Inhomogeneous nucleosynthesis with neutron diffusion

Hannu Kurki-Suonio
Department of Physics and Atmospheric Science, Drexel University, Philadelphia, Pennsylvania 19104

Richard A. Matzner
Center for Relativity and Department of Physics, The University of Texas, Austin, Texas 78712

Joan M. Centrella
Department of Physics and Atmospheric Science, Drexel University, Philadelphia, Pennsylvania 19104

Tony Rothman
Center for Relativity and Department of Physics, The University of Texas, Austin, Texas 78712

James R. Wilson
Lawrence Livermore National Laboratories, Livermore, California 94550
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It was recently proposed that a critical, i.e., closure, baryon density could be compatible with helium and deuterium observations, if this baryon density were suitably inhomogeneous during cosmic nucleosynthesis. In this scenario, neutrons would diffuse out of the high-density regions, which would lead to a nonstandard nucleosynthesis, with reduced helium production and improved deuterium survival, and could restore the agreement with observations, which is lost in *homogeneous* closure models. We have used a numerical model which shows that this proposal is not viable. Our model combines inhomogeneous nucleosynthesis and neutron diffusion self-consistently to study element formation in inhomogeneous situations in which diffusion is important. We find that the proposal fails: it is very difficult to lower the helium production significantly or to raise the final deuterium abundance, because the neutrons diffuse back to the high-density region once nucleosynthesis begins there.

I. INTRODUCTION

Recent developments have cast doubt on one of the basic tenets of cosmology. It had been thought that cosmic nucleosynthesis results demand that the average density of baryonic matter in the Universe is, within a factor of 2, $\rho_b \approx 0.3\times10^{-30}$ g/cm^3 ($\eta \equiv n_b/n_\gamma \approx 4.5\times10^{-10}$) (Ref. 1), which is at least an order of magnitude less than the critical density, $\rho_c = h_0^2 18.7\times10^{-30}$ g/cm^3, where h_0 is the Hubble constant in units of 100 (km/s)/Mpc. There is significant (gravitational) evidence that the total density of the Universe is higher, perhaps even in the range of the critical density, and this discrepancy had led to the postulate that the mass of the Universe is dominated by nonbaryonic matter. Popular theoretical prejudice expects the total density to be just the critical density, but we do not know what this nonbaryonic matter is, and there is no independent evidence for it.

The result from standard homogeneous nucleosynthesis has appeared fairly solid. Attempts at various modifications of it have not been persuasive.[2] An upper limit on the observational cosmic abundance of ^{4}He and a lower limit on ^{2}H both place an upper limit on ρ_b. Adding inhomogeneity tends to raise[3] both ^{2}H and ^{4}He, so that while the limit from ^{2}H is relaxed, the limit from ^{4}He becomes tighter.[4] It has been especially difficult to reduce ^{4}He production in a natural scheme so that higher densities could be allowed.

With this background, the results on nucleosynthesis with neutron diffusion[5–8] appear most remarkable. Prior to nucleosynthesis time, the mean free path of neutrons is much longer than that of protons (because of the Coulomb interaction of protons with the thermal electron-positron plasma). Suppose there were then strong inhomogeneities in the baryon number with a distance scale of present light-hours. [Since the Universe is expanding, distance scales, and densities, have to be specified at a certain moment. We consistently give the baryon density ρ_b at the present moment, $T_\gamma = 2.7$ K. A present light-hour (light-year) corresponds to 180 m (1600 km) at $T = 1$ MeV and to approximately 1 m (10 km) at $T = 100$ MeV.] The neutrons would have diffused out of the high-density regions before nucleosynthesis, whereas the protons would not have had enough time to do so. The low-density regions would have become neutron rich and the high-density regions neutron poor. Nucleosynthesis in these conditions would be very different from the standard case. Applegate, Hogan, and Scherrer[6] have predicted that deuterium would be significantly raised and ^{4}He reduced when compared to a homogeneous model with the same average density. This scenario appeared to naturally reconcile the observed primordial ^{2}H and ^{4}He abundances with a critical baryon density.

A possible cause for the inhomogeneity could be the putative quark-hadron phase transition at $T=100-200$ MeV (Refs. 9, 5, 10, and 11). That just the right distance scale for neutron diffusion to be effective seems to appear "magically" in this transition,[12,13] is another striking coincidence. One reason why no one previously considered diffusion important to nucleosynthesis is that the distance scale involved is so small; much larger-scale inhomogeneities were usually contemplated.[3]

The preceding scheme appears to have some weaknesses. One is the high ^{7}Li abundance that is produced. Another is that the baryon inhomogeneity required is rather dramatic. That the phase transition would be capable of causing some baryon-number separation seems quite possible,[14] but in our opinion[11] no really convincing detailed scenario has been presented which would lead to sufficiently strong inhomogeneity as a final result.

On the other hand, if diffusion in a strongly inhomogeneous model really does bring into agreement a critical ρ_b and observed primordial element abundances, this is remarkable enough to cause us to take seriously the possibility that such inhomogeneity existed (whether caused by the quark-hadron transition or some other process). And from this point of view, the presence of baryon-separating effects and the appearance of the "correct" spatial scale are very suggestive features of the quark-hadron transition.

This whole remarkable but speculative structure is of course based on the correctness of the prediction of agreement between observations, and cosmic nucleosynthesis with critical ρ_b. In contrast with the physics of the quark-hadron transition, the physics at the time of nucleosynthesis is relatively well understood (although some of the reaction rates are not very well known). Nucleosynthesis is a complicated process and the effects of diffusion on it are not immediately obvious. The simple model,[6,7] which treated diffusion and nucleosynthesis separately, is not sufficient for a conclusive result. Therefore it is imperative to do more detailed nucleosynthesis calculations in which the diffusion is consistently combined with nucleosynthesis. We have now done a sequence of such computations and report here on the results.

