Wormholes in spacetime

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Any reasonable theory of quantum gravity will allow closed universes to branch off from our nearly flat region of spacetime. I describe the possible quantum states of these closed universes. They correspond to wormholes which connect two asymptotically Euclidean regions, or two parts of the same asymptotically Euclidean region. I calculate the influence of these wormholes on ordinary quantum fields at low energies in the asymptotic region. This can be represented by adding effective interactions in flat spacetime which create or annihilate closed universes containing certain numbers of particles. The effective interactions are small except for closed universes containing scalar particles in the spatially homogeneous mode. If these scalar interactions are not reduced by sypersymmetry, it may be that any scalar particles we observe would have to be bound states of particles of higher spin, such as the pion. An observer in the asymptotically flat region would not be able to measure the quantum state of closed universes that branched off. He would therefore have to sum over all possibilities for the closed universes. This would mean that the final state would appear to be a mixed quantum state, rather than a pure quantum state.

I. INTRODUCTION

In a reasonable theory of quantum gravity the topology of spacetime must be able to be different from that of flat space. Otherwise, the theory would not be able to describe closed universes or black holes. Presumably, the theory should allow all possible spacetime topologies. In particular, it should allow closed universes to branch off, or join onto, our asymptotic flat region of spacetime. Of course, such behavior is not possible with a real, nonsingular, Lorentzian metric. However, we now all know that quantum gravity has to be formulated in the Euclidean domain. There, it is no problem: it is just a question of plumbing. Indeed, it is probably necessary to include all possible topologies for spacetime to get unitarity.

Topology change is not something that we normally experience, at least, on a macroscopic scale. However, one can interpret the formation and subsequent evaporation of a black hole as an example: the particles that fell into the hole can be thought of as going off into a little closed universe of their own. An observer in the asymptotically flat region could not measure the state of the closed universe. He would therefore have to sum over all possible quantum states for the closed universe. This would mean that the part of the quantum state that was in the asymptotically flat region would appear to be in a mixed state, rather than a pure quantum state. Thus, one would lose quantum coherence.[1,2]

If it is possible for a closed universe the size of a black hole to branch off, it is also presumably possible for little Planck-size closed universes to branch off and join on. The purpose of this paper is to show how one can describe this process in terms of an effective field theory in flat spacetime. I introduce effective interactions which create, or destroy, closed universes containing certain numbers of particles. I shall show that these effective interactions are small, except for scalar particles. There is a serious problem with the very large effective interactions of scalar fields with closed universes. It may be that these interactions can be reduced by supersymmetry. If not, I think we will have to conclude that any scalar particles that we observe are bound states of fermions, like the pion. Maybe this is why we have not observed Higgs particles.

I base my treatment on general relativity, even though general relativity is probably only a low-energy approximation to some more fundamental quantum theory of gravity, such as superstrings. For closed universes of the Planck size, any higher-order corrections induced from string theory will change the action by a factor ~ 1. So the effective field theory based on general relativity should give answers of the right order of magnitude.

In Sec. II, I describe how closed universes or wormholes can join one asymptotically Euclidean region to another, or to another part of the same region. Solutions of the Wheeler-DeWitt equation that correspond to such wormholes are obtained in Sec. III. These solutions can also be interpreted as corresponding to Friedmann universes. It is an amusing thought that our Universe could be just a rather large wormhole in an asymptotically flat space.

In Sec. IV, I calculate the vertex for the creation or annihilation of a wormhole containing a certain number of particles. Section V contains a discussion of the initial quantum state in the closed-universe Fock space. There are two main possibilities: either there are no closed universes present initially, or there is a coherent state which is an eigenstate of the creation plus annihilation operators for each species of closed universe. There will be loss of quantum coherence in the first case, but not the second. This is described in Sec. VI. The interactions between wormholes and particles of different spin in asymptotically flat space are discussed in Sec.

VII. Finally, in Sec. VIII, I conclude that wormholes will have to be taken into account in any quantum theory of gravity, including superstrings.

This paper supercedes earlier work of mine[3-5] on the loss of quantum coherence. These papers were incorrect in associating loss of coherence with simply connected spaces with nontrivial topology, rather than with wormholes.

