PHYSICAL REVIEW D VOLUME 38, NUMBER 5 1 SEPTEMBER 1988

Anomalous magnetic moment of the muon and neutral-current constraints in a supersymmetric $SU(2)_L \times U(1)_{I_3R} \times U(1)_{B-L}$ model inspired by superstring theories

M. Frank and C. S. Kalman
Concordia University Elementary Particle Physics Group, 1455 de Maisoneuve Boulevard, West, Montreal, Quebec, Canada H3G 1M8
(Received 1 February 1988)

An extension of the standard supersymmetric model, $SU(2)_L \times U(1)_{I_3R} \times U(1)_{B-L}$, motivated by superstrings and baryon-number conservation is analyzed. A possible pattern of supersymmetry breaking from E_6 and the phenomenological implications for such a model are studied. In particular it is found that the neutral currents restrict $M_{Z'} \geq 240$ GeV. Constraints on the $g-2$ factor of the muon are also examined.

I. INTRODUCTION

The ultimate theory of matter may well be described in terms of superstrings.[1-5] It is of interest to find models which are compatible with such an ultimate theory and which are susceptible to experimental test. It is assumed that compactification from $D=10$ to $d=4$ dimensions in superstring theories is on a Calabi-Yau manifold. Considerable ambiguity remains in the choice of the particular manifold. Thus the "ultimate" low-energy group G is obscured. It has been argued that G could be $SU(3)_C \times SU(2)_L \times U(1) \times U(1) \times U(1)$ (Ref 6) or $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)$ (Ref. 7). The model presented here could represent a further breakdown of either of the two groups to $SU(3)_C \times SU(2)_L \times U(1)_{I_3R} \times U(1)_{B-L}$. It appears also as a natural extension of the standard model when the additional constraint of baryon-number conservation is introduced. This latter approach has the advantage that no fine-tuning of parameters is required.

In this paper the model is approached from the standard model, as a natural extension, and although a possible symmetry-breaking mechanism from E_6 is established, no constraints on coupling constants other than the ones dictated by the standard model are imposed. The model is applied to an examination of neutral currents and of the anomalous magnetic moment of the muon $g-2$. It is particularly important at this time to obtain good theoretical predictions in both of these areas. Amaldi *et al.*[8] have just published a comprehensive analysis of data pertaining to the weak neutral current. It is interesting to note that while left-handed parameters $\epsilon_L(u)$, $\epsilon_L(d)$ are in close agreement with the standard-model calculations (including radiative corrections), the right-handed $\epsilon_R(u)$, $\epsilon_R(d)$ deviate from these predictions. Such deviations are easily understood in the model presented here, which includes a second Z boson (Z'). Indeed the deviations might be taken to indicate a maximum value of the Z mass of about 1 TeV. It should then be observable with the projected new accelerators. The experimental values also yield a lower limit of 240 GeV. The anomalous magnetic moment of the muon and of the electron are perhaps the most accurately measured quantities in physics [with an uncertainty in the tenth significant figure for $(g-2)_e$]. Supersymmetric models inevitably are used to examine the muon anomaly. Although the experimental measurements are not quite as accurate as in the electron case (the uncertainty is in the eighth significant figure), the contributions of supersymmetry are considerably larger. An improvement in the present level of accuracy in measurements by 1 order of magnitude will yield results that may be compared with the type of theoretical calculation results presented in this paper. This is precisely the kind of results anticipated in the new generation of $g-2$ experiments proposed at BNL and at Novosibirsk.

This paper is organized as follows. In Sec. II the model is described in some detail. In Sec. III the application of the model to neutral-current interactions is considered. In Sec. IV the mixing matrices of the fermions and those of the corresponding scalar partners are considered. In Sec. V the calculations of $(g-2)_\mu$ are presented. Conclusions and prospects are presented in Sec. VI.

II. DESCRIPTION OF THE MODEL

The standard $SU(3) \times SU(2) \times U(1)$ gauge model has been remarkably successful in describing all of particle physics. Certainly this theory must be included within any description of matter. There are, however, certain theoretical difficulties which indicate that the theory is not complete. First, the theory is not stable against large radiative corrections. A supersymmetric extension of the theory is the natural correction of such difficulties. The usual quadratic divergences coming from scalar Higgs particles as shown in Fig. 1 are then canceled by fermionic contributions indicated in Fig. 2. A further difficulty still remains. A typical standard field theory has the form

$$L=\mu\bar{\psi}\phi\psi+\lambda\phi^4 , \qquad (2.1)$$

where ψ is a fermion field and ϕ is a boson field. Suppose one sets $\lambda=0$. At the tree level, one must consider divergent diagrams of the type indicated in Fig. 3. To cancel, one requires a ϕ^4 term in the Lagrangian. In a similar way, consider the full supersymmetric model of the electroweak interaction. The most general superpotential has the form

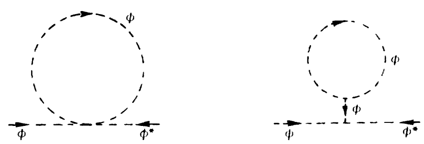

FIG. 1. Quadratic divergences coming from scalar Higgs particles.

$$W=h_u QH_u U^c+h_d QH_d D^c+h_e LH_d E^c +\mu_1 H_u H_d+\mu_2 H_u L+f_{pqr}Q_r L_q D_r^c +\tilde{h}_{[p,q]r}L_p L_q E_r^c+\lambda_1 E^c H_d H_d+\lambda_{[p,q]r}U_p^c D_q D_r^c \ . \tag{2.2}$$

The last term violates baryon-number symmetry and corresponds to a rapid proton decay. The next to last four terms violate lepton number. If the coefficients μ_2, f_{pqr}, $\tilde{h}_{[p,q]r}$, λ_1, and $\lambda_{[p,q]r}$ are set to zero, the terms cannot be regenerated at the tree level. This situation unlike that described in the standard field theory occurs due to the nonrenormalization theorem of supersymmetric field theory. Setting the coefficients to zero corresponds to the "standard" supersymmetric model.[9] Nevertheless, the theory is unsatisfactory as there is no theoretical justification for setting the coefficients to zero.

It is important to establish a model, where no baryon-number-violating terms leading to rapid proton decay or lepton-number-violating terms inconsistent with current experimental limits are allowed. Such a model can be derived as the low-energy limit of $E_8\times E_8$ superstring theories. In the zero-slope limit these theories lead to an anomaly-free 10-dimensional $E_8\times E_8$ super-Yang-Mills theory coupled to supergravity. The six extra dimensions can be compactified[3] to a Calabi-Yau manifold with SU(3) holonomy. This yields an $N=1$ locally supersymmetric four-dimensional grand unified theory based on the $E_8\times E_6$ gauge group. The E_6 group in turn breaks down at the compactification scale to one of its maximal subgroups. Generally[6,9–12] this is taken as $SU(3)\times SU(2)\times U(1)\times U(1)\times U(1)$. Let us assume this group is $SU(3)_C\times SU(2)_L\times U(1)_{I_3R}\times U(1)_{B-L}\times U(1)_N$.

In the E_6 model, there are five neutral fermions[13] denoted by ν, N_R, N_2, N_3, N_4. To give mass to N_2 and N_3, a nonzero vacuum expectation value (VEV) is given to the SO(10)-singlet field component of the **27** Higgs field (denoted by M_H). N_4 receives a mass from an additional singlet S. Consequently N_2, N_3, and N_4 decouple from the low-energy ($E \ll M_H$) spectrum of the model.

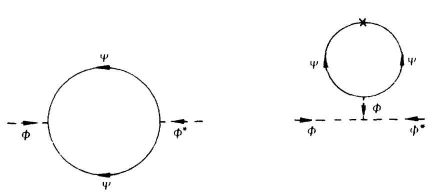

FIG. 2. Quadratic divergences coming from fermionic partners of the Higgs particles.

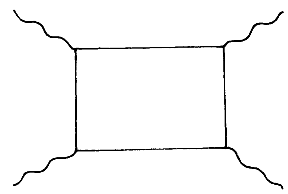

FIG. 3. Typical divergent diagram at the tree level in a fermionic boson field theory without ϕ^4 interaction.

