Decay of mesons in flux-tube quark model

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Strong decays of several light mesons are investigated in a flux-tube quark model, where a $q\bar{q}$ pair is created either in the ${}^3S_1$ state or in the ${}^3P_0$ state within the flux tube. For comparison, decay rates are also calculated in a naive ${}^3P_0$ model, where a $q\bar{q}$ pair is created anywhere in the neighborhood of parent quarks. The sensitivity of the decay width to the radius of the pion, to relativistic factors like (m/E), and to final-state interactions is studied. Both the ${}^3S_1$ and ${}^3P_0$ models can explain the available data; however, the ${}^3S_1$ model requires stronger final-state interactions.

I. INTRODUCTION

Semirelativistic constituent-quark models[1-3] have been quite successful in explaining hadron spectroscopy. Reference 2 contains a large list of citations on this subject. In these models only the degrees of freedom of the valance quark are retained, thus, mesons are treated as interacting $q\bar{q}$ pairs, and baryons have three interacting quarks. The interactions between the quarks are described with potentials based on simple ideas in QCD, the relativistic expression $(m^2+p^2)^{1/2}$ is used for the kinetic energies of the quarks, and the eigenstates of the resulting Hamiltonian are calculated. These models have a somewhat mysterious success. One might have thought that they would work only for the heavy-quark systems such as charmonium, and certainly not for the pion. However, they do give the correct value for the mass difference between π and ρ mesons with interactions whose parameters are adjusted to fit the mass difference between nucleon and the Δ and many other energies.[1-3] The radii of the mesons and baryons obtained from these models are much smaller than the radii obtained from their charge form factors. However, it has been suggested that the difference could be explained by a Lorentz contraction of the hadron in the scattering process,[4] and by the charged-meson cloud of the physical hadrons.[5] Moreover, the constituent quarks of this model are probably not bare quarks, and thus, their size must be considered in obtaining the charge form factor.[4]

In the constituent-quark model a meson can decay into two mesons by the creation of a $q\bar{q}$ pair. The decay of heavy mesons has been studied[6,7] in this model by assuming that the potential $v(\mathbf{r}-\mathbf{r}')$ couples to the color charge density

$$\rho_\alpha(\mathbf{r})=\psi^\dagger(\mathbf{r})\tfrac{1}{2}\lambda_\alpha\psi(\mathbf{r}) \ , \tag{1.1}$$

$$H_I=\tfrac{1}{2}\sum_\alpha\int :\rho_\alpha(\mathbf{r})v(\mathbf{r}-\mathbf{r}')\rho_\alpha(\mathbf{r}'):d^3r\, d^3r' \ . \tag{1.2}$$

This interaction Hamiltonian achieves quark confinement by requiring that $v(\mathbf{r}-\mathbf{r}')$ is linear in $|\mathbf{r}-\mathbf{r}'|$ at large $|\mathbf{r}-\mathbf{r}'|$, however, it fails to incorporate the confinement of color fields. It also leads to long-range van der Waals forces between hadrons.[8]

In the flux-tube model[3] the problem of confining color fields along with the quarks is resolved by assuming that these fields form flux tubes. A flux tube starts from each quark and it may terminate at an antiquark, or three tubes may terminate at a Y junction in a locally gauge-invariant fashion.[3] Thus, the decay of a meson in this model has to be interpreted as the breaking of a flux tube into two pieces by the creation of a $q\bar{q}$ pair in the tube.

General-invariance arguments[9] allow many forms of the amplitude to create the $q\bar{q}$ pair within the flux tube. The simplest of these contain

$$\phi_q^\dagger(\mathbf{r})\boldsymbol{\sigma}\phi_{\bar{q}}(\mathbf{r})\cdot\hat{\mathbf{t}}F(\mathbf{r}) \tag{1.3}$$

or

$$\phi_q^\dagger(\mathbf{r})\boldsymbol{\sigma}\cdot\mathbf{p}_{\bar{q}q}\phi_{\bar{q}}(\mathbf{r})F(\mathbf{r}) \ . \tag{1.4}$$

Here ϕ_q and $\phi_{\bar{q}}$ are the wave functions of the created quark and antiquark, $\hat{\mathbf{t}}$ is a unit vector in the direction of the flux, $\mathbf{p}_{\bar{q}q}$ is the relative momentum of the $\bar{q}q$ pair, and $F(\mathbf{r})$ is one inside the tube and zero outside. The pair is created in the ${}^3S_1$ or ${}^3P_0$ state with the amplitudes 1.3 and 1.4, respectively. The decays of charmonium and Υ states into two heavy mesons have been studied in Ref. 9 and both the models are found to be consistent with the data.

