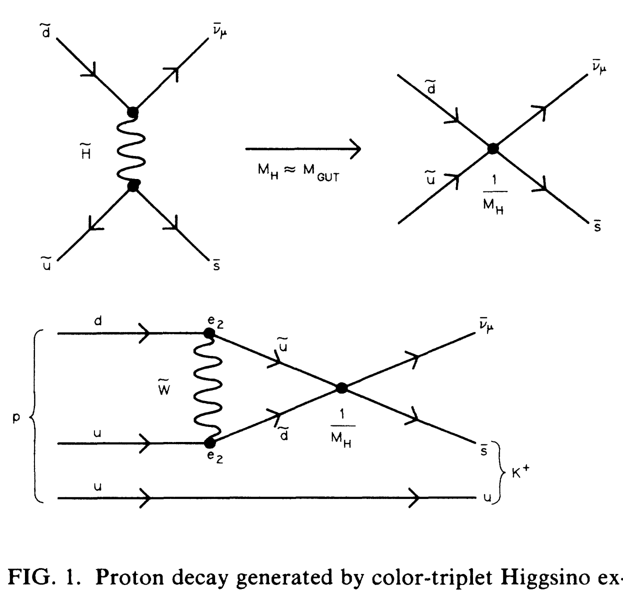

FIG. 1. Proton decay generated by color-triplet Higgsino exchange and W-ino dressing.

$$M_H \lesssim 10^{16}\ \text{GeV}\ , \tag{1.2}$$

a condition that is also well satisfied by superstring models with an intermediate mass scale.[11]

Recently, there has been some interest in supersymmetry models with heavy squarks and heavy gluinos[12–14] (e.g., $m_{\tilde q} \gtrsim 500$ GeV). Such situations can occur in superstring models if one assumes that supersymmetry breaking arises from a gaugino condensate in the hidden E_8' sector, leading to soft-breaking gaugino masses in the physical sector at the compactification scale.[15] In this paper we will leave the squark and photino mass *a priori* arbitrary, and see what constraints proton decay data impose upon them. Remarkably, the existing data strongly restrict $m_{\tilde\gamma}$ and $m_{\tilde q}$. Thus for $m_{\tilde\gamma} \gtrsim 10$ GeV the data imply that the squark mass must generally exceed 1 TeV [and hence will be difficult to detect even at the Superconducting Super Collider[14] (SSC)]. Only for $m_{\tilde\gamma} < 10$ GeV are lighter squarks still possible due to a cancellation between the two W-ino dressing loop integrals from proton decay amplitude.

In Sec. II constraints on the W-ino dressing loop integrals from proton decay data are obtained. In Sec. III these constraints are used to obtain limits on the photino and squark masses. Section IV is devoted to a discussion of the results.

II. PROTON LIFETIME CONSTRAINT

W-ino dressing usually leads to $\bar\nu K$ as the dominant nucleon decay mode in supersymmetry (see Fig. 1), and experimentally,[9] the strongest partial lifetime bound is for $p \to \bar\nu + K^+$. The 90% confidence limits (C.L.) are

$$\tau/B(p \to \bar\nu + K^+) \geq 7\times 10^{31}\ \text{yr}\quad \text{Kamioka}\ , \tag{2.1a}$$

$$\tau/B(p \to \bar\nu + K^+) \geq 1.5\times 10^{31}\ \text{yr}\quad \text{IMB}\ . \tag{2.1b}$$

We shall therefore utilize the lower bounds of Eq. (2.1) to obtain constraints on the SUSY spectrum that enters the decay of Eq. (2.1). The partial width for this mode is

$$\Gamma(p \to \bar\nu K^+) = \sum_i \Gamma(p \to \bar\nu_i K^+)\ , \tag{2.2}$$

where i is the generation index. Dominant contributions arise from $i = \mu$ or τ, and for SU(5) GUT one may write the decay rate as[4]

$$\Gamma(p \to \bar\nu_\mu K^+) \simeq \frac{\beta^2}{M_H^2}\frac{m_N}{32\pi f_\pi^2}\left[1 - \frac{m_K^2}{m_N^2}\right]^2 \times |A_{\nu_\mu K}|^2 |C_{\nu_\mu K}|^2 \times |1 + y^{tK}|^2 A_L^2 (A_S^L)^2\ , \tag{2.3}$$

where

$$A_{\nu_\mu K} = \alpha_2^2 (M_W^2 2\sin 2\alpha_H)^{-1} P_2 m_s m_c V_{21}^\dagger V_{21} V_{22} \times [F(\tilde c;\tilde s;\tilde W) + F(\tilde c;\tilde\mu;\tilde W)]\ , \tag{2.4}$$

$$C_{\nu_\mu K} = \left[1 + \frac{m_N(D+3F)}{3m_B}\right] - \frac{2}{3}\frac{m_N}{m_B}D\ . \tag{2.5}$$