Moreover, in setting $M_H\neq 0$, the symmetry is broken down to $SU(3)_C\times SU(2)_L\times U(1)_{I_3R}\times U(1)_{B-L}$. The most general corresponding super potential in the effective theory is then

$$W=h_u QH_u U^c+h_d QH_d D^c+h_e LH_d E^c +h_\nu LH_u N^c+\mu H_u H_d \ . \tag{2.3}$$

No baryon-number- or lepton-number-violating terms are possible. Finally a nonzero vacuum expectation value is given to the scalar superpartner of the right-handed neutrino ($\langle H_u^0\rangle, \langle H_d^0\rangle < \langle \tilde{N}_R\rangle = V_R \ll M_H$). This breaks the $U(1)_{I_3R}\times U(1)_{B-L}$ down to $U(1)_y$, the weak hypercharge group of the standard model.[14]

The resulting superpotential due to its origin in Eq. (2.3) *a priori* cannot have any of the objectionable terms found in Eq. (2.2).

The superfields for the three groups are as follows:

$SU(2)_L$, $(\lambda_W, \mathbf{W}, D)$ coupling constant g ,

$U(1)_{I_3R}$, $(\lambda_B, \mathbf{B}, D_B)$ coupling constant g_1 ,

TABLE I. Matter and Higgs-boson assignments of supersymmetric gauge model.

Superfield	$SU(2)_L\times U(1)_{I_3(R)}\times U(1)_{B-L}$		
		Matter	
Q	2	0	$\frac{1}{3}$
U_c	1	$-\frac{1}{2}$	$\frac{1}{3}$
D_c	1	$\frac{1}{2}$	$\frac{1}{3}$
L	2	0	-1
E^c	1	$\frac{1}{2}$	-1
N^c	1	$-\frac{1}{2}$	-1
		Higgs boson	
H_u	2	$\frac{1}{2}$	0
H_d	2	$-\frac{1}{2}$	0
		Gauge bosons	
W	3	0	0
B	1	0	0
V	1	0	0

$U(1)_{B-L}$, $(\lambda_v, \mathbf{V}, D_v)$ coupling constant g_2 .

The corresponding assignments of the matter and Higgs particles according to the above gauge groups are shown in Table I. There, Q and L represent the left-handed quarks, and leptons, U^c and D^c are the right-handed quarks, and E^c and N^c are the right-handed leptons. The full Lagrangian is then

$$L = L_{\text{gauge}} + L_{\text{matter}} + L_y - V + L_{\text{soft}} , \tag{2.4}$$

where

$$\begin{aligned}
&SU(2)_L, \quad L_{\text{gauge}} = \tfrac{1}{4}\mathbf{f}^w_{\mu\nu}\cdot\mathbf{f}^w_{\mu\nu} - \tfrac{1}{2}\bar{\lambda}_w\gamma_\mu D^w_\mu\lambda_w ,\\
&U(1)_{I_3R}, \quad -\tfrac{1}{4}f^B_{\mu\nu}f^B_{\mu\nu} - \tfrac{1}{2}\bar{\lambda}_B\gamma_\mu\partial_\mu\lambda_B ,\\
&U(1)_{B-L}, \quad -\tfrac{1}{4}f^V_{\mu\nu}f^V_{\mu\nu} - \tfrac{1}{2}\bar{\lambda}_V\gamma_\mu\partial_\mu\lambda_V ,
\end{aligned} \tag{2.5}$$

$$\begin{aligned}
L_{\text{matter}} = & -\bar{Q}\gamma_\mu\left[\partial_\mu - \frac{ig}{2}\boldsymbol{\tau}\cdot\mathbf{W}_\mu - \frac{ig_2}{6}V_\mu\right]Q - \bar{U}_c\gamma_\mu\left[\partial_\mu + \frac{ig_1}{2}B_\mu + \frac{ig_2}{6}V_\mu\right]U_c\\
& -\bar{D}_c\gamma_\mu\left[\partial_\mu - \frac{ig_1}{2}B_\mu + \frac{ig_2}{6}V_\mu\right]D_c - \bar{L}\gamma_\mu\left[\partial_\mu - \frac{ig}{2}\boldsymbol{\tau}\cdot\mathbf{W}_\mu + \frac{ig_2}{2}V_\mu\right]L\\
& -\bar{\tilde{H}}_\mu\gamma_\mu\left[\partial_\mu - \frac{ig}{2}\boldsymbol{\tau}\cdot\mathbf{W}_\mu - \frac{ig_1}{2}B_\mu\right]\tilde{H}_u - \bar{E}_c\gamma_\mu\left[\partial_\mu - \frac{ig_1}{2}B_\mu - \frac{ig_2}{2}V_\mu\right]E_c\\
& -\bar{\tilde{H}}_d\gamma_\mu\left[\partial_\mu - \frac{ig}{2}\boldsymbol{\tau}\cdot\mathbf{W}_\mu + \frac{ig_1}{2}B_\mu\right]\tilde{H}_d - \bar{N}_c\gamma_\mu\left[\partial_\mu + \frac{ig_1}{2}B_\mu - \frac{ig_2}{2}V_\mu\right]N_c - \bar{\psi}_x\gamma_\mu\partial_\mu\psi_x\\
& -\left|\left[\partial_\mu - \frac{ig}{2}\boldsymbol{\tau}\cdot\mathbf{W}_\mu - \frac{ig_2}{6}V_\mu\right]\tilde{Q}\right|^2 - \left|\left[\partial_\mu + \frac{ig_1}{2}B_\mu + \frac{ig_2}{6}V_\mu\right]\tilde{U}_c\right|^2\\
& -\left|\left[\partial_\mu - \frac{ig_1}{2}B_\mu + \frac{ig_2}{6}V_\mu\right]D_c\right|^2 - \left|\left[\partial_\mu - \frac{ig}{2}\boldsymbol{\tau}\cdot\mathbf{W}_\mu + \frac{ig_2}{2}V_\mu\right]\tilde{L}\right|^2\\
& -\left|\left[\partial_\mu - \frac{ig_1}{2}B_\mu - \frac{ig_2}{2}V_\mu\right]\tilde{E}_c\right|^2 - \left|\left[\partial_\mu + \frac{ig_1}{2}B_\mu - \frac{ig_2}{2}V_\mu\right]\tilde{N}_c\right|^2\\
& -\left|\left[\partial_\mu - \frac{ig}{2}\boldsymbol{\tau}\cdot\mathbf{W}_\mu - \frac{ig_1}{2}B_\mu\right]H_u\right|^2 - \left|\left[\partial_\mu - \frac{ig}{2}\boldsymbol{\tau}\cdot\mathbf{W}_\mu + \frac{ig_1}{2}B_\mu\right]H_d\right|^2\\
& +i\frac{g}{\sqrt{2}}\bar{Q}\boldsymbol{\tau}\cdot\boldsymbol{\lambda}_w\tilde{Q} + i\frac{g_2}{3\sqrt{2}}\bar{Q}\lambda_V\tilde{Q} - i\frac{g_1}{\sqrt{2}}\bar{U}_c\lambda_B\tilde{U}_c - \frac{ig_2}{3\sqrt{2}}\bar{U}_c\lambda_V\tilde{U}_c\\
& +i\frac{g_1}{\sqrt{2}}\bar{D}_c\lambda_B\tilde{D}_c - \frac{i}{3}\frac{g_2}{\sqrt{2}}\bar{D}_c\lambda_V\tilde{D}_c + i\frac{g}{\sqrt{2}}\bar{L}\boldsymbol{\tau}\cdot\boldsymbol{\lambda}_w\tilde{L} - i\frac{g_2}{\sqrt{2}}\bar{L}\lambda_V\tilde{L}\\
& +i\frac{g}{\sqrt{2}}\bar{\tilde{H}}_u\boldsymbol{\tau}\cdot\boldsymbol{\lambda}_w H_u + i\frac{g_1}{\sqrt{2}}\bar{\tilde{H}}_u\lambda_B H_u + i\frac{g_1}{\sqrt{2}}\bar{E}_c\lambda_B\tilde{E}_c + i\frac{g_2}{\sqrt{2}}\bar{E}_c\lambda_V\tilde{E}_c\\
& +i\frac{g}{\sqrt{2}}\bar{\tilde{H}}_d\boldsymbol{\tau}\cdot\boldsymbol{\lambda}_w H_d - i\frac{g_1}{\sqrt{2}}\bar{\tilde{H}}_w\lambda_B H_d - i\frac{g_1}{\sqrt{2}}\bar{N}_c\lambda_B\tilde{N}_c + i\frac{g_2}{\sqrt{2}}\bar{N}_c\lambda_V\tilde{N}_c\\
& +h_u(Q^TC^{-1}\tau_2H_uU_c + Q_L^TC^{-1}\tau_2\tilde{H}_d\tilde{U}_c + \tilde{H}_u^TC^{-1}\tau_2\tilde{Q}U_c) + h_d(u\to d)\\
& +h_e(L^TC^{-1}\tau_2H_dE_c + L^TC^{-1}\tau_2H_d\tilde{E}_c + \tilde{H}_d^TC^{-1}\tau_2\tilde{L}E_c)\\
& +h_\nu(\text{both } H_d\to H_u \text{ and } E_c\to N_c) + \mu\tilde{H}_uC^{-1}\tau_2\tilde{H}_d + \text{H.c.} ,
\end{aligned} \tag{2.6}$$

