BRST-invariant tadpole calculation and diagrammatic formalism in the bosonic string

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Planar and nonorientable tadpole operators are evaluated in the Becchi-Rouet-Stora-Tyutin-(BRST-) invariant formalism. It is shown that the insertion of $(-1)^{N_c+1/2}$ (N_c = ghost-number operator) is needed for keeping the BRST invariance. We shall also discuss the extension of the BRST-invariant diagrammatic calculation to arbitrary diagrams.

A covariant quantization of the bosonic string is given by the Becchi-Rouet-Stora-Tyutin (BRST) formalism.[1] It was shown that the nilpotency of the BRST charge ($Q^2=0$) requires the critical values $D=26$ and $\alpha_0=1$ for the space-time dimension and the intercept of trajectory, respectively. Following this formalism, the ghost interactions corresponding to the Caneshi-Schwimmer-Veneziano (CSV) vertex[2] and the general N-Reggeon vertex[3] have been derived.[4] The BRST invariance of planar one-loop diagrams was also discussed in this formalism.[5] However, no BRST-invariant diagrammatic operator formalism based on Fock spaces has yet been completed. In this paper we shall study a BRST-invariant calculation satisfying unitarity for the planar and nonorientable tadpole operators and show a possible BRST-invariant diagrammatic operator formalism.

The three-string vertex given by Fig. 1 is described in the symmetric form as

$$\langle V_{123}| = \langle V_{123}^X | \langle V_{123}^{\mathrm{gh}} | \ , \tag{1}$$

where V^X and V^{gh}, respectively, stand for the vertices of the X part and the ghost part. Following Ref. 4, the ghost vertex is given by

$$\langle V_{123}^{\mathrm{gh}} | = {}_{123}\langle \tilde{0} | \exp\left[-\sum_{\substack{r\neq 1 \\ (r\neq s)}}^{3} \sum_{s=1}^{3} (c^{(r)} | U_r V_s | b^{(s)}) \right] \times \prod_{n=0,\pm 1} \left[\sum_{s=1}^{3} \sum_{m=0,\pm 1} \mathcal{D}^{(1,0)}(V_s)_{nm} b_m^{(s)} \right] , \tag{2}$$

where ${}_{123}\langle\tilde{0}| \equiv {}_1\langle\tilde{0}|\,{}_2\langle\tilde{0}|\,{}_3\langle\tilde{0}|$, $\langle\tilde{0}|$ is the vacuum with nonzero norm to the SL(2)-invariant vacuum $|0\rangle$ ($\langle\tilde{0}|0\rangle=1$) (Ref. 1),

$$(c^{(r)} | U_r V_s | b^{(s)}) = \sum_n \sum_m c_n^{(r)} \mathcal{D}^{(1,0)}(U_r V_s)_{nm} b_m^{(s)} ,$$

U_r and V_s are defined in conventional forms:[3]

$$V_s = \begin{bmatrix} \infty & 0 & 1 \\ z_{s-1} & z_s & z_{s+1} \end{bmatrix}, \quad U_r = \Gamma V_r^{-1}$$

$$\text{with } \Gamma = \begin{bmatrix} \infty & 0 & 1 \\ 0 & \infty & 1 \end{bmatrix} .$$

c and b are, respectively, the ghost and the antighost oscillators, and $\mathcal{D}^{(1,0)}$ denotes the (1,0) representation of SL(2,R). (The (n,m) element of the $(J,P-J)$ representation is defined by

$$\mathcal{D}^{(J,P-J)}(\Lambda)_{nm} = \frac{1}{(P+m)!} \frac{d^{m+P}}{!dz^{m+P}} \times \left[(cz+d)^{2J} \left[\frac{az+b}{cz+d} \right]^{n+P} \right] \Bigg|_{z=0}$$

where

$$\Lambda = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

and $\det\Lambda = 1$. This is the same as the conventional repre-

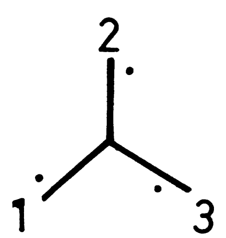

FIG. 1. Symmetric three-string vertex. The dot on each leg indicates the side of the line on which the external particles are to be attached.

