# Solution to the puzzle of $\Gamma(D^0 \to K^+K^-)/\Gamma(D^0 \to \pi^+\pi^-)$ in a unified approach to $D \to \pi\bar{K}$, $K\bar{K}$, and $\pi\pi$ and $K \to \pi\pi$ decays

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A nonperturbative algebraic approach, which deals with long-distance physics in earnest but also maintains a close contact with quark-line diagrams, has previously been applied to the derivation of the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule in the $K \to 2\pi$ decays. Reasonable estimates of the rates of $D^0 \to \pi\bar{K}$ and $\phi\bar{K}^0$ decays in terms of the rate of $K_S^0 \to \pi\pi$ decay have also been obtained, to the approximation in which excited state contributions to the on-mass-shell intermediate states are being neglected for the moment. In this paper, we examine our unified approach to the $D \to \pi\bar{K}$, $K\bar{K}$, and $\pi\pi$ and $K \to \pi\pi$ decays further, now including the four-quark $[QQ][\bar{Q}\bar{Q}]$ meson contribution to the on-mass-shell intermediate states. We find that the result provides us with a solution to the well-known puzzle in the observed rates of various Cabibbo-suppressed decays of the $D^0$ meson, particularly $\Gamma(D^0 \to K^+K^-)/\Gamma(D^0 \to \pi^+\pi^-) \simeq 3.6$, which has never been solved in a systematic manner. In addition, it gives us a significantly improved prediction on $\Gamma(D^0 \to \pi\bar{K})$ in comparison with $\Gamma(K_S^0 \to \pi\pi)$.

## I. INTRODUCTION AND SUMMARY

The $D^0 \to \phi\bar{K}^0$ decay proceeds only through the so-called $W$-exchange diagram[1] in the naive quark model and was expected to be severely suppressed due to the color suppression[2] (in addition to the helicity suppression) in the perturbative QCD approach, if we use the approximation called factorization (or vacuum insertion). However, according to recent experiments,[3] the decay has turned out to have a sizable rate $[B(D^0 \to \phi\bar{K}^0)_{\text{expt}} \simeq 1\%]$.

In Cabibbo-suppressed decays, the $D^0 \to K^0\bar{K}^0$ decay, which can also take place only through the $W$-exchange diagram, seems to be suppressed in comparison with the $D^0 \to K^+K^-$ decay.[4] One may understand the suppression through the cancellation of its amplitudes, which is expected if $SU_f(3)$ symmetry works. However, if $SU_f(3)$ works so well in the nonleptonic decays of $D^0$ meson, how can we understand the observed large ratio[5] (instead of $\simeq 1$) of the decay rates $\Gamma(D^0 \to K^+K^-)/\Gamma(D^0 \to \pi^+\pi^-) \simeq 3.6$? This is the well-known puzzle in the Cabibbo-suppressed decays of $D^0$ meson, which has not been solved to our knowledge in a systematic manner.[6]

The purpose of this paper is to present a possible solution to this puzzle from the same algebraic nonperturbative approach which has been already used by us to give a unified description of other nonleptonic decays,[7–10] $K \to 2\pi$, $D \to \pi\bar{K}$, $D^0 \to \phi\bar{K}^0$, etc.

The method used can be viewed partly as a resurrection of the old current algebra, replacing the problematical soft-meson approximation by the much milder hard-meson extrapolation executed in the infinite-momentum frame (IMF). In the amplitudes thus obtained, *only* the asymptotic *on-mass-shell two-particle* hadronic matrix elements of the effective weak Hamiltonian $H_w$ are found to play a role. Therefore, the problem reduces essentially to the study of these three-point functions.

These asymptotic matrix elements are then argued to be severely constrained in our theoretical framework of "constraint algebras involving the generators of underlying symmetries of QCD plus asymptotic flavor symmetry."

The origin of the famous approximate $|\Delta \mathbf{I}| = \frac{1}{2}$ rule in the $K \to 2\pi$ decays and hyperon nonleptonic decays is then found to be attributable to the fact that the asymptotic on-mass-shell two-particle *ground-state-hadron* matrix elements of $H_w$ are severely constrained to satisfy the *strict* $|\Delta \mathbf{I}| = \frac{1}{2}$ rule. Similarly, other two-particle hadron matrix elements of $H_w$ are also constrained severely. The constraints thus obtained are also found to maintain a close correspondence to the quark-line diagrams.

In this paper we demonstrate further, though not yet completely, that a unified approach to the processes, $K \to \pi\pi$, $D \to \pi\bar{K}$, $D \to K\bar{K}$, and $\pi\pi$ [especially the ratio $\Gamma(D^0 \to K\bar{K})/\Gamma(D^0 \to \pi\pi)$], etc., is indeed possible and that the presence of exotic resonances is strongly favored in this connection.

In Sec. II we present a brief summary of the previous result,[7,10] i.e., the extrapolated amplitudes for the nonleptonic weak decays and also the constraints upon the asymptotic matrix elements of $H_w$ already obtained, which will be used in this paper. In Sec. III, we derive the asymptotic constraints upon the diagonal ground-state-meson two-particle matrix elements of the weak Hamiltonian $H_w=H(-,0)$ (with $\Delta C=-1$ and $\Delta S=0$), which are responsible for the Cabibbo-suppressed decays of $D$ mesons. In Sec. IV, we study the asymptotic constraints imposed upon the nondiagonal matrix elements of various effective weak Hamiltonians, i.e., $H_w=H(0,-)$ with $\Delta C=0$ and $\Delta S=-1$, $H(-,-)$ with $\Delta C=-1$ and $\Delta S=-1$, and $H(-,0)$ with $\Delta C=-1$ and $\Delta S=0$, which now are taken between the $[QQ][\bar{Q}\bar{Q}]$ and $\{Q\bar{Q}\}_{L=0}$ meson states. In Sec. V, we demonstrate that an *improved* unified explanation of the $K_S^0\to\pi^+\pi^-$ and $D^0\to\pi^+K^-$ decays is possible, if we also add the hitherto neglected $[QQ][\bar{Q}\bar{Q}]$ meson contribution to our previous estimate. In Sec. VI, it is shown that the puzzle in the Cabibbo-suppressed decays of $D^0$ meson can now be explained by the *same* effect of the $[QQ][\bar{Q}\bar{Q}]$ meson contribution. In the final section, a short summary is given.

## II. FORMALISM AND THE LIST OF CONSTRAINTS UPON THE ASYMPTOTIC MATRIX ELEMENTS OF $H_w$ ALREADY OBTAINED AND TO BE USED

### A. Decay amplitude in the new hard-pseudoscalar-meson extrapolation

We first give an approximate expression of the amplitude of the weak three-pseudoscalar-meson process such as $K_S^0\to\pi^+\pi^-$:

$$P_1(p_1)\to P_2(p_2)+P_3(q)\ , \tag{2.1}$$

which is symmetrized with respect to the final two pseudoscalar-mesons. By taking a limit $\mathbf{q}\to 0$ in the infinite-momentum frame (IMF), we obtain the extrapolated amplitude for the decay, Eq. (2.1), as (for details, see Refs. 7 and 10)

$$M(P_1\to P_2P_3)\simeq M_{\rm ETC}(P_1\to P_2P_3)+M_S(P_1\to P_2P_3)\ . \tag{2.2}$$

The equal-time-commutator (ETC) part

$$M_{\rm ETC}(P_1\to P_2P_3)=-i\{(2f_{P_2})^{-1}\langle P_3\,|\,[V_{\bar{P}_2},H_w^{\rm PC}]\,|\,P_1\rangle+(2f_{P_3})^{-1}\langle P_2\,|\,[V_{\bar{P}_3},H_w^{\rm PC}]\,|\,P_1\rangle\} \tag{2.3}$$

[where $H_w^{\rm PC}$ ($H^{\rm PV}$) is the parity-conserving (-violating) weak Hamiltonian] now has to be evaluated in the IMF, enabling us conveniently to use asymptotic flavor symmetry which is compatible with the Gell-Mann–Okubo (GMO) mass splittings (including the mixings we desire to retain) of hadrons.

The surviving surface term can also be expressed in terms of the *two-particle on-mass-shell* asymptotic hadron matrix elements of $H_w$ and the axial-vector charges:

$$\begin{aligned}M_S(P_1\to P_2P_3)\equiv&\lim_{\mathbf{q}\to 0,\,\mathbf{p}_1\to\infty} i[(2f_{P_2})^{-1}q_\mu T_\mu^{(2)}+(2f_{P_3})^{-1}q_\mu T_\mu^{(3)}]\\
=&\,i(2f_{P_2})^{-1}\Big[\sum_n[(m_3^2-m_1^2)/(m_n^2-m_1^2)]\langle P_3\,|\,A_{\bar{P}_2}\,|\,n\rangle\langle n\,|\,H_w^{\rm PV}\,|\,P_1\rangle\\
&+\sum_l[(m_3^2-m_1^2)/(m_l^2-m_3^2)]\langle P_3\,|\,H_w^{\rm PV}\,|\,l\rangle\langle l\,|\,A_{\bar{P}_2}\,|\,P_1\rangle\Big]\\
&+i(2f_{P_3})^{-1}\Big[\sum_n[(m_2^2-m_1^2)/(m_n^2-m_1^2)]\langle P_2\,|\,A_{\bar{P}_3}\,|\,n\rangle\langle n\,|\,H_w^{\rm PV}\,|\,P_1\rangle\\
&+\sum_{l'}[(m_2^2-m_1^2)/(m_{l'}^2-m_2^2)]\langle P_2\,|\,H_w^{\rm PV}\,|\,l'\rangle\langle l'\,|\,A_{\bar{P}_3}\,|\,P_1\rangle\Big]\ .\end{aligned} \tag{2.4}$$

Here

$$T_\mu^{(j)}=i\int d^4x\,\langle P_k(p_2)\,|\,T[A_\mu^{(j)}(x),H_w(0)]\,|\,P_1(p_1)\rangle e^{-iqx}$$

($j=2$, $k=3$, and $j=3$, $k=2$). $A_\mu^{(j)}(x)$ denotes the axial-vector current which transforms as $\bar{P}_j$, and $f_{P_j}$ denotes the decay constant of the PS meson $P_i$. The summation $\sum$ is extended over all the possible *on-mass-shell* single-particle hadron states. In Eq. (2.3) we have already used the well-known commutation relation (in the infinite-$W$-boson-mass limit),

$$[A_\alpha,H_w^{\rm PV(PC)}]=[V_\alpha,H_w^{\rm PC(PV)}]\ . \tag{2.5}$$

### B. Constraints upon the asymptotic ground-state-meson matrix elements of $H(0,-)$ and $H(-,-)$ which do satisfy the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule and its charm counterpart

Constraints upon the asymptotic two-particle ground-state-meson matrix elements of $H_w=H(0,-)$, i.e., the *asymptotic* $|\Delta\mathbf{I}|=\frac{1}{2}$ rule, which are listed below, have been obtained from the constraint algebras, Eqs. (3.1a) and (3.1b). Additional asymptotic constraints have also been derived from simpler algebras, Eq. (3.9). Constraints on the asymptotic matrix elements of $H(-,-)$ in broken $SU_f(4)$ have also been obtained[7,10] using the same method called level realization of asymptotic flavor symmetry in these chiral algebras involving the axial-vector charges $A_\alpha$. Here we list only the results which will be used in this paper. The $\mathbf{p}\rightarrow\infty$ limit should always be understood.

