New perturbative calculation of the fermion-boson mass ratio in a supersymmetric quantum field theory

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We apply a recently proposed graphical perturbative calculational scheme to a two-dimensional supersymmetric model. Elsewhere, we reported the results of a second-order calculation of the ground-state energy density E in this model with the result that $E=0$. Here, we calculate the boson-fermion mass ratio R to second order in this new perturbation theory. As in the previous calculation, there is a detailed cancellation between classes of graphs which leads to the unbroken supersymmetric result $R=1$.

A new *artificial* perturbative technique which may be used to solve quantum field theories has been proposed in a series of recent papers.[1-3] This technique involves replacing an interaction term such as $g\phi^4$ with $g(\phi^2)^{1+\delta}$, where δ is to be regarded as a small perturbation parameter. We believe that the resulting series in δ has a finite radius of convergence. Because we are not forcing a physical parameter to play the role of an expansion parameter, we are able to uncover nontrivial dependence on coupling constants and masses. In Refs. 1 and 2 we described how the δ series may be computed by a well-defined graphical method. The Green's functions computed in this manner are dramatically less divergent than the corresponding weak-coupling Green's functions.

In Ref. 3 we applied this technique to the two-dimensional supersymmetric Lagrangian

$$\mathcal{L}=\tfrac{1}{2}(\partial\phi)^2+\frac{i}{2}\bar{\psi}\partial\!\!\!/\psi-\tfrac{1}{2}g(1+\delta)(\phi^2)^{\delta/2}\bar{\psi}\psi+\tfrac{1}{2}g^2(\phi^2)^{1+\delta}\ . \tag{1}$$

We showed that this model possesses unbroken supersymmetry invariance by computing the ground-state energy density through second order in powers of δ; the result of a beautifully nontrivial cancellation between infinite classes of graphs was $E=0$. Here we wish to describe a closely related calculation, that of the ratio R of boson to fermion masses in this model. This calculation is of some interest because now the graphs carry momentum, and wave function as well as mass renormalization is present.

It is illustrative to carry out the calculation to order δ first. To do this, we introduce the provisional Lagrangian

$$\tilde{\mathcal{L}}=\tilde{\mathcal{L}}_0+\tilde{\mathcal{L}}_\alpha\ , \tag{2a}$$

$$\tilde{\mathcal{L}}_0=\tfrac{1}{2}(\partial\phi)^2+\frac{i}{2}\bar{\psi}\partial\!\!\!/\psi+\tfrac{1}{2}g^2\phi^2-\tfrac{1}{2}g(1+\delta)\bar{\psi}\psi\ , \tag{2b}$$

$$\tilde{\mathcal{L}}_\alpha=\tfrac{1}{2}g^2\delta(\phi^2)^{1+\alpha}-\tfrac{1}{4}g\delta(\phi^2)^\alpha\bar{\psi}\psi\ . \tag{2c}$$

We note that the interaction terms in $\tilde{\mathcal{L}}_\alpha$ are proportional to δ so that the corresponding Green's functions $\tilde{G}(\alpha)$ may be calculated in order δ using conventional weak-coupling Feynman rules. We obtain the Green's function corresponding to the original Lagrangian (1) by the rule

$$G_{\text{order }\delta}=\tilde{G}_0+\frac{\partial}{\partial\alpha}\tilde{G}(\alpha)\bigg|_{\alpha=0}\ . \tag{3}$$

Here $\tilde{G}_0$ is the Green's function calculated from the order-δ term in $\tilde{\mathcal{L}}_0$.

The Feynman rules corresponding to (2) are given in Fig. 1. The two diagrams which contribute to the boson mass are shown in Fig. 2(a) and give

$$\tilde{G}^{2,b}(\alpha)=-\frac{g^2\delta}{\sqrt{\pi}}(2I)^\alpha\Gamma(\alpha+\tfrac{1}{2})(1+2\alpha+2\alpha^2)\ , \tag{4}$$

where I is the logarithmically divergent integral

$$I=\frac{1}{(2\pi)^2}\int\frac{d^2p}{p^2+g^2}\ . \tag{5}$$

The boson mass is then obtained from the recipe in (3):