Recently Kokoski and Isgur[10] have studied the decays of very many light mesons with the flux-tube model. They find that the ${}^3P_0$ amplitude (1.4) gives a reasonable explanation of the data, whereas the ${}^3S_1$ amplitude (1.3) gives too large widths for the decays such that $B\rightarrow\omega+\pi$ in which the mesons in the final state are in relative $l=0$ (S-wave) state. The mesons in the final state of the decays considered in Ref. 9 are in $l>0$ states.

The flux tube contains a chromoelectric field $E^\alpha(\mathbf{r})=i\nabla A_4^\alpha(\mathbf{r})$. Using this field and the standard QCD Hamiltonian

$$H_{\rm int}=ig\int d^3r\, \bar{\psi}(\mathbf{r})\gamma_4\tfrac{1}{2}\lambda^\alpha\psi(\mathbf{r})A_4^\alpha(\mathbf{r}) \tag{1.5}$$

we obtain the amplitude (1.3). Thus, this amplitude has a physical appeal in the flux-tube model. On the other hand, using the amplitude (1.4) in this model[10] amounts to assuming that the zero-point oscillations of the flux tube wipe out the amplitude (1.3), and that the breaking of the tube is dominated by vacuum fluctuations. This may be possible, but it is not obvious, and hence, it is interesting to examine if other mechanisms can suppress the large S-wave decay rates obtained with amplitude (1.3).

In this paper we study the decay of several light mesons into pions, as well as the decay of B, A_1, A_2, and A_3 mesons into $\pi+\omega$ and $\pi+\rho$ with this point of view. Both the ${}^3S_1$ and ${}^3P_0$ amplitudes are used and the sensitivity of the decay width to the radius of the pion, to relativistic factors such as (m/E) and to final-state interactions is studied. We find that within the uncertainties in these models, both the amplitudes can explain the available data.

II. THE DECAY AMPLITUDE

The wave functions of the initial and final states are assumed to be

$$|i\rangle=\Phi_i(\mathbf{r}_q-\mathbf{r}_{\bar q},\chi_q,\chi_{\bar q})C(k,l)\ , \tag{2.1}$$

$$|f\rangle=\Psi_R(\mathbf{R}_1-\mathbf{R}_2)\Phi_1(\mathbf{r}_q-\mathbf{r}_{\bar q'},\chi_q,\chi_{\bar q'})\times\Phi_2(\mathbf{r}_{q'}-\mathbf{r}_{\bar q},\chi_{q'},\chi_{\bar q})C(k,j)C(i,l)\ . \tag{2.2}$$

Here Φ_i, Φ_1, and Φ_2 denote the spin, space, and isospin part of the wave functions of the initial and the final two mesons, χ_q and $\chi_{\bar q}$ are the spins of the q and $\bar q$ in the initial meson and $\chi_{q'}$ and $\chi_{\bar q'}$ those of the created $q\bar q$ pair. Labels of isospin states are omitted for brevity, and $C(k,l)$, etc., denote color states. The $\Psi_R(\mathbf{R}_1-\mathbf{R}_2)$ describes the relative motion of mesons 1 and 2 in the final state, $\mathbf{R}_1$ and $\mathbf{R}_2$ are their center of mass:

$$\mathbf{R}_1=\tfrac{1}{2}(\mathbf{r}_q+\mathbf{r}_{\bar q'}),\quad \mathbf{R}_2=\tfrac{1}{2}(\mathbf{r}_{q'}+\mathbf{r}_{\bar q})\ . \tag{2.3}$$

Ψ_R also contains a spin-isospin part that is suppressed here for brevity. The exchange between the quarks q and q' (and also the antiquarks) belonging to mesons 1 and 2 is neglected; however, when 1 and 2 are identical, as in the case of two pion decays, $|f\rangle$ is symmetric under the exchange of the two mesons.

Using the amplitude (1.3), Hamiltonian (1.5), and neglecting the width of the flux tube, we obtain

$$\langle f\,|H_{\text{pair}}|\,i\rangle=-\frac{ig^2}{2m}\frac{4}{3\sqrt{3}}\int d^3r\int_0^r dr'\Psi_R^*\left[\frac{\mathbf{r}}{2}\right]\langle\chi_{q'}|\boldsymbol{\sigma}|\chi_{\bar q'}\rangle\cdot\hat{\mathbf{r}}\Phi_1^\dagger(\mathbf{r}',\chi_q,\chi_{\bar q'})\Phi_2^\dagger(\mathbf{r}-\mathbf{r}',\chi_{q'},\chi_{\bar q})\Phi_i(\mathbf{r},\chi_q,\chi_{\bar q})\ , \tag{2.4}$$