$$V = |F|^2 + \tfrac{1}{2}|D|^2 + V_{\text{soft}} , \tag{2.7}$$

$$\begin{aligned}
|F|^2 = & |h_u\tilde{Q}\tilde{U}_c + h_\nu\tilde{L}\tilde{N}_c + \mu H_d|^2 + |h_d\tilde{Q}\tilde{D}_c + h_e\tilde{L}\tilde{E}_c + \mu H_u|^2\\
& + |h_uH_u\tilde{U}_c + h_dH_d\tilde{D}_c|^2 + h_u^2|\tilde{Q}^T\tau_2H_u|^2 + h_d^2|\tilde{Q}^T\tau_2H_d|^2\\
& + |h_eH_d\tilde{E}_c + h_\nu H_u\tilde{N}_c|^2 + h_e^2|\tilde{L}^T\tau_2H_d|^2 + h_\nu^2|\tilde{L}\tau_2H_u|^2 ,
\end{aligned} \tag{2.8}$$

$$\tfrac{1}{2}|D|^2=\tfrac{1}{8}g^2\sum_a\left|\sum_{A_\psi}A_\psi^\dagger\tau_a A_\psi\right|^2+\tfrac{1}{4}g_1^2\,|-\tfrac{1}{2}\tilde{U}_c^\dagger\tilde{U}_c+\tfrac{1}{2}\tilde{D}_c^\dagger\tilde{D}_c+\tfrac{1}{2}\tilde{E}_c^\dagger E_c-\tfrac{1}{2}\tilde{N}_c^\dagger N_c+\tfrac{1}{2}H_u^\dagger H_u-\tfrac{1}{2}H_d^\dagger H_d\,|^2$$
$$+\tfrac{1}{4}g_2^2\,|\tfrac{1}{3}\tilde{Q}^\dagger\tilde{Q}-\tfrac{1}{3}\tilde{U}_c^\dagger\tilde{U}_c-\tfrac{1}{3}\tilde{D}_c^\dagger\tilde{D}_c-\tilde{L}^\dagger\tilde{L}+\tilde{E}_c^\dagger\tilde{E}_c+\tilde{N}_c^\dagger\tilde{N}_c\,|^2\,, \tag{2.9}$$

where $\psi=\tilde{Q}$, $\tilde{L}$, H_u, or H_d and

$$V_{\rm soft}=m_{3/2}(h_u\tilde{Q}^T\tau_2H_u\tilde{U}_c+h_d\tilde{Q}^T\tau_2H_d\tilde{D}_c+h_\nu\tilde{L}^T\tau_2H_u\tilde{N}_c+h_e\tilde{L}^T\tau_2H_d\tilde{E}_c+\mu H_u^T\tau_2H_d+\text{H.c.})$$
$$+m_Q^2\tilde{Q}^\dagger Q+m_L^2\tilde{L}^\dagger\tilde{L}+m_u^2\tilde{U}_c^*\tilde{U}_c+m_D^2\tilde{D}_c^*\tilde{D}_c+m_N^2\tilde{N}_c^*N_c+m_E^2\tilde{E}_c^*\tilde{E}_c\,, \tag{2.10}$$

and finally

$$L_{\rm soft}=m\lambda_w^TC^{-1}\lambda_w+m'\lambda_B^TC^{-1}\lambda_B+m''\lambda_V^TC^{-1}\lambda_V+\text{H.c.}\ . \tag{2.11}$$

The Lagrangian differs from the standard model only through the extra contributions of Z' the new gauge boson, and the right-handed neutrino. It is important to notice that the extra down-type quark and the extra charged lepton naturally appearing in a SU(2)×U(1)×U(1)-type model pick-up superheavy mass in breaking down from SU(2)×U(1)×U(1)×U(1) and decouple from the low-energy spectrum.

III. NEUTRAL CURRENT

The first low-energy constraints this model is subjected to are neutral-current bounds. Extensive analyses already exist; the model presented here is subjected to the bounds merely to restrict the values of the new coupling constants and the mass of the extra gauge boson Z'.

Corresponding to the original Lagrangian

$$L_{\rm int}=gW_\mu J_\mu^W+g_1B_\mu J_\mu^B+g_2V_\mu J_\mu^V\,, \tag{3.1}$$

described in detail in the preceding section, one obtains the effective Lagrangian

$$L_{\rm eff}=-\frac{G_F}{\sqrt{2}}[\rho J_\mu^zJ_\mu^z+(g_a/g)^2\cos^2\theta_W\rho(M_Z/M_{Z'})^2J_\mu^{z'}J_\mu^{z'}]\,, \tag{3.2}$$

where $g_a=(g_1^2+g_2^2)^{1/2}$, $\rho=[M_W/(M_Z\cos\theta_W)]^2=1$, in the standard model:

$$G_F=g/(4\sqrt{2}M_W^2)\,, \tag{3.3}$$

and

$$J_\mu^z=(\cos^2\theta_WJ_\mu^W-\sin^2\theta_WJ_\mu^V),\quad J_\mu^{z'}=J_\mu^B-(g_1\tan\theta_W/g_2)^2(J_\mu^V+J_\mu^B)\ . \tag{3.4}$$

In the rest of this section the following notation is also used:

$$g_b=g_2\cos\theta_W,\quad \xi=(M_Z/M_{Z'})^2,\quad\text{and }x=\sin^2\theta_W\ . \tag{3.5}$$

From Eq. (2.4), the neutral current is then

$$\begin{aligned}
&\tfrac{1}{2}\bar{u}\gamma(1-\tfrac{4}{3}x)\left[\frac{1-\gamma_5}{2}\right]u-\tfrac{1}{2}\bar{d}\gamma(1-\tfrac{2}{3}x)\left[\frac{1-\gamma_5}{2}\right]d+\tfrac{1}{2}\bar{\nu}\gamma\left[\frac{1-\gamma_5}{2}\right]\nu\\
&\quad-\tfrac{1}{2}\bar{e}\gamma(1-2x)\left[\frac{1-\gamma_5}{2}\right]e-\tfrac{2}{3}x\bar{u}\gamma\left[\frac{1+\gamma_5}{2}\right]u+\tfrac{1}{3}x\,\bar{d}\gamma\left[\frac{1+\gamma_5}{2}\right]d+x\bar{e}\gamma\left[\frac{1+\gamma_5}{2}\right]e\\
&\quad+\frac{g_a^2}{g^2}(1-x)\xi\left[-\frac{e^2}{g_b^2}\frac{1}{6}\bar{u}\gamma\left[\frac{1-\gamma_5}{2}\right]u-\frac{e^2}{g_b^2}\frac{1}{6}\bar{d}\gamma\left[\frac{1-\gamma_5}{2}\right]d+\frac{e^2}{g_b^2}\frac{1}{2}\bar{e}\gamma\left[\frac{1-\gamma_5}{2}\right]e\right.\\
&\quad+\frac{e^2}{g_b^2}\frac{1}{2}\bar{\nu}\gamma\left[\frac{1-\gamma_5}{2}\right]\nu-\tfrac{1}{2}\bar{u}\left[1-\frac{4}{3}\frac{e^2}{g_b^2}\right]\gamma\left[\frac{1+\gamma_5}{2}\right]u\\
&\quad+\tfrac{1}{2}\bar{d}\left[1-\frac{2}{3}\frac{e^2}{g_b^2}\right]\gamma\left[\frac{1+\gamma_5}{2}\right]d\\
&\quad\left.-\tfrac{1}{2}\bar{\nu}\gamma\left[\frac{1+\gamma_5}{2}\right]\nu+\tfrac{1}{2}\bar{e}\left[1-2\frac{e^2}{g_b^2}\right]\gamma\left[\frac{1+\gamma_5}{2}\right]e\right]
\end{aligned} \tag{3.6}$$