In writing Eq. (2.3) we have assumed degeneracy among the d-squarks and degeneracy among the sleptons, and have factored out the second-generation contribution to Eq. (2.3). Thus y^{tK} (defined in Ref. 4) represents the additional third-generation contribution. V_{ij} are the Kobayashi-Maskawa (KM) matrix elements and m_s, m_c, and m_t are the quark masses. f_π, D, F, m_N, and m_B are the chiral Lagrangian factors (defined in Ref. 16), m_K is the K-meson mass, and M_W the W-boson mass. A_L and A_S^L are the long-range and short-range renormalization-group (RG) suppression factors,[17] β is the three-quark matrix element of the nucleon wave function, and P_i are diagonal phases in generation space.[4] The F functions are the triangle loop dressing integrals of Fig. 1 (defined in Ref. 4) and α_H is the Higgs mixing angle describing SU(2)×U(1) breaking[6] and defined in the Appendix.

The quantity β will play an important role in our discussion in Sec. III. A variety of calculations exist in the literature regarding its evaluation. They have the range[18]

$$0.003 \leq \beta \leq 0.03\ \text{GeV}^3\ . \tag{2.6}$$

However, evaluations of β using current algebra,[18] and recent lattice gauge theory calculations[19,20] favor the higher value of β near 0.03 GeV^3. Also as discussed in Sec. I, M_H obeys Eq. (1.2).

The experimental constraint Eq. (2.1a) of Kamioka may now be translated into the following inequality:

$$|B| \leq 3.5\times 10^{-6}\ \text{GeV}^{-1}\left[\frac{M_H}{10^{16}\ \text{GeV}}\right] \times \left[\frac{0.03\ \text{GeV}^3}{\beta}\right]\frac{1}{|1+y^{tK}|}\ , \tag{2.7}$$

where B, the triangle loop integral factor, is defined by (see the Appendix for notation)

$$B = (\sin 2\alpha_H)^{-1}[E \sin\gamma_+ \cos\gamma_- \tilde m_- f(\tilde u, \tilde d, \tilde W_-) + \cos\gamma_+ \sin\gamma_- \tilde m_+ f(\tilde u, \tilde d, \tilde W_+) + \tilde d \to \tilde e]\ , \tag{2.8}$$

where $\tilde u$, etc., is the left-handed u squark, etc.

III. PHOTINO AND SQUARK MASS LIMITS

In this section we investigate the implications of Eq. (2.7). For simplicity, we will assume in the following that the squark and sleptons are all degenerate with a common mass $m_{\tilde q}$. Then from the Appendix, we see that the function B of Eq. (2.8) depends upon the following parameters: $m_{\tilde q}$, μ, $\tilde m_2$, and α_H, where μ is the Higgs mixing parameter. In the following we vary these parameters over the allowed parameter space. $m_{\tilde W}$ is the low-lying W-ino mass ($\tilde m_-$) which may be expressed in terms of μ and $\tilde m_2$, the SU(2) soft-breaking gaugino mass. In general, two values of μ correspond to the same $m_{\tilde W}$. In expressing results below in terms of $m_{\tilde W}$, we of course chose the value of μ which gives the weakest constraints on the superparticle spectrum (e.g., the algebraically largest

TABLE I. B of Eq. (2.7) as a function of the W-ino mass $m_{\tilde{W}}$ for $m_{\tilde{\gamma}}=10$ GeV, $\alpha_H=45°$, and squark masses of 400 and 900 GeV. $M_H=1\times10^{16}$ GeV.