where the vectors $\mathbf{r}$ and $\mathbf{r}'$ are shown in Fig. 1, m is the constituent-quark mass and the factor $4/3\sqrt{3}$ comes from the color part. The strength of this interaction is determined by g^2/m which is taken as the parameter Λ_s. For example, in the case of two pion decays of mesons having $S=1$ and $J=L+1$ [i.e., the $\rho(770)$, $f(1270)$, $g(1690)$, $h(2030)$, $\rho(2350)$, and $r(2510)$ for $J=1-6$, respectively] this matrix element assumes the simple form

$$\langle f\,|H_{\text{pair}}|\,i\rangle=-i^{J+1}[1+(-1)^{T+J}]\frac{\Lambda_s}{18}\left[\frac{3(T+3)(2-T)J}{2J+1}\right]^{1/2}Y_{JM_J}(\hat{\mathbf{k}})I_{2\pi}^J(k)\ , \tag{2.5}$$

$$I_{2\pi}^J(k)=\int_0^\infty r^2dr\,f_J(r)u_{l=J}\left[\frac{kr}{2}\right]\int_0^r dr' f_\pi(r')f_\pi(r-r')\ , \tag{2.6}$$

where $\mathbf{k}$ is the pion momentum, T is the total isospin, and f_J, f_π, and u_l are the radial wave functions of the initial meson, pion, and Ψ_R.

We have also studied the naive ${}^3P_0$ pair creation model[11] without any flux-tube considerations. The decay matrix element in this model is given by

$$\langle f\,|H_{\text{pair}}|\,i\rangle=\gamma_0\int d^3r\int d^3r'\Psi_R^*\left[\frac{\mathbf{r}}{2}\right]\Phi_1^\dagger(\mathbf{r}',\chi_q,\chi_{\bar q'})\Phi_2^\dagger(\mathbf{r}-\mathbf{r}',\chi_{q'},\chi_{\bar q})\sigma_{q'\bar q'}\cdot\mathbf{p}_{q'\bar q'}\Phi_i(\mathbf{r},\chi_q,\chi_{\bar q}) \tag{2.7}$$

and it corresponds to the case $b=0$ of Kokoski and Isgur[10] (KI). We have also studied this model in the narrow flux-tube limit (case $b\to\infty$ of KI) for which the matrix element is

$$\langle f\,|H_{\text{pair}}|\,i\rangle=\bar\gamma_0\int d^3r\int_0^r dr'\Psi_R^*\left[\frac{\mathbf{r}}{2}\right]\Phi_1^\dagger(\mathbf{r}',\chi_q,\chi_{\bar q'})\Phi_2^\dagger(\mathbf{r}-\mathbf{r}',\chi_{q'},\chi_{\bar q})\sigma_{q'\bar q'}\cdot\mathbf{p}_{q'\bar q'}\Phi_i(\mathbf{r},\chi_q,\chi_{\bar q})\ . \tag{2.8}$$

The decay matrix elements are obviously sensitive to the radial wave functions f_J and f_π. These wave functions are calculated in Ref. 3 using the flux-tube Hamiltonian, and are used for all the mesons except the pion. In the flux-tube model the pion has a suspiciously small rms radius $R_\pi=0.16$ fm. In absence of the color-magnetic interaction π, ρ, and ω are degenerate and have $R=0.29$ fm. The color-magnetic interaction lowers the pion energy significantly and decreases its radius to 0.16 fm. Because of the semirelativistic nature of the model it

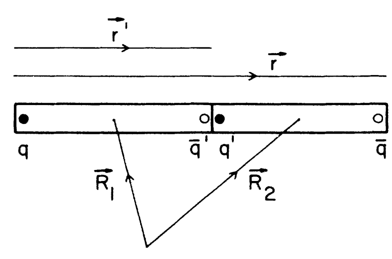

FIG. 1. Picture of meson decay.

is not clear that this decrease is realistic. The radii of other mesons are not significantly influenced by the color-magnetic interaction, that of ρ, for example, increases from 0.29 to 0.32 fm. The radii of mesons having $J \geq 2$ are primarily determined by the tension of the flux tube, and range from 0.46 for $J=2$ $f(1270)$ to 0.77 for the $J=6$ $r(2510)$ meson. Note that the mean distance between the q and $\bar{q}$ is 2 times the rms radius of the meson.

