Photoproduction of delta and Roper resonances in the cloudy bag model

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The photoproduction of delta and Roper resonances has been calculated within the framework of the cloudy bag model. Two alternative formulations, the original pseudoscalar surface coupling and the pseudovector volume coupling of pions, have been used, and the results are compared. The helicity amplitudes are in fair agreement with experimental data, the prediction for the small E_{1+}/M_{1+} ratio for delta production is -1.8% for a bag radius of 1 fm and pseudovector coupling.

I. INTRODUCTION

Over the past 30 years photo- and electroexcitation of nucleon resonances has given a tremendous amount of information about the structure of the nucleon. These data provide crucial tests of the dynamics describing the nucleon in terms of QCD-inspired models. Of particular interest is the mixing of electric quadrupole and magnetic dipole amplitudes for the transition leading from the nucleon, $N(938)$, to its first excited state, $\Delta(1232)$. This ratio $R_\Delta = E_{1+}/M_{1+}$, is directly related to the tensor components in the effective forces between quarks and, consequently, to the intriguing possibility of a quadrupole deformation of elementary particles. In view of the fact that a static deformation, though in principle existent for the Δ and other nucleon resonances with spin $J \geq \frac{3}{2}$, is experimentally hard to observe, the $N\Delta$ transition moment is the only experimental evidence for such an effect.

In the past, experimental evaluations of R_Δ have spanned a large range between practically 0 and a minimum of -5%. The latest analysis of the Particle Data Group[1] gives a value of $R_\Delta = (-1.3 \pm 0.5)\%$. Another recent evaluation[2] of the available data, using a unitarity constraint via Watson's theorem, arrived at a value of $(-1.5 \pm 0.2)\%$. From these analyses it is evident that the ratio is definitely finite in contrast with the predictions of spherical quark models.

Skyrmion models of the nucleon[3] seem to overestimate the effects of quadrupole deformation by obtaining values R_Δ of the order of -5%. A recent study[4] of this ratio in a modified Skyrme model, with the inclusion of stabilizing fourth- and sixth-order terms, predicted values of R_Δ between -2.6% and -4.9%. On the contrary, most other hadronic models have predicted ratios significantly lower than the experimental values. In the nonrelativistic constituent-quark model (CQM), the tensor part of the color hyperfine interaction serves as the effective ingredient to admix D-state components into the bag wave functions. Typical results[5–7] for R_Δ range between -0.08% and -0.5%. Similarly, calculations using relativistic bag models seem to underestimate the quadrupole effect. As a typical example we mention the calculation of Ref. 8 in the framework of the cloudy bag model (CBM). The interaction between the quarks and the surrounding pion cloud leads to an effective tensor force which admixes D states into the baryon wave functions and yields a ratio of $R_\Delta = -0.92\%$ at a bag radius $R = 1$ fm. Further investigations of the effect include a CQM calculation using the color hyperfine interaction and pion exchange at the same time[9] and a recently reported result that nonspherical components are also created by purely relativistic effects without explicit tensor forces.[10]

In view of the crucial importance of the ratio R_Δ for our understanding of bag dynamics, we have repeated the calculation of Ref. 8 using the chiral bag model. In particular, we compare the results of pseudoscalar (PS) surface coupling and pseudovector (PV) volume coupling between bag and pion cloud. Both couplings are related by a chiral transformation[11] similar to the one generating the chiral Lagrangian of Weinberg[12] from the σ model. In practical calculations, the equivalence of the two coupling schemes is usually violated because of a truncation of the Hilbert space and by using MIT bag wave functions without modifications due to the pion pressure. As an example, a calculation of pion-nucleon scattering has shown that the two coupling schemes show a completely different convergence and that about 5–10 excited states have to be included in order to obtain equivalent results for PV and PS coupling.[13] Similarly, the leading Kroll-Rudermann term in pion photoproduction at threshold is only obtained as an infinite sum over sea-quark ($0s_{1/2}$) and valence-quark ($0p_{1/2}$) excitations in the case of PS coupling.[14] Therefore, the comparison of PS and PV coupling can serve as an indication of whether or not a particular truncation of the configuration space is adequate. As we have shown in a previous Letter,[15] the matrix elements of the charge ($C2$) and current ($E2$) quadrupole operators are affected quite differently by such truncations. In fact it turns out that the usually small value for R_Δ in CQM calculations is obtained using the current operator, because of strong cancellations of initial- and final-state deformations in a restricted configuration space. The corresponding matrix element of the charge operator is much more stable against a truncation of the configuration space, which leads to a much larger ratio R_Δ in a good agreement with the data. In the same spirit, we shall evaluate both $C2$ and $E2$ amplitudes in the CBM in order to test the numerical accuracy and stability of the predictions.