$$
\begin{aligned}
&= -\tfrac{1}{2}(\tfrac{1}{2})\bar{u}\gamma\gamma_5 u + \tfrac{1}{2}(\tfrac{1}{2}-\tfrac{4}{3}x)\bar{u}\gamma u + \tfrac{1}{2}(\tfrac{1}{2})\bar{d}\gamma\gamma_5 d - \tfrac{1}{2}(\tfrac{1}{2}-\tfrac{2}{3}x)\bar{d}\gamma d - \tfrac{1}{2}\bar{\nu}\gamma\left[\frac{1-\gamma_5}{2}\right]\nu + \tfrac{1}{2}(\tfrac{1}{2})\bar{e}\gamma\gamma_5 e - \tfrac{1}{2}(\tfrac{1}{2}-2x)\bar{e}\gamma e \\
&\quad + \frac{g_a^2}{g^2}(1-x)\xi\left[-\frac{1}{2}\left[\frac{1}{2}-\frac{1}{2}\frac{e^2}{g_b^2}\right]\bar{u}\gamma u - \frac{1}{2}\left[\frac{1}{2}-\frac{5}{6}\frac{e^2}{g_b^2}\right]\bar{u}\gamma\gamma_5 u\right. \\
&\qquad + \frac{1}{2}\left[\frac{1}{2}-\frac{1}{2}\frac{e^2}{g_b^2}\right]\bar{d}\gamma d + \frac{1}{2}\left[\frac{1}{2}-\frac{1}{6}\frac{e^2}{g_b^2}\right]\bar{d}\gamma\gamma_5 d \\
&\qquad - \frac{1}{2}\left[\frac{1}{2}-\frac{1}{2}\frac{e^2}{g_b^2}\right]\bar{\nu}\gamma\nu - \frac{1}{2}\left[\frac{1}{2}+\frac{1}{2}\frac{e^2}{g_b^2}\right]\bar{\nu}\gamma\gamma_5\nu + \frac{1}{2}\left[\frac{1}{2}-\frac{1}{2}\frac{e^2}{g_b^2}\right]\bar{e}\gamma e \\
&\qquad \left. + \frac{1}{2}\left[\frac{1}{2}-\frac{3}{2}\frac{e^2}{g_b^2}\right]e\gamma\gamma_5 d\right] .
\end{aligned}
\tag{3.7}
$$

The quark couplings are then defined by

$$\epsilon_L^u = \tfrac{1}{2} - \tfrac{2}{3}x + \frac{g_a^2}{g^2}(1-x)\xi\left[-\frac{1}{6}\frac{e^2}{g_b^2}\right] , \tag{3.8}$$

$$\epsilon_L^d = -\tfrac{1}{2} + \tfrac{1}{3}x + \frac{g_a^2}{g^2}(1-x)\xi\left[-\frac{1}{6}\frac{e^2}{g_b^2}\right] , \tag{3.9}$$

$$\epsilon_R^u = -\tfrac{2}{3}x + \frac{g_a^2}{g^2}(1-x)\xi\left[\frac{1}{2}-\frac{2}{3}\frac{e^2}{g_b^2}\right] , \tag{3.10}$$

$$\epsilon_R^d = \tfrac{1}{3}x + \frac{g_a^2}{g^2}(1-x)\xi\left[\frac{1}{2}-\frac{1}{3}\frac{e^2}{g_b^2}\right] . \tag{3.11}$$

The coefficients describing parity-violating interactions are

$$C_{1u} - C_{1d}/2 = 2\sum g_A^e g_V^u - \sum g_A^e g_V^d , \tag{3.12}$$

$$C_{2u} - C_{2d}/2 = 2\sum g_V^e g_A^u - \sum g_V^e g_A^d , \tag{3.13}$$

where

$$g_V^e = -(\tfrac{1}{2}-2x) + \frac{g_a^2}{g^2}(1-x)\xi\left[\frac{1}{2}-\frac{1}{2}\frac{e^2}{g_b^2}\right] , \tag{3.14}$$

$$g_A^e = -\frac{1}{2} - \frac{g_a^2}{g^2}(1-x)\xi\left[\frac{1}{2}-\frac{3}{2}\frac{e^2}{g_b^2}\right] , \tag{3.15}$$

$$g_A^u = \frac{1}{2} + \frac{g_a^2}{g^2}(1-x)\xi\left[\frac{1}{2}-\frac{5}{6}\frac{e^2}{g_b^2}\right] , \tag{3.16}$$

$$g_A^d = -\frac{1}{2} - \frac{g_a^2}{g^2}(1-x)\xi\left[\frac{1}{2}-\frac{1}{6}\frac{e^2}{g_b^2}\right] , \tag{3.17}$$

$$g_V^u = \frac{1}{2} - \frac{4}{3}x - \frac{g_a^2}{g^2}(1-x)\xi\left[\frac{1}{2}-\frac{1}{2}\frac{e^2}{g_b^2}\right] , \tag{3.18}$$

$$g_V^d = -\frac{1}{2} + \frac{2}{3}x + \frac{g_a^2}{g^2}(1-x)\xi\left[\frac{1}{2}-\frac{1}{2}\frac{e^2}{g_b^2}\right] . \tag{3.19}$$

Finally the forward-backward asymmetry is given by

$$h_{AA}^L = \frac{1}{4} + \frac{g_a^2}{g^2}(1-x)\xi\left[\frac{1}{2}-\frac{1}{2}\frac{e^2}{g_b^2}\right] , \tag{3.20}$$

$L=\mu$ or τ. Following Amaldi *et al.*,[8] the experimental values are displayed in Table II. Based on the values in this table, the following constraints on g_a, g_b, and ξ are obtained.

From Eq. (3.8), we find

$$0 \le (g_a/g_b)^2\xi \le 0.78 . \tag{3.21}$$

From Eq. (3.9),

$$0 \le (g_a/g_b)^2\xi \le 0.54 . \tag{3.22}$$

From Eqs. (3.10) and (3.22),

$$0.016 \le (g_a/g)^2\xi \le 0.44 . \tag{3.23}$$

From Eqs. (3.11), and (3.22),

$$0.016 \le (g_a/g)^2\xi \le 0.55 . \tag{3.24}$$

From Eq. (3.14),

$$(g_a/g)^2\xi(1-x) - (g_a/g_b)^2 x\xi(1-x) \le 0.074 . \tag{3.25}$$

TABLE II. Values of the model-independent neutral-current parameters compared with the standard-model predictions for $\sin^2\theta_W = 0.23$

Quantity	Experimental value	Standard-model prediction
$\epsilon_L(u)$	0.339 ± 0.017	0.345
$\epsilon_L(d)$	-0.429 ± 0.014	-0.427
$\epsilon_R(u)$	-0.172 ± 0.014	-0.152
$\epsilon_R(d)$	$-0.011\pm^{0.081}_{0.057}$	0.076
g_A^e	-0.498 ± 0.027	-0.503
g_V^e	-0.044 ± 0.036	-0.045
C_{1u}	-0.249 ± 0.071	-0.191
C_{1d}	0.381 ± 0.064	0.340
$C_{2u}-\frac{1}{2}C_{2d}$	0.19 ± 0.37	-0.039
h_{AA}^{μ}	0.272 ± 0.015	0.25
h_{AA}^{τ}	0.232 ± 0.026	0.25

