Derivation of chiral anomalies and commutator anomalies in a fixed-time regularization method

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A gauge-covariant regularization method is introduced to evaluate current-current and current–electric-field commutator anomalies in chiral gauge theories in two- and four-dimensional spacetime within the Hamiltonian formalism. The results are consistent with those obtained by the Bjorken-Johnson-Low method. Provided that the electric fields commute with each other, a covariant expression is found for the commutator anomaly of the Gauss-law operator. The regularized current is shown to have the covariant anomaly in its divergence due to its noncommutativity with the electric field.

I. INTRODUCTION

During the past several years, the mathematical structure of chiral anomalies has been unveiled.[1,2] Among others, Faddeev's suggestion[2] that gauge symmetry is realized as a projective representation in chiral gauge theories raised hope that chiral gauge theories might be consistently quantized. Much effort has been made along this line.[3,4] Faddeev's suggestion is based on the cohomological argument, and he conjectured that a nontrivial two-cocycle appears as an extra term in the commutator of the Gauss-law operator in chiral gauge theories. From now on we call this term Faddeev's commutator anomaly. This commutator anomaly was confirmed by Jo and by Kobayashi and Sugamoto and one of us (K.S.)[5-7] through the Bjorken-Johnson-Low (BJL) method,[8] which derives the equal-time commutator from the time-ordered product of operators. Meanwhile Jo[9] evaluated the commutator anomaly of the Gauss-law operator by the generalized point-splitting method proposed by Faddeev and Shatashvili.[4] It belongs to the fixed-time method. He obtained a result in $1+3$ dimensions which is not consistent with the one obtained by the BJL method, and he concluded that the BJL method is, at least so far, the only way to verify Faddeev's conjecture.

On the other hand, some people gave formal arguments in the Hamiltonian formalism to connect Faddeev's commutator anomaly with the Berry phase of the fermionic vacuum.[10,11] In $1+3$ dimensions only the work by Niemi and Semenoff[10] evaluates the commutator anomaly within the Hamiltonian formalism and they claim that their result coincides with Faddeev's conjecture. However, their argument is not sufficient, at least, for the following reason: They evaluated only the lowest-order term, the linear term with respect to the gauge field, supposing that higher-order terms, the quadratic and cubic terms with respect to the gauge field, are cohomologically trivial. However, it is known that there exists the cohomologically nontrivial expression consisting of quadratic and cubic terms.[6,7] Therefore, without explicit calculations, we cannot exclude the possibility of the appearance of such cohomologically nontrivial terms. Furthermore, it seems difficult to compute these terms in their formalism.

Towards the settlement of this problem, we proposed a new fixed-time regularization method which respects the gauge symmetry as far as possible, and evaluated the commutator anomalies explicitly within the Hamiltonian formalism.[12]

The advantage of our regularization method is to enable us to avoid the ambiguities in the point-splitting method, which come from the procedure of averaging over the relative coordinate of point-split fields. If we take the commutator of two point-split currents, the canonical term to be separated from the anomalous terms cannot be uniquely identified due to the independent averaging of two currents. In our method any averaging procedure is avoided and once the currents are regularized, the product of any number of currents is also well defined. Furthermore, we can separate the canonical term uniquely from the anomalous terms in the commutator.

Our results on the current-current and the current–electric-field commutator anomalies derived in a fixed-time approach are consistent with those obtained by the BJL method (a spacetime approach). As for the commutator anomaly among the electric fields, we have not yet derived it from first principles. Therefore, the task to confirm Faddeev's commutator anomaly within the Hamiltonian formalism is left for future study.

In this paper we explain the details of the calculations to obtain the final expressions given in a previous paper.[12] In Sec. II we present a gauge-covariant regularization method, and give formal expressions of the current-current and the current–electric-field commutator anomalies. It is shown that they transform covariantly under the gauge transformation. In Sec. III the commutator anomalies are perturbatively evaluated and the results of computation are compared with those obtained by the BJL method. In Sec. IV the chiral anomaly of the current divergence is derived from the commutator anomalies obtained in the previous section. The commutator anomaly of the Gauss-law operator is presented in our covariant regularization scheme, where the electric fields are assumed to commute with each other. With the same assumption, the time derivative of the Gauss-law operator is related to the chiral anomaly. Section V is devoted to conclusions and discussion.

II. CURRENT COMMUTATOR ANOMALIES IN A GAUGE-COVARIANT REGULARIZATION METHOD

We study chiral gauge theories in two- and four-dimensional spacetime, which are described by the following Hamiltonian:

$$\mathcal{H}=\int d^Dx\left[-\tfrac{1}{2}E_i^aE^{ia}+\tfrac{1}{4}F_{ij}^aF^{ij,a}+\bar{\psi}\gamma^k\frac{1+\gamma_5}{2}(-i)(\partial_k+gA_k^at^a)\psi\right], \qquad (2.1)$$

where D is the dimension of space ($D=1$ or 3). The gauge field is treated not as an external field but as a quantum field, and the Weyl gauge ($A_0^a=0$) is taken. In this gauge the electric field is the canonical momentum conjugate to the gauge field, and they satisfy the following canonical commutation relation:

$$[E_i^a(x),A_j^b(y)]=-ig_{ij}\delta_{ab}\delta(x-y)\ . \qquad (2.2)$$

Latin indices from the middle of the alphabet, i, j, k, and so on, are used to specify the space components, while greek indices from the middle of the alphabet, λ, μ, ν, and so on, stand for both the space and the time components. We follow the convention of Bjorken and Drell[13] for the γ matrices and the antisymmetric tensor $\epsilon_{\mu\nu\lambda\rho}$:

$$\mathrm{Tr}(\gamma_5\gamma_0\gamma_1)=-2\epsilon_{01}=-2\ , \qquad (2.3a)$$

$$\mathrm{Tr}(\gamma_5\gamma_0\gamma_1\gamma_2\gamma_3)=4i\epsilon_{0123}=4i\ . \qquad (2.3b)$$

t^a's are anti-Hermitian generator matrices and satisfy $\mathrm{tr}(t^at^b)=-\frac{1}{2}\delta_{ab}$, and $[t^a,t^b]=f_{abc}t^c$. Sometimes the following notation is used in order to simplify mathematical expressions: $A_k=gA_k^at^a$, $E_k=gE_k^at^a$, and $F_{ij}=gF_{ij}^at^a$.

We take the Schrödinger picture and the representation where the gauge field is diagonalized. For each configuration of the gauge field, we consider the eigenvalue problem of a Hermitian operator

$$H(A)=-i\gamma^0\gamma^k\frac{1+\gamma_5}{2}(\partial+A)_k\ ,$$

and denote an eigenvalue and the corresponding eigenfunction by E_n and ϕ_n, respectively. If the fermion field is expanded by this set of eigenfunctions as $\psi=\sum_n\alpha_n\phi_n$, the operators α's satisfy the following canonical anticommutation relations:

$$\{\alpha_n,\alpha_m^\dagger\}=\delta_{nm}\ . \qquad (2.4)$$

The fermion part of the Hamiltonian for a fixed gauge field configuration is formally diagonalized as

$$\mathcal{H}_{\text{fermion}}=\sum_n E_n\alpha_n^\dagger\alpha_n\ . \qquad (2.5)$$

α_n for a positive (negative) E_n is interpreted as an annihilation (creation) operator of a dressed fermion in the presence of a background gauge field.

Since the product of two fermion fields at the same point is not well defined, we regularize any bilinear operator $\psi^\dagger(x)O(x)\psi(x)$ as follows:

$$[\psi^\dagger(x)O(x)\psi(x)]_{\text{reg}}=\sum_{n,m}\alpha_n^\dagger\phi_n^\dagger(x)e^{-(\epsilon/2)E_n^2}O(x)e^{-(\epsilon/2)E_m^2}\phi_m(x)\alpha_m\ , \qquad (2.6)$$

where $O(x)$ is an arbitrary matrix-valued differential operator. The infinitesimal parameter ϵ has a dimension of $(\text{mass})^{-2}$ and plays the role of ultraviolet cutoff. Then the fermion part of the Hamiltonian is regularized as

$$\mathcal{H}_{\text{fermion}}=\sum_n e^{-\epsilon E_n^2}E_n\alpha_n^\dagger\alpha_n\ . \qquad (2.7)$$

First we calculate the commutator of the regularized current defined by

$$j_\mu^a(x)=\sum_{n,m}\alpha_n^\dagger\phi_n^\dagger(x)e^{-(\epsilon/2)E_n^2}t^a\gamma_0\gamma_\mu e^{-(\epsilon/2)E_m^2}\phi_m(x)\alpha_m \qquad (2.8)$$

with the help of the formula

$$\left[\sum_{n,m}\alpha_n^\dagger A_{nm}\alpha_m,\sum_{l,k}\alpha_l^\dagger B_{lk}\alpha_k\right]=\sum_{n,m}\alpha_n^\dagger[A,B]_{nm}\alpha_m\ . \qquad (2.9)$$