Another challenge for dynamical models of the nucleon has been the Roper resonance $N(1440)$. In a naive quark model, this resonance is described by the same spin-isospin structure as the nucleon, but a radial excitation of one of the quarks. This orthogonality of the radial wave functions leads to an underestimation of the $M1$ transition, and not even the sign of the matrix elements is correctly predicted. As has been shown by the authors of Ref. 8, the CBM is quite successful in improving this situation. In fact, the pionic terms are larger than the contributions of the valence quarks by an order of magnitude. We have checked the reliability of such calculations by a comparison of the predictions of PS and PV coupling.

For the sake of completeness and in order to relate our results to previous calculations, we briefly review the cloudy bag model in Sec. II. The definitions of charge and current transition operators are given in Sec. III, followed by analytical expressions for the photoproduction amplitudes corresponding to the various diagrams in Fig. 1. Finally, we present the results of our calculations in Sec. II and conclude with a short summary in Sec. V.

II. THE CLOUDY BAG MODEL

In our calculations we use two different versions of the linear cloudy bag model. In the original one the pion interacts only with the surface of the bag by pseudoscalar (PS) coupling[11]

$$\mathcal{L}_{\rm PS}=(i\bar{q}\partial\!\!\!/ q-B)\theta_V-\tfrac{1}{2}\bar{q}q\Delta_S+\tfrac{1}{2}(\partial_\mu\boldsymbol{\pi})^2-\tfrac{1}{2}m_\pi{}^2\boldsymbol{\pi}^2-\frac{i}{2f}\bar{q}\gamma_5\boldsymbol{\tau}q\cdot\boldsymbol{\pi}\Delta_S\ , \tag{1}$$

where Θ_V is one inside the bag with radius R and zero outside and Δ_S is a Dirac δ function on the surface, $r=R$. In order to satisfy the Weinberg-Tomozawa relationship,[16] Thomas[17] developed an alternative formulation, where the pion interacts with the entire volume of the bag by pseudovector (PV) coupling:

$$\mathcal{L}_{\rm PV}=(i\bar{q}\partial\!\!\!/ q-B)\theta_V-\tfrac{1}{2}\bar{q}q\Delta_S+\tfrac{1}{2}(\partial_\mu\boldsymbol{\pi})^2-\tfrac{1}{2}m_\pi{}^2\boldsymbol{\pi}^2-\frac{\theta_V}{4f^2}\bar{q}\gamma^\mu\boldsymbol{\tau}q\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})+\frac{\theta_V}{2f}\bar{q}\gamma^\mu\gamma_5\boldsymbol{\tau}q\cdot\partial_\mu\boldsymbol{\pi}\ , \tag{2}$$

The quark wave functions are given by the well-known solutions to the Dirac equations for zero-mass quarks in a spherical cavity:

$$q_\alpha(r)=N_{\kappa,n}\begin{bmatrix} j_l\left[\dfrac{\omega_{\kappa,n}r}{R}\right] \\ -\lambda i\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}j_{\bar{l}}\left[\dfrac{\omega_{\kappa,n}r}{R}\right]\end{bmatrix}\chi_\alpha\ , \tag{3}$$

where

$$\chi_\alpha=|(1,1/2)J,\mu\rangle\otimes|1/2\nu\rangle\ ,$$
$$N_{\kappa,n}=\left[\frac{\omega_{\kappa,n}}{2R^3(\omega_{\kappa,n}+\kappa)j_l^2(\omega_{\kappa,n})}\right]^{1/2}, \tag{4}$$
$$\lambda=\mathrm{sgn}(\kappa)\ \text{ and }\ \bar{l}=1-\lambda\ .$$