	$B\times10^6$ GeV^{-1}	
$m_{\tilde{W}}$ (GeV)	400	900
40.4	−119.5	−34.4
47.5	−130.9	−35.3
55.8	−144.4	−36.4
65.3	−159.4	−37.5
73.2	−171.3	−38.3

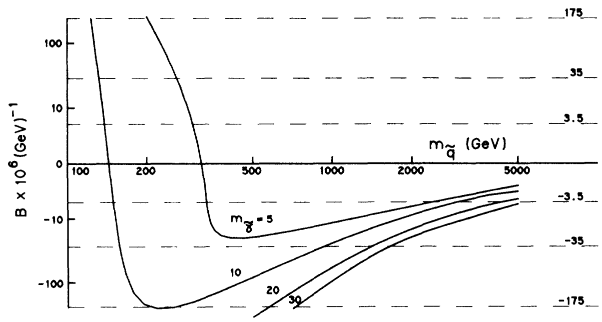

FIG. 2. B of Eq. (2.7) as a function of the squark mass for various values of the photino mass. The curves are plotted for $m_{\tilde{W}}=40$ GeV and $\alpha_H=45°$.

value of μ when $\tilde{m}_2>0$). For models where all the gauginos are degenerate at the GUT (or compactification) scale, $\tilde{m}_2$ is related to the photino mass by[6]

$$m_{\tilde{\gamma}}=(\tfrac{8}{3}\sin^2\theta_W)\tilde{m}_2\ . \quad (3.1)$$

In the following we will use Eq. (3.1) to *define* $m_{\tilde{\gamma}}$ (whether or not it represents the physical photino mass) and express our results in terms of $m_{\tilde{\gamma}}$ rather than $\tilde{m}_2$.

Current experimental bounds on $m_{\tilde{q}}$ are[8]

$$m_{\tilde{q}}\geq45\text{ GeV}\ ,\quad 90\%\text{C.L.}\ , \quad (3.2)$$

or if the squark and gluino are degenerate one has $m_{\tilde{q}}\geq75$ GeV. The UA1 data also allow one to estimate a bound on the W-ino mass for light photinos:[21,22]

$$m_{\tilde{W}}\gtrsim40\text{ GeV}\ . \quad (3.3)$$

One might expect that the large number of unknown parameters entering into the theoretical expression for B, Eq. (2.7), and the rather limited experimental constraints Eqs. (3.2) and (3.3) would make the proton decay condition Eq. (2.7) to be of limited usefulness. We will see, however, that Eq. (2.7) is a rather strong constraint, due in part to the fact that B depends rather strongly on $m_{\tilde{\gamma}}$ and $m_{\tilde{q}}$.

The general dependence of B on the parameters of the theory can be understood qualitatively from Eq. (2.7). For large squark mass B is small and negative since $\gamma_-<0$, $\gamma_+>0$ and the heavier $\tilde{W}_+$ contribution dominates. As the squark mass decreases the loop integral $f(\tilde{u},\tilde{d},\tilde{W})$ increases, and for fixed W-ino mass and fixed α_H, B becomes more negative. Eventually, however, the $\tilde{W}_+$ and $\tilde{W}_-$ contributions become comparable and cancel each other so that B increases passing through zero. Thus there is a domain of parameters leading to a "metastability" of the proton. (This cancellation was first noted for a special case in Ref. 23.) Finally for smaller values of $m_{\tilde{q}}$, B becomes positive and large. In addition, B is a decreasing function of $m_{\tilde{W}}$ and also decreases as α_H moves away from 45°.

The above qualitative behavior can be seen in Tables I and II and Figs. 2 and 3. The tables illustrate the above behavior of B as a function of $m_{\tilde{W}}$ and α_H for fixed $m_{\tilde{q}}$ and $m_{\tilde{\gamma}}$. Figure 2 shows the behavior of B as a function of $m_{\tilde{q}}$. We note that there is only a very narrow "valley of metastability" when $m_{\tilde{\gamma}}\gtrsim10$ GeV, and the metastable region broadens for small photino mass. Figure 3 shows an alternate plot of B as a function of $m_{\tilde{\gamma}}$. These curves become very rapidly varying for $m_{\tilde{q}}\leq500$ GeV.

TABLE II. B of Eq. (2.7) as a function of α_H for $m_{\tilde{W}}\simeq40$ GeV, $m_{\tilde{\gamma}}=10$ GeV, and squark masses of 400 and 900 GeV. $M_H=1\times10^{16}$ GeV.