On account of the regularization factors, the commutator deviates from the canonical one as follows:

$$[j_0^a(x),j_\mu^b(y)]=f_{abc}j_\mu^c(x)\delta(x-y)+\hat{\mathcal{S}}_\mu^{ab}(x,y)\ , \qquad (2.10)$$

with

$$\hat{\mathcal{S}}_\mu^{ab}(x,y)=\sum_{n,m}\alpha_n^\dagger\phi_n^\dagger(x)e^{-(\epsilon/2)E_n^2}t^a\left[\sum_l e^{-\epsilon E_l^2}\phi_l(x)\phi_l^\dagger(y)-\delta(x-y)\right]t^b\gamma_0\gamma_\mu e^{-(\epsilon/2)E_m^2}\phi_m(y)\alpha_m$$
$$-\sum_{n,m}\alpha_n^\dagger\phi_n^\dagger(y)e^{-(\epsilon/2)E_n^2}t^b\gamma_0\gamma_\mu\left[\sum_l e^{-\epsilon E_l^2}\phi_l(y)\phi_l^\dagger(x)-\delta(y-x)\right]t^ae^{-(\epsilon/2)E_m^2}\phi_m(x)\alpha_m\ . \qquad (2.11)$$

Now we evaluate the expectation value of the operator $\hat{\mathcal{S}}_\mu^{ab}$ on the Dirac vacuum in the presence of the background gauge field, $|0\rangle_A$, which is defined by the conditions $\alpha_n|0\rangle_A=0$ for all positive energies E_n and $\alpha_n^\dagger|0\rangle_A=0$ for all negative energies E_n. In terms of the projection operator to the negative-energy eigenfunctions

$$P_-(x,y)=\sum_{E_n<0}\phi_n(x)\phi_n^\dagger(y)\ ,$$

the vacuum expectation value of $\hat{\mathcal{S}}_\mu^{ab}$ in the vanishing limit of ϵ can be expressed as

$$\begin{aligned}\mathcal{S}_\mu^{ab}&=\lim_{\epsilon\to 0}\langle\hat{\mathcal{S}}_\mu^{ab}\rangle_A\\&=\lim_{\epsilon\to 0}\mathrm{Tr}[e^{-\epsilon\Delta_y}P_-(y,x)t^a(e^{-\epsilon\Delta_x}-1)\delta(x-y)t^b\gamma_0\gamma_\mu-e^{-\epsilon\Delta_x}P_-(x,y)t^b\gamma_0\gamma_\mu(e^{-\epsilon\Delta_y}-1)\delta(x-y)t^a]\ ,\end{aligned}\tag{2.12}$$

where $\Delta=H^2$ and the subscript attached to it denotes its argument.

Now we study the transformation property of $\mathcal{S}_\mu^{ab}$ under the gauge transformation $A_i^g=gA_ig^\dagger+g\partial_ig^\dagger$. For later convenience, we introduce a functional $\mathcal{S}_\mu$ defined by

$$\mathcal{S}_\mu[A,u,v]=\int dx\,dy\,u^a(x)v^b(y)\mathcal{S}_\mu^{ab}(x,y)\ ,\tag{2.13}$$

where $u^a(x)$ and $v^b(y)$ are arbitrary functions and $u=u^at^a$, $v=v^at^a$. Since the operator $H(A)$ transforms as

$$H(A^g)=gH(A)g^\dagger\ ,\tag{2.14}$$

the operator $e^{-\epsilon\Delta_x(A)}$ also transforms as

$$e^{-\epsilon\Delta_x(A^g)}=g(x)e^{-\epsilon\Delta_x(A)}g^\dagger(x)\ .\tag{2.15}$$

Meanwhile Eq. (2.14) implies that $H(A^g)$ and $H(A)$ have the same energy spectrum and the eigenfunction of $H(A^g)$ is obtained from that of $H(A)$ by the gauge transformation, that is,

$$H(A^g)(g\phi_n)=E_n(g\phi_n)\ .\tag{2.16}$$

Therefore, the projection operator to the negative-energy eigenfunctions of $H(A^g)$ is equal to

$$\begin{aligned}\sum_{E_n<0}[g(x)\phi_n(x)][g(y)\phi_n(y)]^\dagger&=g(x)\left[\sum_{E_n<0}\phi_n(x)\phi_n^\dagger(y)\right]g^\dagger(y)\\&=g(x)P_-(x,y)g^\dagger(y)\ .\end{aligned}\tag{2.17}$$

Thus we know that $\mathcal{S}_\mu$ transforms gauge covariantly as

$$\mathcal{S}_\mu[A^g,u,v]=\mathcal{S}_\mu[A,g^\dagger ug,g^\dagger vg]\ .\tag{2.18}$$

Next we turn to the current–electric-field commutator. Through the regularization, the current becomes dependent on the gauge field and does not commute with the electric field. The commutator anomaly is formally given by

$$[E_i^a(x),j_\mu^b(y)]=g\hat{\mathcal{T}}_{i\mu}^{ab}(x,y)\tag{2.19}$$

with

$$\hat{\mathcal{T}}_{i\mu}^{ab}(x,y)=-i\psi^\dagger(y)\frac{\delta}{\delta gA^{ia}(x)}\left[e^{-(\epsilon/2)\overleftarrow{\Delta}_y}t^b\gamma_0\gamma_\mu\frac{1+\gamma_5}{2}e^{-(\epsilon/2)\Delta_y}\right]\psi(y)\ ,\tag{2.20}$$

where $\psi^\dagger(y)e^{-(\epsilon/2)\overleftarrow{\Delta}_y}$ is the Hermitian conjugate of $e^{-(\epsilon/2)\Delta_y}\psi(y)$. If we take the vacuum expectation value of $\hat{\mathcal{T}}_{i\mu}^{ab}$ and take the vanishing limit of ϵ, we have

$$\begin{aligned}\mathcal{T}_{i\mu}^{ab}(x,y)=\lim_{\epsilon\to 0}\lim_{z\to y}\mathrm{Tr}\Bigg[(-i)t^b\gamma_0\gamma_\mu\Bigg[&\frac{\delta}{\delta gA^{ia}(x)}(e^{-(\epsilon/2)\Delta_y})P_-(y,z)e^{-(\epsilon/2)\overleftarrow{\Delta}_z}\\&+e^{-(\epsilon/2)\Delta_y}P_-(y,z)\frac{\delta}{\delta gA^{ia}(x)}(e^{-(\epsilon/2)\overleftarrow{\Delta}_z})\Bigg]\Bigg]\ .\end{aligned}\tag{2.21}$$

Like $\mathcal{S}_\mu$ we define a functional $\mathcal{T}_{i\mu}$ by

$$\mathcal{T}_{i\mu}[A,u,v]=\int dx\,dy\,u^a(x)v^b(y)\mathcal{T}_{i\mu}^{ab}(x,y)\ .\tag{2.22}$$

Gauge covariance of $\mathcal{T}_{i\mu}$ can be read from Eqs. (2.15) and (2.17) together with the following fact. If we represent $e^{-(\epsilon/2)\Delta_y}$ and $e^{-(\epsilon/2)\overleftarrow{\Delta}_y}$ by $F(A(y))$, then F transforms as

$$F(A^g(y))=g(y)F(A(y))g^\dagger(y)\ .\tag{2.23}$$

Let us consider w_i^a as an infinitesimal vector function, and we denote $w_i=w_i^at^a$. By substituting $A_i+g^\dagger w_ig$

into A_i on both sides of Eq. (2.23), we obtain the relation

$$F(A^g(y)+w(y))=g(y)F(A(y)+g^\dagger(y)w(y)g(y))g^\dagger(y) . \tag{2.24}$$

If we expand this equation with respect to w, we get

$$\int dx\, w_i^a(x)\frac{\delta F(A)}{\delta A_i^a(x)}\bigg|_{A=A^g} = \int dx\,(g^\dagger w_i g)^a(x) \times g(y)\frac{\delta F(A)}{\delta A_i^a(x)}g^\dagger(y) . \tag{2.25}$$

Since w_i^a is an arbitrary vector function, Eq. (2.25) holds for each vector component, that is,

$$\int dx\, u^a(x)\frac{\delta F(A)}{\delta A_i^a(x)}\bigg|_{A=A^g} = \int dx\,(g^\dagger u g)^a(x) \times g(y)\frac{\delta F(A)}{\delta A_i^a(x)}g^\dagger(y) . \tag{2.26}$$

Thus, it is proved that

$$\mathcal{T}_{i\mu}[A^g,u,v]=\mathcal{T}_{i\mu}[A,g^\dagger ug,g^\dagger vg] . \tag{2.27}$$

III. COMPUTATION OF COMMUTATOR ANOMALIES BY PERTURBATIVE EXPANSION

Now we evaluate the commutator anomalies, whose formal expressions are given in the preceding section, by perturbative expansion. $\mathcal{S}_\mu^{ab}$ and $\mathcal{T}_{i\mu}^{ab}$ involve a D-dimensional δ function, whose mass dimension is D. Then, except for the factor of the δ function, $\mathcal{S}_\mu^{ab}$ and $\mathcal{T}_{i\mu}^{ab}$ have mass dimensions D and $D-1$, respectively, and these numbers are equal to the degree of divergence of $\mathcal{S}$ and $\mathcal{T}$. If we expand the projection operator P_- and the operator $e^{-\epsilon\Delta}$ with respect to the gauge field, the degree of divergence is decreased by one for each factor of the gauge field or for each factor of space derivative acting on the gauge field. Since $\mathcal{T}$ has a factor of ϵ and $\mathcal{S}$ involves the factor of $(e^{-\epsilon\Delta}-1)$, only ultraviolet-divergent integrals survive in the vanishing limit of ϵ. Furthermore, ultraviolet-divergent parts are local functions of the gauge field, and they are at most Dth order for $\mathcal{S}$ and $(D-1)$th order for $\mathcal{T}$ with respect to the gauge field and/or the space derivative acting on the gauge field. Therefore, it is sufficient to expand the projection operator and the operator $e^{-\epsilon\Delta}$ up to Dth order with respect to the gauge field and/or the space derivative.