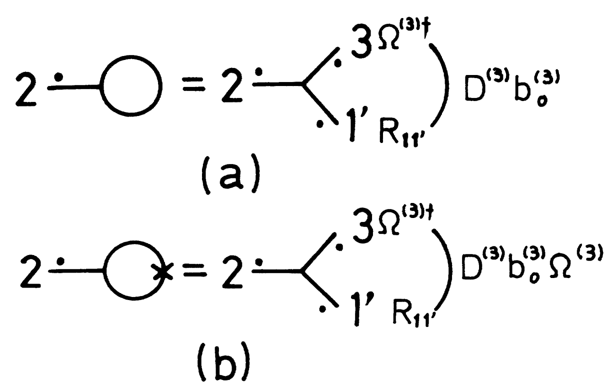

FIG. 2. Diagrams for the (a) planar and the (b) nonorientable tadpole operators, where × stands for the twist.

sentation $D^{(J,P-J)}$ except for the normalization coefficients.) The BRST invariance of $\langle V_{123}|$ is shown by the equation[4]

$$\langle V_{123}|(Q^{(1)}+Q^{(2)}+Q^{(3)})=0\ . \tag{3}$$

Since the BRST charge commutes with all Virasoro operators L_n $(n=-\infty,\ldots,\infty)$, operators written in terms of L_n, e.g., the propagator

$$D\equiv\frac{1}{L_0}=\int_0^1 dx\, x^{L_0-1} \tag{4}$$

and the twist operator

$$\Omega\equiv e^{L_{-1}}(-1)^{L_0}=(-1)^{L_0}e^{-L_{-1}}\ , \tag{5}$$

are BRST invariant, where

$$L_0\equiv L_0^X+L_0^{\rm gh}=\frac{p^2}{2}+\sum_{n=1}^{\infty}n(a_{-n}a_n+b_{-n}c_n+c_{-n}b_n)-1\ . \tag{6}$$

In order to evaluate loop amplitudes illustrated in Fig. 2, we need a reflection operator which transforms a string state $\langle x|$ to $|x\rangle$ in the BRST-invariant way. Such an operator is easily derived from the vertex as follows:

$$|R_{13}\rangle=(\langle V_{123}|\Omega^{(3)\dagger}|0\rangle_2)^\dagger=(b_0^{(1)}-b_0^{(3)})\exp\left[\sum_{n=1}^{\infty}(a_{-n}^{(1)}a_{-n}^{(3)}+c_{-n}^{(1)}b_{-n}^{(3)}+c_{-n}^{(3)}b_{-n}^{(1)})\right]|+\rangle_{13}\ , \tag{7}$$

where $|+\rangle$ is the physical vacuum defined by $|+\rangle=c_0c_1|0\rangle$ (Ref. 1). Since $\langle V|$ and $|0\rangle_2$ are BRST invariant, the BRST invariance of $|R_{13}\rangle$ is trivial, that is,

$$(Q^{(1)}+Q^{(3)})|R_{13}\rangle=0\ . \tag{8}$$

It should, however, be noted that the prefactor $(b_0^{(1)}-b_0^{(3)})$ is required to keep the BRST invariance.