II. DESCRIPTION

The standard homogeneous model of the cosmic nucleosynthesis gives good agreement with observational data on the primordial light-element abundances for a baryon density $\rho_b=0.3\times10^{-30}$ g/cm^3. (All our theoretical models use *three* light neutrino flavors.) A much larger baryon density leads to an overproduction of ^{4}He (because in a dense model nucleosynthesis begins at a higher temperature, when there are more neutrons) and underproduction of deuterium (because deuterium is burned to ^{4}He faster). See Table I.

Inhomogeneity without diffusion does not help in allowing higher baryon densities. A simple inhomogeneous model treats high- and low-density regions independently and obtains the final abundances by averaging the high- and low-density results. Comparing to a homogeneous model with the same average density, the deuterium abundance tends to increase considerably, thus relaxing the upper limit to ρ_b from deuterium. However, ^{4}He is also raised, and a high ρ_b remains unacceptable because of the ^{4}He overproduction.

Let us now consider a specific example. [This example will be the starting point of our diffusion computation. The parameters were chosen to be a representative case of Alcock, Fuller, and Mathews[7] (AFM).] Assume an average baryon density $\rho_b=4.0\times10^{-30}$ g/cm^3, but distributed inhomogeneously so that one quarter of space has a high density $\rho_b=15\times10^{-30}$ g/cm^3, and the remaining three quarters have a low density $\rho_b=0.3\times10^{-30}$ g/cm^3. (This kind of inhomogeneity is of course highly idealized, but in this way the effects of inhomogeneity are revealed the most clearly.) When we do homogeneous nucleosynthesis separately in each region and then find the weighted average, we see that the high-density region dominates the average because of its larger weight, shifting the results toward higher-density homogeneous-model values. ^{2}H is an exception; because ^{2}H increases so dramatically with decreasing ρ_b, there is enough production of deuterium in the low-density region to raise even the averaged value.

In the preceding we assumed that the inhomogeneity remained constant in time. Note that we are discussing

TABLE I. Observational values and predicted values from homogeneous runs. We show the light element abundances for several values of the baryon density ρ_b (given as the present value in units 10^{-30} g/cm^3). The observed values (ranges) are given in parentheses below each heading. We compare three different density "standard" homogeneous models, an inhomogeneous model with no diffusion, and an inhomogeneous "simple-diffusion" model. The last two models are actually just averages from pairs of homogeneous runs.

		^{2}H ($\geq2\times10^{-5}$)	^{3}He (10^{-5}–10^{4})	^{4}He (0.22–0.25)	^{7}Li ($\geq5\times10^{-10}$)
Homogeneous	$\rho_b=0.3$	6.3×10^{-5}	3.0×10^{-5}	0.254	8.5×10^{-10}
	$\rho_b=4.0$	7.4×10^{-9}	9.5×10^{-6}	0.276	3.9×10^{-8}
	$\rho_b=15.0$	8.0×10^{-18}	5.8×10^{-6}	0.287	9.5×10^{-8}
Inhomogeneous	$\rho_{b,\mathrm{avg}}=4.0$	3.5×10^{-6}	7.1×10^{-6}	0.285	8.9×10^{-8}
simple diffusion	$\rho_{b,\mathrm{avg}}=4.0$	5.5×10^{-5}	9.4×10^{-6}	0.226	7.6×10^{-8}

the inhomogeneity in the baryon density, not the energy density. In the early Universe the baryons contributed an insignificant fraction to the total energy density, and thus were dynamically unimportant. Therefore the baryon inhomogeneity evolves only through diffusion. At large scales, e.g., the horizon scale at nucleosynthesis, diffusion is insignificant. If we have a very small scale inhomogeneity, diffusion will eliminate it before nucleosynthesis, and we return to the homogeneous case. However, because neutrons diffuse much faster than protons, there is a large range of intermediate scales where neutrons have time to diffuse out of the high-density regions but protons do not. Dynamical inhomogeneities have been treated elsewhere;[3] they can affect nucleosynthesis through time-scale effects, but the most significant effect usually comes from the inhomogeneity in n_b/n_γ, essentially the baryon inhomogeneity. Only diffusion treats neutrons and protons differently, and thus can directly change the n/p ratio to which nucleosynthesis is sensitive. Thus diffusion can lead to more dramatic results than were obtained in earlier work on inhomogeneous models.

Following Applegate, Hogan, and Scherrer[6] (AHS) and Alcock, Fuller, and Mathews[7] (AFM), let us imagine that diffusion spreads the neutrons evenly just prior to nucleosynthesis, but does not affect the proton distribution. Apply this to our previous example. Before nucleosynthesis the neutron mass fraction is about 15%. Thus, without diffusion, the high-density region would have a proton density 12.75×10^{-30} g/cm^3 and a neutron density 2.25×10^{-30} g/cm^3. The low-density region would have 0.225×10^{-30} g/cm^3 in protons and 0.045×10^{-30} g/cm^3 in neutrons. With diffusion, the proton densities are unaffected, but the neutron density is now 0.6×10^{-30} g/cm^3 everywhere. See Fig. 1. (All densities are given normalized to present values. The actual baryon density at that time is not too far from that of air at room temperature and pressure.) This makes the low-density region (which now has $\rho_b=0.855\times10^{-30}$ g/cm^3) extremely neutron rich, with a 70% neutron fraction; i.e., there are more than twice as many neutrons as protons. The high-density region (now $\rho_b=13.35\times10^{-30}$ g/cm^3) is now neutron poor, with only 4.5% neutrons.

If we assume that homogeneous nucleosynthesis subsequently occurred separately in the two regions (we call this the simple-diffusion model, in contrast with our self-consistent-diffusion model), the neutron-poor high-density region obviously cannot produce more than 9% helium. In the low-density neutron-rich region neutrons outnumber protons by more than two to one; thus, in contrast with the standard model, helium production here is *proton* limited. When nucleosynthesis begins at 10^9 K, the entire proton fraction will be processed into ^{4}He, giving about 60% ^{4}He, and about 40% excess neutrons.