The matrix elements are very sensitive to R_π, but not to the finer details of $f_\pi(r)$. They are practically unchanged when the Coulomb wave function

$$f_{c,\pi}(r)=2\nu_\pi^{3/2}e^{-\nu_\pi r}\ , \tag{2.9}$$

$$\nu_\pi=\frac{\sqrt{3}}{2R_\pi} \tag{2.10}$$

for $R_\pi=0.16$ fm is used instead of the $f_\pi(r)$ of Ref. 3. The calculations with Coulomb wave function are particularly simple because

$$\int_0^r dr' f_{c,\pi}(r') f_{c,\pi}(r-r')=4\nu_\pi^3 r e^{-\nu_\pi r}\ , \tag{2.11}$$

thus, all of our results are obtained with the $f_{c,\pi}$. The decay widths are calculated for two values $R_\pi=0.16$ and 0.29 fm. KI also study decay widths with meson wave functions calculated from flux-tube Hamiltonian,[2] which give $R_\pi=0.16$ fm, as well as harmonic-oscillator wave functions which give $R_\pi=0.3$ fm.

In all the earlier work the radial wave function $u_l(kr/2)$ is taken as the spherical Bessel function $j_l(kr/2)$. This choice amounts to neglecting any interaction between the two mesons in the final state. The factorization of the final-state wave function $|f\rangle$ [Eq. (2.2)] is presumably valid only for $|\mathbf{R}_1-\mathbf{R}_2|\ \rangle$ the sum of meson radii. At very small values of $|\mathbf{R}_1-\mathbf{R}_2|$ the two-quark–two-antiquark system presumably forms a very complex state that cannot be described with the wave function (2.2). One can put this effect into the $|f\rangle$ by assuming that the mesons have a hard-core interaction so that $u_l(kr/2)$ is zero when $r/2<C$. We have used

$$u_l(kr/2)=\begin{cases}\cos\delta_l j_l(kr/2)-\sin\delta_l n_l(kr/2) & \text{for } r/2>C\ , \quad (2.12)\\ 0 \quad \text{for } r/2<C\ , & \quad (2.13)\end{cases}$$

$$\tan\delta_l=\frac{j_l(kC)}{n_l(kC)}\ , \tag{2.14}$$

and C is chosen by fitting the experimental data.

In relativistically covariant theories the matrix element (2.4) has the structure

$$\langle f\,|\,H_{\text{int}}\,|\,i\rangle=\frac{1}{\sqrt{E_i}}\frac{1}{\sqrt{E_1}}\frac{1}{\sqrt{E_2}}$$

$$\text{(a Lorentz-invariant factor)}\ . \tag{2.15}$$

The E_i, E_1, and E_2 are energies of the initial and final two mesons. In relativistic field theories these $1/\sqrt{E}$ factors come naturally from the normalization of quantized Bose fields. They would presumably correctly appear if the mesons are expressed as particle-hole excitations of the QCD vacuum. The present constituent-quark model does not give matrix elements with these $1/\sqrt{E}$ factors; however, a correct calculation must contain them to ensure Lorentz invariance.

These factors are important in many contexts. For example, one of the ω_k in the denominator ω_k^2 of the Yukawa potential comes from the two $1/\sqrt{\omega_k}$ factors at the vertices of the meson-exchange process. We could describe this process as one in which a piece of the flux tube breaks off from one nucleon and attaches to the other. However, if we were to calculate it using nonrelativistic matrix elements like (2.4) we would obtain a potential with denominator ω_k instead of ω_k^2.

The decay widths are calculated with the nonrelativistic (NR) matrix elements given by Eqs. (2.4), (2.7), and (2.8), and "semirelativistic" (SR) matrix elements defined as

$$\langle f\,|\,H_{\text{int}}\,|\,i\rangle_{\text{SR}}=\left[\frac{m_1}{E_1}\right]^{1/2}\left[\frac{m_2}{E_2}\right]^{1/2}\left[\frac{m_i}{E_i}\right]^{1/2}\times\langle f\,|\,H_{\text{int}}\,|\,i\rangle \tag{2.16}$$

in which the $\sqrt{m/\omega}$ factors are introduced by hand with the hopes that they take into account some of the relativistic effects. The SR matrix elements have correct nonrelativistic limit, and they will also give the correct form for the one-pion-exchange potential.