II. WORMHOLES

What I am aiming to do is to calculate the effect of closed universes that branch off on the behavior of ordinary, nongravitational particles in asymptotically flat space at energies low compared to the Planck mass. The effect will come from Euclidean metrics which represent a closed universe branching off from asymptotically flat space. One would expect that the effect would be greater, the larger the closed universe. Thus one might expect the dominant contribution would come from metrics with the least Euclidean action for a given size of closed universe. In the $R=0$ conformal gauge, these are conformally flat metrics:

$$ds^2=\Omega^2 dx^2 ,$$

$$\Omega=1+\frac{b^2}{(x-x_0)^2} .$$

At first sight, this looks like a metric with a singularity at the point x_0. However, the blowing up of the conformal factor near x_0 means that the space opens out into another asymptotically flat region, joined to the first asymptotically flat region by a wormhole of coordinate radius b and proper radius $2b$. The other asymptotic region can be a separate asymptotically flat region of the Universe, or it can be another part of the first asymptotic region. In the latter case, the conformal factor will be modified slightly by the interaction between the two ends of the wormhole, or handle to spacetime.[6] However, the change will be small when the separation of the two ends is large compared to $2b$, the size of the wormhole. Typically, b will be of the order of the Planck length, so it will be a good approximation to neglect the interactions between wormholes. This conformally flat metric is just one example of a wormhole. There are, of course, nonconformally flat closed universes that can join onto asymptotically flat space. Their effects will be similar, but will involve gravitons in the asymptotically flat space. Since it is difficult to observe gravitons, I shall concentrate on conformally flat closed universes.

I shall consider a set of matter fields ϕ in the closed universe. Spin-1 gauge fields are conformally invariant. In the case of matter fields of spin $\frac{1}{2}$ and 0, the effect of any mass will be small for wormholes of the Planck size. I shall therefore take the matter fields ϕ to be conformally invariant. The effect of mass could be included as a perturbation.

In order to find the effect of the closed universe or wormhole on the matter fields ϕ in the asymptotically flat spaces, one should calculate the Green's functions

$$\langle \phi(y_1)\phi(y_2)\cdots\phi(y_r)\phi(z_1)\phi(z_2)\cdots\phi(z_s)\rangle ,$$

where $y_1,\ldots,y_r$ and $z_1,\ldots,z_s$ are points in the two asymptotic regions (which may be the same region). This can be done by performing a path integration over all matter fields ϕ and all metrics $g_{\mu\nu}$ that have one or two asymptotically flat regions and a handle or wormhole connecting them. Let S be a three-sphere, which is a cross section of the closed universe or wormhole. One can then factorize the path integral into a part

$$\langle 0\,|\,\phi(y_1)\cdots\phi(y_r)\,|\,\psi\rangle ,$$

which depends on the fields on one side of S, and a part

$$\langle \psi\,|\,\phi(z_1)\cdots\phi(z_s)\,|\,0\rangle ,$$

which depends on the fields on the other side of S. Strictly speaking, one can factorize in this way only when the regions at the two ends of the wormhole are separate asymptotic regions. However, even when they are the same region, one can neglect the interaction between the ends and factorize the path integral if the ends are widely separated.

In the above $|0\rangle$ represented the usual particle scattering vacuum state defined by a path integral over asymptotically Euclidean metrics and matter fields that vanish at infinity. $|\psi\rangle$ represented the quantum state of the closed universe or wormhole on the surface S. This can be described by a wave function Ψ which depends on the induced metric h_{ij} and the values ϕ_0 of the matter fields on S. The wave function obeys the Wheeler-DeWitt equation

$$\left[-m_P^{-2}G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}}-m_P^2 h^{1/2}\,{}^3R+\tfrac{1}{2}h^{1/2}T^{nn}\left[\phi_0,-i\frac{\delta}{\delta\phi_0}\right]\right]\Psi[h_{ij},\phi_0]=0 ,$$

where

$$G_{ijkl}=\tfrac{1}{2}h^{1/2}(h_{ik}h_{jl}+h_{il}h_{jk}-h_{ij}h_{kl}) .$$

The wave function also obeys the momentum constraint

$$\left[-2im_P^2\left[\frac{\delta}{\delta h_{ij}}\right]_{|j}+T^{ni}\left[\phi_0,-i\frac{\delta}{\delta\phi_0}\right]\right]\Psi[h_{ij},\phi_0]=0 .$$