Hence,

$$(g_a/g)^2\xi \le 0.14 \; . \tag{3.26}$$

From Eq. (3.15),

$$(g_a/g)^2\xi(1-x)-3(g_a/g_b)^2\xi x(1-x) \le 0.064 \; . \tag{3.27}$$

Hence,

$$(g_a/g)^2\xi \le 0.21 \; . \tag{3.28}$$

From the above equations it follows that

$$g_1 \le 0.94g, \quad g_2 \le 0.94g \; , \tag{3.29}$$

$$g_a \le 1.3g, \quad g_b \le 0.83g \; . \tag{3.30}$$

From Eq. (3.12), we find

$$0.30(g_a/g)^2\xi + 0.20(g_a/g_b)^2\xi \le 0.02 \; . \tag{3.31}$$

From Eq. (3.13),

$$0.44(g_a/g)^2\xi + 0.50(g_a/g_b)^2\xi \le 0.14 \; . \tag{3.32}$$

Equations (3.31) and (3.32) imply

$$(g_a/g)^2\xi \le 0.16 \tag{3.33}$$

and

$$(g_a/g_b)^2\xi \le 0.68 \; . \tag{3.34}$$

Equation (3.20) yields

$$0.39(g_a/g)^2\xi - 0.30(g_a/g_b)^2\xi \le 0.12 \quad \text{for } \mu \tag{3.35}$$

$$\le 0.027 \quad \text{for } \tau \; . \tag{3.36}$$

Using Eq. (3.26) implies

$$(g_a/g_b)^2\xi \le 0.30 \quad \text{for } \mu \tag{3.37}$$

$$\le 0.21 \quad \text{for } \tau \; . \tag{3.38}$$

From Eq. (3.25), it then follows that

$$M_{Z'} \ge \left[\frac{x}{1-x} + \frac{0.074}{(1-x)^2(g_a/g_b)^2\xi} \right] \Big/ [(g_a/g)^2\xi]^{1/2} M_Z \; . \tag{3.39}$$

Using Eqs. (3.26) and (3.28) yields

$$M_{Z'} \ge 240 \text{ GeV} \; . \tag{3.40}$$

In the preceding, it is assumed that the supersymmetric corrections include the radiative corrections that the extra terms will generate. In first approximation, these radiative corrections can be assumed to be additive. Note that the accuracy in the measurement of $\epsilon_R(u)$ in Table II is about the same as that of $\epsilon_L(u)$ and $\epsilon_L(d)$. Nonetheless, although $\epsilon_L(u)$ and $\epsilon_L(d)$ are in close agreement with the standard-model calculations (which includes radiative corrections), the experimental value of the right-handed parameter $\epsilon_R(u)$ deviates from the standard-model value. If this deviation is taken seriously then it is interesting to note that either of Eqs. (3.23) or (3.24) and Eq. (3.30) together yield $M_{Z'} < 940$ GeV.

IV. MIXED STATES

In this section the various mixing matrices are listed. The treatment is similar to that of Haber and Kane.[15(a)]

For the sleptons soft supersymmetric breaking occurs via the terms $m_L^2 \tilde{L}^\dagger L$, $m_N^2 \tilde{N}_c^* N_c$, $m_E^2 \tilde{E}_c^* \tilde{E}_c$, and $m_{3/2}(h_e \tilde{L}^T \tau_2 H_d \tilde{E}_c, h_\nu \tilde{L}^T \tau_2 H_u \tilde{N}_c)$. Writing $m_e = h_e \langle H_d \rangle$, it follows that

$$\tilde{e}_1 = \tilde{e}\cos\theta_e + \tilde{E}_c \sin\theta_e \; , \tag{4.1}$$

$$\tilde{e}_2 = -\tilde{e}\sin\theta_e + \tilde{E}_c \cos\theta_e \; , \tag{4.2}$$

where

$$\tan 2\theta_e = 2m_e m_{3/2}/(m_L^2 - m_E^2) \; , \tag{4.3}$$

$$\tilde{M}^2_{e_{1,2}} = m_e^2 + \tfrac{1}{2}\{(m_L^2+m_E^2) \pm [(m_L^2-m_E^2)^2 + 4m_e^2 m_{3/2}^2]^{1/2}\} \; , \tag{4.4}$$

$$\tilde{N}_1 = \tilde{\nu}\cos\theta_\nu + \tilde{N}_c \sin\theta_\nu \; , \tag{4.5}$$

$$\tilde{N}_2 = -\tilde{\nu}\sin\theta_\nu + \tilde{N}_c \cos\theta_\nu \; , \tag{4.6}$$

where

$$\tan 2\theta_\nu = 2m_\nu m_{3/2}/(m_L^2 - m_E^2) \; , \tag{4.7}$$

$$\tilde{M}^2_{N_{1,2}} = m_\nu^2 + \tfrac{1}{2}[(m_L^2+m_N^2) \pm (m_L^2 - m_N^2 + 4m_\nu^2 m_{3/2}^2)^{1/2}] \; . \tag{4.8}$$

For the charged gauginos and Higgsinos, writing

$$\langle H_d \rangle = v_1, \quad \langle H_u \rangle = v_2, \quad 2\lambda_w^+ \lambda_w^- = \lambda_w^1 \lambda_w^1 + \lambda_w^2 \lambda_w^2 \; ,$$

the term in the Lagrangian to consider has the form

$$i(g/\sqrt{2})(v_1 \lambda_w^+ \bar{\tilde{H}}_d + v_2 \lambda_w^- \bar{\tilde{H}}_u) + m' \lambda_w^+ \lambda_w^- - \mu \tilde{H}_u^T C^{-1} \tau_2 \tilde{H}_d \; . \tag{4.9}$$

From here on it will be assumed that $v_1 = v_2 = v$ and $\mu = 0$. Now write

$$\psi_j^+ = (-i\lambda_w^+, \tilde{H}_u), \quad j=1,2 \; , \tag{4.10}$$

$$\psi_J^- = (-i\lambda_w^-, \tilde{H}_d), \quad j=1,2 \; . \tag{4.11}$$

From Eq. (4.9), one can now write

$$-\tfrac{1}{2}(\psi^+ \psi^-) \begin{bmatrix} 0 & x^T \\ x & 0 \end{bmatrix} \begin{bmatrix} \psi^+ \\ \psi^- \end{bmatrix} + \text{H.c.} \; , \tag{4.12}$$

where

$$x = \begin{bmatrix} m' & \sqrt{2}m_W \\ m_W & 0 \end{bmatrix} \tag{4.13}$$

choose unitary matrices U, V such that

$$U^* \times V^{-1} = M_D \; , \tag{4.14}$$

where M_D is a diagonal matrix with non-negative entries. The mass eigenstates are then

$$\chi_i^+ = V_{ij}\psi_j^+, \quad \chi_i^- = U_{ij}\psi_j^- \; , \tag{4.15}$$

and

$$\tilde{M}^2_{\pm}=\tfrac{1}{2}[m'^2+2m_W^2\pm(m'^4+2m_W^2m'^2)^{1/2}] .$$

For the neutral gauginos and Higgsinos, $U^{(1)}_{I_3R}\times U^{(1)}_{B-L}$ breaks down to $U^{(1)}_y$ since $\tilde{N}_c$ acquires a VEV, $\langle\tilde{N}_c\rangle=V_R$. Hence the following equations should be considered:

$$\lambda_{y'}=(-g_1\lambda_B+g_2\lambda_V)/(g_1^2+g_2^2)^{1/2} , \tag{4.16}$$

$$\lambda_y=(g_2\lambda_B+g_1\lambda_V)/(g_1^2+g_2^2)^{1/2} . \tag{4.17}$$

Hence, the part of the Lagrangian to be considered is

$$igV\lambda^3_w(\bar{\tilde{H}}_d-\bar{\tilde{H}}_u)/\sqrt{2}-ig_V V\lambda_y(V_1\bar{\tilde{H}}_d-V_2\bar{\tilde{H}}_u) +m'(\lambda^3_w)^TC^{-1}\lambda^3_w+m_{y'}\lambda_y^TC^{-1}\lambda_y+m_{y'}\lambda^T_{y'}\lambda_{y'} , \tag{4.18}$$

where $g_V=g_1g_2/(g_1^2+g_2^2)^{1/2}$. In writing Eq. (4.18), it was assumed that $\lambda_{y'}$ is much heavier than λ_y, λ^3_w. One can then, as usual, break down to the standard model:

$$L_{\text{matter}}=i(g^2+g_V^2)^{1/2}V\lambda_z(\bar{\tilde{H}}_d-\bar{\tilde{H}}_u)/\sqrt{2} +m'[(\lambda_z)^TC^{-1}\lambda_z+(\lambda_\gamma)^TC^{-1}\lambda_\gamma] . \tag{4.19}$$