(i) The *asymptotic* $|\Delta\mathbf{I}|=\frac{1}{2}$ rule and its charm counterpart:

$$\langle\pi^+|H(0,-)|K^+\rangle+\sqrt{2}\langle\pi^0|H(0,-)|K^0\rangle=0\ , \tag{2.6}$$

$$\langle\pi^+|H(0,-)|K^{*+}\rangle+\sqrt{2}\langle\pi^0|H(0,-)|K^{*0}\rangle=0\ ,\ \text{etc.}\ , \tag{2.7}$$

$$\langle\bar{K}^0|H(-,-)|D^0\rangle+\langle\pi^+|H(-,-)|F^+\rangle=0\ , \tag{2.8}$$

$$\langle\bar{K}^0|H(-,-)|D^{*0}\rangle+\langle\pi^+|H(-,-)|F^+\rangle=0\ ,\ \text{etc.} \tag{2.9}$$

(ii) The SU(6)- and SU(8)-like *asymptotic* constraints:

$$\langle\pi^+|H(0,-)|K^{*+}\rangle=\langle\rho^+|H(0,-)|K^+\rangle\ ,\ \text{etc.}, \tag{2.10}$$

$$\langle\bar{K}^0|H(-,-)|D^{*0}\rangle=\langle\bar{K}^{*0}|H(-,-)|D^0\rangle\ ,\ \text{etc.}, \tag{2.11}$$

and

$$\langle\pi^+|H(0,-)|K^+\rangle=\pm\langle\pi^+|H(0,-)|K^{*+}\rangle\ , \tag{2.12}$$

$$\langle\bar{K}^0|H(-,-)|D^0\rangle=\pm\langle\bar{K}^0|H(-,-)|D^{*0}\rangle\ , \tag{2.13}$$

$$k_0=\pm\sqrt{1/2}\langle\pi^-|A_{\pi^-}|\rho^0\rangle\equiv\pm\sqrt{1/2}H\ , \tag{2.14}$$

where $k_0$ denotes the *fraction* of the ground-state-meson contribution to the complete set of single-particle intermediate states inserted in the left-hand side (LHS) of the single commutator, Eq. (2.5), when it is sandwiched between *various* appropriate ground-state-meson states. $k_0$ is found to be universal. This result certainly supports the idea of level realization.

(iii) *Asymptotic* $SU_f(4)$ parametrization of the two-particle on-mass-shell ground-state-meson matrix elements of weak Hamiltonian:

$$\langle\bar{K}^0|H(-,-)|D^0\rangle=\cot\theta_C\sqrt{2}\langle\pi^0|H(0,-)|K^0\rangle=-\cot\theta_C\langle\pi^+|H(0,-)|K^+\rangle\ , \tag{2.15}$$

$$\langle\pi^+|H(-,-)|F^+\rangle=\cot\theta_C\langle\pi^+|H(0,-)|K^+\rangle\ ,\ \text{etc.} \tag{2.16}$$

They are obtained from the realization of the following constraint algebra (see Appendix A) using asymptotic $SU_f(4)$:

$$[H(-,-),V_{D^0}]=\cot\theta_C H(0,-)\ . \tag{2.17}$$

Here $\theta_c$ denotes the Cabibbo angle, and all the above matrix elements of weak Hamiltonian are evaluated in the IMF.

## III. DERIVATION OF ASYMPTOTIC CONSTRAINTS UPON THE TWO-PARTICLE ON-MASS-SHELL GROUND-STATE-MESON MATRIX ELEMENTS OF $H_w=H(-,0)$

In Sec. II, we have listed the constraint sum rules obtained for the matrix elements of $H_w=H(0,-)$ and $H(-,-)$ taken between the ground-state-meson states with infinite momenta. They explicitly display the *asymptotic* $|\Delta\mathbf{I}|=\frac{1}{2}$ rule and its charm counterpart.

They were derived from the level realization of the commutation relations,

$$[[H_w,A_{\pi^+}],A_{\pi^-}]=[[H_w,V_{\pi^+}],V_{\pi^-}] \tag{3.1a}$$

and

$$[[H_w,A_{\pi^-}],A_{\pi^+}]=[[H_w,V_{\pi^-}],V_{\pi^+}] \tag{3.1b}$$

for the charm-conserving but strangeness-changing Hamiltonian $H_w=H(0,-)$, and also from the realization of

$$[[H_w,A_{K^+}],A_{\pi^-}]=[[H_w,V_{K^+}],V_{\pi^-}] \tag{3.2a}$$

and

$$[[H_w,A_{\pi^-}],A_{K^+}]=[[H_w,V_{\pi^-}],V_{K^+}] \tag{3.2b}$$

for the charm- and strangeness-changing (Cabibbo-favored) Hamiltonian $H_w=H(-,-)$, respectively.

This time we are interested in deriving corresponding *asymptotic* constraints on the charm-changing but strangeness-conserving (the first Cabibbo-suppressed) Hamiltonian $H_w=H(-,0)$, by using exactly the same method as discussed in Sec. II B. We first investigate the constraints upon the *diagonal* on-mass-shell ground-state-meson matrix elements of $H_w=H(-,0)$, $\langle\{Q\bar{Q}\}_0|H_w|\{Q\bar{Q}\}_0\rangle$.

Inserting Eqs. (3.1a) and (3.1b) between appropriate sets of $\{Q\bar{Q}\}_0$ states with infinite momenta; (1) $\langle\pi^+|$ and $|D^+\rangle$; (2) $\langle\pi^0|$ and $|D^0\rangle$; (3) $\langle K^+|$ and $|F^+\rangle$; (4) $\langle\eta|$ and $|D^0\rangle$; (5) $\langle\eta'|$ and $|D^0\rangle$; (6) $\langle\rho^+|$ and $|D^{*+}\rangle$; (7) $\langle\rho^0|$ and $|D^{*0}\rangle$; (8) $\langle K^{*+}|$ and $|F^{*+}\rangle$; (9) $\langle\phi|$ and $|D^{*0}\rangle$; and (10) $\langle\omega|$ and $|D^{*0}\rangle$ (with helicity $\lambda=0$), and applying the same procedure as the above we can again obtain asymptotic constraints:

$$\langle\pi^+|H_w|D^+\rangle-\sqrt{2}\langle\pi^0|H_w|D^0\rangle=0\ , \tag{3.3a}$$

$$\langle\pi^+|H_w|D^+\rangle-\sqrt{2}\langle\rho^0|H_w|D^{*0}\rangle_{\lambda=0}=0\ , \tag{3.3b}$$

$$\langle\rho^+|H_w|D^{*+}\rangle_{\lambda=0}-\sqrt{2}\langle\rho^0|H_w|D^{*0}\rangle_{\lambda=0}=0\ , \tag{3.4a}$$

$$\langle\rho^+|H_w|D^{*+}\rangle_{\lambda=0}-\sqrt{2}\langle\pi^0|H_w|D^0\rangle=0\ . \tag{3.4b}$$

It also follows from Eqs. (3.3a)–(3.4b) that

$$\langle\pi^+|H_w|D^+\rangle=\langle\rho^+|H_w|D^{*+}\rangle_{\lambda=0}\ . \tag{3.5}$$

For the case of $H_w=H_w^{\mathrm{PV}}$, we obtain in a similar way

$$\langle\rho^+|H_w|D^+\rangle-\sqrt{2}\langle\rho^0|H_w|D^0\rangle=0\ , \tag{3.6a}$$

$$\langle\pi^+|H_w|D^{*+}\rangle-\sqrt{2}\langle\rho^0|H_w|D^0\rangle=0\ , \tag{3.6b}$$

$$\langle \pi^+ | H_w | D^{*+} \rangle - \sqrt{2} \langle \pi^0 | H_w | D^{*0} \rangle = 0 \; , \quad (3.7a)$$

$$\langle \rho^+ | H_w | D^+ \rangle - \sqrt{2} \langle \pi^0 | H_w | D^{*0} \rangle = 0 \; , \quad (3.7b)$$

corresponding to Eqs. (3.3a)–(3.4b). From Eqs. (3.6a)–(3.7b) we also obtain

$$\langle \pi^+ | H_w | D^{*+} \rangle = \langle \rho^+ | H_w | D^+ \rangle \; . \quad (3.8)$$

Equations (3.3a), (3.4a), (3.6a), and (3.7a) correspond precisely to the *asymptotic* $|\Delta \mathbf{I}| = \frac{1}{2}$ rule and its charm counterpart, Eqs. (2.6)–(2.9) mentioned in Sec. II, while Eqs. (3.5) and (3.8) to the SU(6)- [or SU(8)-] like relations, Eqs. (2.10) and (2.11).

They suggest the existence of approximate selection rule corresponding to the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule in the $K \to 2\pi$ decays for the present processes. In fact, as will be seen later the suppression of the $D^+ \to \pi^+ \pi^-$ decay is *predicted* in the present approach.

Next, we also investigate the levelwise realizations of the single commutators:

$$[A_\alpha, H_w] = [V_\alpha, H_w] \quad (H_w = H_w^{\rm PC} + H_w^{\rm PV}) \; . \quad (3.9)$$

The same procedure as the one used to obtain the constraints, Eqs. (2.12) and (2.13), leads us to

$$\begin{aligned} \langle \pi^+ | H_w | D^+ \rangle &= \langle \rho^+ | H_w | D^{*+} \rangle_{\lambda=0} \\ &= \pm \langle \pi^+ | H_w | D^{*+} \rangle \\ &= \pm \langle \rho^+ | H_w | D^+ \rangle \; , \end{aligned} \quad (3.10a)$$

$$\begin{aligned} \langle K^+ | H_w | F^+ \rangle &= \langle K^{*+} | H_w | F^{*+} \rangle_{\lambda=0} \\ &= \pm \langle K^+ | H_w | F^{*+} \rangle \\ &= \pm \langle K^{*+} | H_w | F^+ \rangle \; , \end{aligned} \quad (3.10b)$$

$$\alpha_0 = \pm \sqrt{1/2} H \; , \quad (3.11)$$

where $\alpha_0$ denotes again the fraction of the ground-state contribution and is found to be universal, i.e., $\alpha_0$ is equal to $k_0$ obtained in Eq. (2.14). This is an elegant result, though expected.

Similar realization of other commutators, Eq. (3.9) with $\alpha = \pi^-, K^+, K^0, K^-$, and $\bar{K}^0$, leads only to the constraints which have already been obtained above.

All the constraints obtained for the asymptotic two-particle on-mass-shell ground-state-meson matrix elements of various weak Hamiltonian, $H(0,-)$, $H(-,-)$, and $H(-,0)$, do maintain a close correspondence to the quark-line diagrams. Asymptotic $|\Delta \mathbf{I}| = \frac{1}{2}$ rule and its charm counterparts obtained can be associated with exactly the same type of quark-line diagrams.[11]

## IV. ASYMPTOTIC CONSTRAINTS UPON THE NONDIAGONAL TWO-PARTICLE ON-MASS-SHELL MATRIX ELEMENTS OF $H_w = H(0,-)$, $H(-,-)$, AND $H(-,0)$

In order to calculate the excited-state contribution to the surface term $M_S$ in the extrapolated amplitude given in Sec. II, we do need information on the nondiagonal asymptotic two-particle matrix elements of $H_w$, for example, $\langle L' | H_w | L \rangle$ ($L=0$, $L'=1,2,\ldots$, or exotic states, etc.). However, the matrix elements involving orbitally excited states ($L'=1,2,\ldots$) will not be important as will be shown later. In this section we study the constraints on the *asymptotic* nondiagonal matrix elements of $H_w$'s $H(0,-)$, $H(-,-)$, and $H(-,0)$, taken between the $\{Q\bar{Q}\}_0$ and $[QQ][\bar{Q}\bar{Q}]$ states. They can, in fact, be related to each other by using the asymptotic $SU_f(3)$ and $SU_f(4)$ rotations through the commutators Eqs. (A6e) and (A7d) in the Appendix. Moreover, we find below that they can be expressed in the simple form (diagonal matrix element of $H_w$)$\times k_c$. Here $k_c$ denotes the fraction expressing the fractional contribution of the four-quark $[QQ][\bar{Q}\bar{Q}]$ states to the realization of the single commutator, Eq. (3.9) with $H_w = H(0,-)$, $H(-,-)$, and $H(-,0)$, when it is inserted between the appropriate $\{Q\bar{Q}\}_0$ meson states. The values of $k_c$ will be shown to be universal [see Eq. (4.45)], i.e., flavor independent, in support of the idea of level realization. This result is also consistent with the fact that these constraints can also be reproduced by the corresponding spectator-type diagrams, which describe the $[QQ][\bar{Q}\bar{Q}]$ meson contribution, if they satisfy certain simple relations given by Eq. (4.46).