First we carry out the Taylor expansion of the projection operator with respect to the gauge field. If we interpret the eigenfunction of H as the wave function of some state $|n,A\rangle$, that is, $\phi_n(x)=\langle x\,|\,n,A\rangle$, then the projection operator P_- can be rewritten as

$$\begin{aligned} P_-(x,y) &= \sum_{E_n<0}\langle x\,|\,n,A\rangle\langle n,A\,|\,y\rangle \\ &= \sum_n \left\langle x\left|\oint_{C_-}\frac{dE}{2\pi i}\frac{1}{E-E_n}\right|n,A\right\rangle\langle n,A\,|\,y\rangle \\ &= \left\langle x\left|\oint_{C_-}\frac{dE}{2\pi i}\frac{1}{E-H}\right|y\right\rangle , \end{aligned} \tag{3.1}$$

where C_- is a contour surrounding the negative real axis in the complex E plane. Next the operator H is divided into the free part

$$H_0=\gamma^0\gamma^k\frac{1+\gamma_5}{2}(-i\partial_k)$$

and the interaction part

$$V=\gamma^0\gamma^k\frac{1+\gamma_5}{2}(-iA_k) ,$$

and $1/(E-H)$ is expanded with respect to V. The zeroth-order term, denoted by $P_-^{(0)}$, is given by

$$P_-^{(0)}(x,y)=\int\frac{d^Dp}{(2\pi)^D}\frac{1}{2}\left[1-\frac{p\cdot\gamma\gamma^0}{E_p}\right]\frac{1+\gamma_5}{2} \times e^{-ip\cdot(x-y)} , \tag{3.2}$$

where $E_p=|p|$. The contraction of the D-momentum p with a $(D+1)$-vector q is understood as $p\cdot q=p^iq_i$. The higher-order terms up to the third order are explicitly obtained in Appendix A.

Next we expand the operator $e^{-\epsilon\Delta}$ acting on a plane wave $e^{-ip\cdot x}$. Since Δ and P_- are chirality conserving, only the chirality projection $(1+\gamma_5)/2$ in P_- is retained and other projections $(1+\gamma_5)/2$ in Δ are suppressed in the trace of Eq. (2.12). First we notice the identity

$$\begin{aligned} e^{-\epsilon\Delta}e^{-ip\cdot x} &= \exp\left[-\epsilon(\partial+A)^2-\frac{\epsilon}{4}[\gamma^j,\gamma^k]F_{jk}\right]e^{-ip\cdot x} \\ &= e^{-ip\cdot x}\exp\left[-\epsilon(-ip+\partial+A)^2-\frac{\epsilon}{4}[\gamma^j,\gamma^k]F_{jk}\right] \\ &= e^{-ip\cdot x}e^{\epsilon p^2}\exp[2i\epsilon p\cdot(\partial+A)-\epsilon\Delta] . \end{aligned} \tag{3.3}$$

As will be explained later, p is counted as of order $1/\sqrt{\epsilon}$. The third factor of the last expression in Eq. (3.3), denoted by F, is expanded in powers of $\sqrt{\epsilon}$ up to third order as follows:

$$F^{(1)}=2i\epsilon p\cdot A , \tag{3.4a}$$

$$F^{(2)}=-\epsilon(\partial+A)\cdot A-\frac{\epsilon}{4}[\gamma^j,\gamma^k]F_{jk}-2\epsilon^2p\cdot(\partial+A)p\cdot A , \tag{3.4b}$$

$$\begin{aligned} F^{(3)}=&-i\epsilon^2p\cdot(\partial+A)(\partial+A)\cdot A-i\epsilon^2(\partial+A)^2p\cdot A \\ &-\frac{i\epsilon^2}{4}p\cdot(\partial+A)[\gamma^j,\gamma^k]F_{jk}-\frac{i\epsilon^2}{4}[\gamma^j,\gamma^k]F_{jk}p\cdot A-\frac{4i\epsilon^3}{3}p\cdot(\partial+A)p\cdot(\partial+A)p\cdot A . \end{aligned} \tag{3.4c}$$

Then the expanded projection operator and the expanded operator $e^{-\epsilon\Delta}$ are substituted into the formal expression of $\mathcal{S}_\mu^{ab}$ given by Eq. (2.12). We discuss evaluation of terms order by order of the projection operator. First we consider the contributions from the zeroth-order projection operator. Further we take the zeroth order of the expansion of $e^{-\epsilon\Delta}$ as an example. It is the c-number part of $\mathcal{S}_\mu^{ab}$ and is given by

$$\mathrm{Tr}\int \frac{d^Dp\,d^Dq}{(2\pi)^{2D}}\left[e^{\epsilon p^2-ip\cdot(y-x)}\frac{1}{2}\left[1-\frac{p\cdot\gamma\gamma^0}{E_p}\right]\frac{1+\gamma_5}{2}t^a(e^{\epsilon q^2}-1)e^{-iq\cdot(x-y)}t^b\gamma_0\gamma_\mu\right]-(x\leftrightarrow y, a\leftrightarrow b)\,. \tag{3.5}$$

If we change a variable from q to $\xi=q-p$, p decouples from x and y, and it can be regarded as a loop momentum. In $1+3$ dimensions the above expression becomes

$$\mathrm{tr}(t^at^b)\int\frac{d^3\xi}{(2\pi)^3}e^{-i\xi\cdot(x-y)}\int\frac{d^3p}{(2\pi)^3}e^{\epsilon p^2}(e^{\epsilon(p+\xi)^2}-1)\left[g_{\mu 0}-g_{\mu j}\frac{p^j}{E_p}\right]-(x\leftrightarrow y)\,. \tag{3.6}$$

The integral over the loop momentum p is evaluated after Taylor expansion of $e^{\epsilon(p+\xi)^2}$ with respect to the external momentum ξ. If we change a variable from p to $\kappa=\sqrt{\epsilon}p$, the Jacobian gives an overall factor $(1/\sqrt{\epsilon})^3$ and the integral depends only on the dimensionless combination $\sqrt{\epsilon}\xi$. Thus, it is justified to count p as of order $1/\sqrt{\epsilon}$. The zeroth-order terms in ξ are proportional to $(1/\sqrt{\epsilon})^3$, but they are canceled out after subtraction of the terms in which x and y are interchanged. The first-order terms in ξ give the well-known Schwinger term,[14] which is proportional to $1/\epsilon$ and is quadratically divergent in the vanishing limit of ϵ. The second-order terms again compensate each other and the third-order terms give an ultraviolet finite term proportional to the third derivative of the δ function. Finally the expression (3.6) is reduced to

$$g_{\mu j}\,\mathrm{tr}(t^at^b)\left[\frac{i}{12\pi^2\epsilon}\partial^j\delta(x-y)-\frac{i}{20\pi^2}\partial^j\partial^k\partial_k\delta(x-y)\right]. \tag{3.7}$$

When we take the zeroth-order projection operator, we have to pick up terms of the expansion of $e^{-\epsilon\Delta}$ up to third order in $\sqrt{\epsilon}$. Contributions from these higher-order terms are evaluated essentially in the same way. For the reader's convenience, four basic integral formulas which we have used are listed in Appendix B, Eqs. (B1)–(B4).

Next, we consider the contribution from the first-order projection operator. This time we have to retain the terms of the expansion of $e^{-\epsilon\Delta}$ up to second order in $\sqrt{\epsilon}$. Again we take zeroth order as an example, which is given by

$$\mathrm{Tr}\int\frac{d^Dp\,d^Dq\,d^Dr}{(2\pi)^{3D}}\exp(\epsilon p^2-ip\cdot y+iq\cdot x)\frac{-i}{2(E_p+E_q)}\left[\gamma^k\gamma^0+\frac{p\cdot\gamma\gamma^k q\cdot\gamma\gamma^0}{E_pE_q}\right]$$
$$\times\frac{1+\gamma_5}{2}\tilde{A}_k(p-q)t^a(e^{\epsilon r^2}-1)t^b\gamma_0\gamma_\mu e^{-ir\cdot(x-y)}-(x\leftrightarrow y, a\leftrightarrow b)\,. \tag{3.8}$$

With a change of integration variables from q and r to $\xi=q-p$ and $\eta=r-p$, p decouples from x and y. In $1+3$ dimensions the expression (3.8) is transformed as

$$\int\frac{d^3\xi\,d^3\eta}{(2\pi)^6}\exp[i\xi\cdot x-i\eta\cdot(x-y)]\,\mathrm{tr}[\tilde{A}^k(-\xi)t^at^b]\int\frac{d^3p}{(2\pi)^3}e^{\epsilon p^2}(e^{\epsilon(p+\eta)^2}-1)\frac{-i}{E_p+E_q}$$
$$\times\left[g_{\mu k}+(g_{kl}\,g_{m\mu}+g_{km}\,g_{l\mu}-g_{k\mu}g_{lm}+i\epsilon_{lkm\mu})\frac{p^l(p+\xi)^m}{E_pE_q}\right]-(x\leftrightarrow y, a\leftrightarrow b)\,. \tag{3.9}$$