III. WORMHOLE EXCITED STATES

The solutions of the Wheeler-DeWitt equation that correspond to wormholes, that is, closed universes connecting two asymptotically Euclidean regions, form a Hilbert space $\mathcal{H}_w$ with the inner product

$$\langle \psi_1 | \psi_2 \rangle = \int d[h_{ij}] d[\phi_0] \Psi_1^* \Psi_2 \ .$$

Let $|\psi_i\rangle$ be a basis for $\mathcal{H}_w$. Then one can write the Green's function in the factorized form

$$\langle \phi(y_1) \cdots \phi(y_r) \phi(z_1) \cdots \phi(z_s) \rangle = \sum \langle 0 | \phi(y_1) \cdots \phi(y_r) | \psi_i \rangle \langle \psi_i | \phi(z_1) \cdots \phi(z_s) | 0 \rangle \ .$$

What are these wormhole excited states $|\psi_i\rangle$? To find them one would have to solve the full Wheeler-DeWitt and momentum constraint equations. This is too difficult, but one can get an idea of their nature from mode expansions.[7] One can write the three-metric h_{ij} on the surface S as

$$h_{ij} = \sigma^2 a^2 (\Omega_{ij} + \epsilon_{ij}) \ .$$

Here $\sigma^2 = 2/3\pi m_P^2$ is a normalization factor, Ω_{ij} is the metric on the unit three-sphere, and ϵ_{ij} is a perturbation, which can be expanded in harmonics on the three-sphere:

$$\epsilon_{ij} = \sum_{n,l,m} [6^{1/2} a_{nlm} \tfrac{1}{3} \Omega_{ij} Q^n_{lm} + 6^{1/2} b_{nlm} (P_{ij})^n_{lm} + 2^{1/2} c^0_{nlm} (S^0_{ij})^n_{lm} + 2^{1/2} c^e_{nlm} (S^e_{ij})^n_{lm} + 2 d^0_{nlm} (G^0_{ij})^n_{lm} + 2 d^e_{nlm} (G^e_{ij})^n_{lm}] \ .$$

The $Q(x^i)$ are the standard scalar harmonics on the three-sphere. The $P_{ij}(x^i)$ are given by (suppressing all but i,j indices)

$$P_{ij} = \frac{1}{n^2-1} Q_{|ij} + \tfrac{1}{3} \Omega_{ij} Q \ .$$

They are traceless, $P^i_i = 0$. The S^{ij} are defined by

$$S_{ij} = S_{i|j} + S_{j|i} \ ,$$

where S_i are the transverse vector harmonics, $S_i^{\ |i} = 0$. The G_{ij} are the transverse traceless tensor harmonics $G^i_i = G_{ij}^{\ |j} = 0$. Further details about harmonics and their normalization can be found in Ref. 7.

Consider a conformally invariant scalar field ϕ. One can describe it in terms of hyperspherical harmonics on the surface S:

$$\phi_0 = \sigma^{-1} a^{-1} \sum f_n Q_n \ .$$

The wave function Ψ is then a function of coefficients a_n, b_n, c_n, d_n, and f_n and the scale factor a.

One can expand the Wheeler-DeWitt operator to all orders in a and to second order in the other coefficients. In this approximation, the different modes do not interact with each other, but only with the scale factor a. However, the conformal scalar coefficients f_n do not even interact with a. One can therefore write the wave function as a sum of products of the form

$$\Psi = \Psi_0(a, a_i, b_i, c_i, d_i) \prod \psi_n(f_n) \ .$$

The part of the Wheeler-DeWitt operator that acts on ψ_n is

$$-\frac{d^2}{df_n^2} + (n^2+1) f_n^2 \ .$$

It is therefore natural to take them to be harmonic-oscillator wave functions

$$\psi_{nm} = \left[\frac{\beta^2}{\pi 2^{2m} (m!)^2} \right]^{1/4} e^{-\beta^2 f_n^2/2} H_m(\beta f_n) \ ,$$

where $\beta^4 = (n^2+1)$ and H_m are Hermite polynomials. The wave functions ψ_{nm} can then be interpreted as corresponding to the closed universe containing m scalar particles in the nth harmonic mode.