Eventually when half of the excess neutrons decay into protons almost 100% ^{4}He will result in this region. But note that compared to the standard model the total number of neutrons in the Universe available for nucleosynthesis has now been reduced by this "late-decay" process. When the global ^{4}He abundance is computed as twice the n fraction, ^{4}He must then be lowered. Furthermore, since production of part of the helium is delayed, this results in more ^{2}H surviving. Thus we have simultaneously relaxed both (^{4}He and ^{2}H) restrictions to high baryon density. As can be seen from Table I, we can now restore the agreement with observations with a high baryon density. This, in short, is the result of AHS and AFM, illustrated with a representative example.

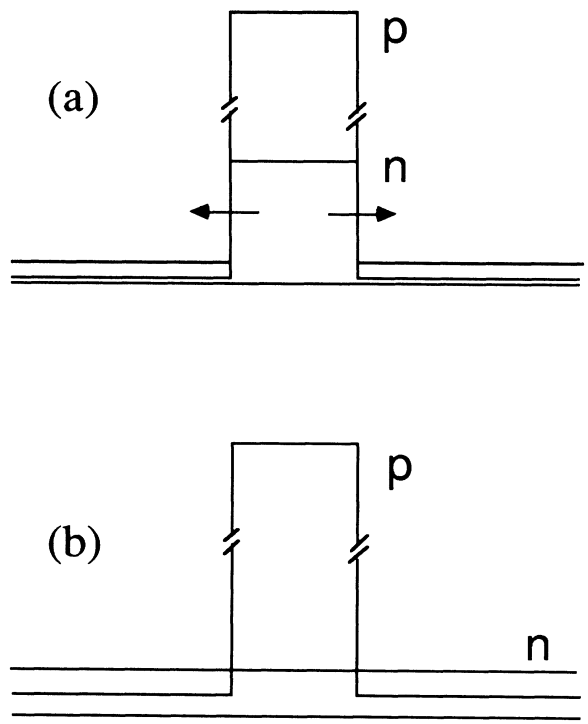

FIG. 1. The simple-diffusion model. Initially we have a strong inhomogeneity in baryon density, but n/p is constant (a). The neutrons then diffuse out of the high-density regions until the neutron distribution becomes homogeneous (b). If the initial inhomogeneity was strong enough, n/p will now exceed 1 in the low-density regions. When nucleosynthesis begins the extra neutrons are not available to form ^{4}He until half of them have decayed to form partners for the remaining ones.

The above simple-diffusion scenario is a drastic simplification because it handles the diffusion and nucleosynthesis separately. First the baryons are allowed to diffuse, with nucleosynthesis turned off, and then the diffusion is ignored after the nucleosynthesis is turned on. In reality both processes are on all the time. We might at first think that with the right distance scale the neutron diffusion would have ceased to be important when nucleosynthesis gets into full swing, since the neutron distribution would have become homogeneous by then (and the proton diffusion would become important only after most of the nucleosynthesis has taken place). However, the neutron-rich–neutron-poor nucleosynthesis will rapidly destroy the (free-)neutron homogeneity, so neutron diffusion remains important. Therefore, what really happens can be found only by doing a self-consistent calculation, with the nucleosynthesis and diffusion handled together. This requires the use of an inhomogeneous nucleosynthesis code.

For several years we have had such a code, which was obtained by combining the nucleosynthesis code developed in Texas by Rothman, Matzner, and Kurki-Suonio[15,16] with the plane-symmetric inhomogeneous cosmology code by Centrella and Wilson.[17] Thus we are not employing the widely used code of Wagoner.[18] The results from our code have been in very good agreement with Wagoner's and others.[1] (The numbers in Table I were obtained with the same version of our code as the results in Table II.) Our value for ^{4}He is slightly higher than that of Ref. 1, due to differences in the value of the neutron lifetime used (we have used $\tau_n = 926$ s) and to a systematic decrease of 0.003 applied to ^{4}He in Ref. 1 to represent higher-order contributions to the weak reaction rates and some other corrections.[19] Our nucleosynthesis code contains all 30 strong reactions listed by Fowler, Caughlan, and Zimmerman[20] and Harris, Fowler, Caughlan, and Zimmerman[21] that involve nuclei with mass numbers $A \leq 7$ only, and their inverse reactions.

We have used this code to study the effect of inhomogeneity in baryon density, energy density, and spacetime curvature on primordial nucleosynthesis.[3,22] Since we had been interested in scales of the order of the nucleosynthesis horizon size, diffusion was not incorporated into the code.

However in order to study the neutron-diffusion scenario we have been forced to include a diffusion scheme. We did not attempt a full general-relativistic diffusion; the diffusive version can be run with a homogeneous background spacetime [Friedmann-Robertson-Walker (FRW)] only. The code now diffuses neutrons and protons and ignores the diffusion of nuclei. That is unlikely to have a large effect, since compound nuclei diffuse even more slowly than protons. On the scales we have run, proton diffusion is fairly unimportant. For more details of the diffusion part of the code, see Matzner, Rothman, Centrella, and Wilson.[23]

The code is plane symmetric with periodic boundary conditions. Thus we imagine the space divided into slabs with different baryon densities. We leave until the Conclusion a discussion of the physical correctness of this approximation, but it is not clear that this is an unnatural geometry of expected baryon inhomogeneity. Here we merely point out that since we do not really know what form the inhomogeneity would have, this is probably as representative as any other simple approximation.