A method to evaluate these integrals is explained in Appendix B. Making use of the formulas (B5)–(B18), we arrive at the following result for the integral (3.9):

$$g_{\mu k}\left[\frac{i}{12\pi^2\epsilon}\mathrm{tr}(A^k[t^a,t^b])\delta(x-y)-\frac{i\ln 2}{24\pi^2}\mathrm{tr}(\partial_l\partial^lA^k-\partial_l\partial^kA^l)[t^a,t^b]\delta(x-y)-\frac{i}{60\pi^2}\mathrm{tr}(2\partial_l\partial^lA^k-\partial_l\partial^kA^l)t^bt^a\delta(x-y)\right.$$
$$+\frac{i}{60\pi^2}\mathrm{tr}[\partial^lA^k\partial_l\delta(x-y)+\partial_lA^l\partial^k\delta(x-y)+\partial^kA^l\partial_l\delta(x-y)]\{t^a,t^b\}$$
$$-\frac{i}{60\pi^2}\mathrm{tr}[6\partial^lA^k\partial_l\delta(x-y)+\partial_lA^l\partial^k\delta(x-y)+\partial^kA^l\partial_l\delta(x-y)]t^bt^a$$
$$\left.+\frac{i}{60\pi^2}\mathrm{tr}[3A^k\partial_l\partial^l\delta(x-y)+A_l\partial^l\partial^k\delta(x-y)][t^a,t^b]\right]+g_{\mu 0}\frac{1}{24\pi^2}\epsilon^{ijk0}\,\mathrm{tr}[\partial_iA_j\partial_k\delta(x-y)\{t^a,t^b\}]\,. \tag{3.10}$$

Contributions from higher-order terms of the expansion of $e^{-\epsilon\Delta}$ are also computed by utilizing the formulas (B5)–(B18).

Evaluation of the contributions from the second-order and the third-order projection operators is carried out with the help of the formulas (B19)–(B28). Collecting all contributions together, we obtain manifestly gauge-covariant expressions for $\mathcal{S}_\mu$ as follows:

$$\mathcal{S}^0=\mathcal{S}^1=-\frac{i}{2\pi}\int dx\ \mathrm{tr}(uD_1v)\quad \text{in } 1+1 \text{ dimensions}\ , \tag{3.11a}$$

$$\mathcal{S}^0=\frac{1}{16\pi^2}\epsilon^{ijk0}\int dx\ \mathrm{tr}[F_{ij}(uD_kv+D_kvu)]\ , \tag{3.11b}$$

$$\mathcal{S}^i=\int dx\left[\frac{i}{12\pi^2\epsilon}\mathrm{tr}(uD^iv)+\left[+\frac{i}{90\pi^2}+\frac{i\ln 2}{24\pi^2}\right]\mathrm{tr}(uD^iD^kD_kv)+\left[-\frac{11i}{360\pi^2}-\frac{i\ln 2}{12\pi^2}\right]\mathrm{tr}(uD_kD^iD^kv)\right.$$
$$\left.+\left[-\frac{11i}{360\pi^2}+\frac{i\ln 2}{24\pi^2}\right]\mathrm{tr}(uD_kD^kD^iv)-\frac{i}{16\pi^2}\left[\frac{\pi}{\epsilon}\right]^{1/2}\epsilon^{ijk0}\mathrm{tr}(F_{jk}[u,v])\right]\quad \text{in } 1+3 \text{ dimensions}\ , \tag{3.11c}$$

where D_k stands for the covariant derivative in the adjoint representation, that is, $D_kv=\partial_kv+[A_k,v]$. In $1+1$ dimensions, $j^0=j^1$ and hence $\mathcal{S}^0=\mathcal{S}^1$ due to the identity

$$\gamma^0\gamma^1\frac{1+\gamma_5}{2}=\frac{1+\gamma_5}{2}\ .$$

Next, the anomaly of the current–electric-field commutator, whose formal expression is given by Eq. (2.21), is perturbatively evaluated. This time we retain the expansion of the projection operator up to $(D-1)$th order, while the expansion of the operators $e^{-(\epsilon/2)\Delta}$ and $e^{-(\epsilon/2)\Delta}$ up to Dth order. With much less labor, we obtain the following results:

$$\mathcal{T}^{10}=\mathcal{T}^{11}=-\frac{1}{2\pi}\int dx\ \mathrm{tr}(uv)\quad \text{in } 1+1 \text{ dimensions}\ , \tag{3.12a}$$

$$\mathcal{T}^{i0}=-\frac{i}{16\pi^2}\epsilon^{ijk0}\int dx\ \mathrm{tr}(F_{jk}\{u,v\})\ , \tag{3.12b}$$

$$\mathcal{T}^{ij}=\int dx\left[\frac{1}{6\pi^2\epsilon}g^{ij}\mathrm{tr}(uv)+\frac{1}{360\pi^2}\mathrm{tr}(-11uD^iD^jv-11uD^jD^iv+4g^{ij}uD_kD^kv)\right.$$
$$\left.-\frac{1}{8\pi^2}\left[\frac{\pi}{\epsilon}\right]^{1/2}\epsilon^{ijk0}\mathrm{tr}(uD_kv)\right]\quad \text{in } 1+3 \text{ dimensions}\ . \tag{3.12c}$$

In order to compare our results with those obtained by the BJL method, we have to take into account the difference of our current and the one used by the authors of Refs. 5–7 in the BJL analyses. As will be shown in the next section, our current has the covariant anomaly[15] in its divergence and is called the covariant current,[16] while the current used by the authors of Refs. 5–7 has the consistent anomaly[17] and is called the consistent current.[16] The difference of the consistent current J^a from our covariant current j^a is known to be[16]

$$\Delta j^a_\mu=J^a_\mu-j^a_\mu=\begin{cases}-\dfrac{i}{4\pi}\epsilon_{\mu\nu}\mathrm{tr}(t^aA^\nu)\quad \text{in } 1+1 \text{ dimensions}\ , & (3.13a)\\[2ex] \dfrac{1}{48\pi^2}\epsilon_{\mu\nu\lambda\rho}\mathrm{tr}[t^a(A^\nu F^{\lambda\rho}+F^{\lambda\rho}A^\nu-A^\nu A^\lambda A^\rho)]\quad \text{in } 1+3 \text{ dimensions}\ . & (3.13b)\end{cases}$$

Then the commutator anomaly for the consistent current is obtained as follows:

$$[J^{0a}(x),J^{0b}(y)]=[j^{0a}(x),j^{0b}(y)]=f_{abc}j^{0c}(x)\delta(x-y)+\mathcal{S}^{0,ab}(x,y)$$
$$=f_{abc}J^{0c}(x)\delta(x-y)+\mathcal{S}^{0,ab}(x,y)-f_{abc}\Delta j^{0c}(x)\delta(x-y)\ , \tag{3.14a}$$

$$[E^{ia}(x),J^{0b}(y)]=[E^{ia}(x),j^{0b}(y)+\Delta j^{0b}(y)]=g\,\mathcal{T}^{i0,ab}(x,y)-i\frac{\delta}{\delta A_i^a(x)}[\Delta j^{0b}(y)]\ . \tag{3.14b}$$

It is remarkable that these anomalies are in complete agreement with those obtained by the BJL method [Eqs. (1.3a), (1.3b), (2.19a), and (2.19b) of Ref. 6] in both $1+1$ and $1+3$ dimensions.

As for the commutator anomalies among the electric fields, we are not able to derive them from first principles, but we expect they are absent so far as we regularize the whole system gauge covariantly. In fact we cannot write down any tensor with odd parity, gauge covariance, locality, and the antisymmetric property under simultaneous exchange of coordinates and indices. Recently, it was suggested by one of us (S.H.) that the electric fields commute with each other in the covariant regularization scheme, while they do not in the consistent regularization scheme.[18]

IV. CHIRAL ANOMALIES AND COMMUTATOR ANOMALIES OF THE GAUSS-LAW OPERATORS IN THE COVARIANT REGULARIZATION SCHEME

In the Schrödinger picture, the time derivative of an operator is given by its commutator with the total Hamiltonian. First, let us take the commutator of the charge density with the fermion part of the Hamiltonian. We transform it as follows:

$$\begin{aligned}[j^{0a}(x),\mathcal{H}_{\text{fermion}}] &= \sum_{n,m} \alpha_n^\dagger \phi_n^\dagger(x) e^{-(\epsilon/2)E_n^2} t^a (E_m e^{-\epsilon E_m^2} - E_n e^{-\epsilon E_n^2}) e^{-(\epsilon/2)E_m^2} \phi_m(x)\alpha_m \\ &= \sum_{n,m} \alpha_n^\dagger \phi_n^\dagger(x) e^{-(\epsilon/2)E_n^2} t^a (E_m - E_n) e^{-(\epsilon/2)E_m^2} \phi_m(x)\alpha_m + \hat{\mathcal{U}}^a(x) \\ &= \sum_{n,m} \alpha_n^\dagger \phi_n^\dagger(x) e^{-(\epsilon/2)E_n^2} (t^a H - \overleftarrow{H} t^a) e^{-(\epsilon/2)E_m^2} \phi_m(x)\alpha_m + \hat{\mathcal{U}}^a(x) \\ &= -i(D_k j^k)^a(x) + \hat{\mathcal{U}}^a(x) \end{aligned} \quad (4.1)$$

with

$$\hat{\mathcal{U}}^a(x) = \sum_{n,m} \alpha_n^\dagger \phi_n^\dagger(x) e^{-(\epsilon/2)E_n^2} t^a [E_m(e^{-\epsilon E_m^2}-1) - E_n(e^{-\epsilon E_n^2}-1)] e^{-(\epsilon/2)E_m^2} \phi_m(x)\alpha_m \ . \quad (4.2)$$

If we take the vacuum expectation value of $\hat{\mathcal{U}}$, it vanishes:

$$\langle \hat{\mathcal{U}}^a \rangle_A = 0 \ . \quad (4.3)$$

In fact, its matrix elements between arbitrary states vanish in the limit of $\epsilon=0$, at least in $1+1$ dimensions.