The decay width $d\Gamma$ is estimated in lowest-order perturbation theory as

$$d\Gamma=2\pi\,|\,\langle f\,|\,H_{\text{int}}\,|\,i\rangle\,|^2 d\rho\ . \tag{2.17}$$

This lowest-order estimate is probably sufficient at the present level of sophistication in this theory since most of the widths are not large compared to the level spacing. Higher-order corrections to $d\Gamma$ have been discussed in Ref. 7. The density of states $d\rho$ is obtained by using the relativistic energies $(m^2+k^2)^{1/2}$ for the final mesons. Thus, in the rest frame of the initial meson $(E_i=m_i)$ we obtain

$$d\rho=k\left[\frac{E_1E_2}{m_i}\right]\frac{d\Omega_k}{(2\pi)^3}\ . \tag{2.18}$$

It has been argued in Ref. 10 that it is inconsistent to use the relativistic density of states (2.16) along with the nonrelativistic matrix element. Thus, they advocate using the nonrelativistic matrix element and density of states:

$$d\rho_{\text{NR}}=k\left[\frac{m_1m_2}{m_i}\right]\frac{d\Omega_k}{(2\pi)^3}\ . \tag{2.19}$$

We note that the expression for the decay width, obtained with our SR matrix element (2.16) and the relativistic $d\rho$,

$$d\Gamma_{\text{SR}}=2\pi\,|\,\langle f\,|\,H_{\text{pair}}\,|\,i\rangle\,|^2 k\left[\frac{m_1m_2}{m_i}\right]\frac{d\Omega_k}{(2\pi)^3} \tag{2.20}$$

is the same as that obtained with the NR matrix element and the NR density of states. However, since the wave vector k is determined from the relativistic energy conservation

$$E_i = m_i = (m_1^2 + k^2)^{1/2} + (m_2^2 + k^2)^{1/2} \tag{2.21}$$

the significance of $d\rho_{\rm NR}$ is not clear.

KI use effective, rather than physical masses $\tilde{m}$ in the calculation of the width. Their expression is

$$d\Gamma_{\rm KI} = 2\pi \, | \langle f \mid H_{\rm pair} \mid i \rangle |^2 k \left[\frac{\tilde{m}_1 \tilde{m}_2}{\tilde{m}_i} \right] \frac{d\Omega_k}{(2\pi)^3} \tag{2.22}$$

and $\tilde{m} \approx m$ for all mesons except pion for which $\tilde{m}_\pi = 5.1 m_\pi$. The calculation of the decay matrix element is certainly correct in the nonrelativistic limit in which the velocities of the emitted mesons are small. Hence, the relativistic factors must become the unit in this limit as our $\sqrt{m/E}$ factors do. In this context the $d\Gamma_{\rm KI}$ does not appear to be satisfactory. We report results with $d\Gamma_{\rm NR}$ [replace $m_1 m_2$ in Eq. (2.20) by $E_1 E_2$], $d\Gamma_{\rm SR}$ [Eq. (2.20)], and $d\Gamma_{\rm KI}$ [Eq. (2.22)].

III. RESULTS

We calculate the 2π decay widths of $\rho(770, {}^3S_1)$, $f(1270, {}^3P_2)$, $g(1690, {}^3D_3)$, $h(2030, {}^3F_4)$, $\rho_5(2350, {}^3G_5)$, and $r(2510, {}^3H_6)$ mesons. Their masses and quark model L,S,J values are given in parentheses, and the pions emitted in the decay are in relative $l = J$ state. The 2π decay widths of ρ, f, g, h, and r are experimentally known.[12,13] We also calculate the known[12] total $\pi+\rho$ decay widths of $A_1(1270, {}^3P_1)$, $A_3(1680, {}^1D_2)$, and $A_2(1320, {}^3P_2)$. The $\pi+\rho$ are emitted in $l=0$ and 2, $l=1$ and 3, and $l=2$ states in these decays. The partial decay widths of $B(1235, {}^1P_1)$ into $l=0$ and $2\pi+\omega$ states are known[12] and calculated. The 2π $l=0$ decay width of $S(975)$ is also calculated assuming that it is a $q\bar{q}$ ${}^3P_0$ state.

The strength of the $H_{\rm pair}$ interaction is chosen in each model to reproduce the decay width of the ρ meson, and the core radii $C_{\pi\pi}$, $C_{\pi\rho}$, and $C_{\pi\omega}$, meant to simulate the final-state interactions, are varied to fit the data. The models are labeled with the expression, NR, SR, or KI for the decay width Γ [Eqs. (2.20) and (2.22)], and that for decay amplitude. The ${}^3S_1$-flux-tube, ${}^3P_0$-flux-tube, and ${}^3P_0$-volume amplitudes given by Eqs. (2.4), (2.7), and (2.8) are, respectively, denoted by S-FT, P-FT, and P-V. The parameters of the models are given in Table I.