The index α stands for the angular momentum l, the total angular momentum $J=l-\lambda/2$ $(\kappa\neq 0)$ with projection μ and the radial quantum number n. From the linear boundary condition at the surface of the bag,

$$j_l(\omega_{\kappa,n})=-\lambda j_{\bar{l}}(\omega_{\kappa,n})\ , \tag{5}$$

the ground state $0s_{1/2}$ and the first excited s state $1s_{1/2}$ have the eigenfrequencies $\omega_0=2.0428$ and $\omega_1=5.3960$.

The pionic field has the representation

$$\boldsymbol{\pi}(r)=\frac{1}{(2\pi)^{3/2}}\int\frac{d^3k}{\sqrt{2\omega}}(\mathbf{a}_{\mathbf{k}}e^{-ik_\mu x^\mu}+\mathbf{a}_{\mathbf{k}}^\dagger e^{ik_\mu x^\mu})\ , \tag{6}$$

with $\mathbf{a}_{\mathbf{k}}$ and $\mathbf{a}_{\mathbf{k}}^\dagger$ the annihilation and creation operators of a pion of four-momentum $k^\mu=(\omega,\mathbf{k})$. In this context we use a spherical notation for the components of the pion field, i.e.,

$$\pi_{\pm1}=\mp\frac{1}{\sqrt{2}}(\pi_x\pm i\pi_y)\ ,\quad \pi_0=\pi_z\ , \tag{7}$$

where π_{+1} is the field which creates a π^+ and destroys a π^-. Note that $(a_{\mathbf{k}}^\dagger)_{\pm1}$ is defined as $\mp(a_x^\dagger\pm ia_y^\dagger)/\sqrt{2}$. Performing the minimal coupling substitution for the

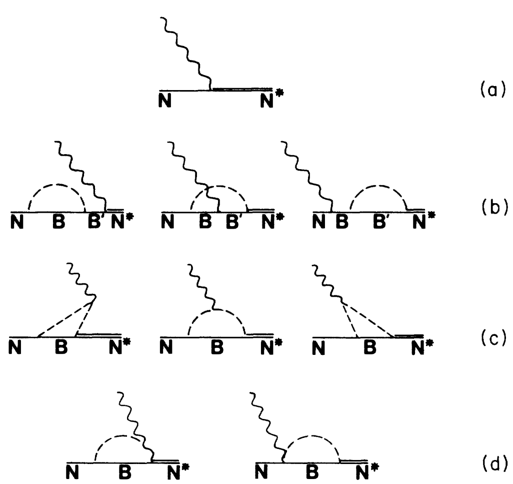

FIG. 1. Diagrams used in the calculation of the $M1$ and $E2$, $C2$ form factors. The intermediate states B and B' represent either nucleon, delta, Roper, or a quadrupole excitation (D state), and N^* stands for delta or Roper.

electromagnetic field A,

$$\partial_\mu q \to \partial_\mu q + ie_q A_\mu q \ , \qquad \partial_\mu \pi_{\pm 1} \to \partial_\mu \pi_{\pm 1} \mp ie A_\mu \pi_{\pm 1} \ , \tag{8}$$

we obtain the electromagnetic interactions of lowest order:

$$\mathcal{L}_{qq\gamma} = -\bar{q} e_q A_\mu \gamma^\mu q \theta_v \ , \tag{9}$$

$$\mathcal{L}_{\pi\pi\gamma} = +ie A_\mu (\pi_{+1}\partial^\mu \pi_{-1} - \pi_{-1}\partial^\mu \pi_{+1}) \ , \tag{10}$$