For example, let us consider the following two-point function:

$$\begin{aligned} G^{ab}(x,y) &= \langle \hat{\mathcal{U}}^a(x) j_0^b(y) \rangle_A \\ &= \langle \hat{\mathcal{U}}^a(x)\rangle_A \langle j_0^b(y)\rangle_A + \mathrm{Tr}\{ e^{-\epsilon\Delta_y} P_-(y,x)[t^a H(e^{-\epsilon\Delta_x}-1) - \overleftarrow{H}(e^{-\epsilon\overleftarrow{\Delta}_x}-1)t^a] e^{-\epsilon\Delta_x} P_+(x,y) t^b \} \ . \end{aligned} \quad (4.4)$$

The first term vanishes. The second term is the connected part of $G^{ab}(x,y)$, and its c-number part is read as

$$\mathrm{tr}\left[\int \frac{dp\,dq}{(2\pi)^2} \exp[-\epsilon p^2 + ip(y-x)] \left[1-\frac{p}{E_p}\right] [t^a q(e^{-\epsilon q^2}-1) - p(e^{-\epsilon p^2}-1)t^a] \exp[-\epsilon q^2 + iq(x-y)] \left[1+\frac{q}{E_q}\right] t^b \right] . \quad (4.5)$$

Changing a variable from q to $\xi = q-p$, we obtain

$$\mathrm{tr}(t^a t^b) \int \frac{d\xi}{2\pi} e^{i\xi(x-y)} \int \frac{dp}{2\pi} \exp[-\epsilon p^2 - \epsilon(p+\xi)]^2 \left[1-\frac{p}{|p|}\right] \left[1+\frac{p+\xi}{|p+\xi|}\right] [(p+\xi)(e^{-\epsilon(p+\xi)^2}-1) - p(e^{-\epsilon p^2}-1)] \ . \quad (4.6)$$

The product of factors coming from two projection operators

$$\left[1-\frac{p}{|p|}\right] \left[1+\frac{p+\xi}{|p+\xi|}\right]$$

cuts off the ultraviolet region, and the integration over p does not yield any ultraviolet divergences. Then we can take the vanishing limit of ϵ in the integrand, and the integrand vanishes due to the factor $(e^{-\epsilon p^2}-1)$ or $(e^{-\epsilon(p+\xi)^2}-1)$. Even if the complete expression of the operator $e^{-\xi\Delta}$ is used, the situation is not changed. Meanwhile if we take the higher-order projection operator, it involves a factor cutting off the ultraviolet region in itself. For example, the first-order projection operator can be rewritten as

$$P_{-}^{(1)}(x,y)=\int \frac{dp\, d\xi}{(2\pi)^2}\frac{i}{2(|p|+|p+\xi|)}\left[1-\frac{p(p+\xi)}{|p|\,|p+\xi|}\right]\exp[ip(x-y)-i\xi y]\tilde{A}_1(-\xi)\frac{1+\gamma_5}{2}$$

$$=\int \frac{dp\, d\xi}{(2\pi)^2}\frac{i}{4(|p|+|p+\xi|)}\left[\left[1-\frac{p}{|p|}\right]\left[1+\frac{p+\xi}{|p+\xi|}\right]+\left[1+\frac{p}{|p|}\right]\left[1-\frac{p+\xi}{|p+\xi|}\right]\right]\exp[ip(x-y)-i\xi y]\tilde{A}_1(-\xi)\frac{1+\gamma_5}{2}\ . \quad (4.7)$$

Roughly speaking, the zeroth-order positive-energy projection operator corresponds to free propagation forward in time of a particle, while the negative one corresponds to free propagation backward in time of a particle. In order for a particle to come back to the time when it started, at least once it has to go forward in time and once backward in time. Thus, higher-order projection operators always involve such a factor as

$$\left[1+\frac{p+\xi}{|p+\xi|}\right]\left[1-\frac{p+\eta}{|p+\eta|}\right]$$

with p a loop momentum and with ξ and η external momenta. Therefore, we can safely take the limit of $\epsilon=0$ in the integrand, and the integrand vanishes due to the factor $(e^{-\epsilon\Delta}-1)$. Thus, the connected part of G is known to vanish when ϵ goes to zero.

This proof can be generalized for n point functions, involving $\hat{\mathcal{U}}$ and arbitrary $(n-1)$ fermion bilinear operators. Any fermion loop with more than two vertices necessarily involves at least one positive-energy projection operator and one negative-energy projection operator and their product cuts off the ultraviolet region. Then the loop integral is ultraviolet finite and the vanishing limit of ϵ can be taken inside the integral. If $\hat{\mathcal{U}}$ is inserted into that loop, the factor $(e^{-\epsilon\Delta}-1)$ of $\hat{\mathcal{U}}$ kills such a loop. Thus it is proved that the connected n point function involving $\hat{\mathcal{U}}$ vanishes in the limit of $\epsilon=0$.

The argument developed here also applies to the case of $\hat{\mathcal{S}}$ and $\hat{\mathcal{T}}$, since the proof given above is not based on the details of $\hat{\mathcal{U}}$ but on the fact $\hat{\mathcal{U}}$ involves a factor $(e^{-\epsilon\Delta}-1)$. $\hat{\mathcal{S}}$ involves the same factor and $\hat{\mathcal{T}}$ has a factor of ϵ instead.

In 1 + 3 dimensions, the structure of the projection operator is much more complicated, and we have not yet proved that any connected n point function involving $\hat{\mathcal{S}}$ or $\hat{\mathcal{T}}$ or $\hat{\mathcal{U}}$ vanishes in the limit of $\epsilon=0$.

In the Weyl gauge, the anomaly of the current divergence arises from noncommutativity of the charge density with the electric field. With the aid of Eqs. (3.12a) and (3.12b), we obtain the chiral anomaly as follows:

$$\dot{j}^{0a}(x)=-i[j^{0a}(x),\mathcal{H}]=-(D_k j^k)^a(x)+\mathcal{A}^a(x)\ , \quad (4.8)$$

with

$$\mathcal{A}^a(x)=\frac{i}{2}\int dy\{j^{0a}(x),E^{ib}(y)E_i^b(y)\}=-\frac{ig}{2}\int dy\{\mathcal{T}^{i0,ba}(y,x),E_i^b(y)\}$$

$$=\begin{cases}-\dfrac{i}{2\pi}\mathrm{tr}(t^aE^1)\ \text{ in } 1+1 \text{ dimensions}\ , & (4.9\text{a})\\[2ex] -\dfrac{1}{16\pi^2}\epsilon^{i0jk}\,\mathrm{tr}(t^a\{E_i,F_{jk}\})\ \text{ in } 1+3 \text{ dimensions}\ . & (4.9\text{b})\end{cases}$$

These anomalies are gauge covariant and their normalizations including signs agree with those obtained by the path-integral method.[15]

Next we examine the commutators among the Gauss-law operators defined by

$$G^a(x)=\partial^i E_i^a(x)+gf_{abc}A^{ib}(x)E_i^c(x)+igj^{0a}(x)\ . \quad (4.10)$$

The commutator of G^a with the current is found from Eqs. (3.11a)–(3.12c) as follows:

$$\int dx\, dy\, u^a(x)v^b(y)\{[G^a(x),j_\mu^b(y)]-igf_{abc}j_\mu^c(x)\delta(x-y)\}$$

$$=-g\sum_i \mathcal{T}_{i\mu}[A,D^iu,v]+ig\mathcal{S}_\mu[A,u,v]$$

$$=\begin{cases} 0 \text{ in } 1+1 \text{ dimensions}\,, & (4.11\text{a})\\ 0 \text{ for } \mu=0 \text{ in } 1+3 \text{ dimensions}\,, & (4.11\text{b})\\ g\int dx\left[\frac{1}{12\pi^2\epsilon}\mathrm{tr}(uD_jv)+\frac{\ln 2}{24\pi^2}\mathrm{tr}(-uD_jD_kD^kv+2uD_kD_jD^kv-uD^kD_kD_jv)\right] \text{ for } \mu=j \text{ in } 1+3 \text{ dimensions}\,. & (4.11\text{c})\end{cases}$$

Thus, the charge density has a canonical commutator with G^a. As for the commutator among the electric fields, we assume that they vanish in the covariant regularization scheme. Then the commutator anomaly for the Gauss-law operator takes the following form in the covariant regularization scheme:

$$\begin{aligned}\int dx\,dy\,u^a(x)v^b(y)\{[G^a(x),G^b(y)]-igf_{abc}G^c(x)\delta(x-y)\}\\ &=\int dx\,dy\,u^a(x)v^b(y)\{[G^a(x),(D^jE_j)^b(y)]-igf_{abc}(D^jE_j)^c(x)\delta(x-y)\}\\ &=\int dx\,dy\,u^a(x)v^b(y)[igj^{0a}(x),(D^jE_j)^b(y)]=ig^2\sum_j \mathcal{T}_{j0}[A,D^jv,u]\\ &=-g^2\mathcal{S}_0[A,v,u]=g^2\mathcal{S}_0[A,u,v]\,, \qquad (4.12)\end{aligned}$$

where use has been made of Eqs. (4.11a) and (4.11b).