In the evaluation of the loops we also have to define the trace of the inner product such as $O_{3\bar 1}=\langle V_{1'23}|O^{(3)}|R_{11'}\rangle$, which may be defined in terms of the coherent states as

$$\begin{aligned}{\rm Tr}^{13}[O_{3\bar1}]\equiv&\int\prod_{n=1}^{\infty}(d^2\xi^{(1)}d^2\xi^{(3)}e^{-(\bar\xi_n^{(1)}\xi_n^{(1)}+\bar\xi_n^{(3)}\xi_n^{(3)})})_A\,\langle\bar\xi^{(3)}|\otimes{}_1\langle\bar\xi^{(1)}|O_{3\bar1}^X|\xi^{(3)}\rangle_3\otimes|\xi^{(1)}\rangle_A\\&\times\int\prod_m^{\infty}(d\bar\beta_m^{(1)}d\gamma_m^{(1)}d\bar\beta_m^{(3)}d\gamma_m^{(3)}e^{-(\bar\beta_m^{(1)}\gamma_m^{(1)}+\bar\beta_m^{(3)}\gamma_m^{(3)})})\\&\times\int\prod_l^{\infty}(d\bar\gamma_l^{(1)}d\beta_l^{(1)}d\bar\gamma_l^{(3)}d\beta_l^{(3)}e^{-(\bar\gamma_l^{(1)}\beta_l^{(1)}+\bar\gamma_l^{(3)}\beta_l^{(3)})})\\&\times{}_A\langle\bar\beta^{(3)},\bar\gamma^{(3)}|\otimes{}_1\langle-\bar\beta^{(1)},-\bar\gamma^{(1)}|O_{3\bar1}^{\rm gh}|\beta^{(3)}\gamma^{(3)}\rangle\otimes|\beta^{(1)},\gamma^{(1)}\rangle_A\ ,\end{aligned} \tag{9}$$

where $O_{3\bar1}\equiv O_{3\bar1}^XO_{3\bar1}^{\rm gh}$, the coherent states follow the usual conventions,[4] and $|\xi\rangle_A$ and $|\beta,\gamma\rangle_A$ represent the coherent states written by auxiliary oscillators $a_n^{(A)}$, $b_n^{(A)}$, and $c_n^{(A)}$ which do not couple with any other oscillators. The contraction in the auxiliary Fock spaces automatically yields the identification of $a_n^{(1)}=a_n^{(3)}$, $b_n^{(1)}=b_n^{(3)}$, and $c_n^{(1)}=c_n^{(3)}$. It is noted that the minus sign of $\langle-\bar\beta^{(1)},-\bar\gamma^{(1)}|$ must be introduced to derive the right trace over the ghost states.

(Although the identity operator of the coherent states for one fermionic operator ψ is give by $\int d\bar\alpha\, d\alpha\, e^{-\bar\alpha\alpha}|\alpha\rangle\langle\bar\alpha|$ where $|\alpha\rangle\equiv e^{-\alpha\psi^\dagger}|0\rangle$ and $\bar\alpha|=\langle0|e^{\bar\alpha\psi}$ with $\langle0|0\rangle=1$, the trace of O must be done $\int d\bar\alpha\, d\alpha\, e^{-\bar\alpha\alpha}\langle-\bar\alpha|O|\alpha\rangle$ because of the antiperiodic boundary condition for ψ. The consistency of the definition is easily checked by setting $O=1$, i.e., $\int d\bar\alpha\, d\alpha\, e^{-\bar\alpha\alpha}\langle-\alpha|\alpha\rangle=2$ gives the right answer, while $\int d\bar\alpha\, d\alpha\, e^{-\bar\alpha\alpha}\langle\bar\alpha|\alpha\rangle=0$ is wrong.)

Following the definition (9), we can easily derive the well-known result

$$\begin{aligned}&{\rm Tr}^{13}\left[{}_3\left\langle\bar0\right|\exp\left[\sum_{y=a,b,c}(y^{(1)\dagger}A_yy^{(3)}+y^{(1)\dagger}B_y+C_yy^{(3)})\right]\left|\bar0\right\rangle_1 :O^X(a^{(3)\dagger}a^{(3)})::O^{\rm gh}(b^{(3)\dagger}c^{(3)};c^{(3)\dagger},b^{(3)}):\right]\\&\qquad={\rm Tr}\left[:\exp\left[\sum_{y=a,b,c}[y^\dagger(A_y-1)y+y^\dagger B_y+C_yy]\right]::O^X(a^\dagger a)::O^{\rm gh}(b^\dagger,c;c^\dagger,b):\right]\ ,\end{aligned} \tag{10}$$

where $|\bar0\rangle_1$ and ${}_3\langle\bar0|$ should be chosen as $\langle\bar0|\bar0\rangle=1$, i.e., $\langle\bar0|0\rangle$ or $\langle+|-\rangle$ for the ghost oscillators, the creation and annihilation operators and the normal-ordered product have to be defined on the vacuum $|\bar0\rangle$ and the coefficient matrices A_y and also the coefficient vectors B_y and C_y do not contain any oscillator variables.