In Secs. II and III the constraints on the diagonal asymptotic matrix elements of $H_w$ have already been obtained from the realization of the double commutators

$$[[H_w, A_\alpha], A_\beta] = [[H_w, V_\alpha], V_\beta] \; . \quad (4.1)$$

However, in the case when higher excited states are involved, we have to find a correct prescription.

Therefore, as for the derivation of the constraints on the *nondiagonal* asymptotic two-particle on-mass-shell matrix elements of $H_w$ which will be used in Secs. V and VI, we choose to derive them from the realization of single commutators, Eq. (3.9), since a unique prescription is already at our disposal.

As an excited state, one may, first, consider the orbitally excited $Q\bar{Q}$ meson, $\{Q\bar{Q}\}_L$, $L \geq 1$. However, the two-particle on-mass-shell matrix elements of $H_w$,

$$\langle \{Q\bar{Q}\}_L | H_w | \{Q\bar{Q}\}_0 \rangle \text{ and } \langle \{Q\bar{Q}\}_0 | H_w | \{Q\bar{Q}\}_L \rangle \text{ with } L \geq 1 \; , \quad (4.2)$$

are proportional to the value of the wave function of $\{Q\bar{Q}\}_L$ meson at the origin, $\Psi_L(0)$, which vanishes in the nonrelativistic limit if $L \neq 0$, so that they are expected[9,11] to be small.

However, the presence of spectator diagrams depicted by Fig. 1 implies in the present formalism the presence of the contributions of $\{QQ\bar{Q}\bar{Q}\}$ mesons. In fact, recent experiments[12] and theoretical analysis[13] provide some evidences for the existence of four-quark mesons. If these $\{QQ\bar{Q}\bar{Q}\}$ exotics exist, they are certainly entitled to contribute to the on-mass-shell hadron intermediate states, as will be discussed in this section.

Four-quark $\{QQ\bar{Q}\bar{Q}\}$ meson can be classified into four classes: (i) $[QQ][\bar{Q}\bar{Q}]$; (ii) $(QQ)(\bar{Q}\bar{Q})$; (iii) $(QQ)[\bar{Q}\bar{Q}]$; and

(iv) $[QQ](\bar{Q}\bar{Q})$, where the two quarks (or antiquarks) in ( ) and [ ] are symmetric and antisymmetric, respectively, with respect to the exchange of their flavors. The first two can have $J^{PC}=0^{++}$ and may contribute significantly to the on-mass-shell intermediate states in Eq. (2.4). However, in this paper we investigate the contribution of *only* the $[QQ][\bar{Q}\bar{Q}]$ mesons and neglect that of the $(QQ)(\bar{Q}\bar{Q})$ meson for the time being, without affecting the result seriously (see Sec. VII).

According to Jaffe's analysis,[14] $[QQ][\bar{Q}\bar{Q}]$ mesons with $J^{PC}=0^{++}$ are classified into two classes with different combinations of color degrees of freedom and the observed scalar mesons were actually identified with the members of the lighter $[QQ][\bar{Q}\bar{Q}]$ multiplet. However, they have large recoupling coefficients to two PS mesons and are expected to be too unstable,[15] in general, except for the $\delta$ and $S^*$ which lie just below the $K\bar{K}$ threshold. However, there is an argument by Peaslee and Schnitzer[16] (from the study of spin effects) that favors the $\{Q\bar{Q}\}$ assignment for the $\delta$ and $S^*$. In the present algebraic approach, the $\delta$ and $S^*$ can also be accommodated as the members of the $\{Q\bar{Q}\}_{L=1}$ meson multiplet which mixes with a glueball.[17] Therefore, it is not certain whether the lighter $[QQ][\bar{Q}\bar{Q}]$ mesons of $J^{PC}=0^{++}$ can exist as resonances.

On the other hand, a new resonance[18] at 1.48 GeV has been found recently in the $\bar{p}n$ annihilations into $\pi^-(\rho^0\rho^0)$. It can be interpreted[19] as an $I=0$ component of heavier $[QQ][\bar{Q}\bar{Q}]$ meson multiplet without an $s$ quark.

For this reason, we pick up only the heavier $[QQ][\bar{Q}\bar{Q}]$ meson multiplet, i.e., an *ideally mixed* **20″⊕15⊕1=36** multiplet. We list the ideally mixed **36**-plet of $[QQ][\bar{Q}\bar{Q}]$ mesons in Table I.

Now, we study asymptotic constraints on the *nondiagonal* matrix elements:

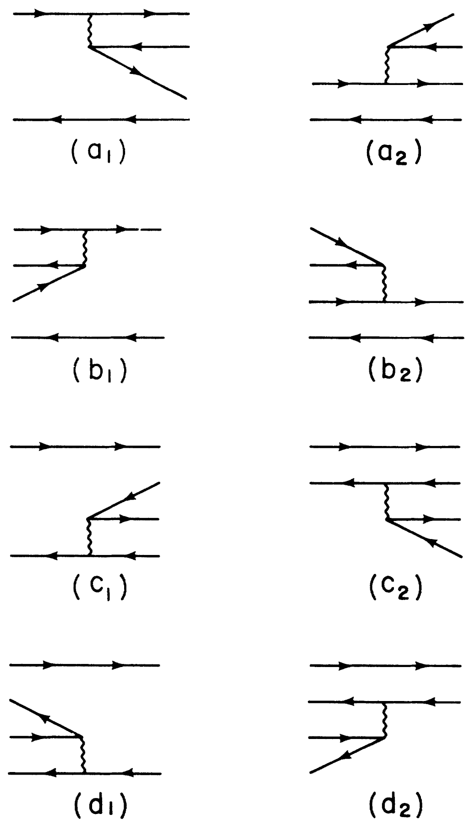


FIG. 1. The spectator diagrams: (a$_1$) $S_1$, (a$_2$) $S_2$, (b$_1$) $\hat{S}_1$, (b$_2$) $\hat{S}_2$, (c$_1$) $\bar{S}_1$, (c$_2$) $\bar{S}_2$, (d$_1$) $\hat{\bar{S}}_1$, and (d$_2$) $\hat{\bar{S}}_2$. The quark in (a) and (b) and the antiquark in (c) and (d) are decaying, while the antiquark in (a) and (b) and the quark in (c) and (d) are the spectators.

$$\langle [QQ][\bar{Q}\bar{Q}] \,|\, H_w \,|\, \{Q\bar{Q}\}_0 \rangle \text{ and } \langle \{Q\bar{Q}\}_0 \,|\, H_w \,|\, [QQ][\bar{Q}\bar{Q}] \rangle \;, \tag{4.3}$$

by extending the prescription used in Secs. II and III to the $[QQ][\bar{Q}\bar{Q}]$ contribution.

In the case of $H_w=H(0,-)$, we insert, for example, Eq. (3.9) with $\alpha=\pi^+$, between $\langle \pi^+ |$ and $| K^0 \rangle$ with infinite momenta and extract $[QQ][\bar{Q}\bar{Q}]$ meson contribution to the intermediate states on the LHS:

$$\langle \pi^+ | A_{\pi^+} | \hat{\sigma} \rangle \langle \hat{\sigma} | H(0,-) | K^0 \rangle - \langle \pi^+ | H(0,-) | \hat{\kappa}^+ \rangle \langle \hat{\kappa}^+ | A_{\pi^+} | K^0 \rangle + \cdots$$
$$= -\{ \langle \pi^+ | H(0,-) | K^+ \rangle + \sqrt{2} \langle \pi^0 | H(0,-) | K^0 \rangle \} \;, \tag{4.4}$$

where the ellipsis denotes contributions of the ground state and other excited states (other than the $[QQ][\bar{Q}\bar{Q}]$ mesons under consideration) such as $(QQ)(\bar{Q}\bar{Q})$ mesons, radially excited $\{Q\bar{Q}\}$ states, etc.

The asymptotic $|\Delta \mathbf{I}|=\frac{1}{2}$ rule already obtained, Eq. (2.6), implies that the RHS of Eq. (4.4) vanishes. Using the $SU_f(3)$ parametrization for the asymptotic matrix elements of the axial-vector charges $A_{\pi^{\pm,0}}$ obtained from the algebras, $[V_\alpha, A_\beta]=if_{\alpha\beta\gamma}A_\gamma$ (we are dealing with the ideally mixed **36**-plet),

$$\begin{aligned}\langle \hat{\kappa}^+ | A_{\pi^+} | K^0 \rangle &= \langle \hat{\kappa}^0 | A_{\pi^-} | K^+ \rangle \\ &= 2\langle \hat{\kappa}^+ | A_{\pi^0} | K^+ \rangle = -2\langle \hat{\kappa}^0 | A_{\pi^0} | K^0 \rangle \\ &= -\langle \pi^+ | A_{\pi^+} | \hat{\sigma} \rangle = -\langle \pi^- | A_{\pi^-} | \hat{\sigma} \rangle = -\sqrt{2}\langle \pi^0 | A_{\pi^0} | \hat{\sigma} \rangle = -2A \;,\end{aligned} \tag{4.5}$$

and applying to Eq. (4.4) the ansatz of "level realization" (a straightforward extension of the procedure used in Sec. II to the $[QQ][\bar{Q}\bar{Q}]$ meson contributions), we obtain

TABLE I. Ideally mixed $20''\oplus 15\oplus 1=36$ multiplet $[QQ][\bar{Q}\bar{Q}]$ mesons. Antiparticles involved in the 36-plet are dropped.

| $C$ | $S$ | $I=1$ | $I=\frac{1}{2}$ | $I=0$ | Estimated mass (GeV) |
|---|---|---|---|---|---|
| +1 | +1 | $\hat{F}_I^{++},\hat{F}_I^{+},\hat{F}_I^{0}$ | | $\hat{F}^{+}$ | 3.15 |
| | 0 | | $\hat{D}^{+},\hat{D}^{0}$<br>$\hat{D}_s^{+},\hat{D}_s^{0}$ | | 2.95 |
| | −1 | | | $\hat{F}^{0}$ | 3.15 |
| 0 | +1 | | $\hat{\kappa}^{+},\hat{\kappa}^{0}$<br>$\hat{\kappa}_c^{+},\hat{\kappa}_c^{0}$ | | 1.60 |
| | 0 | $\hat{\delta}_s^{+},\hat{\delta}_s^{0},\hat{\delta}_s^{-}$<br>$\hat{\delta}_c^{+},\hat{\delta}_c^{0},\hat{\delta}_c^{-}$ | | $\hat{\sigma}$<br>$\hat{\sigma}_s$<br>$\hat{\sigma}_c$<br>$\hat{\sigma}_{sc}$ | 1.45<br>1.80 |

$$\langle\hat{\sigma}\,|\,H(0,-)\,|\,K^0\rangle+\langle\pi^+\,|\,H(0,-)\,|\,\hat{\kappa}^+\rangle=0\ , \tag{4.6}$$

when $A\neq 0$ is assumed.