Finally we examine the time derivative of the Gauss-law operator G^a by taking its commutator with the Hamiltonian. In 1 + 3 dimensions as well as in 1 + 1 dimensions, we have proved that

$$[(D^kE_k)^a(x),\mathcal{H}_{\text{fermion}}]=-g(D^kj_k)^a(x)\,. \qquad (4.13)$$

Provided that the electric fields commute with each other, we obtain the relation

$$\begin{aligned}\dot{G}^a(x)&=-i[G^a(x),\mathcal{H}]\\ &=-i[(D^kE_k)^a(x),\mathcal{H}_{\text{fermion}}]-i[igj^{0a}(x),\mathcal{H}]\\ &=ig[(D^kj_k)^a(x)+\dot{j}^{0a}(x)]=ig\mathcal{A}^a(x)\,. \qquad (4.14)\end{aligned}$$

This relation was first noticed by Fujikawa[19] and has been investigated more closely by one of us (S.H.) (Ref. 18).

V. CONCLUSIONS AND DISCUSSION

We succeeded in evaluating the current-current and the current–electric-field commutator anomalies by making use of a new covariant regularization method in 1 + 3 dimensions as well as in 1 + 1 dimensions. The commutator anomalies take covariant forms since our currents are regularized gauge covariantly. The chiral anomaly of the current divergence was derived by taking the commutator of the charge density with the total Hamiltonian. The anomalous contribution from the commutator among the currents is canceled by the one from the commutator of the charge density with the kinetic term of the fermion. Thus, the commutator of the charge density with the fermion part of the Hamiltonian gives only the canonical term, the covariant divergence of the space components of the current. The chiral anomaly arises solely from noncommutativity of the charge density with the electric field. The resultant expressions have right signs and normalizations of the covariant anomaly in both 1 + 1 and 1 + 3 dimensions.

Our results on commutator anomalies of the covariant current are translated into those of the consistent current, which are in agreement with those obtained by the BJL method so far as the charge density is concerned. As for the commutator anomalies among the electric fields, we do not know their origin, but it is reasonable to assume they vanish in the covariant regularization scheme from dimensional and symmetry consideration.[20] On this assumption, the commutator anomaly of the Gauss-law operator is related to the current commutator anomaly, and time derivative of the Gauss-law operator is identified with the chiral anomaly.

In order to verify Faddeev's commutator anomaly, we have to establish a translation rule between the electric fields in the covariant regularization scheme and those in the consistent regularization scheme.

Finally two comments are in order. First we evaluated the vacuum expectation values of $\hat{\mathcal{S}}$ and $\hat{\mathcal{T}}$. As explained in Sec. IV, arbitrary matrix elements of $\hat{\mathcal{S}}$ and $\hat{\mathcal{T}}$ are equal to the vacuum expectation values of $\hat{\mathcal{S}}$ and $\hat{\mathcal{T}}$ in the vanishing limit of ϵ in 1 + 1 dimensions, that is,

$$\lim_{\epsilon\to 0}\langle\hat{\mathcal{S}}\cdots\rangle_A=\lim_{\epsilon\to 0}\langle\hat{\mathcal{S}}\rangle_A\langle\cdots\rangle_A\,, \qquad (5.1)$$

where the ellipses mean a product of arbitrary operators. Therefore, as operator equations, we can replace $\hat{\mathcal{S}}$ and $\hat{\mathcal{T}}$ by their vacuum expectation values $\mathcal{S}$ and $\mathcal{T}$ in the right-hand side of Eqs. (2.10) and (2.19) at least in 1 + 1 dimensions. In $1+3$ dimensions, we are not convinced of this point. For a finite ϵ, Eqs. (2.10) and (2.19) are well-defined operator equations, and the Jacobi identity holds for three currents and/or electric fields. However, if we replace $\hat{\mathcal{S}}$ and $\hat{\mathcal{T}}$ by their vacuum expectation values, the Jacobi identity is no longer satisfied. This fact suggests that it might not be allowed to replace $\hat{\mathcal{S}}$ and $\hat{\mathcal{T}}$ by their vacuum expectation values in Eqs. (2.10) and (2.19) as operator equations in $1+3$ dimensions. This problem is now under investigation.

The other comment is on reliability of the fixed-time method. Almost 2 decades ago, the Schwinger term[14] of the current commutator was evaluated by several methods.[21] In the Abelian case our result for the Schwinger term $\mathcal{S}^i$ is reduced to the c-number expression (3.7) with $\mathrm{tr}(t^a t^b)$ replaced by unity. The coefficient of the third derivative term $1/(20\pi^2)$ is different from that obtained by the BJL method: $1/(12\pi^2)$. We do not consider this discrepancy to be serious, since the Schwinger term itself has no physical meaning or is not an observable quantity. In the non-Abelian case, our result for $\mathcal{S}^i$ does not agree with that obtained by the BJL method[5] either, even if we take into account the difference of the covariant current and the consistent current. Again $\mathcal{S}^i$ itself has no physical meaning nor is it related to any cohomological quantity, and it may depend on the regularization scheme. On the other hand, the commutator anomaly with a physical meaning or a definite topological origin should be reproduced in any well-defined regularization scheme, even if it belongs to the fixed-time method.

Note added

Recently, Mickelsson and Rajeev [Commun. Math. Phys. **116**, 365 (1988)] have constructed a method to obtain a representation for $(1+d)$-dimensional current algebra in terms of the infinite-dimensional Grassmannian manifold. They have not been able to prove the unitarizability of their representation for $1+d$ dimensions $(d>1)$.

In contrast with their approach, our field-theoretical method automatically satisfies the unitarity condition since the current operators are Hermitian by construction. However, we have not been able to prove the commutation relation as an operator relation for $1+d$ dimensions $(d>1)$.

In a recent paper by G. Semenoff [Phys. Rev. Lett. **60**, 680 (1988); **60**, 1590(E) (1988)], he claims that the algebra of the Gauss-law operators has trivial two-cocycles (contrary to the perturbative analyses) if the properties of the electric fields are correctly taken into account. Much investigation on the commutator of the electric fields seems to be necessary in order to settle the controversy.

After submission of our paper, we became aware of a paper by S. Ghosh and R. Banerjee [Saha Institute of Nuclear Physics Report No. SINP-TNP/87-26, 1987 (unpublished)] in which they calculated the same commutator anomalies as ours by the point-splitting method. Their results are in agreement with ours.

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APPENDIX A: PERTURBATIVE EXPANSION OF THE PROJECTION OPERATOR

We denote an eigenvalue of H_0 and the corresponding eigenfunction by $E_n^{(0)}$ and $\phi_n^{(0)}$, respectively. According to the notation introduced in the text, we can write $\phi_n^{(0)}(x)=\langle x \mid n,0\rangle$. The projection operator to the positive-energy eigenfunctions of H_0 and the one to the negative-energy eigenfunctions are given by

$$P_+^{(0)}(x,y)=\sum_{E_n^{(0)}>0}\phi_n^{(0)}(x)\phi_n^{(0)\dagger}(y)=\int\frac{d^{D+1}p}{(2\pi)^D}\theta(p^0)\delta(p^2)\not{p}\gamma^0\frac{1+\gamma_5}{2}e^{-ip\cdot(x-y)}$$
$$=\int\frac{d^Dp}{(2\pi)^D}\frac{1}{2}\left[1+\frac{p\cdot\gamma\gamma^0}{E_p}\right]\frac{1+\gamma_5}{2}e^{-ip\cdot(x-y)}\,, \tag{A1}$$

$$P_-^{(0)}(x,y)=\sum_{E_n^{(0)}<0}\phi_n^{(0)}(x)\phi_n^{(0)\dagger}(y)=\int\frac{d^{D+1}p}{(2\pi)^D}\theta(-p^0)\delta(p^2)(-\not{p}\gamma^0)\frac{1+\gamma_5}{2}e^{-ip\cdot(x-y)}$$
$$=\int\frac{d^Dp}{(2\pi)^D}\frac{1}{2}\left[1-\frac{p\cdot\gamma\gamma^0}{E_p}\right]\frac{1+\gamma_5}{2}e^{-ip\cdot(x-y)}\,. \tag{A2}$$

The first-order correction to $P_-(x,y)$ is formally written as

$$P_-^{(1)}(x,y)=\sum_{n,m}\oint_{C_-}\frac{dE}{2\pi i}\left\langle x\left|\frac{1}{E-E_n^{(0)}}\right|n,0\right\rangle\langle n,0\mid V\mid m,0\rangle\left\langle m,0\left|\frac{1}{E-E_m^{(0)}}\right|y\right\rangle\,. \tag{A3}$$