A. Planar tadpole operator (T)

The simplest formula for computing the planar tadpole operator is given by

$$\langle T_2|={\rm Tr}^{13}[\langle V_{1'23}|\Omega^{(3)\dagger}D^{(3)}b_0^{(3)}(-1)^{N_c^{(3)}+1/2}|R_{11'}\rangle]\ , \tag{11}$$

where the ghost-number operator[1] is defined by

$$N_c=\tfrac{1}{2}(c_0b_0-b_0c_0)+\sum_{n=1}^{\infty}(c_{-n}b_n-b_{-n}c_n) \tag{12}$$

and $b_0^{(3)}$ has to be inserted because the expectation values between ${}_3\langle x\,|$ and $|\,x\rangle_1$ should have no ghost number. The insertion of $(-1)^{N_c+1/2}$ may be understood as the change of the antiperiodic boundary condition to the periodic one for the ghost.[6] It is noted that $(-1)^{N_c+1/2}$ anticommutes with $Q^{(3)}$, $b^{(3)}$, and $c^{(3)}$, and commutes with all operators constructed from $L_n^{(3)}$ and the oscillators except $b^{(3)}$ and $c^{(3)}$, whereas $b_0^{(3)}$ does not anticommute with $Q^{(3)}$. This indicates that the b_0 insertion is not BRST invariant. We can, however, show that in the tadpoles it is effectively BRST invariant by using Feynman's tree theorem. Following the argument of Freeman and Olive,[5] we can write $\langle T_2\,|\,Q^{(2)}$ as

$$\begin{aligned}\langle T_2\,|\,O^{(2)}&=\mathrm{Tr}^{13}[\delta_{L_0^{(3)},0}\langle V_{1'23}\,|\,Q^{(2)}\Omega^{(3)\dagger}b_0^{(3)}(-1)^{N_c^{(3)}+1/2}\,|\,R_{11'}\rangle]\\&=\mathrm{Tr}^{13}[\delta_{L_0^{(3)},0}\langle V_{1'23}\,|\,\Omega^{(3)\dagger}b_0^{(3)}(-1)^{N_c^{(3)}+1/2}(-Q^{(1)}-Q^{(3)})\,|\,R_{11'}\rangle]\\&\quad+\mathrm{Tr}^{13}[\delta_{L_0^{(3)},0}\langle V_{1'23}\,|\,\Omega^{(3)\dagger}L_0^{(3)}(-1)^{N_c^{(3)+1/2}}\,|\,R_{11'}\rangle]\,,\end{aligned} \tag{13}$$

where the Eqs. (3) and (8) are used in the derivation of the first term and the last term with $L_0^{(3)}$ derived from $\{Q^{(3)},b_0^{(3)}\}=L_0^{(3)}$ vanishes because of $\delta_{L_0^{(3)},0}$. By making use of the definition of the trace (9) we can change $Q^{(1)}$ to $-Q^{(3)}$ as

$$\begin{aligned}\mathrm{Tr}^{13}[\delta_{L_0^{(3)},0}\langle\cdots Q^{(1)}\,|\,R_{11'}\rangle]&=\mathrm{Tr}^{13}[\delta_{L_0^{(3)},0}(-Q^{(1)})\langle\cdots\rangle]\\&=\mathrm{Tr}^{13}[\delta_{L_0^{(3)},0}\langle\cdots(-Q^{(3)})\,|\,R_{11'}\rangle]\,.\end{aligned}$$

Then we have

$$\langle T_2\,|\,Q^{(2)}=0\,. \tag{14}$$

That is, the tadpole operator defined by (11) is BRST invariant. It should be stressed that the insertion of $(-1)^{N_c^{(3)}+1/2}$ is essential to keep the BRST invariance.