The mass eigenstates are then

$$\tilde{\chi}^0_1=-i\lambda_z\cos\phi+(\tilde{H}^0_d-\tilde{H}^0_u)\sin\phi/\sqrt{2} , \tag{4.20}$$

$$-i\tilde{\chi}^0_2=i\lambda_z\sin\phi+(\tilde{H}^0_d-\tilde{H}^0_u)\cos\phi/\sqrt{2} , \tag{4.21}$$

where

$$\cos\phi=[\tilde{M}_2/(\tilde{M}_1+\tilde{M}_2)]^{1/2} \tag{4.22}$$

and

$$\tilde{M}_{1,2}=(M_Z^2+m'^2/4)^{1/2}\pm m'/2 . \tag{4.23}$$

If a singlet fermion ψ_x is included the other combination $(\tilde{H}^0_d+\tilde{H}^0_u)/\sqrt{2}$ mixes with ψ_x and both acquire a mass $m_x=\lambda_v$, otherwise $(\tilde{H}^0_d+\tilde{H}^0_u)$ remains massless. For a general expression for the neutralino masses see Ref. 15(b).

An explanation is needed for our choice of the ψ_x mass (μ). Without including supergravity we do not have a way of estimating masses. We choose $\mu\simeq 0$ for simplicity of calculation, but also because we expect μ, which is associated with the axion mass to be very small (the invisible axion). Note that even in supergravity μ can be small ($\mu>6$ GeV in renormalization-group models). In general this choice simplifies calculations but makes the photino become massive and decouple from the neutralino mass matrix. We do not consider this to be a serious problem since not much is known about $\tilde{m}_\gamma$. Note that even for $\mu\neq 0$ we will not have a significant contribution to $g-2$ from the singlet of mass μ since we assume μ to be small.

For the squarks the results will be similar to Eqs. (4.1)–(4.8) inclusive. Thus

$$\tilde{u}_1=\tilde{u}\cos\theta_u+\tilde{U}_c\sin\theta_u , \tag{4.24}$$

$$\tilde{u}_2=-\tilde{u}\sin\theta_u+\tilde{U}_c\cos\theta_u , \tag{4.25}$$

where

$$\tan 2\theta_u=2m_um_{3/2}(m_Q^2-m_U^2) , \tag{4.26}$$

$$m_u=h_u\langle H_u\rangle , \tag{4.27}$$

and

$$\tilde{d}_1=\tilde{d}\cos\theta_d+\tilde{d}_c\sin\theta_d , \tag{4.28}$$

$$\tilde{d}_2=-\tilde{d}\sin\theta_d+\tilde{d}_c\cos\theta_d , \tag{4.29}$$

where

$$\tan 2\theta_d=2m_dm_{3/2}/(m_Q^2-m_D^2) , \tag{4.30}$$

$$m_d=h_d\langle H_d\rangle . \tag{4.31}$$

V. $g-2$ OF THE MUON

The collection of additional graphs due to the present model are displayed in Fig. 4. Consider the term calculated from the graphs of the form

$$\frac{ie}{2m_\mu}F(q^2)\bar{u}\sigma_{\alpha\beta}q^\beta u . \tag{5.1}$$

Then the muon anomaly is defined by

$$a_\mu\equiv(g-2)_\mu/2=F(0) . \tag{5.2}$$

Since the anomalous magnetic moments are so accurately measured, it is essential for any major change in particle theory that one must check that the addition of new particles will not adversely affect the present theoretical success. The present experimental value[16] from the last CERN $g-2$ experiment is

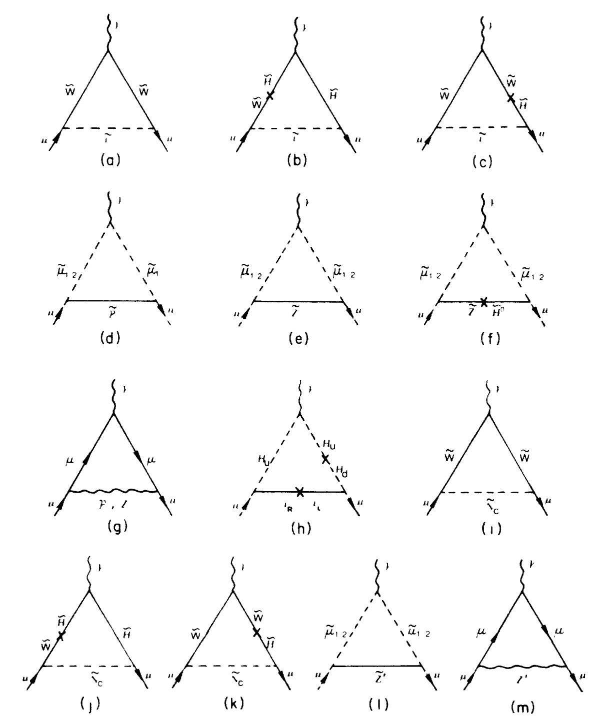

FIG. 4. One-loop diagrams contributing to $(g-2)_\mu$.

$$a_\mu(\text{expt}) = 1\,165\,922(9)\times 10^{-9} , \tag{5.3}$$

where the number in parentheses represents the error in the last significant figure. The Weinberg-Salam standard-model contribution is[17]

$$a_\mu(\text{WS}) = 1.95(1)\times 10^{-9} . \tag{5.4}$$

Contributions from supersymmetry should be of the same order of magnitude as the standard-model contributions.[18] Thus this should be detectable by the new generation of $(g-2)_\mu$ experiments proposed at BNL and Novosibirsk.

The graph in Fig. 4(d) was first considered by Fayet.[19] In addition, the graphs in Figs. 4(a) and 4(e) are considered by Ellis, Hagelin, and Nanopoulos[20] and by Grifols and Mendez.[21] Additionally, the graph in Fig. 4(f) is considered by Barbieri and Maiani.[22] Finally Kosower, Krauss, and Sakai[23] also considered the graphs in Figs. 4(b) and 4(c). Figure 4(g) is the QED and standard-model contributions.[24] Figure 4(h) can be neglected since it involves the neutrino mass and the free parameter μ which can be assumed to be small. The new contributions of this model are given by Figs. 4(i), 4(j), 4(k), 4(l), and 4(m). The results are as follows.