By expansion to partial fractions, the contour integration can be carried out, and it is transformed as

$$P_-^{(1)}(x,y)=i\int\frac{d^{D+1}p\,d^{D+1}q}{(2\pi)^{2D}}\delta(p^2)\delta(q^2)\frac{\theta(-p^0)\theta(q^0)-\theta(p^0)\theta(-q^0)}{p^0-q^0}\not{p}\gamma^0\gamma^0\gamma^k\frac{1+\gamma_5}{2}\tilde{A}_k(p-q)\not{q}\gamma^0\exp(-ip\cdot x+iq\cdot y)$$

$$=-i\int\frac{d^Dp\,d^Dq}{(2\pi)^{2D}}\frac{1}{2(E_p+E_q)}\left[\gamma^k\gamma^0+\frac{p\cdot\gamma\gamma^k q\cdot\gamma\gamma^0}{E_pE_q}\right]\frac{1+\gamma_5}{2}\tilde{A}_k(p-q)\exp(-ip\cdot x+iq\cdot y)\ , \tag{A4}$$

where $\tilde{A}(p)$ is the Fourier transform of $A(x)$. With the help of the formulas

$$\oint_{C_-}\frac{dE}{2\pi i}\frac{1}{(E-E_n)(E-E_m)(E-E_l)}$$
$$=\frac{1}{(E_n-E_m)(E_n-E_l)}[\theta(-E_n)\theta(E_m)\theta(E_l)-\theta(E_n)\theta(-E_m)\theta(-E_l)]+(\text{two permuted terms})\ , \tag{A5}$$

$$\oint_{C_-}\frac{dE}{2\pi i}\frac{1}{(E-E_n)(E-E_m)(E-E_l)(E-E_k)}$$
$$=\frac{1}{(E_n-E_m)(E_n-E_l)(E_n-E_k)}[\theta(-E_n)\theta(E_m)\theta(E_l)\theta(E_k)-\theta(E_n)\theta(-E_m)\theta(-E_l)\theta(-E_k)]$$
$$+(\text{three permuted terms})$$
$$+\frac{E_l+E_k-E_n-E_m}{(E_n-E_l)(E_n-E_k)(E_m-E_l)(E_m-E_k)}[\theta(-E_n)\theta(-E_m)\theta(E_l)\theta(E_k)-\theta(E_n)\theta(E_m)\theta(-E_l)\theta(-E_k)]$$
$$+(\text{two permuted terms})\ , \tag{A6}$$

the second-order and the third-order corrections are obtained as

$$P_-^{(2)}(x,y)=\int\frac{d^Dp\,d^Dq\,d^Dr}{(2\pi)^{3D}}\frac{1}{2(E_p+E_q)(E_p+E_r)(E_q+E_r)}$$
$$\times\left[(\gamma^j\gamma^k r\cdot\gamma+\gamma^j q\cdot\gamma\gamma^k+p\cdot\gamma\gamma^j\gamma^k)\gamma^0+\frac{E_p+E_q+E_r}{E_pE_qE_r}p\cdot\gamma\gamma^j q\cdot\gamma\gamma^k r\cdot\gamma\gamma^0\right]\frac{1+\gamma_5}{2}$$
$$\times\tilde{A}_j(p-q)\tilde{A}_k(q-r)e^{-ip\cdot x+ir\cdot y}\ , \tag{A7}$$

$$P_-^{(3)}(x,y)=i\int\frac{d^Dp\,d^Dq\,d^Dr\,d^Ds}{(2\pi)^{4D}}\frac{1}{2(E_p+E_q)(E_p+E_r)(E_p+E_s)(E_q+E_r)(E_q+E_s)(E_r+E_s)}$$
$$\times\Big[(E_pE_qE_r+E_pE_qE_s+E_pE_rE_s+E_qE_rE_s)\gamma^j\gamma^k\gamma^l\gamma^0$$
$$+(E_p+E_q+E_r+E_s)(\gamma^j\gamma^k r\cdot\gamma\gamma^l s\cdot\gamma+\gamma^j q\cdot\gamma\gamma^k\gamma^l s\cdot\gamma+\gamma^j q\cdot\gamma\gamma^k r\cdot\gamma\gamma^l$$
$$+p\cdot\gamma\gamma\gamma^j\gamma^k\gamma^l s\cdot\gamma+p\cdot\gamma\gamma^j\gamma^k r\cdot\gamma\gamma^l+p\cdot\gamma\gamma^j q\cdot\gamma\gamma^k\gamma^l)\gamma^0$$
$$+\frac{1}{E_pE_qE_rE_s}[E_p^2(E_q+E_r+E_s)+E_q^2(E_p+E_r+E_s)+E_r^2(E_p+E_q+E_s)$$
$$+E_s^2(E_p+E_q+E_r)+2(E_pE_qE_r+E_pE_qE_s+E_pE_rE_s+E_qE_rE_s)]$$
$$\times p\cdot\gamma\gamma^j q\cdot r\gamma^k r\cdot\gamma\gamma^l s\cdot\gamma\gamma^0\Big]\frac{1+\gamma_5}{2}\tilde{A}_j(p-q)\tilde{A}_k(q-r)\tilde{A}_l(r-s)\exp(-ip\cdot x+is\cdot y)\ . \tag{A8}$$

APPENDIX B: INTEGRAL FORMULAS

In order to evaluate the contribution from the zeroth-order projection operator, we have to pick up terms of the Taylor expansion of the operator of $e^{-\epsilon\Delta}$ up to third order in $\sqrt{\epsilon}$, which involve 1, ϵp_i, ϵ, $\epsilon^2p_ip_j$, ϵ^2p_i, or $\epsilon^3p_ip_jp_k$. Taking into account the parity, we reduce all of the integrals to the following basic integrals:

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2} = \frac{1}{(2\pi)^3} \left[\frac{\pi}{\epsilon}\right]^{3/2} , \tag{B1}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2} \epsilon^2 p_i p_j = -\frac{1}{(2\pi)^3} \left[\frac{\pi}{\epsilon}\right]^{3/2} \frac{\epsilon}{2} g_{ij} , \tag{B2}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2} \frac{\epsilon p_i p_j}{E_p} = -\frac{1}{12\pi^2 \epsilon} g_{ij} , \tag{B3}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2} \frac{\epsilon^3 p_i p_j p_k p_l}{E_p} = \frac{1}{30\pi^2}(g_{ij}g_{kl}+g_{ik}g_{jl}+g_{il}g_{jk}) . \tag{B4}$$

A typical integral which appears in the contribution from the first-order projection operator is

$$I(\xi^2) = \int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}(e^{\epsilon p^2}-1)\frac{1}{|p|+|p+\xi|} . \tag{B5}$$

$I(\xi^2)$ itself and its first derivative at $\xi^2=0$ are well defined but its higher derivatives are not. In fact its asymptotic behavior at $\xi^2=0$ turns out to be

$$I(\xi^2) = \frac{1}{\epsilon}\{C_0 + C_1 \epsilon \xi^2 + \epsilon^2 \xi^4 [C_2^1 + C_2^2 \ln(\epsilon \xi^2)] + O(\epsilon^3)\} . \tag{B6}$$

Then only the first two terms are relevant in the vanishing limit of ϵ. The constant C_0 is easily evaluated as

$$C_0 = \epsilon I(0) = \epsilon \int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}(e^{\epsilon p^2}-1)\frac{1}{2|p|} = -\frac{1}{16\pi^2} . \tag{B7}$$

As for the constant C_1, we differentiate Eqs. (B5) and (B6) twice with respect to ξ and we set $\xi=0$. From Eq. (B5) we obtain

$$\left.\frac{\partial^2 I(\xi^2)}{\partial \xi^i \partial \xi^j}\right|_{\xi=0} = \int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}(e^{\epsilon p^2}-1)\frac{1}{4|p|^3}\left[g_{ij} + 2\frac{p_i p_j}{|p|^2}\right] = \tfrac{1}{12} g_{ij} \int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}(e^{\epsilon p^2}-1)\frac{1}{|p|^3} . \tag{B8}$$

From Eq. (B6) we get

$$\left.\frac{\partial^2 I(\xi^2)}{\partial \xi^i \partial \xi^j}\right|_{\xi=0} = 2C_1 g_{ij} . \tag{B9}$$

Then the constant C_1 is related to the integral on the right-hand side of Eq. (B8) as follows:

$$\begin{aligned} C_1 = \tfrac{1}{24}\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}(e^{\epsilon p^2}-1)\frac{1}{|p|^3} &= \frac{1}{24}\frac{1}{4\pi^2}\int_0^\infty dt \frac{1}{t} e^{-t}(e^{-t}-1) \\ &= \frac{1}{96\pi^2}\lim_{\epsilon\to 0}\int_0^\infty dt\, t^{\epsilon-1}(e^{-2t}-e^{-t}) = \frac{1}{96\pi^2}\lim_{\epsilon\to 0}[(\tfrac{1}{2})^\epsilon - 1]\Gamma(\epsilon) = -\frac{\ln 2}{96\pi^2} . \end{aligned} \tag{B10}$$

Similarly we can derive the formulas

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}(e^{\epsilon p^2}-1)\frac{p_i p_j}{|p|\,|p+\xi|(|p|+|p+\xi|)} = \frac{1}{48\pi^2\epsilon} g_{ij} + \frac{\ln 2}{96\pi^2} g_{ij}\xi^2 - \frac{\ln 2}{24\pi^2}\xi_i \xi_j + O(\epsilon) , \tag{B11}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}(e^{\epsilon p^2}-1)\frac{p_i}{|p|\,|p+\xi|(|p|+|p+\xi|)} = \frac{\ln 2}{16\pi^2}\xi_i + O(\epsilon) . \tag{B12}$$

This time, terms of the Taylor expansion of the operator $e^{-\epsilon\Delta}$ up to second order in $\sqrt{\epsilon}$ should be retained and they involve ϵp_i, ϵ, or $\epsilon^2 p_i p_j$. Therefore, the relevant formulas are