The explicit expression of $\langle T_2\,|$ is given by

$$\langle T_2\,|=\int_0^1\frac{dx}{x}\langle T_2^X\,|\,\langle T_2^{\mathrm{gh}}\,|\;, \tag{15}$$

where $\langle T_2^X\,|$ and $\langle T^{\mathrm{gh}}\,|$ stand for the contributions of the X part and ghost part, respectively, and are evaluated as follows:

$$\langle T_2^X\,|=\left[\frac{2\pi}{\ln x}\right]^{D/2}\prod_{n=1}^{\infty}(1-x^n)^{-D}{}_2\langle 0\,|\,e^{(a^{(2)}|\,A(x)\,|\,a^{(2)})} \tag{16}$$

with

$$A(x)_{nm}=\frac{1}{2\sqrt{nm}\,\ln x}+\frac{1}{2}\left[M_+\frac{(x)}{1-(x)}M_-+M_-^T\frac{(x)}{1-(x)}M_+\right]_{nm}$$

for $n,m\geq 1$ and

$$\begin{aligned}\langle T^{\mathrm{gh}}\,|&=\frac{1}{x}\prod_{n=1}^{\infty}(1-x^n)^2{}_n\langle +\,|\,\exp\{-[c^{(2)}\,|\,F(x)\,|\,b^{(2)}]\}\\&\quad-\sum_{n,m=2}C_n^{(2)}[G_{nm}(x)mb_0^{(2)}+H_{nm}(x)b_1^{(2)}]+\frac{x}{1-x}\sum_{n=2}\left[b_0^{(2)}+(b_0^{(2)}-b_1^{(2)})+\left[\frac{n}{2}\right]\right]c_n^{(2)}+b_0^{(2)}\sum_{n=1}c_n^{(2)}\end{aligned} \tag{17}$$

with

$$F(x)_{nm}=\left[M_+\frac{(x)}{1-(x)}M_-+M_-^T\frac{(x)}{1-(x)}M_+\right]_{nm}\,,$$

$$G(x)_{nm}=\left[M_+\frac{(x)}{1-(x)}+M_-^T\frac{(x)}{1-(x)}\right]_{nm}\,,$$

and

$$H(x)_{nm}=\left[M_-^T\frac{(x)}{1-(x)}\right]_{nm}\begin{bmatrix}m\\2\end{bmatrix}-\left[M_+\frac{(x)}{1-(x)}\right]_{nm}\begin{bmatrix}m+1\\2\end{bmatrix},$$

for $n,m\geq 2$. The notation in (16) follows that of Ref. 7 as

$$(a\mid A\mid a)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}a_nD^{(0,0)}(A)_{nm}a_m\ ,$$

$$D^{(0,0)}(M_+)_{nm}=\sqrt{m/n}\begin{bmatrix}n+m-1\\m\end{bmatrix},$$

$$D^{(0,0)}(M_-)_{mn}=(-1)^m\sqrt{m/n}\begin{bmatrix}n\\m\end{bmatrix},$$

$$D^{(0,0)}(M_-^T)_{nm}=(-1)^n\sqrt{n/m}\begin{bmatrix}m\\n\end{bmatrix},$$

where $D^{(0,0)}$ differs from $\mathcal{D}^{(0,0)}$ only by the normalization, while, in (17),

$$(c\mid F\mid b)=\sum_{n=2}^{\infty}\sum_{m=2}^{\infty}c_n\mathcal{D}^{(1,0)}(F)_{nm}b_m\ ,$$

$$\mathcal{D}^{(1,0)}(M_+)_{nm}=\begin{bmatrix}n+m-1\\m+1\end{bmatrix},$$

$$\mathcal{D}^{(1,0)}(M_-)_{nm}=(-1)^m\begin{bmatrix}n+1\\m+1\end{bmatrix}$$

$$\mathcal{D}^{(1,0)}(M_-^T)_{n,m}=(-1)^n\begin{bmatrix}m-2\\n-2\end{bmatrix}.$$

In both equations $((x)/[1-(x)])_{nm}=[x^n/(1-x^n)]\delta_{nm}$ is used.