From the commutation relations, Eq. (3.9) with $\alpha=\pi^{0,-}$, we also obtain

$$2A\langle\pi^+\,|\,H(0,-)\,|\,\hat{\kappa}^+\rangle=(c_{\pi^0})\langle\pi^+\,|\,H(0,-)\,|\,K^+\rangle\ , \tag{4.7}$$

$$2A\{\sqrt{2}\langle\hat{\sigma}\,|\,H(0,-)\,|\,K^0\rangle-\langle\pi^0\,|\,H(0,-)\,|\,\hat{\kappa}^0\rangle\}=(c_{\pi^0})\langle\pi^+\,|\,H(0,-)\,|\,K^+\rangle\ , \tag{4.8}$$

$$2A\langle\pi^0\,|\,H(0,-)\,|\,\hat{\kappa}^0\rangle=(c_{\pi^-})\langle\pi^0\,|\,H(0,-)\,|\,K^0\rangle\ , \tag{4.9}$$

$$2A\langle\hat{\sigma}\,|\,H(0,-)\,|\,K^0\rangle=\sqrt{2}(c_{\pi^-})\langle\pi^0\,|\,H(0,-)\,|\,K^0\rangle\ , \tag{4.10}$$

where $c_{\pi^0}$ and $c_{\pi^-}$ denote the fractions of the $[QQ][\bar{Q}\bar{Q}]$ meson contributions to the LHS of Eq. (3.9) with $\alpha=\pi^0$ and $\pi^-$, respectively. On the RHS of Eq. (4.9) we have used the asymptotic $|\Delta\mathbf{I}|=\frac{1}{2}$ rule, Eq. (2.6), which is used again to solve the constraint equations, Eqs. (4.6)–(4.10).

The result is

$$\langle\pi^+\,|\,H(0,-)\,|\,\hat{\kappa}^+\rangle=-\sqrt{2}\langle\pi^0\,|\,H(0,-)\,|\,\hat{\kappa}^0\rangle=-\langle\hat{\sigma}\,|\,H(0,-)\,|\,K^0\rangle$$
$$=(c_{\pi^0}/2A)\langle\pi^+\,|\,H(0,-)\,|\,K^+\rangle\ , \tag{4.11}$$

and, as we might expect,

$$c_{\pi^0}=c_{\pi^-}\ . \tag{4.12}$$

Equation (4.11) shows that the matrix elements, Eq. (4.3) with $H_w=H(0,-)$, also satisfy the *asymptotic* $|\Delta\mathbf{I}|=\frac{1}{2}$ rule.

We have gotten all the necessary constraints on the matrix elements of Eq. (4.3) with $H_w=H(0,-)$ from the constraint algebras, Eq. (3.9) with $\alpha=\pi^+$, $\pi^0$, and $\pi^-$.

Other constraint algebras, Eq. (3.9) with $\alpha=K^+$, $K^0$, $K^-$, and $\bar{K}^0$, are found to give no new constraints on these matrix elements in the present case. However, these algebras could, in general, produce stronger constraints than the algebras involving only $A_\pi$'s, since the former involve $A_K$'s which are the generators of $\mathrm{SU}_f(3)_L\times\mathrm{SU}_f(3)_R$ rather than $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$.

Next we study the constraints upon the nondiagonal matrix elements, Eq. (4.3) with $H_w=H(-,-)$ in the same way. In the case of $H_w=H(-,-)$, however, we cannot obtain a sufficient number of constraints on the matrix elements, Eq. (4.3), from the algebras, Eq. (3.9) with $\alpha=\pi^0$ and $\pi^-$ alone. [Equation (3.9) with $\alpha=\pi^+$ is trivial in the case of $H_w=H(-,-)$.] Therefore, we also use additional constraint algebras involving $A_K$'s, Eq. (3.9) with $\alpha=K^+$ and $K^0$. Then, we obtain the following set of constraint equations:

$$A\{\langle\hat{\bar{\kappa}}^0\,|\,H(-,-)\,|\,D^0\rangle+\langle\bar{K}^0\,|\,H(-,-)\,|\,\hat{D}^0\rangle\}=(C_{\pi^0})\langle\bar{K}^0\,|\,H(-,-)\,|\,D^0\rangle\ , \tag{4.13}$$

$$-\sqrt{2}A\langle\pi^+\,|\,H(-,-)\,|\,\hat{F}_I^+\rangle=(C_{\pi^0})\langle\pi^+\,|\,H(-,-)\,|\,F^+\rangle\ , \tag{4.14}$$

$$2A\langle\bar{K}^0\,|\,H(-,-)\,|\,\hat{D}^0\rangle=(C_{\pi^-})\langle\bar{K}^0\,|\,H(-,-)\,|\,D^0\rangle\ , \tag{4.15}$$

$$2A\langle\hat{\bar{\kappa}}^0\,|\,H(-,-)\,|\,D^0\rangle=(C_{\pi^-})\langle\bar{K}^0\,|\,H(-,-)\,|\,D^0\rangle\ , \tag{4.16}$$

$$2A\langle\pi^0|H(-,-)|\hat{F}_I^0\rangle=\sqrt{2}(C_{\pi^-})\langle\pi^+|H(-,-)|F^+\rangle\ , \tag{4.17}$$

$$\sqrt{2}A\{\sqrt{2}\langle\hat{\kappa}^0|H(-,-)|D^0\rangle-\langle\pi^+|H(-,-)|\hat{F}_I^+\rangle-\langle\pi^+|H(-,-)|\hat{F}^+\rangle\}=0\ , \tag{4.18}$$

$$\sqrt{2}A\{\langle\pi^+|H(-,-)|\hat{F}^+\rangle-\langle\pi^+|H(-,-)|\hat{F}_I^+\rangle\}=(C_{K^0})\langle\pi^+|H(-,-)|F^+\rangle\ , \tag{4.19}$$

$$2A\{\langle\hat{\kappa}^0|H(-,-)|D^0\rangle+\sqrt{2}\langle\pi^0|H(-,-)|\hat{F}_I^0\rangle\}=-(C_{K^0})\langle\bar{K}^0|H(-,-)|D^0\rangle\ , \tag{4.20}$$

where $C_\alpha$ ($\alpha=\pi^{0,-},K^0$) denotes the fractions of the $[QQ][\bar{Q}\bar{Q}]$ meson contribution to the intermediate states on the LHS of the constraint algebras, Eq. (3.9) with $\alpha=\pi^{0,-},K^0$, respectively, when they are sandwiched between appropriate states. [Equation (4.18) was obtained from Eq. (3.9) with $\alpha=K^+$.] We have used the result from the asymptotic SU(2) rotation of the matrix elements of $H(-,-)$, Eq. (A8), given in Appendix A and also the asymptotic $SU_f(4)$ parametrization for the matrix elements of the axial-vector charges $A_\pi$'s and $A_K$'s obtained from the constraint algebras $[V_\alpha,A_\beta]=if_{\alpha\beta\gamma}A_\gamma$:

$$\langle\hat{F}_I^0|A_{\pi^-}|F^+\rangle=-\langle\hat{D}^0|A_{\pi^-}|D^+\rangle=-\sqrt{2}\langle\hat{F}_I^+|A_{K^+}|D^0\rangle=-\sqrt{2}\langle\hat{F}^+|A_{K^+}|D^0\rangle=-\langle\hat{\kappa}^0|A_{K^+}|\pi^-\rangle$$
$$=-\langle\delta_s^+|A_{K^+}|\bar{K}^0\rangle=\sqrt{2}\langle\hat{\sigma}_s|A_{K^+}|K^-\rangle=\sqrt{2}\langle\hat{\sigma}_s|A_{K^0}|\bar{K}^0\rangle=\cdots=2A\ . \tag{4.21}$$

Although we can obtain extra constraint equations in addition to Eqs. (4.13)–(4.20) from the commutation relations involving Eq. (3.9) with $\alpha=K^-$ and $\bar{K}^0$, we dropped them since they involve the matrix elements of $H_w=H(-,-)$ which are irrelevant to the $D\to\pi\bar{K}$ decays in which we are interested. Corresponding to Eq. (4.11), we now get the following constraints on the matrix elements of Eq. (4.3) with $H_w=H(-,-)$ from Eqs. (4.13)–(4.20):

$$\langle\hat{\kappa}^0|H(-,-)|D^0\rangle=\langle\bar{K}^0|H(-,-)|\hat{D}^0\rangle=\sqrt{1/2}\langle\pi^+|H(-,-)|\hat{F}_I^+\rangle$$
$$=-\sqrt{1/2}\langle\pi^0|H(-,-)|\hat{F}_I^0\rangle=(C_{\pi^0}/2A)\langle\bar{K}^0|H(-,-)|D^0\rangle\ , \tag{4.22}$$

$$\langle\pi^+|H(-,-)|\hat{F}^+\rangle=0\ , \tag{4.23}$$

$$C_{\pi^0}=C_{\pi^-}=C_{K^0}\ . \tag{4.24}$$

We now derive the asymptotic constraints on the nondiagonal matrix elements, Eq. (4.3) with $H_w=H(-,0)$, in exactly the same way as we carried out for the cases of $H_w=H(0,-)$ and $H(-,-)$ discussed above. We insert the constraint algebras, Eq. (3.9) with $\alpha=\pi^{\pm,0}$, $K^{\pm,0}$, and $\bar{K}^0$, between all the appropriate pairs of external ground-state-meson states with infinite momenta and then extract $[QQ][\bar{Q}\bar{Q}]$ meson contribution to the intermediate states on the LHS. Using asymptotic $SU_f(4)$ parametrization of the matrix elements of the axial-vector charges $A_\pi$'s and $A_K$'s given by Eq. (4.21), we then obtain

$$A\{\langle\hat{\sigma}|H(-,0)|D^0\rangle+\langle\pi^+|H(-,0)|\hat{D}^+\rangle\}=0\ , \tag{4.25}$$

$$-2A\langle\pi^+|H(-,0)|\hat{D}^+\rangle=(e_{\pi^0})\langle\pi^+|H(-,0)|D^+\rangle\ , \tag{4.26}$$

$$2A\{\sqrt{2}\langle\hat{\sigma}|H(-,0)|D^0\rangle+\langle\pi^0|H(-,0)|\hat{D}^0\rangle\}=(e_{\pi^0})\langle\pi^0|H(-,0)|D^0\rangle\ , \tag{4.27}$$

$$-2A\{\langle\hat{\kappa}^+|H(-,0)|F^+\rangle+\sqrt{2}\langle K^+|H(-,0)|\hat{F}_I^+\rangle\}=(e_{\pi^0})\langle K^+|H(-,0)|F^+\rangle\ , \tag{4.28}$$

$$2A\langle\hat{\sigma}|H(-,0)|D^0\rangle=\sqrt{2}(e_{\pi^-})\langle\pi^0|H(-,0)|D^0\rangle\ , \tag{4.29}$$

$$2A\langle\pi^0|H(-,0)|\hat{D}^0\rangle=(e_{\pi^-})[\langle\pi^0|H(-,0)|D^0\rangle-\sqrt{2}\langle\pi^+|H(-,0)|D^+\rangle]\ , \tag{4.30}$$