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}\frac{\epsilon}{|p|+|p+\xi|} = \frac{1}{8\pi^2} + O(\epsilon) , \tag{B13}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}\frac{\epsilon p_i}{|p|+|p+\xi|} = -\frac{1}{48\pi^2}\xi_i + O(\epsilon) , \tag{B14}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2} \frac{\epsilon^2 p_i p_j}{|p|+|p+\xi|} = -\frac{1}{24\pi^2} g_{ij} + O(\epsilon) , \tag{B15}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2} \frac{\epsilon p_i p_j}{|p|\,|p+\xi|(|p|+|p+\xi|)} = -\frac{1}{24\pi^2} g_{ij} + O(\epsilon) , \tag{B16}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2} \frac{\epsilon p_i p_j p_k}{|p|\,|p+\xi|(|p|+|p+\xi|)} = \frac{1}{80\pi^2} (g_{ij}\xi_k + g_{ik}\xi_j + g_{jk}\xi_i) + O(\epsilon) , \tag{B17}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2} \frac{\epsilon^2 p_i p_j p_k p_l}{|p|\,|p+\xi|(|p|+|p+\xi|)} = \frac{1}{120\pi^2} (g_{ij}g_{kl} + g_{ik}g_{jl} + g_{il}g_{jk}) + O(\epsilon) . \tag{B18}$$

In the following, we list the formulas to be utilized in evaluation of the contributions from the second-order and the third-order projection operators:

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}(e^{\epsilon p^2}-1) \frac{p_i}{D_3(p,\xi,\eta)} = \frac{\ln 2}{96\pi^2} (\xi+\eta)_i + O(\epsilon) , \tag{B19}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}(e^{\epsilon p^2}-1) \frac{1}{D_3(p,\xi,\eta)} = -\frac{\ln 2}{32\pi^2} + O(\epsilon) , \tag{B20}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}(e^{\epsilon p^2}-1) \frac{p_i p_j p_k(|p|+|p+\xi|+|p+\eta|)}{|p|\,|p+\xi|\,|p+\eta|\,D_3(p,\xi,\eta)} = -\frac{\ln 2}{96\pi^2} [g_{ij}(\xi+\eta)_k + g_{ik}(\xi+\eta)_j + g_{jk}(\xi+\eta)_i] + O(\epsilon) , \tag{B21}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}(e^{\epsilon p^2}-1) \frac{p_i p_j(|p|+|p+\xi|+|p+\eta|)}{|p|\,|p+\xi|\,|p+\eta|\,D_3(p,\xi,\eta)} = \frac{\ln 2}{32\pi^2} g_{ij} + O(\epsilon) , \tag{B22}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2} \frac{\epsilon p_i p_j}{D_3(p,\xi,\eta)} = -\frac{1}{96\pi^2} g_{ij} + O(\epsilon) , \tag{B23}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2} \frac{\epsilon p_i p_j p_k p_l(|p|+|p+\xi|+|p+\eta|)}{|p|\,|p+\xi|\,|p+\eta|\,D_3(p,\xi,\eta)} = \frac{1}{160\pi^2} (g_{ij}g_{kl} + g_{ik}g_{jl} + g_{il}g_{jk}) + O(\epsilon) , \tag{B24}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}(e^{\epsilon p^2}-1) \frac{|p|\,|p+\xi|\,|p+\eta|\,|p+\zeta|}{D_4(p,\xi,\eta,\zeta)} \left[\frac{1}{|p|} + \frac{1}{|p+\xi|} + \frac{1}{|p+\eta|} + \frac{1}{|p+\zeta|} \right] = -\frac{\ln 2}{64\pi^2} + O(\epsilon) , \tag{B25}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}(e^{\epsilon p^2}-1) \frac{p_i p_j(|p|+|p+\xi|+|p+\eta|+|p+\zeta|)}{D_4(p,\xi,\eta,\zeta)} = \frac{\ln 2}{192\pi^2} g_{ij} + O(\epsilon) , \tag{B26}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}(e^{\epsilon p^2}-1) \frac{p_i p_j p_k p_l}{D_4(p,\xi,\eta,\zeta)} \left[\frac{1}{|p|} + \frac{1}{|p+\xi|} + \frac{1}{|p+\eta|} + \frac{1}{|p+\zeta|} \right] = -\frac{\ln 2}{960\pi^2} (g_{ij}g_{kl} + g_{ik}g_{jl} + g_{il}g_{jk}) + O(\epsilon) , \tag{B27}$$

$$\int \frac{d^3p}{(2\pi)^3} e^{\epsilon p^2}(e^{\epsilon p^2}-1) \frac{p_i p_j p_k p_l}{D_4(p,\xi,\eta,\zeta)} \left[\frac{|p+\eta|+|p+\zeta|}{|p|\,|p+\xi|} + \frac{|p+\xi|+|p+\zeta|}{|p|\,|p+\eta|} + \frac{|p+\xi|+|p+\eta|}{|p|\,|p+\zeta|} + \frac{|p|+|p+\zeta|}{|p+\xi|\,|p+\eta|} + \frac{|p|+|p+\eta|}{|p+\xi|\,|p+\zeta|} + \frac{|p|+|p+\xi|}{|p+\eta|\,|p+\zeta|} \right] = -\frac{\ln 2}{320\pi^2} (g_{ij}g_{kl} + g_{ik}g_{jl} + g_{il}g_{jk}) + O(\epsilon) , \tag{B28}$$

where $D_3(p,\xi,\eta)$ and $D_4(p,\xi,\eta,\zeta)$ are defined by

$$D_3(p,\xi,\eta) = (|p|+|p+\xi|)(|p|+|p+\eta|)(|p+\xi|+|p+\eta|) , \tag{B29}$$

$$D_4(p,\xi,\eta,\zeta) = (|p|+|p+\xi|)(|p|+|p+\eta|)(|p|+|p+\zeta|) \times (|p+\xi|+|p+\eta|)(|p+\xi|+|p+\zeta|)(|p+\eta|+|p+\zeta|) . \tag{B30}$$

[1]B. Zumino, Y.-S. Wu, and A. Zee, Nucl. Phys. **B239**, 477 (1984); B. Zumino, in *Relativity, Groups and Topology II*, edited by B. S. DeWitt and R. Stora (North-Holland, Amsterdam, 1984); R. Stora, in *Recent Progress in Gauge Theories*, Cargese, 1983, edited by H. Lehmann (Plenum, New York, 1984).
[2]L. D. Faddeev, Phys. Lett. **145B**, 81 (1984); L. D. Faddeev and S. L. Shatashvili, Theor. Math. Phys. **60**, 770 (1984).
[3]R. Jackiw and R. Rajaraman, Phys. Rev. Lett. **54**, 1219 (1985); **54**, 2060(E) (1985); R. Jackiw, MIT Report No. CTP 1436, 1986 (unpublished).
[4]L. D. Faddeev and S. L. Shatashvili, Phys. Lett. **167B**, 225 (1986).
[5]M. Kobayashi and A. Sugamoto, Phys. Lett. **159B**, 315 (1985); S.-G. Jo, Nucl. Phys. **B259**, 616 (1985).
[6]S.-G. Jo, Phys. Lett. **163B**, 353 (1985).
[7]M. Kobayashi, K. Seo, and A. Sugamoto, Nucl. Phys. **B273**, 607 (1986).
[8]J. D. Bjorken, Phys. Rev. **148**, 1467 (1966); K. Johnson and F. E. Low, Prog. Theor. Phys. Suppl. **37-38**, 74 (1966).
[9]S.-G. Jo, Phys. Rev. D **35**, 3179 (1987).
[10]A. Niemi and G. Semenoff, Phys. Rev. Lett. **56**, 1019 (1986); H. Sonoda, Phys. Lett. **156B**, 220 (1985); Nucl. Phys. **B266**, 410 (1986).
[11]P. Nelson and L. Alvarez-Gaume, Commun. Math. Phys. **99**, 103 (1985); A. Niemi and G. Semenoff, Phys. Rev. Lett. **55**, 925 (1985); S. Hosono, Nucl. Phys. **B300** [FS22], 238 (1988).
[12]S. Hosono and K. Seo, Mod. Phys. Lett. **A3**, 691 (1988).
[13]J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).
[14]T. Goto and I. Imamura, Prog. Theor. Phys. **14**, 196 (1955); J. Schwinger, Phys. Rev. Lett. **3**, 296 (1959).
[15]K. Fujikawa, Phys. Rev. D **21**, 2848 (1980); **31**, 341 (1985).
[16]W. A. Bardeen and B. Zumino, Nucl. Phys. **B244**, 421 (1984).
[17]W. A. Bardeen, Phys. Rev. **184**, 1848 (1969); J. Wess and B. Zumino, Phys. Lett. **37B**, 95 (1971).
[18]S. Hosono, Nagoya University Report No. DPNU-87-33, 1987 (unpublished).
[19]K. Fujikawa, Phys. Lett. B **171**, 424 (1986).
[20]Throughout this paper, we took gauge covariance as a guiding principle, and it leads to the assumption of the vanishing electric-field-commutator anomaly. This assumption has a further support from the following consideration. If we regularize the electric fields so that they satisfy the Jacobi identity, then their commutator anomaly must be a two-cocycle on the gauge-field configuration space. However, this two-cocycle is trivial, since the gauge configuration space is affine and has no nontrivial cohomology. Therefore, we are able to redefine the electric fields so that they commute with each other.
[21]R. A. Brandt, Phys. Rev. **166**, 1795 (1968); R. Jackiw and K. Johnson, *ibid.* **182**, 1459 (1969); D. G. Boulware and R. Jackiw, *ibid.* **186**, 1442 (1969); M. Chanowitz, Phys. Rev. D **2**, 3016 (1970).