In the above calculation we can derive the following useful formulas. The multiple law of an operator O written in terms of L_0 and $L_{\pm 1}$ to $\langle V\mid$ is simply described as

$$\langle V_{123}\mid O^{(i)}={}_{123}\langle\tilde{0}\mid\exp\left[-\sum_{\substack{r=1\\(r\neq s)}}^{3}\sum_{s=1}^{3}(c^{(r)}\mid\tilde{U}_r\tilde{V}_s\mid b^{(s)})\right]\times\prod_{n=0,\pm1}\left[\sum_{s=1}^{3}\sum_{m=0,\pm1}\mathcal{D}^{(1,0)}(\tilde{V}_s)_{nm}b_m^{(s)}\right],\tag{18}$$

where $\tilde{U}_t=U_t$ and $\tilde{V}_t=V_t$ for $t\neq i$ and $\tilde{U}_t=O^{(i)\dagger}U_t$ and $\tilde{V}_t=V_tO^{(i)}$ for $t=i$. The trace for the operator with $(-1)^{N_c+1/2}$ can be reduced to a simple form

$$\mathrm{Tr}[O(-1)^{N_c+1/2}O']=\int\prod_n(d\bar{\beta}_nd\gamma_ne^{-\bar{\beta}_n\gamma_n})\prod_m(d\bar{\gamma}_md\beta_me^{-\bar{\gamma}_m\beta_m})\times\langle\bar{\beta}\bar{\gamma}\mid O'O\mid\beta\gamma\rangle\tag{19}$$

by inserting the identity operator between O and $(-1)^{N_c+1/2}$. This equation shows that the BRST-invariant trace over the ghosts with the insertion of $(-1)^{N_c+1/2}$ may be represented with the trace formula defined by the coherent states with the periodic boundary condition. It is well known in the statistical mechanics of covariant gauge theory.[6]

B. Nonorientable tadpole operator (^NT)

The nonorientable tadpole operator shown in Fig. 2 is evaluated as

$$\langle {}^NT_2\mid=\mathrm{Tr}^{13}[\langle V_{1'23}\mid\Omega^{(3)\dagger}D^{(3)}b_0^{(3)}\Omega^{(3)}\times(-1)^{N_c+1/2}\mid R_{11'}\rangle]\ .\tag{20}$$

The BRST invariance of ^NT is easily proved in terms of Feynman's tree theorem as was done in A. The operator ^NT is also described as

$$\langle {}^NT_2\mid=\int_0^1\frac{dx}{x}\langle {}^NT_2^X\mid\langle {}^NT_2^{\mathrm{gh}}\mid\ ,\tag{21}$$

where

$$\langle {}^NT_2^X\mid=\left[\frac{2\pi}{\ln x}\right]^{D/2}\prod_{n=1}[1-(-x)^n]^{-D}\times{}_2\langle 0\mid e^{(a^{(2)}\mid B(-x)\mid a^{(2)})}$$

with

$$B(x)_{nm}=(1+x)^nA(-x)_{nm}(1+x)^m\tag{22}$$

and

$$\langle {}^NT_2^{\mathrm{gh}}\mid=\frac{1}{x(1+x)}\prod_{n=1}[1-(-x)^n]^2\times{}_2\langle +\mid\exp[-(c^{(2)}\mid J(x)\mid b^{(2)})-(c^{(2)}\mid B_0)b_0^{(2)}-(c^{(2)}\mid B_1)b_1^{(2)}+b_0^{(2)}(1+x)c_1^{(2)}]\tag{23}$$