$$-2A[\langle\hat{\kappa}^+|H(-,0)|F^+\rangle+\langle K^0|H(-,0)|\hat{F}_I^0\rangle]=(e_{\pi^-})\langle K^+|H(-,0)|F^+\rangle\ , \tag{4.31}$$

$$\langle\hat{\sigma}_s|H(-,0)|D^0\rangle-\langle\hat{\delta}_s^0|H(-,0)|D^0\rangle+\langle K^+|H(-,0)|\hat{F}_I^+\rangle+\langle K^+|H(-,0)|\hat{F}^+\rangle=0\ , \tag{4.32}$$

$$\sqrt{2}A[\sqrt{2}\langle\hat{\delta}_s^+|H(-,0)|D^+\rangle+\langle K^+|H(-,0)|\hat{F}_I^+\rangle-\langle K^+|H(-,0)|\hat{F}^+\rangle]$$
$$=(e_{K^0})[\langle\pi^+|H(-,0)|D^+\rangle-\langle K^+|H(-,0)|F^+\rangle]\ , \tag{4.33}$$

$$A[\langle\hat{\delta}_s^0|H(-,0)|D^0\rangle+\langle\hat{\sigma}_s|H(-,0)|D^0\rangle+\sqrt{2}\langle K^0|H(-,0)|\hat{F}_I^0\rangle]=\sqrt{2}(e_{K^0})\langle\pi^+|H(-,0)|D^+\rangle\ , \tag{4.34}$$

$$2A[\langle\hat{\sigma}_s|H(-,0)|D^0\rangle-\langle\hat{\delta}_s^0|H(-,0)|D^0\rangle]$$
$$=(e_{K^-})[\langle\pi^0|H(-,0)|D^0\rangle+\langle\eta_0|H(-,0)|D^0\rangle-\sqrt{2}\langle\eta_s|H(-,0)|D^0\rangle]\ , \quad (4.35)$$

$$-2A[\langle\hat{\delta}_s^+|H(-,0)|D^+\rangle-\langle\bar{K}^0|H(-,0)|\hat{F}^0\rangle]=(e_{K^-})\langle\pi^+|H(-,0)|D^+\rangle\ , \quad (4.36)$$

$$2A[\langle\hat{\kappa}^+|H(-,0)|F^+\rangle-\sqrt{2}\langle\pi^0|H(-,0)|\hat{D}_s^0\rangle]=(e_{K^-})[\langle K^+|H(-,0)|F^+\rangle-\sqrt{2}\langle\pi^0|H(-,0)|D^0\rangle]\ , \quad (4.37)$$

$$2A[\langle\hat{\sigma}_s|H(-,0)|D^0\rangle+\langle\hat{\delta}_s^0|H(-,0)|D^0\rangle-\sqrt{2}\langle\bar{K}^0|H(-,0)|\hat{F}^0\rangle]$$
$$=-(e_{\bar{K}^0})[\langle\pi^0|H(-,0)|D^0\rangle-\langle\eta_0|H(-,0)|D^0\rangle+\sqrt{2}\langle\eta_s|H(-,0)|D^0\rangle]\ , \quad (4.38)$$

$$-2A[\langle\hat{\kappa}^+|H(-,0)|F^+\rangle-\langle\pi^+|H(-,0)|\hat{D}_s^+\rangle]=(e_{\bar{K}^0})[\langle\pi^+|H(-,0)|D^+\rangle-\langle K^+|H(-,0)|F^+\rangle]\ , \quad (4.39)$$

where we have assumed the ideal $\eta-\eta'$ mixing for simplicity, since our final result is not affected by this assumption. The above equations cannot be solved immediately. Therefore, we use the constraint on the diagonal matrix elements obtained in Sec. III, Eq. (3.3a), and also the results on diagonal and nondiagonal matrix elements given in Appendix A obtained from asymptotic $SU_f(3)$ rotations, Eqs. (A9), (A12), and (A18)–(A20). Then we obtain the solution as

$$\langle K^0|H(-,0)|\hat{F}_I^0\rangle=\sqrt{2}\langle K^+|H(-,0)|\hat{F}_I^+\rangle=\langle\bar{K}^0|H(-,0)|\hat{F}^0\rangle$$
$$=-2\langle\pi^+|H(-,0)|\hat{D}^+\rangle=2\langle\pi^+|H(-,0)|\hat{D}_s^+\rangle=-2\sqrt{2}\langle\pi^0|H(-,0)|\hat{D}^0\rangle$$
$$=2\sqrt{2}\langle\pi^0|H(-,0)|\hat{D}_s^0\rangle=-2\langle\hat{\kappa}^+|H(-,0)|F^+\rangle=2\langle\hat{\delta}_s^+|H(-,0)|D^+\rangle$$
$$=2\sqrt{2}\langle\hat{\delta}_s^0|H(-,0)|D^0\rangle=2\langle\hat{\sigma}|H(-,0)|D^0\rangle=-2\sqrt{2}\langle\hat{\sigma}_s|H(-,0)|D^0\rangle$$
$$=(e_{\pi^0}/A)\langle\pi^+|H(-,0)|D^+\rangle\ , \quad (4.40)$$

$$\langle K^+|H(-,0)|\hat{F}^+\rangle=0\ , \quad (4.41)$$

$$e_{\pi^0}=e_{\pi^-}=e_{K^0}=e_{\bar{K}^0}=e_{K^-}\ . \quad (4.42)$$

Here we examine the relations among the three fractions $c_{\pi^0}$, $C_{\pi^0}$, and $e_{\pi^0}$. If we substitute the constraints on the nondiagonal matrix elements of $H_w=H(0,-)$ and $H(-,-)$, Eqs. (4.11) and (4.22), into the asymptotic $SU_f(4)$ relation, Eq. (A21), then we find

$$c_{\pi^0}=C_{\pi^0}\ . \quad (4.43)$$

Substitution of the solutions, Eqs. (4.22) and (4.40), of the constraint equations for the nondiagonal matrix elements of $H(-,-)$ and $H(-,0)$ into Eq. (A14) and the use of Eq. (A11) then lead to

$$e_{\pi^0}=C_{\pi^0}\ . \quad (4.44)$$

Therefore, *all* the fractional $[QQ][\bar{Q}\bar{Q}]$ contributions to the intermediate states appearing on the LHS of all the single commutators involving $H_w$ are found to be equal (i.e., universal),

$$k_c\equiv c_{\pi^0}=C_{\pi^0}=e_{\pi^0}\ , \quad (4.45)$$

which implies that the fractions are flavor independent—a very reasonable and pretty result.

Here, we add a brief comment on the result obtained above from a simple diagrammatical point of view. The nondiagonal matrix elements of $H_w=H(0,-)$, $H(-,-)$, and $H(-,0)$, Eq. (4.3), can be associated with appropriate combinations of eight independent *spectator* diagrams, (a₁) $S_1$, (a₂) $S_2$, (b₁) $\hat{S}_1$, (b₂) $\hat{S}_2$, (c₁) $\bar{S}_1$, (c₂) $\bar{S}_2$, (d₁) $\hat{\hat{S}}_1$, and (d₂) $\hat{\hat{S}}_2$, shown in Fig. 1, when the diagrams involving *disconnected* quark-antiquark pair creation (and annihilation) are neglected. Then, paying attention to the relative signs of the matrix elements of $H(-,0)$ coming from the Cabibbo mixing [see Eq. (A4c)], we find that we can reproduce perfectly the relations among the nondiagonal matrix elements of $H_w$ obtained in each of Eqs. (4.11), (4.22), (4.23), (4.40), and (4.41), if we impose simple relations among the above eight independent spectator diagrams, i.e.,

$$(S_1-S_2)=(\hat{S}_1-\hat{S}_2)$$
and (4.46)
$$(\hat{S}_1-\hat{S}_2)=-(\hat{\hat{S}}_1-\hat{\hat{S}}_2)\ .$$

Furthermore, we find that Eq. (4.46) itself is, in fact, also satisfied by our asymptotic $SU_f(4)$ relations of the nondiagonal matrix elements of $H_w=H(-,-)$, given by Eqs. (A23) and (A24).

## V. ESTIMATE OF THE RATIO OF THE DECAY RATES $\Gamma(D^0\to\pi^+K^-)$ RELATIVE TO $\Gamma(K_S^0\to\pi^+\pi^-)$

In previous papers[7,10] we have calculated the ratio of the decay rates $\Gamma(D^0\to\pi^+K^-)/\Gamma(K_S^0\to\pi^+\pi^-)$ under the simplifying approximation in which only the ground-state-meson contribution to the surface term $M_S$ has been retained. This first approximation result was taken[7,10] to be reasonable in comparison with the old data which contained large uncertainties. However, the recent more improved data[5] have produced the value of the ratio larger by a factor of 3–4 than our previous estimate.[7,10] In this paper we now take into account the $[QQ][\bar{Q}\bar{Q}]$ meson contribution as the next step.

We now substitute the asymptotic constraints on the matrix elements of $H_w$ given in Secs. III and IV into the amplitude presented in Sec. II. The explicit expressions of the amplitudes for the $K_S^0 \to \pi^+\pi^-, D^0 \to \pi^+ K^-$ and $\pi^0 \bar{K}^0$ decays are then given by

$$M(K_S^0 \to \pi^+\pi^-) \simeq (i/2f_\pi)\sqrt{2}\langle \pi^+ | H(0,-) | K^+\rangle \{1 - [(m_K^2 - m_\pi^2)/(m_{K^*}^2 - m_\pi^2)]k_0 + [2(m_K^2 - m_\pi^2)/(m_{\hat{\sigma}}^2 - m_K^2) + (m_K^2 - m_\pi^2)/(m_{\hat{\kappa}}^2 - m_\pi^2)]k_c\} , \tag{5.1}$$

$$M(D^0 \to \pi^+ K^-) \simeq (i/2f_\pi)\langle \bar{K}^0 | H(-,-) | D^0\rangle \{1 - (m_D^2 - m_\pi^2)/(m_{F^*}^2 - m_\pi^2)k_0 - [(2m_D^2 - m_K^2 - m_\pi^2)/(m_D^2 - m_{\hat{\kappa}}^2) - (m_D^2 - m_\pi^2)/(m_{\hat{F}_I}^2 - m_\pi^2)]k_c\} , \tag{5.2a}$$

$$M(D^0 \to \pi^0 \bar{K}^0) \simeq (i/2\sqrt{2}f_\pi)\langle \bar{K}^0 | H(-,-) | D^0\rangle \{1 - [(m_D^2 - m_K^2)/(m_{D^*}^2 - m_K^2)]k_0 - [(2m_D^2 - m_K^2 - m_\pi^2)/(m_D^2 - m_{\hat{\kappa}}^2) + (m_D^2 - m_K^2)/(m_{\hat{D}}^2 - m_K^2) - (m_D^2 - m_\pi^2)/(m_{\hat{F}_I}^2 - m_\pi^2)]k_c\} , \tag{5.2b}$$

where we have assumed $f_\pi \simeq f_K$, since the above amplitudes are not so sensitive to the ratio $f_\pi / f_K$. We have again used the same asymptotic $SU_f(4)$ parametrization of the axial-vector charges. The above amplitudes, Eqs. (5.1) and (5.2), involve the masses of $[QQ][\bar{Q}\bar{Q}]$ mesons and also the *universal* fraction $k_c$ of the $[QQ][\bar{Q}\bar{Q}]$ meson contribution [see Eq. (4.45)] in the (asymptotic) levelwise realization of the *single* commutators considered. The masses of $[QQ][\bar{Q}\bar{Q}]$ mesons and the value of the fraction $k_c$ are still unknown.