The treatment for spin-$\frac{1}{2}$ and -1 fields is similar. The appropriate data for the fields on S can be expanded in harmonics on the three-sphere. The main difference is that the lowest harmonic is not the $n=0$ homogeneous mode, as in the scalar case, but has $n=\frac{1}{2}$ or 1. Again, the coefficients of the harmonics appear in the Wheeler-DeWitt equation to second order only as fermionic[8] or bosonic harmonic oscillators, with a frequency independent of a. One can therefore take the wave functions to be fermion or boson harmonic-oscillator wave functions in the coefficients of the harmonics. They can then be interpreted as corresponding to definite numbers of particles in each mode.

In the gravitational part of the wave function, Ψ_0, the coefficients a_n, b_n, and c_n reflect gauge degrees of freedom. They can be made zero by a diffeomorphism of S and suitable lapse and shift functions. The coefficients d_n correspond to gravitational wave excitations of the closed universe. However, gravitons are very difficult to observe. I shall therefore take these modes to be in their ground state.

The scale factor a appears in the Wheeler-DeWitt equation as the operator

$$\frac{\partial^2}{\partial a^2} - a^2 \ .$$

I shall assume that the zero-point energies of each mode are either subtracted or canceled by fermions in a supersymmetric theory. The total wave function Ψ will then satisfy the Wheeler-DeWitt equation if the gravitational part Ψ_0 is a harmonic-oscillator wave function in a with unit frequency and level equal to the sum E of the energies of the matter-field harmonic oscillators.

The wave function Ψ_0 will oscillate for $a < r_0 = (2E)^{1/2}$. In this region one can use the WKB approximation[7,9,10] to relate it to a Lorentzian solution of the classical field equations. This solution will be a $k = +1$ Friedmann model filled with conformally invariant matter. The maximum radius of the Friedmann model will be $a = r_0$. For $a > r_0$, the wave function will be exponential. Thus, in this region it will correspond to a Euclidean metric. This will be the wormhole metric described in Sec. II, with $b = 1/2\sigma r_0$. These excited state solutions were first found in Ref. 11, but their significance as wormholes was not realized. Notice that the wave function is exponentially damped at large a, whereas the cosmological wave functions described in Refs. 7, 9, and 10 tend to grow exponentially at large a. The difference here is that one is looking at the closed universe from an asymptotically Euclidean region, instead of from a compact Euclidean space, as in the cosmological case. This changes the sign of the trace K surface term in the gravitational action.

IV. THE WORMHOLE VERTEX

One now wants to calculate the matrix element of the product of the values of ϕ at the points $y_1, y_2, \ldots, y_r$ between the ordinary, flat-space vacuum $\langle 0|$ and the closed-universe state $|\psi\rangle$. This is given by the path integral

$$\langle 0|\phi(y_1)\cdots\phi(y_r)|\psi\rangle = \int d[h_{ij}]d[\phi_0]\Psi[h_{ij},\phi_0]\int d[g_{\mu\nu}]d[\phi]\phi(y_1)\cdots\phi(y_r)e^{-I[g,\phi]} .$$

The gravitational field is required to be asymptotically flat at infinity, and to have a three-sphere S with induced metric h_{ij} as its inner boundary. The scalar field ϕ is required to be zero at infinity, and to have the value ϕ_0 on S.

In general, the positions of the points y_i cannot be specified in a gauge-invariant manner. However, I shall be concerned only with the effects of the wormholes on low-energy particle physics. In this case the separation of the points y_i can be taken to be large compared to the Planck length, and they can be taken to lie in flat Euclidean space. Their positions can then be specified up to an overall translation and rotation of Euclidean space.

Consider first a wormhole state $|\psi\rangle$ in which only the $n = 0$ homogeneous scalar mode is excited above its ground state. The integral over the wave function Ψ of the wormhole can then be replaced in the above by

$$\int da\, df_0\, \psi_E(a)\psi_{0m}(f_0) .$$

The path integral will then be over asymptotically Euclidean metrics whose inner boundary is a three-sphere S of radius a and scalar fields with the constant value f_0 on S. The saddle point for the path integral will be flat Euclidean space outside a three-sphere of radius a centered on a point x_0 and the scalar field

$$\phi = \frac{a\sigma f_0}{(x - x_0)^2}$$

(the energy-momentum tensor of this scalar field is zero). The action of this saddle point will be $(a^2 + f_0^2)/2$. The determinant Δ of the small fluctuations about the saddle point will be independent of f_0. Its precise form will not be important.