$$m_b^2=g^2+g^2\delta[\psi(\tfrac{3}{2})+\ln(2I)]+O(\delta^2)\ . \tag{6}$$

The two diagrams contributing to the fermion mass are shown in Fig. 2(b); the second contributes to $\tilde{G}^{2,f}(\alpha)$,

$$\tilde{G}^{2,f}(\alpha)=\frac{g\delta}{2\sqrt{\pi}}(2I)^\alpha\Gamma(\alpha+\tfrac{1}{2})\ , \tag{7a}$$

while the first gives

$$\tilde{G}_0^{2,f}=g\delta\ . \tag{7b}$$

Equation (3) then gives the fermion mass

$$m_f=g+\frac{g\delta}{2}[\psi(\tfrac{3}{2})+\ln(2I)]+O(\delta^2)\ . \tag{8}$$

The boson and fermion masses, (6) and (8), are identical through order δ.

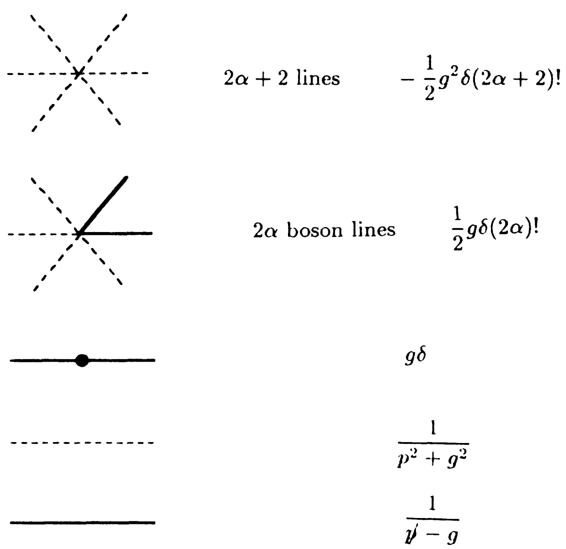

FIG. 1. Feynman rules for the Lagrangian $\tilde{\mathcal{L}}$ in (2). These rules determine the Green's functions to order δ.

The order-δ^2 calculation is considerably more complex. We use the second-order provisional Lagrangian $\tilde{\mathcal{L}}=\tilde{\mathcal{L}}_0+\tilde{\mathcal{L}}_{\alpha,\beta}$, where $\tilde{\mathcal{L}}_0$ is still given by (2b), but now

$$\tilde{\mathcal{L}}_{\alpha,\beta}=\tfrac{1}{2}g^2(\delta+\delta^2)(\phi^2)^{1+\alpha}+\tfrac{1}{2}g^2(\delta^2-\delta)(\phi^2)^{1+\beta}$$
$$-\tfrac{1}{8}g(2\delta+3\delta^2)(\phi^2)^{\alpha}\bar{\psi}\psi+\tfrac{1}{8}g(2\delta+\delta^2)(\phi^2)^{\beta}\bar{\psi}\psi\ . \tag{9}$$

The corresponding Feynman rules are given in Fig. 3. The Euclidean-space boson and fermion propagators are

$$\Delta(x)=\frac{1}{(2\pi)^2}\int\frac{d^2p}{p^2+g^2}e^{ip\cdot x}\ , \tag{10a}$$

$$\Delta_f(x)=\frac{1}{(2\pi)^2}\int\frac{d^2p}{\not p-g}e^{ip\cdot x}=(i\not\partial-g)\Delta(x)\ . \tag{10b}$$

There are 13 types of graphs which contribute to the boson two-point function to second order in δ. (This count does not include α,β interchanges.) These graphs are shown in Fig. 4. When all these graphs are calculated, we apply the differential operator

$$D=\frac{1}{2}\left[\frac{\partial}{\partial\alpha}-\frac{\partial}{\partial\beta}\right]+\frac{1}{4}\left[\frac{\partial^2}{\partial\alpha^2}+\frac{\partial^2}{\partial\beta^2}\right]\ , \tag{11}$$

and evaluate the result at $\alpha=\beta=0$. The contribution of each graph to $G^{2,b}(p)$ is as follows:

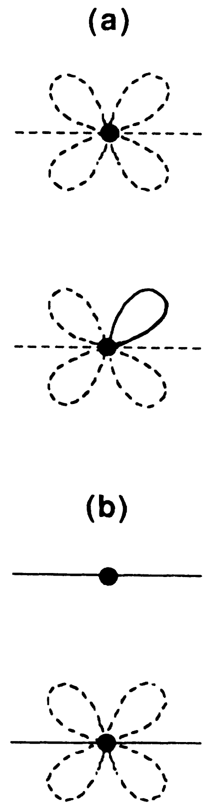

FIG. 2. Diagrams contributing to the (a) boson and (b) fermion two-point function to order δ.

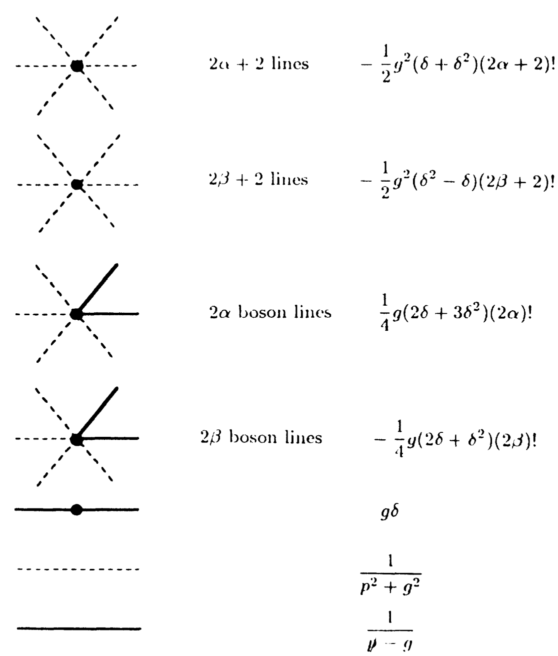

FIG. 3. Feynman rules for the Lagrangian $\tilde{\mathcal{L}}$ in (9). These rules determine the Green's functions to order δ^2.

$$-g^2\delta[1+\psi(\tfrac{3}{2})+\ln 2I]-\frac{g^2\delta^2}{2}\{[\psi(\tfrac{3}{2})+\ln 2I][\psi(\tfrac{3}{2})+\ln 2I+2]+\psi'(\tfrac{3}{2})\}\ , \tag{12a}$$

$$g^2\delta+\frac{g^2\delta^2}{2}[\psi(\tfrac{3}{2})+\ln 2I]\ , \tag{12b}$$

$$-g^2\delta^2\left[-1+2g^2I\int\frac{\Delta^2(x)}{I^2}d^2x\right]\ , \tag{12c}$$

$$-\frac{g^4\delta^2\sqrt{\pi}}{2}\sum_{l=2}^{\infty}\frac{(l-2)!}{l\Gamma(l+\frac{1}{2})}I\int\frac{\Delta^{2l}(x)}{I^{2l}}d^2x+2g^4\delta^2[1+\psi(\tfrac{3}{2})+\ln 2I]I\int\frac{\Delta^2(x)}{I^2}d^2x -g^4\delta^2[1+\psi(\tfrac{3}{2})+\ln 2I]I\int\frac{\Delta^2(x)}{I^2}d^2x\ , \tag{12d}$$

$$-\frac{g^4\delta^2\sqrt{\pi}}{2}\sum_{l=2}^{\infty}\frac{(l-2)!}{\Gamma(l+\frac{1}{2})}I\int\frac{\Delta^{2l}(x)}{I^{2l}}d^2x+2g^4\delta^2[-1+\psi(\tfrac{3}{2})+\ln 2I]\int\frac{\Delta^2(x)}{I^2}d^2x +g^4\delta^2[3-\psi(\tfrac{3}{2})-\ln 2I]I\int\frac{\Delta^2(x)}{I^2}d^2x\ , \tag{12e}$$

$$-\frac{g^4\delta^2\sqrt{\pi}}{2}\sum_{l=1}^{\infty}\frac{(l-1)!}{l\Gamma(l+\frac{1}{2})}I\int\frac{\Delta^{2l}(x)}{I^{2l}}d^2x\ , \tag{12f}$$