However, in the "levelwise" realization of *single* commutators considered in the previous sections, the sum of all the fractional contributions from each "level" which is inserted between the factors $A_\alpha$ and $H_w$ should sum up to unity up to the overall sign, i.e.,

$$f_{L=0} + \sum_{L \geq 1} f_L + f_{[QQ][\bar{Q}\bar{Q}]} + \cdots = \pm 1 , \tag{5.3}$$

where the ellipsis denotes the contributions of the radially excited states and also the $(QQ)(\bar{Q}\bar{Q})$ meson states, etc. According to our previous analyses, negative value of $f_{L=0}$ [$\equiv k_0$ in Eq. (2.14)] is favored,[21] i.e., $k_0 = \sqrt{1/2}\langle \pi^- | A_{\pi^-} | \rho^0\rangle \simeq -0.7$, when the positive sign is chosen in Eqs. (2.12) and (2.13). Therefore, we choose again negative values of $k_0$ in this paper. Diagrammatical analysis[9,11] (which is also consistent with the result obtained by algebraic method) given in Sec. IV says that the contribution of orbitally excited $\{Q\bar{Q}\}$ meson states to the single commutators, Eq. (3.9), will be small, i.e., $\sum_{L\geq 1} f_L \simeq 0$. Therefore, we expect that the fractional contributions of exotic states and the radially excited $L=0$ states, etc., amount to about 30%.

Thus we obtain an upper limit for $-k_c$,

$$-k_c \equiv -f_{[QQ][\bar{Q}\bar{Q}]} \simeq 1 + (f_{L=0} + \cdots) \lesssim 0.3 , \tag{5.4}$$

where we have used $f_{L=0} (\equiv k_0) \simeq -0.7$.

Masses of four-quark mesons have been calculated on the basis of the bag model by Jaffe[14] and some of their possible candidates are observed at the values not far from the predicted ones mentioned in Sec. IV. Our amplitudes are sensitive to the masses $m_{\hat{\sigma}_s}$ and $m_{\hat{\kappa}}$ as is seen in Eq. (5.2) and will also be seen later, if their values lie near $m_D$. However, they are not so sensitive to the masses of the $[QQ][\bar{Q}\bar{Q}]$ states involving charm quark(s). Therefore we use the values of noncharm $[QQ][\bar{Q}\bar{Q}]$ mesons [$C(9^*)$ in Jaffe's notation[14]] and estimate the charm $[QQ][\bar{Q}\bar{Q}]$ meson masses crudely by using the quark counting with $m_c - m_u \simeq 1.5$ GeV and $m_s - m_u \simeq 0.2$ GeV. In Table I we list the masses of $[QQ][\bar{Q}\bar{Q}]$ mesons thus estimated.

We show in Table II the branching ratios $B(D^0 \to \pi^+ K^-)$ and $B(D^0 \to \pi^0 \bar{K}^0)$, calculated by using the above values of $[QQ][\bar{Q}\bar{Q}]$ meson masses, the possible and reasonable values of the fraction in the range $0 \lesssim -k_c \lesssim 0.3$ according to Eq. (5.4) and the observed lifetime,[5] $\tau(D^0) \simeq 4.4 \times 10^{-13}$ sec, and the decay rate,[20] $\Gamma(K_S^0 \to \pi^+\pi^-) \simeq 0.7689 \times 10^{10}$ sec$^{-1}$, as the input. It is seen that our result can be greatly improved for *reasonable* values of $k_c$ compared with the previous one obtained *without* including the effect of exotic states ($k_c = 0$).

TABLE II. Branching ratios for the two-body decays of $D$ mesons calculated by taking into account the $[QQ][\bar{Q}\bar{Q}]$-meson contributions to the surface term. ($k_c=0$ means no contribution from the $[QQ][\bar{Q}\bar{Q}]$ mesons.)

| Decay | Theory (%) $\lvert k_c \rvert =0.0$ | $\lvert k_c \rvert =0.10$ | $\lvert k_c \rvert =0.20$ | Experiment (%) |
|---|---|---|---|---|
| $D^0 \to \pi^+ K^-$ | 1.5 | 3.4 | 6.1 | $4.2\pm0.4\pm0.3$[a] |
| $D^0 \to \pi^0 \bar{K}^0$ | 1.2 | 2.6 | 4.8 | $1.9\pm0.5\pm0.4$[b] |
| $D^0 \to \pi^+ \pi^-$ | 0.10 | 0.17 | 0.27 | $0.14\pm0.04\pm0.03$[a] |
| $D^0 \to K^+ K^-$ | 0.053 | 0.46 | 1.3 | $0.51\pm0.09\pm0.06$[a] |
| $D^0 \to K^0 \bar{K}^0$ | 0 | 0 | 0 | $\leq 0.46$ at 90% CL[a] |
| $D^+ \to \pi^+ \pi^0$ | 0 | 0 | 0 | $<0.5$[c] |
| $D^+ \to K^+ \bar{K}^0$ | 0.13 | 1.1 | 3.0 | $1.01=0.32\pm0.18$[a] |

[a]Given by Hitlin in Ref. 5.
[b]Estimated from the value $B(D^0\to\pi^0\bar{K}^0)/B(D^0\to\pi^+K^-)=0.45\pm0.08\pm0.05$ given by Hitlin in Ref. 5. The values of errors are tentative.
[c]Reference 20.

## VI. CABIBBO-SUPPRESSED DECAYS OF $D$ MESONS: RESOLUTION OF THE PUZZLE OF $\Gamma(D^0\to K^+K^-)/\Gamma(D^0\to\pi^+\pi^-)$

Substituting the constraints obtained for the matrix elements of $H_w=H(-,0)$, Eqs. (3.3a), (3.10a), (4.40), and (4.41) together with Eq. (A9), into the general form of the two-body decay amplitude, Eqs. (2.2)–(2.4), and neglecting the contributions of the radially excited $\{Q\bar{Q}\}$ and also of the $(QQ)(\bar{Q}\bar{Q})$ mesons, etc., we write down explicitly the amplitudes for the Cabibbo-suppressed two-body decays of $D$ mesons, $D^0\to\pi^+\pi^-$, $D^0\to K^+K^-$, $D^0\to K^0\bar{K}^0$, $D^+\to\pi^+\pi^0$, and $D^+\to K^+\bar{K}^0$, as

$$M(D^0\to\pi^+\pi^-)\simeq-(i/2f_\pi)\langle\pi^+|H(-,0)|D^+\rangle\{1-[(m_D^2-m_\pi^2)/(m_{D^*}^2-m_\pi^2)]k_0 -[2(m_D^2-m_\pi^2)/(m_D^2-m_{\hat{\sigma}}^2)-(m_D^2-m_\pi^2)/(m_{\hat{D}}^2-m_\pi^2)]k_c\}\ , \qquad (6.1)$$

$$M(D^0\to K^+K^-)\simeq(i/2f_K)\langle\pi^+|H(-,0)|D^+\rangle\{1-[(m_D^2-m_K^2)/(m_{F^*}^2-m_K^2)]k_0 -[2(m_D^2-m_K^2)/(m_D^2-m_{\hat{\sigma}_s}^2)-(m_D^2-m_K^2)/(m_{\hat{F}_I}^2-m_K^2)]k_c\}\ , \qquad (6.2)$$

$$M(D^0\to K^0\bar{K}^0)\simeq0\ , \qquad (6.3)$$

$$M(D^+\to\pi^+\pi^0)\simeq0\ , \qquad (6.4)$$

$$M(D^+\to K^+\bar{K}^0)\simeq(i/2f_K)\langle\pi^+|H(-,0)|D^+\rangle\{1-[(m_D^2-m_K^2)/(m_{F^*}^2-m_K^2)]k_0 -(m_D^2-m_K^2)[2/(m_D^2-m_{\hat{\delta}_s}^2)-2/(m_{\hat{F}^0}^2-m_K^2)+1/(m_{\hat{F}_I}^2-m_K^2)]k_c\}\ , \qquad (6.5)$$

where we have assumed $m_{\hat{\sigma}_s}=m_{\hat{\delta}_s}$ (which has been predicted by Jaffe[14] and can be derived also in the present theoretical framework, if the 36-plet is ideally mixed) and have used the result of asymptotic $SU_f(4)$ parametrization of the matrix elements of axial-vector charges $A_\pi$'s and $A_K$'s.

Equation (6.3) does reflect the cancellation of the amplitudes for the $D^0\to K^0\bar{K}^0$ decay in the $SU_f(3)$-symmetry limit as was mentioned in Sec. I, although we have used *asymptotic* $SU_f(3)$ symmetry [not *exact* $SU_f(3)$ symmetry] *only* for the two-particle matrix elements of $H_w$ (and $A_\alpha$'s). We emphasize that all our sum rules are broken-flavor-symmetry sum rules, compatible with the GMO splittings of flavor multiplets and all the particles involved are on mass shell.

Equation (6.4) corresponds precisely to the suppression of the $K^+\to\pi^+\pi^0$ decay due to the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule, valid up to the present consideration of the ground-state-meson and the exotic $[QQ][\bar{Q}\bar{Q}]$-meson contributions to $M_S$.

Keeping *only* the ground-state-meson contribution to the surface term, we obtain $\Gamma(D^0\to K^+K^-)/$

$\Gamma(D^0 \to \pi^+\pi^-) \simeq 0.50$, where the 50% deviation from unity [expected in the $SU_f(3)$-symmetry limit] is due to the ratio of the decay constants, $f_K/f_\pi \simeq 1.2$, and the phase-space difference. However, by substituting into Eqs. (6.1), (6.2), and (6.5) the values of the $[QQ][\bar{Q}\bar{Q}]$-meson masses listed in Table I and the value of the fraction $k_c$ of the $[QQ][\bar{Q}\bar{Q}]$-meson contribution to the "levelwise" realization of the single commutators involving $H_w = H(-,0)$ which have already been estimated and used in the previous section, we obtain the results shown in Table II, where we have used the observed lifetime,[5] $\tau(D^+) \simeq 10.3 \times 10^{-13}$ sec, as well as $\tau(D^0)_{\rm expt}$ and $\Gamma(K_S^0 \to \pi^+\pi^-)_{\rm expt}$ used in the previous section as the input. From Table II, it is seen that our calculation reproduces the observed ratio of the decay rates $\Gamma(D^0 \to K^+K^-)/\Gamma(D^0 \to \pi^+\pi^-) \simeq 3.6$ fairly well for the very reasonable value of $k_c$, $-k_c \simeq 0.1$, together with the reasonable predicted values of the rates of $\Gamma(D^+ \to K^+\bar{K}^0)$, $\Gamma(D^0 \to K^+K^-)$, and $\Gamma(D^0 \to \pi^+\pi^-)$.

Therefore, the inclusion of the effect of $[QQ][\bar{Q}\bar{Q}]$ mesons is important and helps to explain the $D \to \pi\bar{K}$, $K\bar{K}$, and $\pi\pi$ and also $K \to \pi\pi$ decays in a consistent and a unified manner.

## VII. SUMMARY AND COMMENTS

We have demonstrated in Sec. VI a possible solution to the well-known puzzle in the Cabibbo-suppressed decays of $D^0$ meson (which has never been solved in a systematic manner) by evaluating systematically the contributions of the ground-state $\{Q\bar{Q}\}_0$ and the exotic $[QQ][\bar{Q}\bar{Q}]$ mesons to the surface term $M_S$ of the amplitude. It indicates that the contribution of four-quark meson states could be sizable in the nonleptonic decays of $D$ mesons, while it is much less important in the $K \to 2\pi$ decays as can be seen explicitly from Eq. (5.1). From this reason, the unified description of $K_S^0 \to 2\pi$ and $D^0 \to \pi^+K^-$ decays, which has been calculated in previous papers[7,10] without including the excited-state contributions, has also been *improved* significantly by the inclusion of $[QQ][\bar{Q}\bar{Q}]$-meson contribution as discussed in Sec. V.