The integral over the coefficient f_0 of the $n = 0$ scalar harmonic will contain a factor of

$$\int df_0 f_0^r e^{-f_0^2} H_m(f_0) .$$

This will be zero when m, the number of particles in the mode $n = 0$, is greater than r, the number of points y_i in the correlation function. This is what one would expect, because each particle in the closed universe must be created or annihilated at a point y_i in the asymptotically flat region. If $r > m$, particles may be created at one point y_i and annihilated at another point y_j without going into the closed universe. However, such matrix elements are just products of flat-space propagators with matrix elements with $r = m$. It is sufficient therefore to consider only the case with $r = m$.

The integral over the radius a will contain a factor

$$\int da\, a^m e^{-a^2} H_E(a)\Delta(a) ,$$

where $E = m$ is the level number of the radial harmonic oscillator. For small m, the dominant contribution will come from $a \sim 1$, that is, wormholes of the Planck size. The value $C(m)$ of this integral will be ~ 1.

The matrix element will then be

$$D(m)\prod \frac{\sigma}{(y_i - x_0)^2} ,$$

where $D(m)$ is another factor ~ 1. One now has to integrate over the position x_0 of the wormhole, with a measure of the form $m_P^4 dx_0^4$, and over an orthogonal matrix O which specifies its orientation with respect to the points y_i. The $n = 0$ mode is invariant under O, so this second integral will have no effect, but the integral over x_0 will ensure the energy and momentum are conserved in the asymptotically flat region. This is what one would expect, because the Wheeler-DeWitt and momentum constraint equations imply that a closed universe has no energy or momentum.

The matrix element will be the same as if one was in flat space with an effective interaction of the form

$$F(m)m_P^{4-m}\phi^m(c_{0m}+c_{0m}^\dagger)\ ,$$

where $F(m)$ is another coefficient ~ 1 and c_{0m} and $c_{0m}^\dagger$ are the annihilation and creation operators for a closed universe containing m scalar particles in the $n=0$ homogeneous mode.

In a similar way, one can calculate the matrix elements of products of ϕ between the vacuum and a closed-universe state containing m_0 particles in the $n=0$ mode, m_1 particles in the $n=1$ mode, and so on. The energy-momentum tensor of scalar fields with higher harmonic angular dependence will not be zero. This will mean that the saddle-point metric in the path integral for the matrix element will not be flat space, but will be curved near the surface S. In fact, for large particle numbers, the saddle-point metric will be the conformally flat wormhole metrics described in Sec. II. However, the saddle-point scalar fields will have a Q_n angular dependence and a $\sigma^{n+1}/(x-x_0)^{n+2}$ radial dependence in the asymptotic flat region. This radial decrease is so fast that the closed universes with higher excited harmonics will not give significant matrix elements, except for that containing two particles in the $n=1$ modes. By the constraint equations, or, equivalently, by averaging over the orientation O of the wormhole, the matrix element will be zero unless the two particles are in a state that is invariant under O. The matrix element for such a universe will be the same as that produced by an effective interaction of the form

$$\nabla\phi\nabla\phi(c_{12}+c_{12}^\dagger)$$

with a coefficient ~ 1.

In a similar way one can calculate the matrix elements for universes containing particles of spin $\frac{1}{2}$ or higher. Again, the constraint equations or averaging over O mean that the matrix element is nonzero only for closed-universe states that are invariant under O. This means that the corresponding effective interactions will be Lorentz invariant. In particular, they will contain even numbers of spinor fields. Thus, fermion number will be conserved mod 2: the closed universes are bosons.