The original code was not written with the diffusion problem in mind; thus our present code is not optimal for this problem and we limited the scope of this study. We did a fairly small number of runs (when compared to the large parameter space of different possibilities), and for most of the runs we used a 20-zone grid. We did a few runs with a 40-zone grid to find that the inaccuracy due to the coarse grid is about 0.001 (absolute) in ^{4}He, and about 10% (relative) in other isotopes. We[24] are in the process of writing a new code specifically aimed at studying the diffusive nucleosynthesis in detail. In the present code small-distance scales lead to a very small time step and therefore computer time placed a limit on the smallness of the length scales we could study. Fortunately we were able to go to distance scales small enough to arrive at clear conclusions.

III. RESULTS

All our runs were for the same standard background spacetime and for the same physical constants. (We assume three massless neutrino flavors.) The runs differ only in the distance covered by the grid and in the baryon density (both its average value and its distribution). Our aim was to see whether we can accommodate a critical baryon density with observed abundances. First we did a series of runs with an average baryon density $\rho_{avg} = 10 \times 10^{-30}$ g/cm^3. This is the critical density for a "compromise" Hubble constant $H_0 = 73$ (km/s)/Mpc. All these runs produced too much helium and too little deuterium to be consistent with accepted observational values. Therefore we did another series of runs with $\rho_{avg} = 4 \times 10^{-30}$ g/cm^3, which is about the smallest density that could be critical—corresponding to $H_0 = 46$ (km/s)/Mpc. (This appears to be close to the AFM value.)

Table II shows the results from the runs. ρ_{avg} is the average baryon density in units of 10^{-30} g/cm^3 (at present). Most of the runs used a simple initial inhomogeneity, where the grid was divided into two regions: one with a constant high density and the other with a constant low density. R is the ratio between the high and low density and f_V is the volume fraction of the high-density region (and 1-f_V that of the low-density region). This is the same notation as in AFM. In run Nos. 13 and 14 the grid was divided into several high- and low-density regions. In run Nos. 6 and 11 we used a rounded density profile with three bumps (the same that was used in Ref. 23) and since their density profile is different from that of the rest of the runs, the quantities R and f_V are not well defined for Nos. 6 and 11, hence an asterisk in the table. d is the distance covered by the grid and thus gives the scale of the inhomogeneity. We give it as a fraction of the initial horizon size. Our runs start, at $T = 3 \times 10^{10}$ K, when the age of the Universe is $t = 0.11$ s. Thus the initial horizon size is 0.22 light-seconds (at that time), which corresponds to 100 light-years at present. In the next column we give this distance scale in present light-years (a) or light-hours (h). A missing entry in the table means that the value directly above was used. The last four columns give the final averaged mass fractions of the isotopes produced in the nucleosynthesis. All runs used a grid of 20 zones, except run Nos. 24b and 26b, which used a finer grid of 40 zones.

Let us focus on the sequence of runs (Nos. 21–28) with $\rho_{avg} = 4 \times 10^{-30}$ g/cm^3. These runs differ from each other only in the distance scale which we made progressively smaller from one run to the next. They all had the same initial density profile: A five-zones-wide region at the center had a high density of $\rho_b = 15 \times 10^{-30}$ g/cm^3 and the remaining 15 zones had a low density of $\rho_b = 0.3 \times 10^{-30}$ g/cm^3. Thus in AFM notation we used $f_V = 0.25$, $R = 50$, which is close to the best case of their Fig. 2(a) in producing the least ^{4}He and the most ^{2}H.

Figures 2–5 are from run Nos. 21, 24b, 26b, and 28. In these figures, plot (a) shows the neutron density, in-

TABLE II. Average nuclear mass fraction abundances resulting from inhomogeneous runs with self-consistent diffusion. A fiducial square density distribution is used with R = ratio of high to low density; f_V = volume fraction of high-density region. Where no entry is given, the entry above is used. The asterisk in two places in the R, f_V columns indicate that those models used a density distribution that did not fit this scheme; see text. Other entries are ρ_{avg}, the averaged baryon density measured as current value in units 10^{-30} g/cm^3, d, the fraction of the horizon covered when the simulation began (at $T = 30\times10^9$ K) and the present scale of the structure measured in light hours (h) or light years (a).

No.	ρ_{avg}	R	f_V	d	Present scale	^{2}H	^{3}He	^{4}He	^{7}Li
6	10.0	*	*	0.0005	200h	3.7×10^{-7}	8.1×10^{-6}	0.286	1.4×10^{-7}
7		100	0.05	0.0005	500h	1.5×10^{-7}	9.0×10^{-6}	0.296	1.3×10^{-6}
8				0.0002	200h	1.4×10^{-7}	9.3×10^{-6}	0.290	1.1×10^{-6}
9				0.00005	50h	1.2×10^{-7}	9.6×10^{-6}	0.277	7.0×10^{-7}
10				0.00002	20h	5.5×10^{-8}	9.7×10^{-6}	0.273	6.6×10^{-7}
11		*	*	0.00005	20h	2.4×10^{-8}	8.6×10^{-6}	0.278	1.5×10^{-7}
12		100	0.15	0.00005	50h	3.7×10^{-7}	1.2×10^{-5}	0.277	3.9×10^{-7}
13			0.3		25h	1.4×10^{-7}	1.1×10^{-5}	0.277	2.8×10^{-7}
14		50	0.25	0.00005	10h	1.3×10^{-8}	9.9×10^{-6}	0.278	3.5×10^{-7}
15		50	0.05	0.00005	50h	1.5×10^{-8}	8.9×10^{-6}	0.279	5.7×10^{-7}
16			0.15		50h	1.9×10^{-7}	1.0×10^{-5}	0.279	3.6×10^{-7}
17			0.25		50h	2.5×10^{-7}	9.8×10^{-6}	0.280	2.4×10^{-7}
20		50	0.25	0.00001	10h	1.9×10^{-8}	9.5×10^{-6}	0.278	2.9×10^{-7}
21	4.0	50	0.25	0.5	50a	3.5×10^{-6}	7.1×10^{-6}	0.285	8.9×10^{-8}
22				0.05	5a	3.5×10^{-6}	7.1×10^{-6}	0.285	9.0×10^{-8}
23				0.005	0.5a	3.5×10^{-6}	8.3×10^{-6}	0.285	1.1×10^{-7}
24				0.0005	500h	3.4×10^{-6}	1.0×10^{-5}	0.282	1.0×10^{-7}
25				0.0002	200h	3.2×10^{-6}	1.1×10^{-5}	0.278	9.6×10^{-8}
26				0.00005	50h	1.7×10^{-6}	1.3×10^{-5}	0.268	1.0×10^{-7}
27				0.00002	20h	8.1×10^{-7}	1.3×10^{-5}	0.271	1.2×10^{-7}
28				0.00001	10h	3.6×10^{-7}	1.2×10^{-5}	0.274	1.3×10^{-7}
24b				0.0005	500h	3.3×10^{-6}	9.1×10^{-6}	0.282	9.3×10^{-8}
26b				0.00005	50h	1.7×10^{-6}	1.2×10^{-5}	0.268	8.8×10^{-8}