$$-\frac{g^4\delta^2\sqrt{\pi}}{2}\sum_{l=1}^{\infty}\frac{(l-1)!}{\Gamma(l+\frac{1}{2})}I\int\frac{\Delta^{2l}(x)}{I^{2l}}d^2x\ , \tag{12g}$$

$$\frac{g^2\delta^2\sqrt{\pi}}{4}\sum_{l=2}^{\infty}\left[-\frac{(l-2)!}{\Gamma(l+\frac{1}{2})}+2g^2\frac{l(l-2)!}{\Gamma(l+\frac{1}{2})}I\int\frac{\Delta^{2l}(x)}{I^{2l}}d^2x\right]+g^2\delta^2[\psi(\tfrac{3}{2})+\ln 2I-2]\left[1-2g^2I\int\frac{\Delta^2(x)}{I^2}d^2x\right] -\frac{g^2\delta^2}{2}[\psi(\tfrac{3}{2})+\ln 2I-2]\left[1-2g^2I\int\frac{\Delta^2(x)}{I^2}d^2x\right]\ , \tag{12h}$$

$$\frac{g^4\delta^2\sqrt{\pi}}{2}\sum_{l=1}^{\infty}\frac{(l-1)!}{l\Gamma(l+\frac{3}{2})}I\int\frac{\Delta^{2l+1}(x)}{I^{2l+1}}e^{-i\mathbf{p}\cdot\mathbf{x}}d^2x\ , \tag{12i}$$

$$g^4\delta^2\sqrt{\pi}\sum_{l=1}^{\infty}\frac{(l-1)!}{\Gamma(l+\frac{3}{2})}I\int\frac{\Delta^{2l+1}(x)}{I^{2l+1}}e^{-i\mathbf{p}\cdot\mathbf{x}}d^2x\ , \tag{12j}$$

$$\frac{g^4\delta^2\sqrt{\pi}}{2}\sum_{l=1}^{\infty}\frac{l!}{\Gamma(l+\frac{3}{2})}I\int\frac{\Delta^{2l+1}(x)}{I^{2l+1}}e^{-i\mathbf{p}\cdot\mathbf{x}}d^2x\ , \tag{12k}$$