The matrix elements for universes containing spin-$\frac{1}{2}$ particles will be equivalent to effective interactions of the form

$$m_P^{4-3m/2}\psi^m d_m+\text{c.c.}\ ,$$

where ψ^m denotes some Lorentz-invariant combination of m spinor fields ψ or their adjoints $\bar{\psi}$, and d_m is the annihilation operator for a closed universe containing m spin-$\frac{1}{2}$ particles in $n=\frac{1}{2}$ modes. One can neglect the effect of closed universes with spin-$\frac{1}{2}$ particles in higher modes.

In the case of spin-1 gauge particles, the effective interaction would be of the form

$$m_P^{4-2m}[(F_{\mu\nu})^m(g_m+g_m^\dagger)]\ ,$$

where g_m is the annihilation operator for a closed universe containing m spin-1 particles in $n=1$ modes. As before, the higher modes can be neglected.

V. THE WORMHOLE INITIAL STATE

What I have done is introduce a new Fock space $\mathcal{F}_w$ for closed universes, which is based on the one wormhole Hilbert space $\mathcal{H}_w$. The creation and annihilation operators $c_{nm}^\dagger$, c_{nm}, etc., act on $\mathcal{F}_w$ and obey the commutation relations for bosons. The full Hilbert space of the theory, as far as asymptotically flat space is concerned, is isomorphic to $\mathcal{F}_p\otimes\mathcal{F}_w$, where $\mathcal{F}_p$ is the usual flat-space particle Fock space.

The distinction between annihilation and creation operators is a subtle one because the closed universe does not live in the same time as the asymptotically flat region. If both ends of the wormhole are in the same asymptotic region, one can say that a closed universe is created at one point and is annihilated at another. However, if a closed universe branches off from our asymptotically flat region, and does not join back on, one would be free to say either (1) it was present in the initial state and was annihilated at the junction point x_0, (2) it was not present initially, but was created at x_0 and is present in the final state, or (3) as Sidney Coleman (private communication) has suggested, one might have a coherent state of closed universes in both the initial and final states, in such a way that they were both eigenstates of the annihilation plus creation operators $c_{nm}+c_{nm}^\dagger$, etc., with some eigenvalue q.

In this last case, the closed-universe sector of the state would remain unchanged and there would be no loss of quantum coherence. However, the initial state would contain an infinite number of closed universes. Such eigenstates would not form a basis for the Fock space of closed universes.

Instead, I shall argue that one should adopt the second possibility: there are no closed universes in the initial state, but closed universes can be created and appear in the final state. If one takes a path-integral approach, the most natural quantum state for the Universe is the so-called "ground" state, or, "no boundary" state.[8] This is the state defined by a path integral over all compact metrics without boundary. Calculations based on minisuperspace models[7–11] indicate that this choice of state leads to a universe like we observe, with large regions that appear nearly flat. One can then formulate particle scattering questions in the following way: one asks for the conditional probability that one observes certain particles on a nearly flat surface S_2 given that the region is nearly asymptotically Euclidean and is in the quantum state defined by conditions on the surfaces S_1 and S_3 to either side of S_2, and at great distance from it in the positive and negative Euclidean-time directions, respectively. One then analytically continues the position of S_2 to late real time. It then measures the final state in the nearly flat region. One continues the positions of both S_1 and S_3 to early real time. One gives the time coordinate of S_1 a small positive imaginary part, and the time coordinate of S_3 a small negative imaginary part. The initial state is then defined by data on the surfaces S_1 and S_3.

If one adopts the formulation of particle scattering in terms of conditional probabilities, one would impose the conditions on the surfaces S_1 and S_3 in the nearly flat region. However, one would not impose conditions on any closed universes that branched off or joined on between S_1 and S_3, because one could not observe them. Thus, the initial or conditional state would not contain any closed universes. A closed universe that branched off between S_1 and S_2 (or between S_2 and S_3) would be regarded as having been created. If it joined up again between S_1 and S_2 (S_2 and S_3, respectively), it would be regarded as having been annihilated again. Otherwise, it would be regarded as part of the final state. An observer in the nearly flat region would be able to measure only the part of the final state on S_2 and not the state of the closed universe. He would therefore have to sum over all possibilities for the closed universes. This summation would mean that the part of the final state that he could observe would appear to be in a mixed state rather than in a pure quantum state.