cluding the neutrons in nuclei, as a function of spacetime. Plot (b) shows the ^{4}He mass fraction.

Run No. 21 had the largest scale, the grid covering 50 (present) light-years. The results are almost identical to the inhomogeneous (nondiffusion) result in Table I. This is because diffusion is unimportant in this scale. Thus we were justified in ignoring diffusion in our previous work on large spatial scale inhomogeneous nucleosynthesis.[3] The behavior of the model is shown in Fig. 2. Initially the neutron density drops because of decay into protons. This stops when nucleosynthesis takes place and the neutrons find safe haven in helium nuclei [Fig. 2(a)]. From the helium mass fraction [Fig. 2(b)] we see that nucleosynthesis happens much earlier in the high-density region (around $t = 140$ s) than in the low-density region (around $t = 260$ s), and produces a higher ^{4}He abundance.

One has to reduce the inhomogeneity scale by 3 orders of magnitude, to 500 light-hours, before diffusion begins to have a significant effect (Fig. 3). Now we notice a highly increased ^{4}He-fraction produced outside, but close to, the high-density region [Fig. 3(b)]. This is because of the extra neutrons that have diffused here from the high-density region. Because the neutron density is still below the proton density almost everywhere, no significant reduction in final averaged ^{4}He is expected and is not seen.

When the inhomogeneity scale is reduced by another factor of 10, to 50 light-hours (this corresponds to 50 m at $T = 100$ MeV), over half of the neutrons diffuse out of the high-density region (Fig. 4), raising the n/p ratio to about 1.4 in the low-density region. In our smallest scale run, 10 light-hours, the neutron diffusion is complete, making the neutron density homogeneous before nucleosynthesis begins (Fig. 5). According to the simple-diffusion scenario discussed earlier, we would now expect that half of the extra neutrons decay and that almost 100% ^{4}He is produced in the low-density region. What we actually observe is something completely different.

The difference is clearest in this smallest-scale run where the low-density regions produce nowhere near 100% ^{4}He. In fact, they produce only about 10% ^{4}He, much less than in the case without diffusion [Fig. 5(b)]. What happened? We can see that from the neutron density [Fig. 5(a)]. Instead of dropping by half of the n-p excess in the low-density region, it has dropped there by 2 orders of magnitude. Most of the neutrons have gone back to the high-density region. There they have concentrated near its surface, producing a high ^{4}He fraction there.

In retrospect, what has happened is obvious. Neutron diffusion always works towards evening out the distribution of *free* neutrons. Nucleosynthesis begins first in the high-density region. As the neutrons are incorporated into nuclei, they are removed from the distribution. The neutrons in the low-density region are still free and now begin to diffuse back into the high-density region, now low in free neutrons (Fig. 6). Neutrons diffusing back are readily absorbed in forming ^{4}He and thus the high-density regions remain an unsatiable sink, absorbing most of the neutrons that first diffused out—in our smallest-scale run, even most of the original low-density neutrons.

Diffusion actually makes the time interval between the onset of high- and low-density nucleosynthesis shorter, because the density contrast (especially in neutron density) is reduced. However, if the main interest is in the total ^{4}He produced, it is not the back-diffusion time versus this time difference, but versus the neutron decay time that is decisive.

It had already been noted earlier that back diffusion of neutrons would occur,[23,25] although it was not realized how drastic its effects would be. In fact, it was proposed that this might solve the problem of lithium overproduction. (High-density models produce a large primordial ^{7}Li abundance, which is difficult to reconcile with observations. Inhomogeneity just makes this slightly worse, and the simple-diffusion scenario does not help much.) This overproduction is due to the high amounts of ^{7}Be (which later becomes ^{7}Li through β^+ decay or e^- capture) produced in the high-density region. It was suggested that the returning neutrons would destroy some of this ^{7}Be (Ref. 26). If this happens, the effect is small (see Fig. 7). Instead, the back-diffusing neutrons destroy the whole scenario.

In Fig. 8 we plot the final averaged abundances as a function of the inhomogeneity scale. ^{3}He and ^{7}Li are not strongly affected by diffusion effects. When the scale is below $500h$ (i.e., $d=0.0005$), we see effects on ^{4}He and ^{2}H. At first ^{4}He goes down because of decay of out-diffused neutrons. When the scale is reduced further, the back diffusion begins to raise ^{4}He again. At our smallest scale ^{4}He is almost at its homogeneous-model value. Thus there is an intermediate scale where we get the maximum effect depressing the ^{4}He abundance. This corresponds to partial diffusion, where the distance scale (about 50 m at the quark-hadron phase transition, about 50 light-hours now) is small enough to allow many neutrons to diffuse out, but large enough that some of these decay before diffusing back into the high-density region. For ^{2}H, we do not see any increase at all over the mass fraction value obtained in a no-diffusion inhomogeneous model. Instead the ^{2}H abundance begins to drop towards the homogeneous-model value as the scale is made smaller and diffusion becomes important.