We may add here a comment to the approximate selection rules found in Eqs. (6.3) and (6.4). They can, in fact, be violated rather significantly by the inclusion of the neglected contribution to $M_S$ from $(QQ)(\bar{Q}\bar{Q})$-type exotics, glueballs and, in general, less importantly radially excited $\{Q\bar{Q}\}$ mesons.

Glueballs with $J^{PC} = 0^{++}$, if they exist, may contribute to the Cabibbo-suppressed decays of $D$ mesons and the magnitude of their interactions can be studied by future observation of the $D^0 \to K^0\bar{K}^0$ decay. This decay proceeds only through the $W$-exchange diagram and its amplitudes cancel each other in the $SU_f(3)$-symmetry limit as was mentioned in Sec. I. Indeed as seen in Eq. (6.3), $M_S$ of this decay receives no contribution from the ground-state $\{Q\bar{Q}\}$ meson and $[QQ][\bar{Q}\bar{Q}]$ exotics. The $(QQ)(\bar{Q}\bar{Q})$-type exotics also cannot take part in $M_S$ as far as the $s$ channel is concerned. Their contribution to $M_S$ in the crossed channel will take the form [as seen from Eq. (6.2)], $(i/2f_K)\langle \pi^+ | H(-,0) | D^+ \rangle \times k_s \times$(mass-dependent factor). The first term is the expression of $M_{\rm ETC}$ and $k_s$ is the fraction of the $(QQ)(\bar{Q}\bar{Q})$-meson contribution analogous to the $k_c$ of the $[QQ][\bar{Q}\bar{Q}]$ exotics which is estimated to be $|k_s| \lesssim 0.1$. The mass-dependent factor is less than one. Therefore, the magnitude of the $D^0 \to K^0\bar{K}^0$ amplitude will be less than $\frac{1}{10}$ of that of the typical Cabibbo-suppressed decays as long as the glueball contribution is neglected. Therefore, if $B(D^0 \to K^0\bar{K}^0) \gtrsim 10^{-4}$ is indicated by future experiment, it may imply the presence of $0^{++}$ glueball. In fact, possible presence of $0^{++}$ glueball was argued[17] by Teshima and Oneda in the present algebraic approach from the study of $0^{++}$ meson spectrum.

As to the $D^+ \to \pi^+\pi^0$ decay, the contribution of the $(QQ)(\bar{Q}\bar{Q})$ mesons to $M_S$ could enhance the first forbidden amplitude, Eq. (6.4), significantly. Indeed the nonstrange $(QQ)(\bar{Q}\bar{Q})$ mesons can take part in $M_S$ in the $s$ channel and their masses are expected to be close to the $D$-meson mass. Future observation of the $D^+ \to \pi^+\pi^0$ rate is very important in assessing the size of the $(QQ)(\bar{Q}\bar{Q})$-meson contribution to the Cabibbo-suppressed decays of the $D$ meson.

The direct contribution of radially excited states $\{Q\bar{Q}\}_{\rm rad}$ to $M_S$ is, in general, expected to be less important than that of exotics. $\{Q\bar{Q}\}_{\rm rad}$ have the same flavor multiplet structure as $\{Q\bar{Q}\}_0$. The axial-vector charges $A_\alpha(A_\pi, \ldots)$ connect the $\{Q\bar{Q}\}_0$ states to the $\{Q\bar{Q}\}_0$ and $\{Q\bar{Q}\}_{L=1}$ states with standard strength but with much less strength to the $\{Q\bar{Q}\}_{\rm rad}$. For example, $\rho' \to \pi\pi$ coupling strength is much weaker than $\rho \to \pi\pi$ coupling. Therefore, barring the accidental mass degeneracy in the mass-dependent factors in Eq. (2.4) which may cause enhancement, the fraction of the radially excited meson contribution to $M_S$ will be small ($<0.1$) compared with the ground-state-meson contribution $|k_0| \simeq 0.7$.

However, the following effect is more important. In asymptotic flavor symmetry, the effect of symmetry breaking manifests itself, besides the mass splittings, through the mixings in the asymptotic limit. In this paper the mixings with the radially excited states, i.e., the leakage to the radially excited $0^{-+}$ and $1^{--}$ states through the flavor charges [$V_K$'s in $SU_f(3)$, $V_D$ and $V_F$ in $SU_f(4)$] is not considered. The crude estimate of the effect is as follows. From $f_+^{K\pi}(0)$ in the $K \to \pi e^+\nu$ decay, the leakage through $V_K$'s is certainly very small. From $f_+^{KD}(0) \simeq 0.73 \pm 0.05$ in the $D \to Ke^+\nu$ decay, the leakage in the values of the asymptotic matrix elements of the $SU_f(4)$ charges $V_D$ and $V_F$ through inter- (not intra) multiplet mixing is expected to be around[22] 20–30%. Therefore, the neglect of the leakage may produce an overall 20–30% errors in the amplitudes when the $D \to \pi\bar{K}$ and $K \to 2\pi$ decays are compared.

From the present work it is clear that the old soft-PS-meson extrapolation, which drops the contribution of the surface term $M_S$ completely, is a too drastic extrapolation. Our method utilizes much milder extrapolation and the surviving surface term is found to be still manageable, since it involves only the *asymptotic on-mass-shell two-particle hadron matrix elements* of the effective weak Hamiltonian and the well-known axial-vector charges.

The constraints upon these asymptotic matrix elements of $H_w$ which we have derived have a close correspondence to the quark-line diagrams as briefly mentioned in Secs. III and IV. We may perhaps hope that our approach has achieved a kind of synthesis of current- and constituent-quark physics, which is difficult in the pertubative QCD approach. The interpretations of quark-line diagrams are quite different between the two contrasting approaches. We also stress that in the present formulation, virtual particles are never involved. All the external and the intermediate states used are *physical* (i.e., "in" or "out") particles and the effect of long-distance physics can, in principle, be fully accommodated. We do not need to resort to the picture of final-state interaction.

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## APPENDIX A: ASYMPTOTIC $SU_f(N)$ ($N=2,3,4$) ROTATION FOR THE MATRIX ELEMENTS OF $H_w$

In the weak-boson mass $m_W \to \infty$ limit, the effective weak Hamiltonian $H_w$ responsible for the nonleptonic decays can be written in the form of the products of left-handed currents in the standard model

$$H_w=(G/2\sqrt{2})\{[J_\mu(\Delta C=0)+J_\mu(\Delta C=1)] \times[J_\mu(\Delta C=0)+J_\mu(\Delta C=1)]^\dagger+\text{H.c.}\}\ , \tag{A1}$$

where the charm-conserving and -changing currents are defined by

$$J_\mu(\Delta C=0)\equiv\cos\theta_C J_\mu^{\pi^+}+\sin\theta_C J_\mu^{K^+} \tag{A2a}$$

and

$$J_\mu(\Delta C=1)\equiv-\sin\theta_c J_\mu^{D^+}+\cos\theta_C J_\mu^{F^+}\ , \tag{A2b}$$

respectively, in the Glashow-Iliopoulos-Maiani scheme.[23] Here, $\theta_C$ is the Cabibbo angle and we have used, for simplicity, the following abbreviations:

$$J_\mu^{\pi^+}=\bar u\gamma_\mu(1+\gamma_5)d\ , \tag{A3a}$$

$$J_\mu^{K^+}=\bar u\gamma_\mu(1+\gamma_5)s\ , \tag{A3b}$$

$$J_\mu^{D^+}=\bar c\gamma_\mu(1+\gamma_5)d\ , \tag{A3c}$$

$$J_\mu^{F^+}=\bar c\gamma_\mu(1+\gamma_5)s\ . \tag{A3d}$$

The $H(0,-)$, $H(-,-)$, and $H(-,0)$ can then be expressed as

$$H(0,-)\equiv H_w(\Delta C=0,\Delta S=-1) =(G/2\sqrt{2})\sin\theta_C\cos\theta_C(J_\mu^{\pi^+}J_\mu^{K^-}-J_\mu^{D^+}J_\mu^{F^-})\ , \tag{A4a}$$

$$H(-,-)\equiv H_w(\Delta C=-1,\Delta S=-1) =(G/2\sqrt{2})\cos^2\theta_C(J^{\pi^+}J^{F^-})\ , \tag{A4b}$$

$$H(-,0)\equiv H_w(\Delta C=-1,\Delta S=0) =(G/2\sqrt{2})\sin\theta_C\cos\theta_C(-J_\mu^{\pi^+}J_\mu^{D^-}+J_\mu^{K^+}J_\mu^{F^-})\ . \tag{A4c}$$

Therefore, we can obtain commutation relations from which we can derive $SU_f(N)$ ($N=2,3,4$) rotations of the on-mass-shell asymptotic matrix elements of $H_w$'s, by using asymptotic $SU_f(N)$:

I. $SU_I(2)$ commutation relations:

$$[H(-,-),V_{\pi^+}]=0\ , \tag{A5a}$$

$$[[H_w,V_{\pi^+}],V_{\pi^+}]=0\ ,\quad [H_w=H(0,-)\text{ and }H(-,0)]\ . \tag{A5b}$$

II. $SU_f(3)$ commutation relations:

$$[H(0,-),V_{\bar K^0}]=0\ , \tag{A6a}$$

$$[[H_w,V_{K^-}],V_{K^-}]=[[H_w,V_{\pi^-}],V_{\pi^-}]\ ,\quad \{H_w=H(0,-)+[H(0,-)]^\dagger\}\ , \tag{A6b}$$

$$[[H(-,-),V_{K^0}],V_{\bar K^0}]=2H(-,-)\ , \tag{A6c}$$

$$[[H(-,0),V_{\bar K^0}],V_{K^0}]=2H(-,0)\ , \tag{A6d}$$

$$[H(-,0),V_{\bar K^0}]=2\tan\theta_C H(-,-)\ . \tag{A6e}$$

III. $SU_f(4)$ commutation relations:

$$[H(-,-),V_{F^-}]=0\ , \tag{A7a}$$

$$[[H(-,-),V_{D^0}],V_{\bar D^0}]=2H(-,-)\ , \tag{A7b}$$

$$[[H_w,V_{F^-}],V_{F^-}]=[[H_w,V_{\pi^-}],V_{\pi^-}]\ ,\quad \{H_w=H(-,-)+[H(-,-)]^\dagger\}\ , \tag{A7c}$$

$$[H(-,-),V_{D^0}]=\cot\theta_C H(0,-)\ . \tag{A7d}$$

We now examine asymptotic $SU_f(N)$ ($N=2,3,4$) rotations for the on-mass-shell matrix elements of $H_w$ which are used in the text.

### 1. Asymptotic $SU_I(2)$ relation

We, first, investigate asymptotic $SU_I(2)$ relations of the nondiagonal matrix elements, Eq. (4.3) with $H_w=H(-,-)$. Insertion of Eq. (A5a) between the states $\langle\pi^+|$ and $|\hat F_I^0\rangle$ with infinite momenta leads to

$$\langle\pi^+|H(-,-)|\hat F_I^+\rangle+\langle\pi^0|H(-,-)|\hat F_I^0\rangle=0\ . \tag{A8}$$

We have no other useful $SU_I(2)$ relations of the nondiagonal matrix elements of Eq. (4.3) with $H_w=H(-,-)$.