In our opinion the SR expression for the width Γ is most realistic, and the results obtained with it and the S-FT, P-FT, and P-V amplitudes are shown in Figs. 2–4, respectively. The widths obtained with these models are not too different from each other. All models overestimate the 2π decay width of $S(975)$ by a factor of 10 or more. As discussed in Ref. 10 it is probably wrong to assume that $S(975)$ is a ${}^3P_0$ $q\bar{q}$ state; it probably is a narrow $K\bar{K}$ state riding on an unidentified, broad $q\bar{q}$ ${}^3P_0$ state. All models also seem to overestimate the width of the $A_3(1680)$ by a factor of 3 to 4, and favor pion wave functions with $R_\pi = 0.29$ fm. Decays in which two pions are emitted in $l \geq 2$ states are particularly sensitive to R_π.

We note that for 2π decays

$$\Gamma_{\rm KI}(2\pi) \sim (\tilde{m}_\pi / m_\pi)^2 \Gamma_{\rm SR} \sim (5.1)^2 \Gamma_{\rm SR} \tag{3.1}$$

and for $\pi+\omega$ and $\pi+\rho$ decays

$$\Gamma_{\rm KI}(\pi+\omega \text{ or } \pi+\rho) \sim (\tilde{m}_\pi / m_\pi) \Gamma_{\rm SR} \,. \tag{3.2}$$

Thus, the fit to 2π decay widths with the KI expression requires that we keep $C_{\pi\pi}$ the same as in SR models and just reduce the interaction strength by a factor of ~ 5.1. If we also keep $C_{\pi\rho}$ and $C_{\pi\omega}$ the same as in SR models the $\pi+\rho$ and $\pi+\omega$ decay widths are smaller by a factor of ~ 5 than in SR models. However, much better fits can be obtained by reducing $C_{\pi\omega}$ and $C_{\pi\rho}$. The repulsive core has a much stronger effect on states with smaller values of l and hence, the widths of A_3, A_2, and B (D wave) reduce when $\Gamma_{\rm KI}$ is used instead of $\Gamma_{\rm SR}$. We note that with the KI prescription the ${}^3P_0$-volume model gives the best account of all the data considered in this work without any final-state interactions. The optimum values of $C_{\pi\pi}$, $C_{\pi\rho}$, and $C_{\pi\omega}$ are zero in this model when $R_\pi = 0.29$. The P-FT model works marginally better with

TABLE I. Parameters of the models.

Model	Λ_s (fm)	R_π (fm)	$C_{\pi\pi}$ (fm)	$C_{\pi\omega}$ (fm)	$C_{\pi\rho}$ (fm)
SR, S-FT	158.0	0.16	0.27	0.51	0.58
SR, S-FT	62.2	0.29	0.18	0.43	0.51
KI, S-FT	11.7	0.29	0.18	0.31	0.39
NR, S-FT	22.7	0.16	0.14	0.35	0.42
NR, S-FT	17.5	0.29	0.00	0.33	0.41
	$\bar{\gamma}_0$ (fm^2)				
SR, P-FT	2.24	0.16	0.20	0.36	0.42
SR, P-FT	1.48	0.29	0.06	0.27	0.33
KI, P-FT	0.279	0.29	0.06	0.12	0.19
	γ_0				
SR, P-V	7.55	0.16	0.15	0.34	0.41
SR, P-V	1.97	0.29	0.00	0.17	0.25
KI, P-V	0.370	0.29	0.00	0.00	0.00

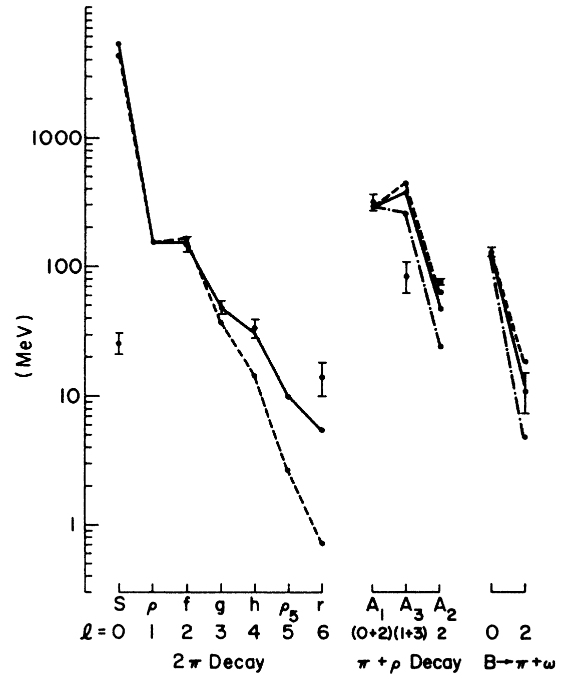

FIG. 2. Decay widths in the $^3S_1$-flux-tube model. Solid and dashed lines give results obtained with $R_\pi=0.29$ and 0.16 fm, respectively, and the SR expression for decay width. The results obtained with KI expression and $R_\pi=0.29$ fm are shown with dashed-dotted lines. The KI and SR expressions give the same 2π decay widths.