where $e_q = e(1+3\tau_0)/6$ and $e = \sqrt{(4\pi\alpha)}$ are the quark and proton charges, respectively. Moreover, the pseudovector coupling contains a contribution of the contact term:

$$\mathcal{L}_{\pi q q \gamma} = +\frac{\theta_V}{2f} \bar{q} A_\mu \gamma^\mu \gamma_5 ie(\tau_{-1}\pi_{+1} - \tau_{+1}\pi_{-1}) q \ , \tag{11}$$

where τ and π are defined as spherical tensors [Eq. (7)]. The first-order interactions between quarks and pions differ in the two coupling schemes. Introducing special form factors, they can be cast into the form

$$H_{qq\pi} = \langle \chi_f \,|\, \hat{H}_{qq\pi} \,|\, \chi_i \rangle \ , \tag{12}$$

with

$$\hat{H}_{qq\pi} = \frac{i}{6f} \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega}} \mathbf{\Sigma}\cdot\mathbf{k}[a_{fi}^\dagger(k)\tau\cdot\mathbf{a}_k^\dagger - a_{\bar{f}\bar{i}}(k)\tau\cdot\mathbf{a}_k] \ . \tag{13}$$

In this form only p-wave pion emission and absorption is considered, other partial waves do not enter our calculations. In detail for specific quark transitions we find

$$\mathbf{\Sigma} = \boldsymbol{\sigma} \quad \text{for } s_{1/2} \leftrightarrow s_{1/2}$$

and

$$\mathbf{\Sigma} = -\sqrt{(8\pi)}[\boldsymbol{\sigma} \times Y_2(\hat{\mathbf{r}})]^{[1]} \quad \text{for } s_{1/2} \leftrightarrow d_{3/2} \ . \tag{14}$$

The pseudoscalar form factors for pion absorption $(-)$ and emission $(+)$ are identical,

$$a_{ab}^{\mathrm{PS},+} = a_{ab}^{\mathrm{PS},-} = \frac{3j_1(kR)}{kR} \left[\frac{\omega_a \omega_b}{(\omega_a + \kappa_a)(\omega_b + \kappa_b)} \right]^{1/2} \times \mathrm{sgn}[j_{l_a}(\omega_a) j_{l_b}(\omega_b)] \ , \tag{15}$$

while the pseudovector form factors differ for transitions involving different eigenfrequencies, $\omega_a \neq \omega_b$,

$$a_{ab}^{\mathrm{PV},\pm} = a_{ab}^{\mathrm{PS},\pm} + \frac{3}{kR}(\omega_a - \omega_b \pm \omega R) N_a N_b \int_0^R dr\, r^2 j_1(kr) \left[\lambda_b j_{l_a}\left(\frac{\omega_a r}{R}\right) j_{\bar{l}_b}\left(\frac{\omega_b r}{R}\right) - \lambda_a j_{\bar{l}_a}\left(\frac{\omega_a r}{R}\right) j_{l_b}\left(\frac{\omega_b r}{R}\right) \right] \ . \tag{16}$$

III. THE PHOTOPRODUCTION AMPLITUDES

Photoproduction of the $\Delta(1232)$ and $N(1440)$ resonances proceeds mainly through the $M1$ transition.[18] In the case of the Δ resonance an additional $E2$ transition is possible. Furthermore, if the resonance is excited by virtual photons, the longitudinal multipoles $C0$ and $C2$ contribute for Roper and delta, respectively. While the monopole contribution $C0$ vanishes at the photon point, the $C2$ transition of the $\Delta(1232)$ is related to the $E2$ in the long-wavelength limit. From general arguments the NN^* electromagnetic transition currents can be written in the c.m. frame by nonrelativistic approximation as follows.[19]