	$B\times10^6$ GeV^{-1}	
α_H (deg)	400	900
45	−119.5	−34.4
40	−123.7	−35.1
35	−135.9	−37.3
30	−158.7	−41.4
25	−200.6	−48.4

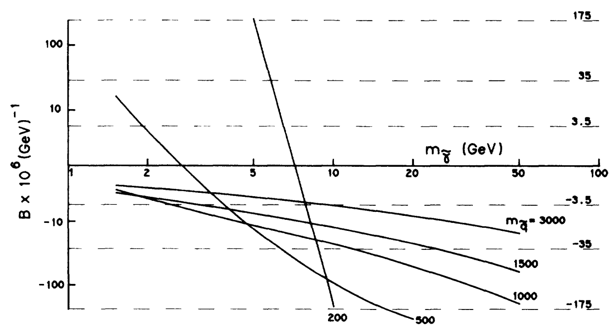

FIG. 3. B of Eq. (2.7) as a function of the photino mass for various values of the squark mass. The curves are plotted for $m_{\tilde{W}}=40$ GeV and $\alpha_H=45°$.

TABLE III. (a) The minimum value m_3 of squark mass satisfying $|B|<B_0$ for the three cases of Eqs. (4.1), (4.2), and (4.3). $M_H=1\times10^{16}$ GeV. (b) The minimum value m_3 of squark mass satisfying $|B|<B_0$ for the three cases (1), (2), and (3) of Sec. IV with $M_H=2\times10^{16}$ GeV.

(a)

$m_{\tilde{\gamma}}$ (GeV)	m_3 (GeV) Eq. (4.1)	Eq. (4.2)	Eq. (4.3)
10	3000	895	250
15	3750	1130	445
20	4300	1300	560
30	5300	1650	710

(b)

$m_{\tilde{\gamma}}$ (GeV)	m_3 (GeV) (1)	(2)	(3)
10	2120	590	
15	2640	780	280
20	3050	920	370
30	3730	1150	480

IV. DISCUSSION OF RESULTS

The physical constraints produced by Eq. (2.7) depend upon the value of the parameter β and the effects of the third generation (given by y^{tK}). In the following we will take $M_H=1\times10^{16}$ GeV and consider three cases.

(1) $\beta=0.03$, $y^{tK}\approx0$. Here β takes on its preferred value and we assume a small third-generation contribution. One has from Eqs. (2.7) then,

$$|B|\leq3.5\times10^{-6}\ \text{GeV}^{-1}\ ,\quad \beta=0.03\ ,\quad y^{tK}\approx0\ . \tag{4.1}$$

(2) $\beta=0.003$, $y^{tK}\approx0$. This choice represents the extreme limit for β in Eq. (2.7) and implies

$$|B|\leq35\times10^{-6}\ \text{GeV}^{-1}\ ,\quad \beta=0.003\ ,\quad y^{tK}\approx0\ . \tag{4.2}$$

(3) $\beta=0.003$, $|1+y^{tK}|=0.2$. This represents an extreme case where β has its smallest value and we assume there is a destructive interference between the third and second generations.[4] This yields the largest reasonable value of B (a constructive interference from the third generation only decreases $|B|$):

$$|B|\leq175\times10^{-6}\ \text{GeV}^{-1}\ ,\qquad \beta=0.003\ ,\quad |1+y^{tK}|=0.2\ . \tag{4.3}$$

The three bounds are represented by the horizontal lines in Figs. 2 and 3. As one can see from Fig. 2, a given line $|B|=B_0$ generally can intersect a given squark-photino contour at most three times. If there are three intersections: $m_{\tilde{q}}=m_i$, $i=1,2,3$ with $m_i<m_j$ for $i<j$, then the allowed domains of squark mass for $|B|\leq B_0$ are

$$m_{\tilde{q}}>m_3\ ,\quad m_1<m_{\tilde{q}}<m_2\ . \tag{4.4}$$

We examine first the situation of a "large" photino mass, i.e., $m_{\tilde{\gamma}}>10$ GeV. Here, as can be seen from Fig. 2, the allowed band m_2-m_1 is so narrow that it would require an unnatural dialing of parameters for $m_{\tilde{q}}$ to lie in this band. We will therefore discuss only the first inequality of Eq. (4.4). As discussed in Sec. III and seen in Tables I and II, the curves of Fig. 2 actually are upper bounds obtained when varying $m_{\tilde{W}}$ and α_H. Thus m_3 is actually a *lower* bound on the squark mass for $|B|\leq B_0$. The values of m_3 for the three cases of (1)–(3), are given in Table III(a) for $M_H=1\times10^{16}$ GeV and in Table III(b) for[24] $M_H=2\times10^{16}$ GeV.