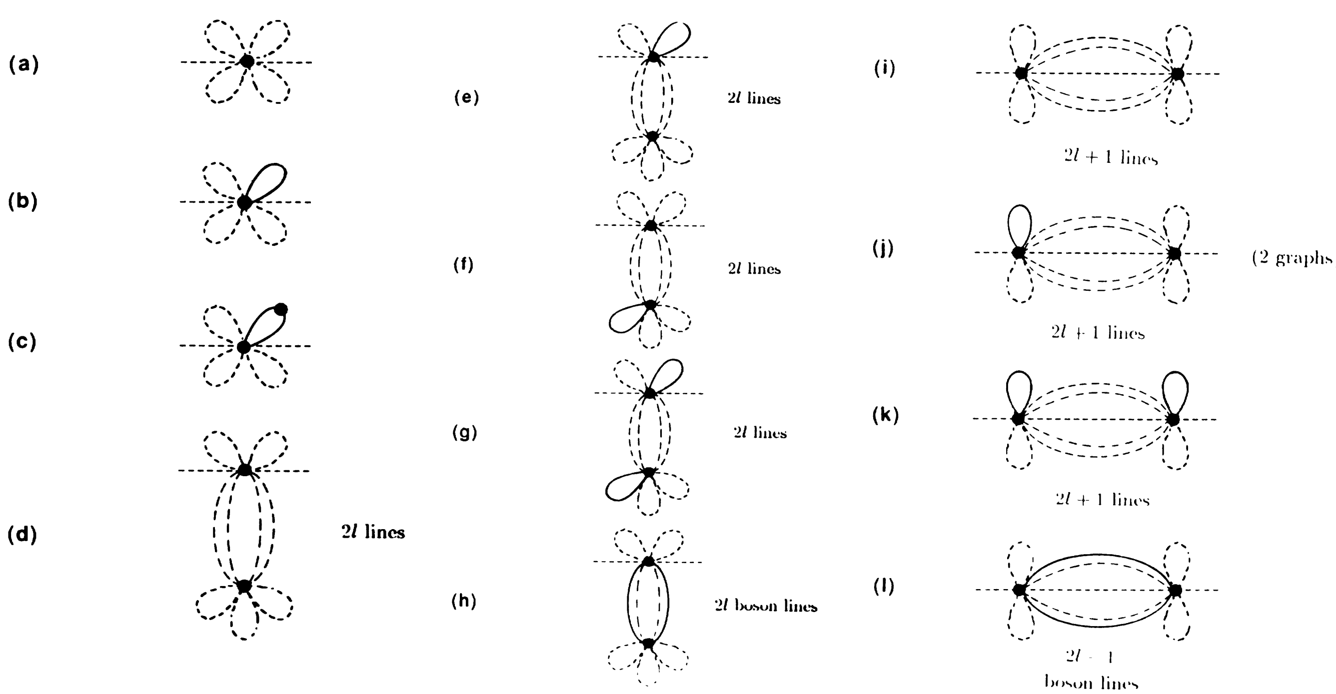

FIG. 4. Classes of graphs which contribute to the boson two-point function to order δ^2.

$$-\frac{g^2\delta^2\sqrt{\pi}}{4}\sum_{l=1}^{\infty}\frac{(l-1)!}{\Gamma(l+\frac{1}{2})}\left[-\frac{1}{l}+g^2\frac{2l+1}{l}I\int\frac{\Delta^{2l+1}(x)}{I^{2l+1}}e^{-ip\cdot x}d^2x+\frac{p^2}{l(2l+1)}I\int\frac{\Delta^{2l+1}(x)}{I^{2l+1}}e^{-ip\cdot x}d^2x\right]. \tag{12l}$$

The evaluation of this last diagram, which involves a fermion loop connecting the two vertices, made use of the identity (a generalization of one given in Ref. 3)

$$\begin{aligned}\mathrm{Tr}\int d^2x\,\Delta^n(x)\Delta_f(x)\bar{\Delta}_f(x)e^{-ip\cdot x}&=-\frac{2}{n+1}I^{n+1}+2g^2\frac{n+2}{n+1}\int\Delta^{n+2}(x)e^{-ip\cdot x}d^2x\\&\quad+\frac{2p^2}{(n+1)(n+2)}\int\Delta^{n+2}(x)e^{-ip\cdot x}d^2x\,.\end{aligned}\tag{13}$$

There are five classes of diagrams that contribute to the fermion two-point function. There are shown in Fig. 5, and result in the following contributions to $G^{2,f}(p)$:

$$g\delta\,, \tag{14a}$$

$$\frac{g\delta}{2}[\psi(\tfrac{3}{2})+\ln 2I-2]+\frac{g\delta^2}{8}\{[\psi(\tfrac{3}{2})+\ln 2I-2][\psi(\tfrac{3}{2})+\ln 2I+2]+\psi'(\tfrac{3}{2})+4\}\,, \tag{14b}$$

$$\begin{aligned}&\frac{g^3\delta^2\sqrt{\pi}}{4}\sum_{l=2}^{\infty}\frac{(l-2)!}{l\Gamma(l+\frac{1}{2})}I\int\frac{\Delta^{2l}(x)}{I^{2l}}d^2x-\frac{g^3\delta^2}{2}[2\psi(\tfrac{3}{2})+2\ln 2I-1]I\int\frac{\Delta^2(x)}{I^2}d^2x\\&\qquad+\frac{g^3\delta^2}{2}[\psi(\tfrac{3}{2})+\ln 2I-2]I\int\frac{\Delta^2(x)}{I^2}d^2x\,,\end{aligned}\tag{14c}$$

$$\frac{g^3\delta^2\sqrt{\pi}}{4}\sum_{l=1}^{\infty}\frac{(l-1)!}{l\Gamma(l+\frac{1}{2})}I\int\frac{\Delta^{2l}(x)}{I^{2l}}d^2x\,, \tag{14d}$$

$$-\frac{g^3\delta^2\sqrt{\pi}}{4}\sum_{l=1}^{\infty}\frac{(l-1)!}{l\Gamma(l+\frac{1}{2})}I\int\frac{\Delta^{2l+1}(x)}{I^{2l+1}}e^{-ip\cdot x}d^2x-\not{p}\frac{g^2\delta^2\sqrt{\pi}}{4}\sum_{l=1}^{\infty}\frac{(l-1)!}{l(2l+1)\Gamma(l+\frac{1}{2})}I\int\frac{\Delta^{2l+1}(x)}{I^{2l+1}}e^{-ip\cdot x}d^2x\,. \tag{14e}$$