(i) $\Delta(\frac{3}{2}^+, \frac{3}{2}; 1232)$

$$\hat{\rho}_{\Delta N} = \frac{e\sqrt{5}G_{C2}^{\Delta N}}{m_N(m_\Delta - m_N)} (\sigma_{\Delta N}^{[2]} \times q^{[2]})^{[0]} \ , \tag{17}$$

$$\hat{\mathbf{j}}_{\Delta N} = i\frac{eG_{M1}^{\Delta N}}{2m_N} \boldsymbol{\sigma}_{\Delta N} \times \mathbf{q} - e(\tfrac{5}{3})^{1/2} \frac{G_{E2}^{\Delta N}}{m_N} (\sigma_{\Delta N}^{[2]} \times q^{[1]})^{[1]} \ . \tag{18}$$

(ii) $N(1/2^+, 1/2; 1440)$

$$\hat{\rho}_{RN} = \frac{-e(m_R + m_N)\mathbf{q}^2 G_{C0}^{RN}}{2m_N(m_R - m_N)^2} \ , \tag{19}$$

$$\hat{\mathbf{j}}_{RN} = \frac{-e(m_R + m_N)G_{E0}^{RN}}{2m_N(m_R - m_N)} \mathbf{q} + i\frac{eG_{M1}^{RN}}{2m_N} \boldsymbol{\sigma} \times \mathbf{q} \ , \tag{20}$$

where m_N, m_Δ, and m_R are the nucleon, delta, and Roper masses, respectively, and $\mathbf{q}$ is the momentum of the transferred (real or virtual) photon. The transition spin matrices for the delta are defined by their reduced matrix elements:

$$\langle \Delta \| \sigma_{\Delta N}^{[1]} \| N \rangle = 2 \ , \quad \langle \Delta \| \sigma_{\Delta N}^{[2]} \| N \rangle = \sqrt{(10)} \ . \tag{21}$$

Furthermore, we use the definition $q^{[2]} = (q^{[1]} \times q^{[1]})^{[2]}$. In comparison with Ref. 19 we include an additional factor $\frac{1}{2}$ in the definition of $G_{E2}^{\Delta N}$, which is now defined in accordance with standard literature.[20] At resonance energies, $\omega = m_{N^*} - m_N$, the electric and Coulomb form factors are related by current conservation: $G_{C0} = G_{E0}$ and $G_{C2} = G_{E2}$. Strictly speaking, this equivalence is only valid in the long-wavelength limit, $q \to 0$ (Siegert theorem[21]).

To connect with experimental quantities we define the radiative decay width of resonances:

$$\Gamma^{p,n}(N^* \to N\gamma) = \frac{\omega_\gamma^{\,2}}{\pi} \frac{m_N}{m_N^*} \frac{2}{2J_{N^*}+1} (\,|A_{1/2}^{p,n}|^2 + |A_{3/2}^{p,n}|^2) \ , \tag{22}$$

with ω_γ the photon energy in the rest frame of the resonance of spin J_{N^*} and mass m_{N^*}. The helicity amplitudes are given by

$$A^{p,n}_{1/2} = \pm \frac{1}{\sqrt{2\omega_\gamma}} \langle N^*; J^*, +\tfrac{1}{2} \,|\, -\mathcal{L}_{\rm em} \,|\, N; \tfrac{1}{2}, -\tfrac{1}{2} \rangle \,, \tag{23}$$

$$A^{p,n}_{3/2} = \pm \frac{1}{\sqrt{2\omega_\gamma}} \langle N^*; J^*, +\tfrac{3}{2} \,|\, -\mathcal{L}_{\rm em} \,|\, N; \tfrac{1}{2}, +\tfrac{1}{2} \rangle \,, \tag{24}$$

where the sign ($\pm$) is determined by the pion photoproduction amplitudes.[22] Specifically, for N^* being the Roper (R) or delta (Δ) resonances the sign is positive.

The electromagnetic interaction is obtained by minimal coupling to the model Lagrangian and can be written as

$$-\mathcal{L}_{\rm em} = \mathcal{H}_{\rm em} = j^\mu A_\mu = \rho\phi - \mathbf{j}\cdot\mathbf{A} \,. \tag{25}$$

Using definitions (17)–(24) and a right-handed photon polarization vector $\epsilon = -1/\sqrt{2}(1,i,0)$, we can express the helicity amplitudes in terms of the following form factors.