*2. Asymptotic $SU_f(3)$ relations*

In order to get asymptotic $SU_f(3)$ relations of the ground-state-meson matrix elements of $H_w = H(-,0)$, we insert Eq. (A6d) between the states $\langle \pi^+ |$ and $| D^+(D^{*+}) \rangle$ with infinite momenta. Using the $SU_f(4)$ parametrization of the vector charges $V_{K^0}$ and $V_{\bar{K}^0}$, we have

$$\langle \pi^+ | H(-,0) | D^+(D^{*+}) \rangle + \langle K^+ | H(-,0) | F^+(F^{*+}) \rangle = 0 \ . \quad \text{(A9)}$$

Sandwiching Eq. (A6e) between (a) $\langle \pi^+ |$ and $| F^+ \rangle$ and (b) $\langle \bar{K}^0 |$ and $| D^0 \rangle$ with infinite momenta, we are led to

$$\langle \pi^+ | H(-,0) | D^+ \rangle - \langle K^+ | H(-,0) | F^+ \rangle = -2\tan\theta_C \langle \pi^+ | H(-,-) | F^+ \rangle \quad \text{(A10a)}$$

and

$$\sqrt{1/2}\{ \langle \pi^0 | H(-,0) | D^0 \rangle - \langle \eta_0 | H(-,0) | D^0 \rangle + \sqrt{2} \langle \eta_s | H(-,0) | D^0 \rangle \} = 2\tan\theta_C \langle \bar{K}^0 | H(-,-) | D^0 \rangle \ , \quad \text{(A10b)}$$

where we again used $SU_f(4)$ Clebsch-Gordan coefficients *only* in the asymptotic limit. With the help of Eq. (A9) and the charm counterpart of the asymptotic $|\Delta \mathbf{I}| = \frac{1}{2}$ rule, Eq. (2.8) in the text, we obtain, from Eq. (A10a),

$$\langle \pi^+ | H(-,0) | D^+ \rangle = \tan\theta_C \langle \bar{K}^0 | H(-,-) | D^0 \rangle \ . \quad \text{(A11)}$$

Substituting Eq. (A11) into Eq. (A10b) and using Eq. (3.3a) of the text, we obtain

$$\sqrt{2} \langle K^+ | H(-,0) | F^+ \rangle = -\sqrt{2} \langle \pi^+ | H(-,0) | D^+ \rangle = \langle \pi^0 | H(-,0) | D^0 \rangle + \langle \eta_0 | H(-,0) | D^0 \rangle - \sqrt{2} \langle \eta_s | H(-,0) | D^0 \rangle \ . \quad \text{(A12)}$$

Next, we study asymptotic $SU_f(3)$ relations for the nondiagonal matrix elements of Eq. (4.3) with $H_w = H(-,0)$. Here we show typical asymptotic $SU_f(3)$ relations which are used in the text. We insert the commutation relation, Eq. (A6e), between the states $\langle \pi^+ |$ and $| \hat{F}^+ \rangle$ and use Eq. (4.23). Then we have

$$\sqrt{2} \langle K^+ | H(-,0) | \hat{F}^+ \rangle - \langle \pi^+ | H(-,0) | \hat{D}^+ \rangle - \langle \pi^+ | H(-,0) | \hat{D}_s^+ \rangle = 0 \ . \quad \text{(A13)}$$

Insertion of Eq. (A6e) between $\langle \pi^+ |$ and $| \hat{F}_I^+ \rangle$ leads to

$$\langle \pi^+ | H(-,0) | \hat{D}_s^+ \rangle - \langle \pi^+ | H(-,0) | \hat{D}^+ \rangle + \sqrt{2} \langle K^+ | H(-,0) | \hat{F}_I^+ \rangle = 2\sqrt{2}\tan\theta_C \langle \pi^+ | H(-,-) | \hat{F}_I^+ \rangle \ . \quad \text{(A14)}$$

Inserting the commutation relation, Eq. (A6d), for example, between $\langle \hat{\kappa}^+ |$ and $| F^+ \rangle$, $\langle K^+ |$ and $| \hat{F}^+ \rangle$, and $\langle K^+ |$ and $| \hat{F}_I^+ \rangle$, and using the asymptotic $SU_f(4)$ parametrization for the matrix elements of vector charges $V_{K^0}$ and $V_{\bar{K}^0}$, we obtain

$$\langle \hat{\kappa}^+ | H(-,0) | F^+ \rangle + \langle \hat{\delta}_s^+ | H(-,0) | D^+ \rangle = 0 \ , \quad \text{(A15)}$$

$$\sqrt{2} \langle K^+ | H(-,0) | \hat{F}^+ \rangle + \langle \pi^+ | H(-,0) | \hat{D}^+ \rangle + \langle \pi^+ | H(-,0) | \hat{D}_s^+ \rangle = 0 \ , \quad \text{(A16)}$$

$$\sqrt{2} \langle K^+ | H(-,0) | \hat{F}_I^+ \rangle + \langle \pi^+ | H(-,0) | \hat{D}^+ \rangle - \langle \pi^+ | H(-,0) | \hat{D}_s^+ \rangle = 0 \ . \quad \text{(A17)}$$

Substitution of Eq. (A13) into Eq. (A16) leads to a useful result

$$\langle K^+ | H(-,0) | \hat{F}^+ \rangle = 0 \ . \quad \text{(A18)}$$

From Eqs. (A16), (A17), and (A18), we obtain

$$\langle K^+ | H(-,0) | \hat{F}_I^+ \rangle + \sqrt{2} \langle \pi^+ | H(-,0) | \hat{D}^+ \rangle = 0 \quad \text{(A19)}$$

and

$$\langle \pi^+ | H(-,0) | \hat{D}^+ \rangle + \langle \pi^+ | H(-,0) | \hat{D}_s^+ \rangle = 0 \ . \quad \text{(A20)}$$

*3. Asymptotic $SU_f(4)$ relations*

In order to obtain asymptotic $SU_f(4)$ relations among the nondiagonal matrix elements, Eq. (4.3) with $H_w = H(-,-)$ and $H(0,-)$ in the text we insert commutation relation, Eq. (A7d), between the states $\langle \pi^+ |$ and $| \hat{\kappa}^+ \rangle$. The result is

$$\langle \pi^+ | H(-,-) | \hat{F}^+ \rangle - \langle \pi^+ | H(-,-) | \hat{F}_I^+ \rangle = \sqrt{2}\cot\theta_C \langle \pi^+ | H(0,-) | \hat{\kappa}^+ \rangle \ . \quad \text{(A21)}$$

Substituting Eq. (4.23) into the LHS of Eq. (A21) we get

$$\langle \pi^+ | H(-,-) | \hat{F}_I^+ \rangle = -\sqrt{2}\cot\theta_C \langle \pi^+ | H(0,-) | \hat{\kappa}^+ \rangle \ . \quad \text{(A22)}$$

Insertions of the commutation relation, Eq. (A7c), between the pairs of external states $\langle \hat{\kappa}^- |$ and $| D^+ \rangle$ and $\langle \pi^- |$ and $| \hat{F}^+ \rangle$ lead us to

$$\langle \bar{K}^0 | H(-,-) | \hat{D}^0 \rangle = \langle \hat{\bar{\kappa}}^0 | H(-,-) | D^0 \rangle \quad \text{(A23)}$$

and

$$\langle \pi^+ | H(-,-) | \hat{F}^+ \rangle = 0 \ , \quad \text{(A24)}$$

respectively, which are compatible with the result from the level realization of single commutator given in the text [see Eqs. (4.22) and (4.23)].

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[1]I. Bigi and M. Fukugita, Phys. Lett. **91B**, 121 (1980).
[2]N. Cabibbo and L. Maiani, Phys. Lett. **73B**, 418 (1980).
[3]ARGUS Collaboration, H. Albrecht *et al.*, Phys. Lett. **158B**, 525 (1985); Mark III Collaboration, R. Baltrusaitis *et al.*, Phys. Rev. Lett. **55**, 150 (1985); CLEO Collaboration, C. Bebek *et al.*, *ibid.* **56**, 1893 (1986).
[4]R. H. Schindler, in *Proceedings of the XXII International Conference on High Energy Physics,* Leipzig, 1984, edited by A. Meyer and E. Wieczorek (Akademie der Wissenschaften der DDR, Institute für Hochenergiephysik, Zeuthen, Berlin, 1984), p. 171; D. Hitlin, Caltech Report No. CALT-68-1230, 1985 (unpublished).
[5]R. H. Schindler *et al.*, Phys. Rev. D **24**, 78 (1981); R. M. Baltrusaitis *et al.*, Phys. Rev. Lett. **55**, 150 (1985). For the recent review on the experiments and their implications on the conventional theoretical ideas, see, for example, D. Hitlin, Report No. CALT-68-1420, 1987 (unpublished).
[6]See, however, A. I. Sanda, Phys. Rev. D **22**, 2814 (1980).
[7]T. Tanuma, S. Oneda, and K. Terasaki, Phys. Rev. D **29**, 444 (1984); K. Terasaki, S. Oneda, and T. Tanuma, *ibid.* **29**, 456 (1984).
[8]K. Terasaki and S. Oneda, Phys. Rev. Lett. **48**, 1715 (1982); K. Terasaki and S. Oneda, Phys. Rev. D **29**, 466 (1984).
[9]K. Terasaki and S. Oneda, Phys. Rev. D **34**, 2778 (1986).
[10]S. Oneda and K. Terasaki, Prog. Theor. Phys. Suppl. No. **82**, 1 (1985).
[11]K. Terasaki, Nuovo Cimento **92**, 384 (1986).
[12]T. Tsuru, in *Proceedings of the Second International Conference on Hadron Spectroscopy,* KEK, Tsukuba, Japan, 1987, edited by Y. Oyanagi, K. Takamatsu, and T. Tsuru (KEK, Tsukuba, 1987), p. 138, and the references quoted therein.
[13]K. F. Liu, in *Proceedings of the Second International Conference on Hadron Spectroscopy* (Ref. 12), p. 138, and the references quoted therein.
[14]R. J. Jaffe, Phys. Rev. D **15**, 267 (1977); **15**, 281 (1977).
[15]J. Weinstein and N. Isgur, Phys. Rev. D **27**, 588 (1983); B. A. Li and K. F. Liu, *ibid.* **30**, 613 (1984).
[16]D. C. Peaslee, Surv. High Energy Phys. **4**, 213 (1984); H. J. Schnitzer, in *Hadron Spectroscopy—1985,* College Park, Maryland, edited by S. Oneda (AIP Conf. Proc. No. 132) (AIP, New York, 1985), p. 404.
[17]T. Teshima and S. Oneda, Phys. Rev. D **33**, 1974 (1986); see also C. A. Dominguez and N. Paver, Report No. DESY 87-132, 1987 (unpublished).
[18]D. Bridges *et al.*, Phys. Rev. Lett. **56**, 211 (1986); **56**, 215 (1986).
[19]K. F. Liu, Report No. ITP-SB-86-48, 1986 (unpublished).
[20]Particle Data Group, Phys. Lett. **170B**, 1 (1986).
[21] $|\langle \pi^- | A_{\pi^-} | \rho^0 \rangle|$ is estimated from $\Gamma(\rho \to 2\pi)_{\rm expt} \simeq 160$ MeV by using PCAC (partial conservation of axial-vector current).
[22]Y. Koide and S. Oneda, Phys. Rev. D **36**, 2876 (1987).
[23]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285 (1970).