In general, squarks will be detectable at the Fermilab Tevatron if[25] $m_{\tilde{q}}\leq150-175$ GeV, while it will probably be difficult to detect squarks with mass $m_{\tilde{q}}\geq1$ TeV at the SSC (Ref. 14). From Table III we see that for *$m_{\tilde{\gamma}}>10$ GeV, the proton decay constraint of Kamioka is now so strong, that squarks would not be observable at the Tevatron* and would probably not be observable even at the SSC unless the lower values of β and/or a generation cancellation is realized. In this domain, supersymmetry sig-

TABLE IV. (a) Values of m_i (GeV) of Eq. (4.4) for light photinos when $|B|<B_0$ for the cases of Eqs. (4.1), (4.2), and (4.3) for $m_{\tilde{W}}=40$ GeV and $\alpha_H=45°$. $M_H=1\times10^{16}$ GeV. (b) Values of m_i (GeV) of Eq. (4.4) for light photinos when $|B|<B_0$ for the cases (1), (2), and (3) of Sec. IV for $m_{\tilde{W}}=40$ GeV and $\alpha_H=45°$. $M_H=2\times10^{16}$ GeV.

(a)

		Eq. (4.1)			Eq. (4.2)		Eq. (4.3)
$m_{\tilde{\gamma}}$ (GeV)	m_1	m_2	m_3	m_1	m_2	m_3	m_1
8	187	190	2700	177	204	750	151
4	376	408	1800	309			220
1.5	695			430			270

(b)

		(1)			(2)		(3)
$m_{\tilde{\gamma}}$ (GeV)	m_1	m_2	m_3	m_1	m_2	m_3	m_1
8	186	191	1880	169	230	465	133
4	364	431	1185	273			182
1.5	618			356			214

TABLE V. Values of m_i (GeV) of Eq. (4.4) for Kamioka bound Eq. (2.1a) and IMB bound Eq. (2.1b) for cases (1), (2), and (3) of Sec. IV. ($m_{\tilde{\gamma}}=10$ GeV, $m_{\tilde{W}}=40$ GeV, $\alpha_H=45°$, and $M_H=1\times10^{16}$ GeV.)

	Kamioka			IMB		
Case	m_1	m_2	m_3	m_1	m_2	m_3
(1)	143	144	3000	142.5	144.5	2050
(2)	138.5	149.5	895	133.5	158	560
(3)	124	220	250	111		

nals could still arise from a light W-ino ($m_{\tilde{W}}<M_W$) or from gluinos.[26]

For light photino masses, $m_{\tilde{\gamma}}<10$ GeV, the situation is more complicated. If all three intersections can occur for a specific value of B_0, then the allowed domains are as in Eq. (4.4). Since the curves of Fig. 2 decrease with $m_{\tilde{W}}$ and $|\alpha_H-45°|$, then m_3 increases and $m_{1,2}$ decrease as $m_{\tilde{W}}$ and $|\alpha_H-45°|$ increases. Further, the band $m_1<m_{\tilde{q}}<m_2$, the "valley of metastability" in parameter space becomes broader as $m_{\tilde{\gamma}}$ decreases. If the minimum of the contour lies *above* $-B_0$, then only the intersection m_1 exists and all squark masses $M_{\tilde{q}}>m_1$ are allowed. Table IV gives values of m_i for light photinos. This domain can accommodate squarks that could be detected at the Tevatron, but only for large values of $|B_0|$, and preferably for heavier $m_{\tilde{W}}$ and for α_H different from 45°.