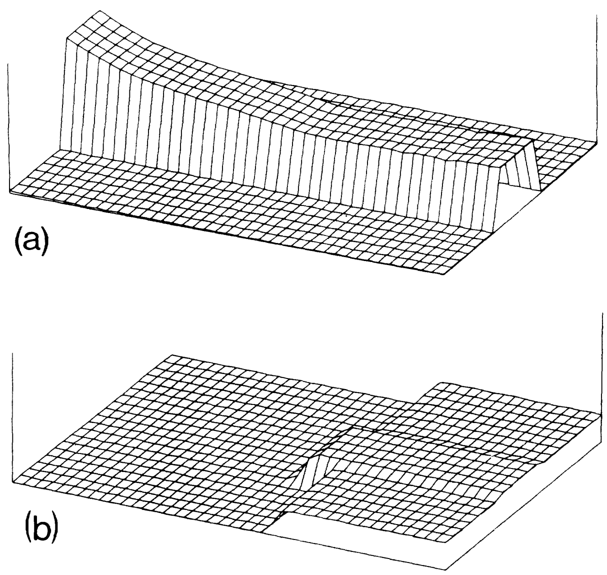

FIG. 2. These plots are from run No. 21. Time goes from left to right (the long edge) and we use a logarithmic scale, so that the left edge corresponds to $t=1$ s, the right edge $t=10\,000$ s, and $t=100$ s is in the middle. The other horizontal direction is space in the direction of inhomogeneity, which we have divided into 20 zones. These zones are comoving, i.e., they expand with the Universe, but we use comoving coordinates here, so the expansion does not show. The left edge has a physical length of 0.3 light-seconds and the right edge 30 light-seconds. These distances are too long for neutrons to diffuse in the time shown. (a) shows the neutron density (vertical scale given by the spikes at the corners of the plot, the height of which corresponds to approximately 6×10^{-30} g/cm^3, present density), i.e., number of neutrons including those bound in nuclei per comoving volume. In our initial data we have divided the space into a high-density region (5 zones) and a low-density region (remaining 15 zones) with a density contrast of 50:1. In the ^{4}He plots (b), the spike height corresponds to 120%. See discussion in the main text.

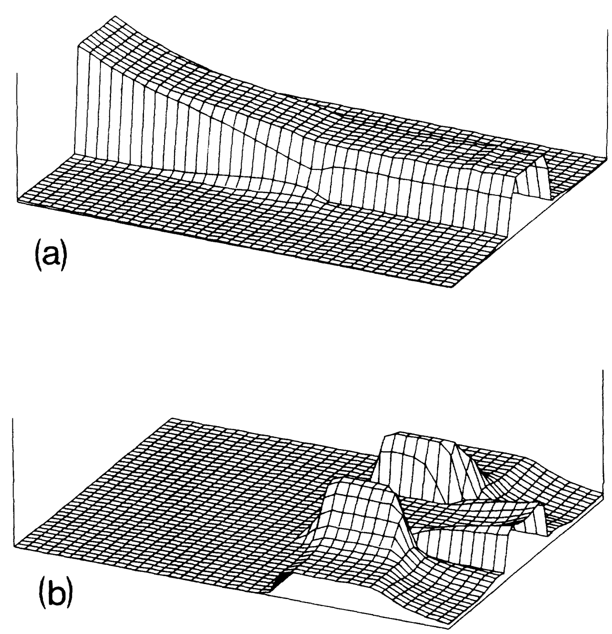

FIG. 3. In run No. 24b, the scale was reduced to $\frac{1}{1000}$ of run No. 21. This run used 40 zones for better resolution. In the neutron density (a) we now see some diffusion. n/p is raised close to 1 in those low-density zones nearest to the high-density region. This has a prominent effect on the ^{4}He fraction (b). In addition to neutrons, our code also diffuses protons, but not the nuclei (with $A\geq2$). The proton diffusion shows up towards the end of the run. It totally changes all local mass fractions [see the effect on ^{4}He in plot (b)] although no reactions take place and thus the average abundances are not affected. Since in reality the nuclei would also diffuse at this point, the end of the plot is not real, but it serves to reveal, approximately, the actual ^{4}He density (instead of mass fraction) produced earlier.

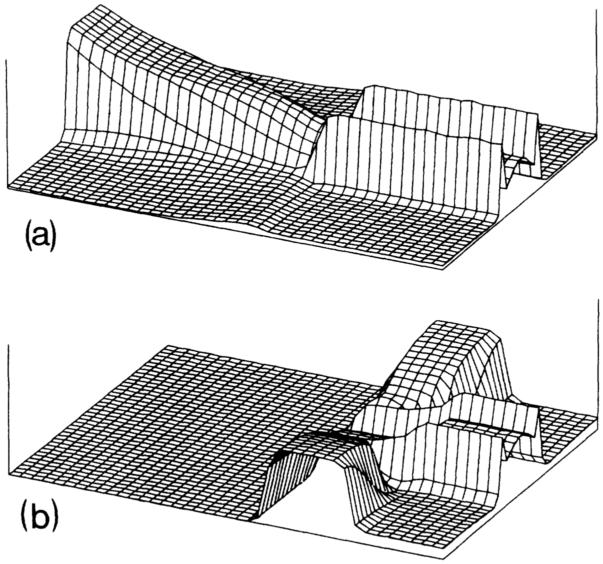

FIG. 4. Run No. 26b. With distance scale another factor of 10 smaller, neutron diffusion becomes very prominent (a). At the time that nucleosynthesis begins in the high-density region, the n/p ratio in the low-density region is approximately 1.4. However, many of the neutrons diffuse back into the high-density region during the time interval between high- and low-density nucleosynthesis. Close to the high-density region n/p thus drops below 1 again before nucleosynthesis begins there and we get much less than 100% ^{4}He (b). Farther away n/p is still above 1 and we get almost 100% ^{4}He, and some of the extra neutrons decay, so we get a little bit of the simple-diffusion effect. Most of the extra neutrons, however, diffuse back into the high-density region before decaying.