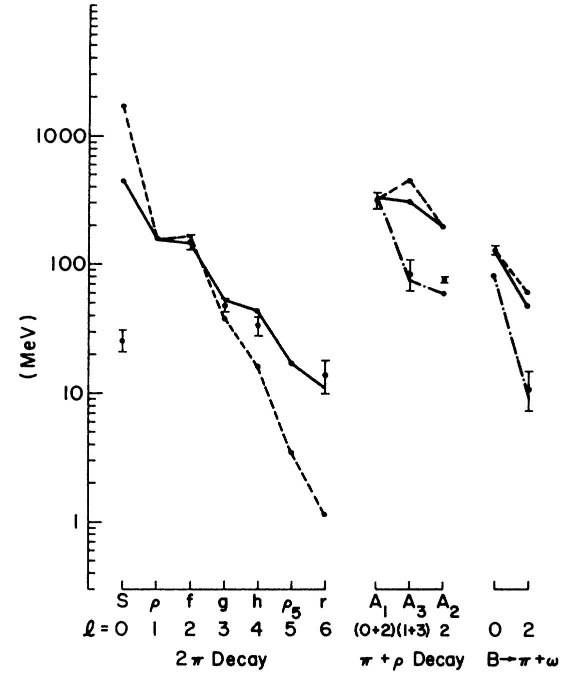

FIG. 4. Decay widths in the $^3P_0$-volume model. See Fig. 2 for notation.

KI prescription instead of the SR, while there is little improvement in the fit with the S-FT model.

The results of the NR S-FT model are shown in Fig. 5. This model would fit the 2π decay widths with $R_\pi \sim 0.2$ fm, but it overestimates the A_3 decay width by a factor of ~ 5.

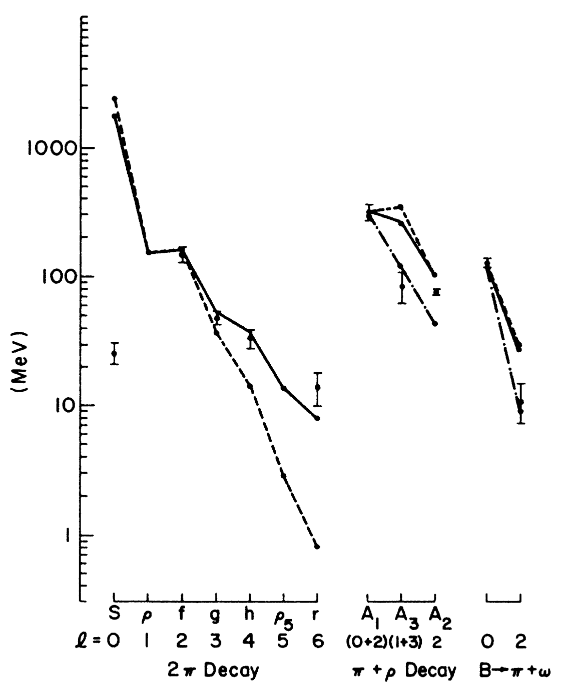

FIG. 3. Decay widths in the $^3P_0$-flux-tube model. See Fig. 2 for notation.

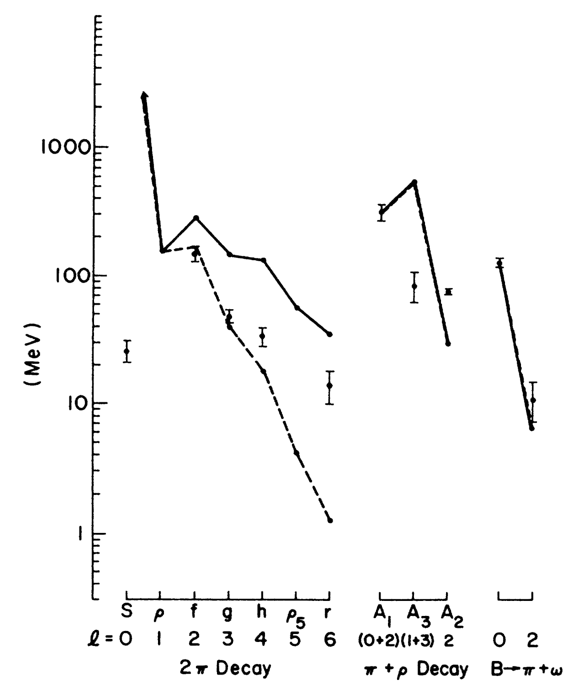

FIG. 5. Decay widths in the $^3S_1$-flux-tube model. Solid and dashed lines give results obtained with $R_\pi=0.29$ and 0.16 fm, respectively, and the nonrelativistic expression for decay width.