From Figs. 4(a) and 4(i),

$$a_\mu^{(\tilde{\nu}L\tilde{\nu})} = \frac{g^2}{96\pi^2}\{\cos\phi_-\cos\phi_+[\cos^2\alpha_\nu F'(x_{11})+\sin^2\alpha_\nu F'(x_{21})]+\sin\phi_-\sin\phi_+[\cos^2\alpha_\nu F'(x_{12})+\sin^2\alpha_\nu F'(x_{22})]\} , \tag{5.5}$$

where

$$F'(x_{km}) = \frac{m^2}{m^2_{\tilde{\chi}m}}\left[\frac{1-5x_{km}-2x_{km}^2}{(1-x_{km})^3}-\frac{6x_{km}^2}{(1-x_{km})^4}\ln x_{km}\right], \quad x_{km} = \frac{m^2_{\tilde{\nu}_K}}{m^2_{\tilde{\chi}_m}} . \tag{5.6}$$

From Figs. 4(b), 4(c), 4(j), and 4(k),

$$a_\mu^{(\tilde{\nu},\tilde{W},\tilde{H})} = -\frac{gh_d}{8\pi^2}\{\sin_+\phi_-\cos\phi[\cos^2\alpha_\nu F(x_{12})+\sin^2\alpha_\nu F(x_{22})]+\sin\phi_-\cos\phi_+[\cos^2\alpha_\nu F(x_{11})+\sin^2\alpha_\nu F(x_{21})]\} , \tag{5.7}$$

where

$$F(x_{km}) = \frac{m}{m_{\tilde{\chi}_m}}\left[\frac{1-3x_{km}}{(1-x_{km})^2}-\frac{2x_{km}^2}{(1-x_{km})^3}\ln x_{km}\right], \quad x_{km} = \frac{m^2_{\tilde{\nu}_k}}{m^2_{\tilde{\chi}_m}} . \tag{5.8}$$

From Fig. 4(e),

$$a_\mu^{(\tilde{Z})} = -\frac{g^2}{24\pi^2\cos^2\theta_W}\{\cos^2\beta[\cos^2\alpha_\mu G'(x_{11})+\sin^2\alpha_\mu G'(x_{21})]$$
$$+\sin^2\beta[\cos^2\alpha_\mu G'(x_{12})+\sin^2\alpha_\mu G'(x_{22})](\tfrac{1}{4}-\sin^2\theta_W+2\sin^4\theta_W)\} , \tag{5.9}$$

where

$$G'(x_{km}) = \frac{m^2}{m^2_{\tilde{\chi}_m}}\left[\frac{2+5x_{km}-x_{km}^2}{2(1-x_{km})^3}+\frac{3x_{km}}{(1-x_{km})^4}\ln x_{km}\right], \quad x_{km} = \frac{m_{\tilde{\mu}_k}}{m^2_{\tilde{\chi}_m}} . \tag{5.10}$$

From Fig. 4(f),

$$a_\mu^{(\tilde{Z},\tilde{H})} = +\frac{gh_d}{32\pi^2\cos\theta_W}\sin 2\beta\{(\cos^2\alpha_\mu\cos 2\theta_W+\sin^2\alpha_\mu 2\sin^2\theta_W)[G(x_{11})+G(x_{12})]$$
$$+(\sin^2\alpha_\mu\cos 2\theta_W+\cos^2\alpha_\mu 2\sin^2\theta_W)[G(x_{21})+G(x_{22})]\} , \tag{5.11}$$

where

$$G(x_{km}) = \frac{m}{m_{\tilde{\chi}_m}}\left[\frac{1+x_{km}}{(1-x_{km})^2}+\frac{2x_{km}}{(1-x_{km})^3}\ln x_{km}\right], \quad x_{km} = \frac{m^2_{\tilde{\upsilon}_k}}{m^2_{\tilde{\chi}_m}} . \tag{5.12}$$

From Fig. 4(l),

$$a_\mu^{(\tilde{Z})} = -\frac{g_1^2+g_2^2}{48\pi^2}\left\{\left[\frac{g}{g_1}\right]^4\tan^4\theta_W[\cos^2\alpha_\mu G'(x_1)+\sin^2\alpha_\mu G'(x_2)]+\left[\left[\frac{g}{g_1}\right]^2 2\tan^2\theta_W-1\right]^2\right\}$$
$$\times[\sin^2\alpha_\mu G'(x_1)+\cos^2\alpha_\mu G'(x_2)] , \tag{5.13}$$

where

$$G'(x_k)=\frac{m^2}{m_{\tilde{Z}}^2}\left[\frac{2+5x_k-x_k^2}{2(1-x_k)^3}+\frac{3x_k}{(1-x_k)^4}\ln x_k\right],\quad x_k=\frac{m_{\tilde{\mu}_k}^2}{m_{\tilde{Z}}^2}\ . \tag{5.14}$$

From Fig. 4(d),

$$a_\mu^{(\tilde{\gamma})}=-\frac{\alpha}{12\pi}\left[\frac{1}{m_{\mu_1}^2}+\frac{1}{m_{\mu_2}^2}\right]\ . \tag{5.15}$$

From Fig. 4(g),

$$a_\mu^{(Z)}=\frac{g^2}{48\pi^2\cos^2\theta_W}\frac{m^2}{m_Z^2}(-1-2\sin^2\theta_W+4\sin^4\theta_W)\ . \tag{5.16}$$

From Fig. 4(m),

$$a_\mu^{(Z')}=\frac{g_1^2+g_2^2}{48\pi^2}\frac{m^2}{m_Z^2}\left[\left[\frac{g}{g_1}\right]^4\tan^4\theta_W+\left[\frac{g}{g_1}\right]^2\tan^2\theta_W-1\right]\ . \tag{5.17}$$

Adding these contributions we obtain the following.

The total W-ino–charged-Higgsino contribution is

$$\begin{aligned}a_\mu^{W\tilde{W}\tilde{H}}=\frac{5g^2}{96\pi^2}\Bigg[&\frac{m^2}{m_W^2}+\tfrac{1}{5}\{\cos\phi_+\cos\phi_-[\cos^2\alpha_\nu F(x_{11})F(x_{11})+F(x_{21})\sin^2\alpha_\nu]\\&+\sin\phi_+\sin\phi_-[\cos^2\alpha_\nu F(x_{12})+F(x_{22})\sin^2\alpha_\nu]\}\\&-\tfrac{6}{5}\{\sin\phi_+\cos\phi_-[\cos^2\alpha_\nu F(x_{12})+\sin^2\alpha_\nu F(x_{22})]+\sin\phi_-\cos\phi_+[\cos^2\alpha_\nu F(x_{11})+\sin^2\alpha_\nu F(x_{21})]\}\Bigg]\ .\end{aligned}$$

Note that this contribution becomes zero in the exact supersymmetric limit. This contribution is what is expected in the ordinary SU(2)×U(1) SUSY. The restrictions of the masses of $m_{\tilde{\nu}}$ and $m_{\tilde{W}}$ are the same as in Ref. 22.

Similarly the Z-ino–neutral-Higgsino contribution is

$$\begin{aligned}a_\mu^{Z\tilde{Z}\tilde{H}}=\frac{g^2}{48\pi^2\cos^2\theta_W}\Bigg[&\frac{m^2}{m_Z^2}(-1-2\sin^2\theta_W+4\sin^4\theta_W)\\&-2\{\cos^2\beta[\cos^2\alpha_\mu G(x_{11})+\sin^2\alpha_\mu G(x_{21})]\\&+\sin^2\beta[\cos^2\alpha_\mu G(x_{12})+\sin^2\alpha_\mu G(x_{22})]\}(\tfrac{1}{4}-\sin^2\theta_W+2\sin^2\theta_W)\\&+\tfrac{3}{2}\sin2\beta(\cos^2\alpha_\mu\cos^2\theta_W+\sin^2\alpha_\mu 2\sin^2\theta_W)\\&\times\{[G(x_{11})+G(x_{12})]+(\sin^2\alpha_\mu\cos^2\theta_W+\cos^2\alpha_\mu 2\sin^2\theta_W)[G(x_{21})+G(x_{22})]\}\Bigg]\ .\end{aligned}$$