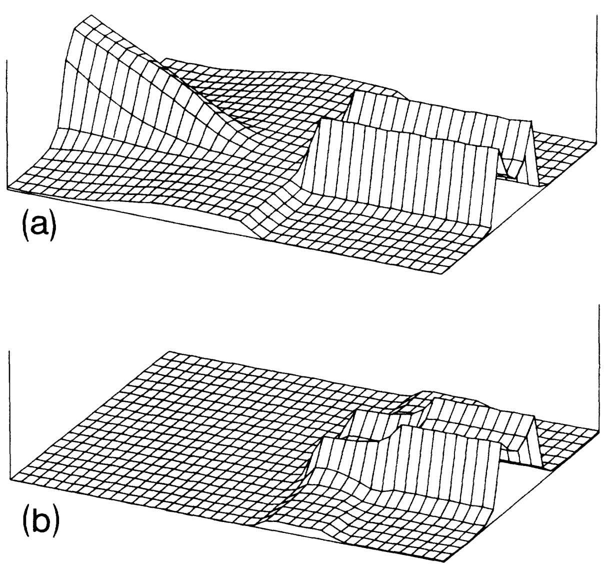

FIG. 5. Run No. 28. This is our smallest-scale run. Because of the small scale, neutron diffusion is rapid and has completely homogenized the neutron density before nucleosynthesis begins. Once nucleosynthesis consumes the free neutrons in the high-density region, the back diffusion from the low-density region is also very rapid and hardly any neutrons are left in the low-density region. There is no opportunity for any decay of extra neutrons.

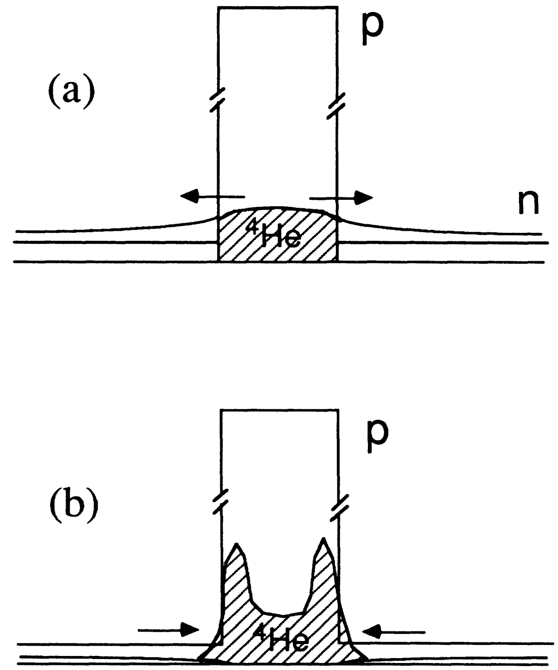

FIG. 6. What really happens (compare to Fig. 1). Nucleosynthesis begins first in the high-density region, and all neutrons there are absorbed in ^{4}He. This reverses the direction of neutron diffusion. The neutrons flow back into the high-density region. When they reenter the high-density region they form ^{4}He. If there is enough time before nucleosynthesis begins in the low-density region, it can be drained of neutrons almost completely.

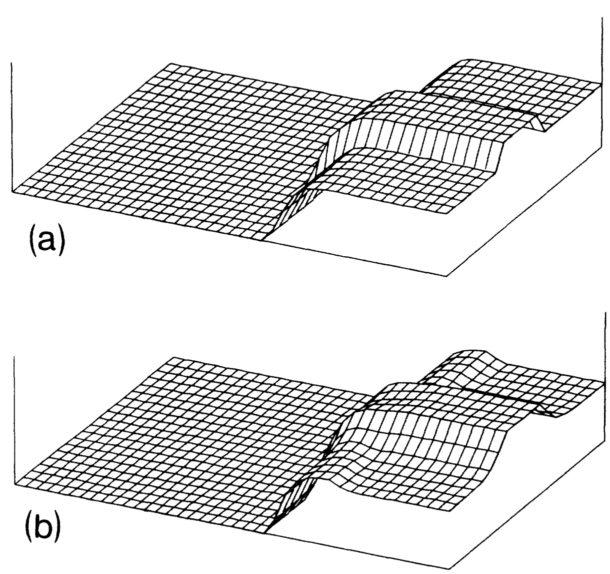

FIG. 7. These plots show the ^{7}Be mass fraction in the largest scale (No. 21) (a) and smallest-scale (No. 28) (b) runs. The vertical scale is logarithmic and extends from 10^{-15} to 10^{-3}. No destruction of ^{7}Be due to back-diffusing neutrons is visible here.

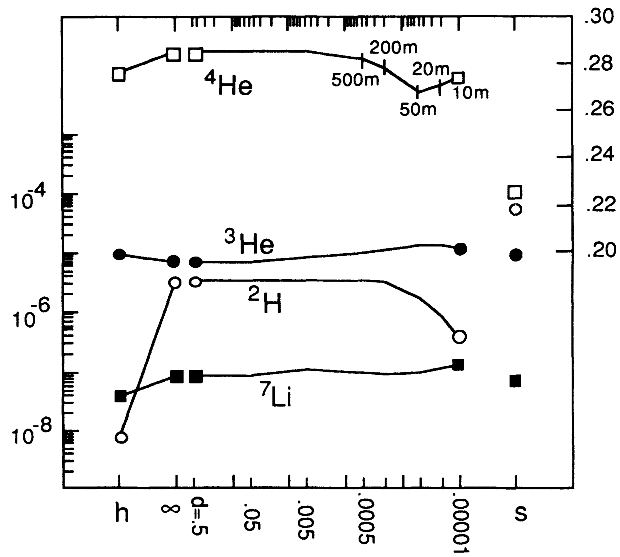

FIG. 8. The final averaged mass fractions produced in runs Nos. 21–28 and in some simpler models. All cases are for an average $\rho_b = 4.0 \times 10^{-30}$ g/cm^3. h is the homogeneous model. ∞ is the simple inhomogeneous case ($R = 50$, $f_V = 0.25$) without diffusion. s is the simple-diffusion model, with the large reduction in ^{4}He and increase in ^{2}H. Note that we have labeled the helium plot with the inhomogeneity length scales at $T = 100$ MeV (after the quark-hadron phase transition) for the particular models.