Let us now write

$$G^{2,b}(p)=A(p^2)p^2+B(p^2) \tag{15a}$$

and similarly

$$G^{2,f}(p^2)=a(p^2)\not{p}+b(p^2)\,, \tag{15b}$$

where the explicit p^2 and $\not{p}$ factors appear in (12l) and (14e), respectively. Then, the mass squared of the boson is given by the zero of

$$p^2+\frac{g^2-B(p^2)}{1-A(p^2)}\approx p^2+g^2-B(p^2)+g^2A(p^2)\,, \tag{16}$$

because, from (12l), $A(p^2)=O(\delta^2)$. Similarly, the fermion mass squared is given by the zero of

$$\begin{aligned}p^2+\frac{[g+b(p^2)]^2}{[1-a(p^2)]^2}&\approx p^2+g^2+2gb(p^2)\\&\quad+b(p^2)^2+2g^2a(p^2)\,.\end{aligned}\tag{17}$$

The comparison of boson and fermion mass operators is now very simple. The contribution to (17) involving

$$I\int\frac{\Delta^{2l+1}(x)}{I^{2l+1}}e^{-ip\cdot x}d^2x$$

from (14e) is seen to be precisely the same as the similar contribution to (16) of (12i), (12j), (12k), and (12l). Likewise, the contribution to (17) involving

$$I\int\frac{\Delta^{2l}(x)}{I^{2l}}d^2x\,,\quad l\geq 2\,,$$

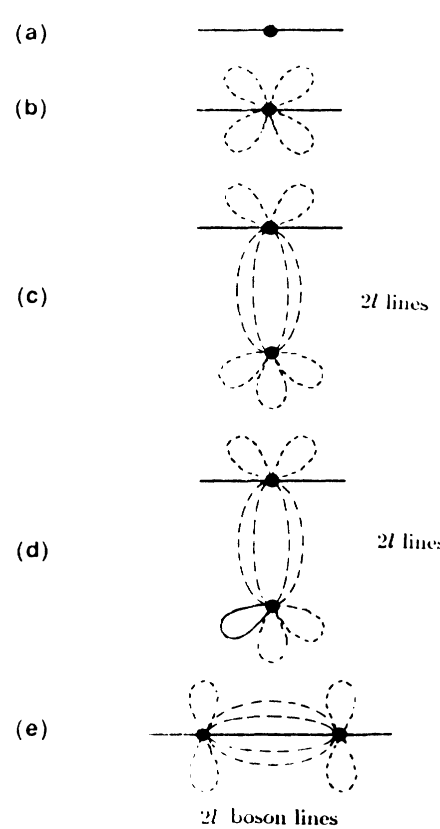

FIG. 5. Classes of graphs which contribute to the fermion two-point function to order δ^2. Note that graph (a) contributes through $\tilde{G}_0^{2,f}$ in (3).

from (14c) and (14d) is identical to the corresponding contribution to (16) from (12d), (12e), (12f), (12g), and (12h). The contributions to (17) involving

$$I\int \frac{\Delta^2(x)}{I^2}d^2x$$

from (14c) and (14d) are the same as those from (12c), (12d), (12e), (12f), (12g), and (12h) to (16). Finally, the remaining numerical contributions from (14a) and (14b) are just the same as those arising from (12a), (12b), (12c), (12h), and (12l) when we recognize the identity[3]

$$\sum_{l=2}^{\infty} \frac{(l-2)!}{(l-1)\Gamma(l+\frac{1}{2})} = \frac{2}{\sqrt{\pi}}\psi'(\tfrac{3}{2}) . \tag{18}$$

Thus we have established the supersymmetric result

$$m_f = m_b \tag{19}$$

through order δ^2.

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