TABLE II. The D/S ratio in $B \rightarrow \omega+\pi$. Columns 1,2, 3,4, and 5,6 give results obtained with the S-FT, P-FT,and P-V models, respectively, with $R_\pi=0.29$ fm.

$C_{\pi\omega}$ (fm)	D/S	$C_{\pi\omega}$ (fm)	D/S	$C_{\pi\omega}$ (fm)	D/S
0.43	0.30	0.27	0.46		
0.31	0.20	0.12	0.26	0.17	0.61
0	0.054	0	0.17	0	0.35

The ratio D/S of the D-wave to S-wave amplitude of $B \rightarrow \omega+\pi$ decay, depends only upon the assumed core radius $C_{\pi\omega}$ and the decay model. Experimentally[12] it is 0.29 ± 0.05, and the calculated values are listed in Table II. Since the repulsive core has a much larger effect on S-wave final states the D/S ratio increases with the core size. With the S-FT (P-FT) models and $R_\pi=0.29$ fm, the experimental value is reached when $C_{\pi\omega}\sim0.43$ (0.14) fm, while the P-V model gives D/S values >0.35 in the present calculations.

There is no experimental information on the short-range interaction between the mesons, and none of the theoretical models are sufficiently developed to estimate values of $C_{\pi\pi}$, $C_{\pi\rho}$, and $C_{\pi\omega}$. However, it is well known that there is a repulsive core $C_{NN}\sim0.5$ fm in the nucleon-nucleon interaction. In the analysis of low-energy S-wave π-nucleon scattering data[14] it is also useful to assume a repulsive core $C_{\pi N}=0.42\pm0.11$ fm. It is possible that these repulsive cores are associated with composite character of hadrons, and the value of C_{xy} should be of order R_x+R_y, where R_x and R_y are the radii of the hadrons. From this point of view the cores required to explain the widths with the S-FT model do not seem unreasonable.

There are other final-state interactions associated with hadron exchange and resonant scattering terms[14] that we have neglected in this work. These interactions involve breaking and rejoining flux tubes, and theoretically they occur when the present calculation is carried out to higher orders. As a matter of fact, when higher-order terms are included, the theory calculates π-π scattering amplitudes. There is experimental data on low-energy π-π phase shifts,[15] and a particularly stringent test of the various models is provided by the S-wave π-π phase shift in isospin-zero state. This phase shift is positive indicating an attractive interaction. In order to obtain a positive phase shift in the present models the attraction from the strong coupling of the pions to the $^3P_0$ $q\bar{q}$ state must overwhelm the repulsion from the hard-core interaction.

The main conclusions of this work are as follows. (i) It appears that the decay widths of light mesons cannot be easily explained with an $R_\pi=0.16$ fm as given by semi-relativistic flux-tube model.[2,3] A larger value ($R_\pi\sim0.3$ fm) is necessary to fit the decay widths. This suggests that the treatment of color-magnetic interactions in these models is too crude. (ii) By including repulsive core type final-state interaction effects it may be possible to qualitatively explain the data with either $^3S_1$- or $^3P_0$-pair creation models. (iii) Treatment of relativistic effects in this basically nonrelativistic approach causes an uncertainty in the calculation of widths. If one introduces these effects by m/E factors so as to preserve the nonrelativistic limit, and the form of one-pion-exchange interaction, then the flux-tube $^3P_0$ and $^3S_1$ models give qualitatively similar results, and the naive $^3P_0$ model gives a little poorer fit. All models fail to explain the width of A_3 by more than a factor of 2, while the naive $^3P_0$ also overestimates the widths of A_2 and $B\rightarrow(\pi+\omega)_D$ by more than a factor of 2. (iv) If the KI $\tilde{m}/E$ factors are used, the nonrelativistic limit is violated, however, a much better fit is obtained with the naive $^3P_0$ model without any final-state interactions. (v) The $q\bar{q}$ $^3P_0$ 0^+ meson has a much stronger coupling to the 2π state than the $S(975)$ in all models we have studied.

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