IV. CONCLUSIONS

The mechanisms for producing baryon inhomogeneity in the early Universe remain speculative. The prime candidate, the quark-hadron phase transition, is very poorly understood (as is any other possible mechanism). It seems to have the potential for achieving baryon separation, but certainly it has not been conclusively demonstrated that the end result would indeed be strongly inhomogeneous in the baryon-number density. When it was suggested that the baryon inhomogeneity could bring the predicted cosmic nucleosynthesis with a critical baryon density into agreement with observations, the possibility of these prenucleosynthesis inhomogeneities became very attractive.

We have demonstrated that such inhomogeneities can indeed affect the resulting element abundances, but to a much smaller degree than was at first thought. Most importantly, it seems very unlikely that observed abundances could be produced with a critical baryon density. The large reduction in produced ^{4}He, expected to occur because of the decay of the extra neutrons which diffused into the low-density regions, does not happen because these neutrons diffuse back into the high-density regions. Also the increase in ^{2}H due to delayed nucleosynthesis (waiting for the decay of the surplus neutrons) does not then materialize. The back diffusion was thought to help by reducing the value of ^{7}Li, which is overproduced, compared to accepted observations, in inhomogeneous scenarios, but we found no significant effect. Thus, the nucleosynthesis results do not lend any special support to the idea of baryon inhomogeneity.

The value of ρ_b obtained from standard nucleosynthesis was one of the few quantities in cosmology that we liked to think we knew. The possibility of a drastic change here must have appeared unsettling to some cosmologists. Much work has been based on the standard-model value of ρ_b and on the implication that the mass of the Universe is dominated by some kind of nonbaryonic matter. Although the possibility of baryon inhomogeneity can make the value of ρ_b less certain, our results seriously weaken the case that had been put forth for a high ρ_b.

Clearly we have studied a very limited sample of the possible inhomogeneity configurations. Strictly, we have just shown that only certain earlier propositions were wrong. We have run a model with critical density, which was said to produce the observed abundances of ^{4}He and ^{2}H, and we found it did not. It might be argued that because we studied just a small part of a large parameter space, we cannot be sure that the critical baryon density would not be successful with some other density contrasts or high-density volume fractions. Making the high-density volume fraction very small could make diffusion out of it easier and the diffusion back more difficult. However, that would require a larger density contrast, increasing the time difference between the onset of nucleosynthesis in the two regions, and thus increasing the time available for back diffusion.

One may argue that our plane symmetric geometry is somewhat unrealistic. However, that does not appear to be so. First, the quark-hadron phase transition, to provide the inhomogeneities, presumably proceeds by the nucleation of normal (mesonic) matter bubbles in the quark plasma. These normal bubbles will grow, approximately spherically. The quark plasma will occupy smaller volumes of three-space as the normal bubbles expand. Thus, toward the end of the quark-hadron phase transition, the quark plasma, which still carries the baryon number, is confined to surfaces (such as the surfaces in a pile of soap bubbles). Hence the baryons, in this picture, are reasonably approximated as being distributed in planar sheets when nucleosynthesis begins.

Second, there is the behavior we have just seen for the diffusion and the nucleosynthesis. We suppose there is an upper limit (say 100 to 1, as predicted by the quark-hadron phase transition) on the density contrast. Then there is a maximum spacing for the high-density inhomogeneities given a specified spatial size. We can consider then three different choices for the size and spacing. If the high-density baryon lumps are very large and well separated, then the evolution near their surface will be like our planar results. If the lumps are very small and thus closely spaced, then diffusion of neutrons out of, and back into, the high proton regions will be fast, and the small-scale simulations described here will be relatively accurate. Then the question arises of whether, if the inhomogeneities do somehow turn out to be roughly spherical, there might be some intermediate scale where the difference between planar and other geometry becomes important. Regardless of our prejudice that there is no such scale, we[24] are implementing a diffusion code that will allow the treatment of geometries other than planar, to settle this issue.

In any case, the mechanism of back diffusion will always be there. If the neutrons are able to diffuse out, they will also be able to diffuse back in. Even if an inhomogeneity configuration were found where the diffusion would lead to sufficient decay of neutrons, it would still require at least a fine-tuning of the distance scale (to something like 40–100 km at $T=100$ keV, or 20–50 m at $T=100$ MeV) to allow sufficient out diffusion, but negligible back diffusion.

ACKNOWLEDGMENTS

We have been informed of the ongoing work of others, in Santa Cruz[27] and in Livermore,[28] who apparently have found the effect of neutron back diffusion also. At the time of this writing, we have not yet seen their results, so we do not know how similar their approach is to ours and whether there is an agreement of results. R.M. acknowledges conversations with Dr. G. Mathews, and continuing dialogue with Professor G. Shields on nucleosynthesis, both of which contributed to our early interest in this work. T.R. would like to thank Tsvi Piran for initially bringing to his attention this problem. The computations were done with the supercomputers at The University of Texas System Center for High Performance Computing and at the National Center for Supercomputing Applications. This research was supported in part by National Science Foundation Grants Nos. PHY84-04931, PHY84-51732, and PHY87-06315.

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