with

$$J(x)_{nm}=(1+x)^nF(-x)_{nm}(1+x)^m\ ,$$

$$(B_0)_n=\sum_{m=2}\left\{(1+x)^n\left[M_+\frac{(-x^2)}{1-(-x)}M_-\right]_{nm}\left[\frac{1+x}{x}\right]^m\left[m-x-mx^{m+1}-\frac{1-x-x^2}{1+x}x^m\begin{bmatrix}m+1\\2\end{bmatrix}\right]+(1+x)^n\left[M_-^T\frac{(-1)}{1-(-x)}M_-^T\right]_{nm}\left[\frac{x}{1+x}\right]^n(m+x)\right\}+\left\{1-\frac{x}{1+x}-\sum_{m=2}(M_+)_{nm}x^m\left[mx+\frac{1-x-x^2}{1+x}\begin{bmatrix}m+1\\2\end{bmatrix}\right]\right\},$$

$$
\begin{aligned}
(B_1)_n = \sum_{m=2} \Bigg[& -(1+x)^n \left[M_+ \frac{(-x^2)}{1-(-x)} M_- \right]_{nm} \left[\frac{1+x}{x} \right]^m \left[1-\frac{x^2}{1+x} \right] \begin{bmatrix} m+1 \\ 2 \end{bmatrix} \\
& +(1+x)^n \left[M_-^T \frac{(-1)}{1-(-x)} M_-^T \right]_{nm} \left[\frac{x}{1+x} \right]^m \begin{bmatrix} m \\ 2 \end{bmatrix} \Bigg] \\
& + \frac{1}{1+x} \left[x \begin{bmatrix} n \\ 2 \end{bmatrix} + \sum_{m=2} (M_+)_{nm} x^m \begin{bmatrix} m+1 \\ 2 \end{bmatrix} \right] ,
\end{aligned}
$$

and the same matrices as were used in (17) should be taken for $M_\pm$ and M_-^T.

It is noticeable that attaching an N-tachyon state constructed on the vacuum $|-\rangle$ to the tadpole operators derives the well-known one-loop N-point formulas as illustrated in Fig. 3. Since the details of the derivation were given in the paper by Gross and Schwarz,[8] we do not give them here. It is noted, however, that in their derivation the definitions of the variables should be changed as follows:

$$
\begin{aligned}
&\zeta_j = 1-(1-\omega)\rho_j \rightarrow \zeta_j = 1-\rho_j \text{ for the planar loop ,} \\
&\zeta_j = 1-(1-\omega^2)\rho_j \rightarrow \zeta_j = 1-(1+x)\rho_j \\
&\qquad\qquad \text{for the nonorientable loop .}
\end{aligned} \tag{24}
$$

It is noted that the factor $(1+x)^{-1}$ in ${}^N T^{\mathrm{gh}}$ cancels out the Jacobian factor $(1+x)$ associated with the above change of variables in the nonorientable loop. Both loops, of course, have the right measures.

In the extension of the BRST-invariant calculation to general diagrams a difficulty arises from the b_0 insertion, because b_0 effectively anticommutes with Q only in one-loop amplitudes where $\delta_{L_0,0}$ in the Feynman tree theorem saves it. Remembering the relations

$$\{Q, b_0 - b_1\} = L_0 - L_1$$

and

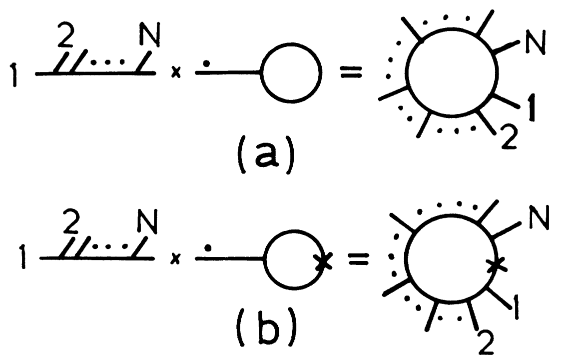

FIG. 3. Diagrams where an N-tachyon state is joined to the (a) planar and the (b) nonorientable tadpoles. They coincide with the usual one-loop N-point formulas.