Figure 3 exhibits the inverse information of the possible values of $m_{\tilde{\gamma}}$ obeying $|B|<B_0$ for a fixed value of $m_{\tilde{q}}$. Thus for the constraint of Eq. (4.1) to hold one sees that $m_{\tilde{\gamma}}<2.3$ is required for $m_{\tilde{q}}=1000$ GeV, while for $m_{\tilde{q}}<500$ GeV, the curves are so vertical that this constraint can be satisfied only for a very narrow band of photino masses. Condition (4.2) relaxes these constraints somewhat. Thus for $m_{\tilde{q}}=1000$ GeV, this condition can be satisfied for $m_{\tilde{\gamma}}\le 12.5$ GeV. For $m_{\tilde{q}}=500$ GeV, one requires $m_{\tilde{\gamma}}\le 6$ GeV, and again only a narrow band of photino masses (centered around $m_{\tilde{\gamma}}\simeq 7$ GeV) is allowed when $m_{\tilde{q}}=200$ GeV. Thus one sees again that if the squark mass is light, the proton decay data require the photino to be very light and lie in a narrow band of values.

The above results can only be circumvented if by fine-tuning one assumes that the top quark has a mass allowing the third- and second-generation contributions to proton decay to cancel each other, i.e., $y^{tk}=-1$.

The above analysis was based on the experimental bound Eq. (2.1a) of Kamioka. If instead, one adopts the IMB experimental bound of Eq. (2.1b), the constraints on the squark and photino masses are relaxed somewhat. A comparison of the results from these two experiments is illustrated for $m_{\tilde{\gamma}}=10$ GeV in Table V. As can be seen from this table the case of Eq. (4.2) can now accommodate squarks that could be observable at the SSC.

The above discussion shows that proton decay data already have begun to put strong constraints on the low-energy superparticle mass spectrum for supersymmetry models that are viewed as the low-energy residue of a unified theory. From the strong constraints that already exist, it is clear that the next generation of proton decay experiments will sharply delineate such supersymmetry models. Further, an accurate lattice gauge determination of the parameter β will greatly aid in testing these predictions of supersymmetry.

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APPENDIX

We define here the notation used in Sec. II. Further details may be found in Ref. 4. We assume that the low-energy theory depends on one pair of Higgs doublets H^α, H'_α, $\alpha=1,2$ and α_H is defined by $\tan\alpha_H=\langle H'_2\rangle/\langle H^2\rangle$. The $\tilde{m}_\pm$ refer to the W-ino mass eigenvalues

$$\tilde{m}_\pm=\tfrac{1}{2}\,|\,[4v_+^2+(\mu-\tilde{m}_2)^2]^{1/2}\pm[4v_-^2+(\mu+\tilde{m}_2)^2]^{1/2}\,| \ , \tag{A1}$$

where

$$\sqrt{2}v_\pm=M_W(\cos\alpha_H\pm\sin\alpha_H)\ . \tag{A2}$$

In Eq. (A1) μ is the Higgs mixing parameter which appears in the superpotential in the form $\mu H'_\alpha H^\alpha$ and $\tilde{m}_2$ is the soft-breaking SU(2) gaugino mass. The $\gamma_\pm$ are defined by $\gamma_\pm=\beta_+\pm\beta_-$, where

$$\sin 2\beta_\pm=(\mu\mp\tilde{m}_2)/[4v_\pm^2+(\mu\mp\tilde{m}_2)^2]^{1/2}\ . \tag{A3}$$

E in Eq. (2.8) is defined by $E=(-1)^\theta$, where

$$\begin{aligned}\theta&=0\ ,\quad \sin 2\alpha_H>\frac{\mu\tilde{m}_2}{M_W^2}\ ,\\ \theta&=1\ ,\quad \sin 2\alpha_H<\frac{\mu\tilde{m}_2}{M_W^2}\ .\end{aligned} \tag{A4}$$

The dressing loop integral f_{abc} is given by

$$f_{abc}=\frac{m_c}{m_b^2-m_c^2}\left[\frac{m_b^2}{m_a^2-m_b^2}\ln\frac{m_a^2}{m_b^2}-\frac{m_c^2}{m_a^2-m_c^2}\ln\frac{m_a^2}{m_c^2}\right]\ . \tag{A5}$$

[1]Y. Totsuka, in *Proceedings of the 1985 International Symposium on Lepton and Photon Interactions at High Energies,* Kyoto, Japan, 1985, edited by M. Konuma and K. Takahashi (Research Institute for Fundamental Physics, Kyoto University, Kyoto, 1986); H. Meyer, in *Neutrino '86: Neutrino Physics and Astrophysics,* proceedings of the 12th International Conference, Sendai, Japan, 1986, edited by T. Kitagaki and H. Yuta (World Scientific, Singapore, 1986).