VI. THE LOSS OF QUANTUM COHERENCE

Let $|\alpha_i\rangle$ be a basis for the flat-space Fock space $\mathcal{F}_p$ and $|\beta\rangle_j$ be a basis for the wormhole Fock space $\mathcal{F}_w$. In case (2) above, in which there are no wormholes initially, the initial, or conditional, state can be written as the state

$$\lambda^i |\alpha_i\rangle |O\rangle_w ,$$

where $|O\rangle_w$ is the zero closed-universe state in $\mathcal{F}_w$. The final state can be written as

$$\mu^{ij} |\alpha_i\rangle |\beta_j\rangle .$$

However, an observer in the nearly flat region can measure only the states $|\alpha_i\rangle$ on S_2, and not the closed-universe states $|\beta_j\rangle$. He would therefore have to sum over all possible states for the closed universes. This would give a mixed state in the $\mathcal{F}_p$ Fock space with density matrix

$$\rho^i_k = \mu^{ij}\bar{\mu}_{kj} .$$

The matrix ρ^{ik} will be Hermitian and positive semidefinite, if the final state is normalized in $\mathcal{H}$:

$$\mathrm{tr}\rho = \mu^{ij}\bar{\mu}_{ij} = 1 .$$

These are the properties required for it to be interpreted as the density matrix of a mixed quantum state. A measure of the loss quantum coherence is

$$1-\mathrm{tr}(\rho^2) = 1-\mu^{ij}\mu^{kl}\bar{\mu}_{il}\bar{\mu}_{kj} .$$

This will be zero if the final state is a pure quantum state. Another measure is the entropy which can be defined as

$$-\mathrm{tr}(\rho \ln \rho) .$$

This again will be zero for a pure quantum state.

If case (3) above is realized, the initial closed-universe state is not the no-wormhole state $|O\rangle_w$, but a coherent state $|q\rangle_w$ such that

$$(c_{nm}+c^\dagger_{nm})|q\rangle_w = q_{nm}|q\rangle_w .$$

The effective interactions would leave the closed-universe sector in the same coherent state. Thus the final state would be the product of some state in $\mathcal{F}_p$ with the coherent state $|q\rangle_w$. There would be no loss of quantum coherence, but one would have effective ϕ^m and other interactions whose coefficients would depend on the eigenvalues q_{nm}, etc. It would seem that these could have any value.

VII. WORMHOLE EFFECTIVE INTERACTIONS

There will be no significant interaction between wormholes, unless they are within a Planck length of each other. Thus, the creation and annihilation operators for wormholes are practically independent of the positions in the asymptotically flat region. This means that the effective propagator of a wormhole excited state is $\delta^4(p)$. Using the propagator one can calculate Feynman diagrams that include wormholes, in the usual manner.

The interactions of wormholes with m scalar particles in the $n=0$ mode are alarmingly large. The $m=1$ case would be a disaster; it would give the scalar field a propagator that was independent of position because a scalar particle could go into a wormhole whose other end was at a great distance in the asymptotically flat region. Suppose, however, that the scalar field were coupled to a Yang-Mills field. One would have to average over all orientations of the gauge group for the closed universe. This would make the matrix element zero, except for closed-universe states that were Yang-Mills singlets. In particular, the matrix element would be zero for $m=1$. A special case is the gauge group Z_2. Such fields are known as twisted scalars. They can reverse sign on going round a closed loop. They will have zero matrix elements for m odd because one will have to sum over both signs.

Consider now the matrix element for the scalar field, and its complex conjugate, between the vacuum and a closed universe containing a scalar particle and antiparticle in the $n=0$ mode. This will be nonzero, because a particle-antiparticle state contains a Yang-Mills singlet. It would give an effective interaction of the form

$$m_P^2\,\mathrm{tr}(\phi\bar{\phi})(c_{011}+c^\dagger_{011}) ,$$

where c_{011} is the annihilation operator for a closed universe with one scalar particle and one antiparticle in the $n=0$ mode. This again would be a disaster; with two of these vertices one could make a closed loop consisting of a closed universe [propagator, $\delta^4(p)$] and a scalar particle (propagator, $1/p^2$). This closed loop would be infrared divergent. One could cut off the divergence by giving the scalar particle a mass, but the effective mass would be the Planck mass. One might be able to remove this mass by renormalization, but the creation of closed universes would mean that a scalar particle would lose quantum coherence within a Planck length. The

$m=4$ matrix element will give a large ϕ^4 effective vertex.