This is again the standard supersymmetric SUSY contribution and it is negligible ($\sim10^{-19}$) for all $m_{\tilde{Z}}, m_{\tilde{H}}, m_{\tilde{\mu}}, \geq 20$ GeV. Note that these contributions appear to be the same order of magnitude as the usual Weinberg-Salam contributions ($\sim10^{-9}$) but have the opposite sign. The only new contribution is the one from the $Z'Z'$ graph:

$$\begin{aligned}a_\mu^{Z'\tilde{Z}'}=\frac{g^2}{48\pi^2}\Bigg[&g''^4[\cos^2\alpha_\mu G(x_1)+\sin^2\alpha_\mu G(x_2)]+(2g''^2-1)[\sin^2\alpha_\mu G(x_1)+\cos^2\alpha_\mu G(x_2)]\\&+\tfrac{3}{2}g''^2(1-2g''^2)[G(x_1)+G(x_2)]+\frac{m^2}{M_Z^2}(g''^4+g''^2-1)\Bigg]\ ,\end{aligned}$$

where $g''=(g/g_1)\tan\theta_W$. We have studied this contribution for $M_{Z'}\approx 250$ GeV in two extreme cases $\cos\alpha_\mu=1$ (one smuon dominating) and

$$\cos\alpha_\mu=\frac{1}{\sqrt{2}}(m_{\tilde{\mu}_1}=m_{\tilde{\mu}_2}) .$$

In both cases the contribution $a_\mu^{Z'\tilde{Z}'}$ is very small, less than 10^{-9} for $m_{\tilde{\mu}}=20-200$ GeV, $m_{\tilde{Z}}=20-200$ GeV. This is due in part to the smallness of $m_{\tilde{\mu}}/m_{\tilde{Z}}$, but mostly to cancellations that occur between the $\tilde{Z}'$ and the Z' parts, cancellations that become exact in the unbroken-supersymmetry limit. It is expected that because of the nature of the cancellations, the smallness of $a_\mu^{Z'\tilde{Z}'}$ is a feature common to all SU(2)×U(1) supersymmetric models.

VI. CONCLUSIONS AND PROSPECTS

This paper presents an extension of the standard supersymmetric model. It has a few attractive features: it can be obtained from breaking down E_6. In addition, it provides a satisfactory solution to the two problems that plague superstring phenomenology: baryon-number conservation and smallness of neutrino mass.[13] The model is presented in some detail and then subjected to some rigorous constraints of the low-energy phenomenology: neutral currents bounds and the anomalous magnetic moment of the muon.

While the former imposes the bound $M_{Z'}\geq 240$ GeV (and possibly, the more speculative one $M_{Z'}\leq 1$ TeV) the latter presents no restrictions on the parameters of the model.

Further work on the restrictions imposed on the model is in progress, as well as investigations about the interesting signatures of this type of model in e^+e^- and $p\bar{p}$.

ACKNOWLEDGMENTS

The authors would especially like to thank Dr. R. N. Mohapatra for his kindness in discussing the topic of this paper at length during two separate visits to the University of Maryland. One of us (M.F.) wishes to thank the Carleton Particle Physics Group (in particular P. Kalyniak, M. K. Sundaresan, and P. Watson) for discussions and suggestions. The authors would also like to thank the Natural Sciences and Engineering Research Council of Canada for financial support through Grants Nos. A 0358 and UO 529.

[1] M. Green and J. Schwarz, Phys. Lett. **149B**, 117 (1984); **151B**, 21 (1985).

[2] D. Gross, J. Harvey, E. Martinec, and R. Rohm, Phys. Rev. Lett. **54**, 502 (1985); Nucl. Phys. **B256**, 251 (1985).

[3] P. Candelas, G. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. **B518**, 46 (1985).

[4] E. Witten, Nucl. Phys. **B258**, 75 (1985).

[5] For phenomenological studies of these models, see Witten (Ref. 4); M. Dine, V. Kaplunovsky, M. Mangano, C. Nappi, and N. Seiberg, Nucl. Phys. **B529**, 549 (1985); J. Breit, B. Ovrut, and G. Segré, Phys. Lett. **158B**, 33 (1985); J. P. Derendinger, L. Ibanez, and H. P. Nilles, *ibid.* **169B**, 354 (1986); F. del Aguila, G. Blair, M. Daniel, and G. G. Ross, Nucl. Phys. **B272**, 413 (1986); S. Cecotti, J. P. Derendinger, S. Ferrara, L. Girardello, and M. Roncadelli, Phys. Lett. **156B**, 318 (1985); C. Nappi and V. Kaplunovsky, Comments Nucl. Part. Phys. **16**, 57 (1986); P. Binetruy, S. Dawson, I. Hinchliffe, and M. Sher, Nucl. Phys. **B263**, 413 (1986). For earlier work on E_6 grand unification, see F. Gürsey, P. Sikivie, and P. Ramond, Phys. Lett. **60B**, 177 (1976); F. Gürsey and M. Serdaroglu, Lett. Nuovo Cimento **21**, 28 (1978); Y. Achiman and B. Stech, Phys. Lett. **77B**, 389 (1978); Q. Shafi, *ibid.* **79B**, 301 (1979); J. Rosner, Comments Nucl. Part. Phys. **15**, 195 (1986).

[6] J. Ellis, K. Enqvist, D. V. Nanopoulos, and F. Zwirner, Nucl. Phys. **B276**, 14 (1986).

[7] E. Ma, Phys. Rev. D **36**, 274 (1987).

[8] U. Amaldi, A. Bohm, L. S. Durkin, P. Langacker, A. K. Mann, W. J. Marciano, A. Sirlin, and H. H. Williams, Phys. Rev. D **36**, 1385 (1987).

[9] R. Mohapatra, *Unification and Supersymmetry, The Frontiers of Quark-Lepton Physics* (Springer, New York, 1986).

[10] V. Barger, N. G. Deshpande, and K. Whisnant, Phys. Rev. Lett. **56**, 30 (1986); Phys. Rev. D **33**, 1912 (1986).

[11] V. Barger, R. J. N. Phillips, and K. Whisnant, Phys. Rev. Lett. **57**, 48 (1986).

[12] J. L. Hewett, T. G. Rizzo, and J. A. Robinson, Phys. Rev. D **33**, 1476 (1986); R. W. Robinett, *ibid.* **33**, 1908 (1986); P. M. Fishbane, R. E. Norton, and M. J. Rivard, *ibid.* **33**, 2632 (1986).

[13] F. Gürsey, P. Ramond, and P. Sikivie, Phys. Lett. **60B**, 177 (1977); Y. Achiman and B. Stech, *ibid.* **77B**, 389 (1977); P. K. Mohapatra, R. N. Mohapatra, and P. Pol, Phys. Rev. D **33**, 2010 (1986); J. Rosner, Comments Nucl. Part. Phys. **17**, 93 (1987).

[14] The particular calculations reported in this paper do not depend on h_ν or $\langle\bar{\nu}_R\rangle\neq 0$, although the latter aspect may be needed to solve the neutrino mass problem, see R. N. Mohapatra, Phys. Rev. Lett. **56**, 561 (1986).

[15] (a) H. E. Haber and G. L. Kane, Phys. Rep. **117**, 76 (1984); (b) T-C Yuan, R. Arnowitt, A. H. Chamseddine, and Pran Nath, Z. Phys. C **26**, 407 (1984).

[16] J. Bailey *et al.*, Phys. Lett. **68B**, 191 (1977); F. J. M. Farley and E. Picasso, Annu. Rev. Nucl. Part. Sci. **29**, 243 (1979).

[17] T. Kinoshita, B. Nize, and Y. Okamoto, Phys. Rev. Lett. **52**, 717 (1984).

[18] R. Arnowitt and P. Nath, Northeastern University Report No. NUB 2697, 1986 (unpublished).

[19] P. Fayet, in *Unification of the Fundamental Particle Interactions*, proceedings of the Europhysics Study Conference, Erice, Italy, 1980, edited by S. Ferrara, J. Ellis, and P. Van Nieuwenhuizen (Ettore Majorana International Science Series: Physical Science, Vol. 7) (Plenum, New York, 1980).

[20] J. Ellis, J. Hagelin, and D. V. Nanopoulos, Phys. Lett. **116B**, 283 (1982).

[21] J. A. Grifols and A. Mendez, Phys. Rev. D **26**, 1809 (1982).

[22] R. Barberi and L. Maiani, Phys. Lett. **117B**, 203 (1982).

[23] D. A. Kosower, L. M. Krauss, and N. Sakai, Phys. Lett. **133B**, 305 (1983).

[24] R. Jackiw and S. Weinberg, Phys. Rev. D **5**, 2396 (1972); I. Bars and M. Yoshimura, *ibid.* **6**, 374 (1972); W. A. Bardeen, R. Gastmans, and B. Lautrup, Nucl. Phys. **B46**, 319 (1972); K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D **6**, 2923 (1972).