$$(L_0 - L_1) \int_0^1 \frac{dx}{x(1-x)} P(x) ,$$

where $P(x) \equiv x^{L_0} \Omega (1-x)^W$ with $W = L_0 - L_1$, we find out one possibility of the BRST-invariant b_0 insertion by using

$$D^T \equiv (b_0 - b_1) \int_0^1 \frac{dx}{x(1-x)} P(x)$$

instead of Db_0, which was used in the recent paper by DiVecchia, Frau, Lerda, and Sciuto.[7] This operator D^T is nothing but the multiple of the $(b_0 - b_1)$ factor to the twisted propagator. In this choice the tadpole operators are expressed as

$$\langle T_2 | = \mathrm{Tr}^{13} [\langle V_{1'23} | D^{T(3)}(x) (-1)^{N_c + 1/2} | R_{11'} \rangle] , \tag{25}$$

$$\langle {}^N T_2 | = \mathrm{Tr}^{13} [\langle V_{1'23} | D^{T(3)}(x) \Omega^{(3)} (-1)^{N_c + 1/2} | R_{11'} \rangle] . \tag{26}$$

Equivalence of (25) and (26) to our (11) and (20), may be shown in the following consideration: We have an operator identity in SL(2,R)

$$(b_0 - b_1) P(x) = \Omega^\dagger (-z)^{L_0} b_0 \Omega , \tag{27}$$

where $z = x/(1-x)$. Since $\Omega = (-1)^{L_0} e^{-L_{-1}}$, we rewrite (27) as

$$(b_0 - b_1) P(x) = \Omega^\dagger (z)^{L_0} b_0 e^{-L_{-1}} .$$

Considering that the factor $e^{-L_{-1}}$ may be removed because it represents a conformal transformation continuously connected to the identity, we may consider that D^T is effectively equivalent to

$$\Omega^\dagger \int_0^1 dz \, z^{L_0 - 1} b_0 = \Omega^\dagger D b_0$$

which is nothing but the operator used in our formulas. Note that the replacement of D^T with $\Omega^\dagger D b_0$ makes calculations very simple as was done in our evaluations.

Now we can calculate arbitrary diagrams in terms of the operators $\langle V_{123} |$, D^T, Ω, $(-1)^{N_c + 1/2}$, and $| R_{13} \rangle$. The details for the BRST-invariant diagrammatic oscillator formalism will be discussed in a future paper.

After the completion of our evaluations, we found the paper by DiVecchia, Frau, Lerda, and Sciuto.[7] In their work and also that by LeClair,[4] however, the necessity of the $(-1)^{N_c + 1/2}$ insertion is not discussed and the BRST invariance of the definition of the reflection, i.e., $b_n \rightarrow b_n^\dagger$,

$c_n \rightarrow -c_n^\dagger$, and $\langle \tilde{0} | \rightarrow | \tilde{0} \rangle$, in Ref. 7 and the b_0 insertion to general diagrams by LeClair are not clear.

Recently Cristofano, Nicodemi, and Pettorino derived the planar tadpole formula[9] by using the method given in Ref. 7. There is some difference between their formula and our Eq. (17), especially in the expression for the ghost modes. The difference due to that between the technical terms used there is not essential, e.g., the difference between the propagators and also that between the vacuum $\langle \tilde{0} |$ and $\langle + |$. It should, however, be noted that the BRST invariance of their formula is only recovered when their formula is attached to physical states, whereas ours is manifestly BRST invariant as was shown in (14). The main difference for the expression of b_0 and b_1 modes in the exponent arises from this point. Of course, both formulas derive the same results for arbitrary physical quantities. An example has already been shown by the derivation of the one-loop N-point formula, where the difference between the definition of the variables noted in (24) has to be taken into account.

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