[2]K. Enqvist, A. Masiero, and D. V. Nanopoulos, Phys. Lett. **156B**, 209 (1985).

[3]For a review of earlier work on SUSY grand unified theories see W. Lucha, Fortschr. Phys. **33**, No. 10 (1985).

[4]R. Arnowitt, A. H. Chamseddine, and P. Nath, Phys. Lett. **156B**, 215 (1985); P. Nath, A. H. Chamseddine, and R. Arnowitt, Phys. Rev. D **32**, 2348 (1985).

[5]R. Arnowitt and P. Nath, Phys. Rev. D **36**, 3423 (1987).

[6]For reviews of supergravity models see P. Nath, R. Arnowitt, and A. H. Chamseddine, *Applied* $N=1$ *Supergravity* (World Scientific, Singapore, 1984); H. P. Nilles, Phys. Rep. **110**, 1 (1984); H. E. Haber and G. Kane, *ibid.* **117**, 75 (1985); H. Lahanas and D. V. Nanopoulos, *ibid.* **145**, 1 (1986).

[7]N. Sakai and T. Yanagida, Nucl. Phys. **B197**, 533 (1982); S. Weinberg, Phys. Rev. D **26**, 287 (1982).

[8]UA1 Collaboration, C. Albajar *et al.*, Phys. Lett. B **198**, 261 (1987).

[9]R. Barloutaud, in Proceedings of the 8th Workshop on Grand Unification, Syracuse, New York, 1987, edited by K. Wali (World Scientific, Singapore, to be published).

[10]These include missing-partner models as well as models where the gauge hierarchy is achieved through fine-tuning.

[11]B. Greene, K. H. Kirlin, P. J. Miron, and G. G. Ross, Phys. Lett. B **180**, 69 (1986); Nucl. Phys. **B274**, 574 (1986).

[12]H. Baer, V. Barger, D. Karatas, and X. Tata, Phys. Rev. D **36**, 96 (1987).

[13]H. Baer, M. Barnett, M. Drees, J. Gunion, H. Haber, D. Karatas, and X. Tata, Int. J. Mod. Phys. **A2**, 1131 (1987).

[14]H. Baer, M. Drees, D. Karatas, and X. Tata, Reports Nos. MAD/PH/362, 1987 and CP-87-88, 1987 (unpublished).

[15]J. Ellis, K. Enqvist, D. V. Nanopoulos, and F. Zwirner, Nucl. Phys. **B276**, 14 (1986); Mod. Phys. Lett. **A1**, 57 (1986).

[16]S. Chadha and M. Daniels, Nucl. Phys. **B229**, 105 (1983).

[17]J. Ellis, D. V. Nanopoulos, and S. Rudaz, Nucl. Phys. **B202**, 43 (1982).

[18]J. Brodsky, J. Ellis, S. Hagelin, and C. T. Sachradja, Nucl. Phys. **B238**, 561 (1984).

[19]Y. Hara, S. Itoh, Y. Iwasaki, and T. Yoshie, Phys. Rev. D **34**, 3399 (1986).

[20]K. C. Bowler, D. Daniel, T. D. Kieu, D. G. Richards, and C. J. Scott, Nucl. Phys. **B296**, 431 (1988).

[21]A. H. Chamseddine, R. Arnowitt, and P. Nath, Phys. Lett. B **174**, 399 (1986).

[22]H. Baer, K. Hagiwara, and X. Tata, Phys. Rev. D **35**, 1598 (1987).

[23]K. Enqvist and D. V. Nanopoulos, Prog. Part. Nucl. Phys. **16**, 1 (1986).

[24]For the fine-tuning model of Ref. 2, we find the upperbound $M_H < \sqrt{2} M_{\rm GUT} \lesssim 2\times 10^{16}$ GeV, using Eq. (1.1). In the missing partner model $M_H < M_{\rm GUT}$.

[25]H. Baer and E. L. Berger, Phys. Rev. D **34**, 1361 (1986); E. Reya and D. P. Roy, Z. Phys. C **32**, 615 (1986).

[26]For a review of supersymmetry signals at colliders see R. Arnowitt and P. Nath, Proceedings of the Eighth Vanderbilt High Energy Physics Conference, Nashville, 1987, edited by R. Panvini (World Scientific, Singapore, to be published).