There seems to be four possibilities in connection with wormholes containing only scalar particles in the $n=0$ mode.

(1) They may be reduced or canceled in a supersymmetric theory.

(2) The scalar field may be absorbed as a conformal factor in the metric. This could happen, however, only for one scalar field that was a Yang-Mills singlet.

(3) It may be that any scalar particle that we observe is a bound state of particles of higher spin, such as the pion.

(4) The universe may be in a coherent state $|q\rangle_w$ as described above. However, one would then have the problem of why the eigenvalues q should be small or zero. This is similar to the problem of why the θ angle should be so small, but there are now an infinite number of eigenvalues.

In the case of particles of spin $\frac{1}{2}$, the exclusion principle limits the occupation numbers of each mode to zero or 1. Averaging over the orientation O of the wormhole will mean that the lowest-order interaction will be for a wormhole containing one fermion and one antifermion. This would give an effective interaction of the form

$$m_P\psi\bar{\psi}(d_{11}+d_{11}^{\dagger}) ,$$

where d_{11} is the annihilation operator for a closed universe containing a fermion and an antifermion in $n=\frac{1}{2}$ modes. This would give the fermion a mass of the order of the Planck mass. However, if the fermion were chiral, this interaction would cancel out under averaging over orientation and gauge groups. This is because there is no two-chiral-fermion state that is a singlet under both groups. This suggests that supersymmetry might ensure the cancellation of the dangerous interactions with wormholes containing scalar particle in the $n=0$ mode. Conformally flat wormholes, such as those considered in this paper, should not break supersymmetry.

For chiral fermions, the lowest-order effective interaction will be of the four-Fermi form

$$m_P^{-2}\,\mathrm{tr}(\psi_1\gamma^{\mu}\bar{\psi}_1\psi_2\gamma_{\mu}\bar{\psi}_2)(d_{1111}+d_{1111}^{\dagger}) ,$$

where d_{1111} is the annihilation operator for a wormhole containing a fermion and an antifermion each of species 1 and 2. This would lead to baryon decay, but with a lifetime $\sim 10^{50}$ yr. There will also be Yukawa-type effective interactions produced by closed universes containing one scalar particle, one fermion, and one antifermion.

VIII. CONCLUSION

It would be tempting to dismiss the idea of wormholes by saying that they are based on general relativity, and we now all know that string theory is the ultimate theory of quantum gravity. However, string theory, or any other theory of quantum gravity, must reduce to general relativity on scales large compared to the Planck length. Even at the Planck length, the differences from general relativity should be only ~ 1. In particular, the ultimate theory of quantum gravity should reproduce classical black holes and black-hole evaporation. It is difficult to see how one could describe the formation and evaporation of a black hole except as the branching off of a closed universe. I would therefore claim that any reasonable theory of quantum gravity, whether it is supergravity, or superstrings, should allow little closed universes to branch off from our nearly flat region of spacetime.

The effect of these closed universes on ordinary particle physics can be described by effective interactions which create or destroy closed universes. The effective interactions are small, apart from those involving scalar fields. The scalar field interactions may cancel because of supersymmetry. Or, any scalar particles that we observe may be bound states of particles of higher spin. Near a wormhole of the Planck size, such a bound state would behave like the higher-spin particles of which it was made. A third possibility is that the universe is in a coherent $|q\rangle_w$ state. I do not like this possibility because it does not seem to agree with the "no boundary" proposal for the quantum state of the Universe. There also would not seem to be any way to specify the eigenvalues q. Yet the values of the eigenvalues for large particle numbers cannot be zero if these interactions are to reproduce the results of semiclassical calculations on the formation and evaporation of macroscopic black holes.

The effects of little closed universes on ordinary particle physics may be small, apart, possibly, for scalar particles. Nevertheless, it raises an important matter of principle. Because there is no way in which we could measure the quantum state of closed universes that branch off from our nearly flat region, one has to sum over all possible states for such universes. This means that the part of the final state that we can measure will appear to be in a mixed quantum state, rather than a pure state. I think even Gross[12] will agree with that.

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