(i) $\Delta \to N\gamma$

$$\begin{aligned} A_{1/2} &= A_{1/2}(M1) + A_{1/2}(E2) \\ &= -\left[\frac{\alpha\pi\omega_\gamma}{6m_N{}^2}\right]^{1/2} (G^{\Delta N}_{M1} - 3G^{\Delta N}_{E2}) \,, \end{aligned} \tag{26}$$

$$\begin{aligned} A_{3/2} &= A_{3/2}(M1) + A_{3/2}(E2) \\ &= -\left[\frac{\alpha\pi\omega_\gamma}{2m_N{}^2}\right]^{1/2} (G^{\Delta N}_{M1} + G^{\Delta N}_{E2}) \,. \end{aligned} \tag{27}$$

(ii) $R \to N\gamma$

$$A^{p,n}_{1/2} = -\left[\frac{\alpha\pi\omega_\gamma}{m_N{}^2}\right]^{1/2} G^{RN}_{M1} \,. \tag{28}$$

Finally, we obtain the ratio of electric quadrupole and magnetic dipole transitions in standard high-energy notation:

$$\begin{aligned} R_\Delta = \frac{E_{1+}}{M_{1+}} &= \frac{A_{1/2} - \frac{1}{\sqrt{3}} A_{3/2}}{A_{1/2} + \sqrt{3} A_{3/2}} \\ &= -\frac{A_{3/2}(E2)}{A_{3/2}(M1)} = -\frac{G^{\Delta N}_{E2}}{G^{\Delta N}_{M1}} \,. \end{aligned} \tag{29}$$

For the Roper resonance we define the ratio

$$R_R = \frac{A^p_{1/2}}{A^n_{1/2}} = \frac{G^{R^+p}_{M1}}{G^{R^0 n}_{M1}} \,. \tag{30}$$

For an evaluation of these quantities in the cloudy bag model we have considered the diagrams depicted in Figs. 1(a)–1(d). Diagram 1(a) is the direct absorption term and the others are pionic terms coupled to N, Δ, R intermediate states denoted generally by B. In the case of $\Delta \to N\gamma$ we have omitted intermediate Roper states but have also included excitations of intermediate D states.

The wave functions of nucleon, delta, and Roper are constructed as direct products of a symmetric coordinate space part, a symmetric spin-isospin part, and the antisymmetric color-singlet state. Omitting the color degree of freedom we find the overall symmetrical wave functions

$$\begin{aligned} |N\rangle &= R_{0s}(\mathbf{r}_1)R_{0s}(\mathbf{r}_2)R_{0s}(\mathbf{r}_3)\frac{1}{\sqrt{2}}(\phi_{MS}\chi_{MS} + \phi_{MA}\chi_{MA}) \,, \\ |\Delta\rangle &= R_{0s}(\mathbf{r}_1)R_{0s}(\mathbf{r}_2)R_{0s}(\mathbf{r}_3)\phi_s\chi_s \,, \\ |R\rangle &= \frac{1}{\sqrt{3}}(R_{1s}(\mathbf{r}_1)R_{0s}(\mathbf{r}_2)R_{0s}(\mathbf{r}_3) + \text{permutations}) \\ &\quad \times \frac{1}{\sqrt{2}}(\phi_{MS}\chi_{MS} + \phi_{MA}\chi_{MA}) \,. \end{aligned} \tag{31}$$

The spin and isospin wave functions are denoted according to their symmetry behavior, given in standard angular momentum algebra by

$$\begin{aligned} \chi_s &= |[(\tfrac{1}{2}\tfrac{1}{2})1\tfrac{1}{2}]\tfrac{3}{2}\mu\rangle \,, \\ \chi_{MS} &= |[(\tfrac{1}{2}\tfrac{1}{2})1\tfrac{1}{2}]\tfrac{1}{2}\mu\rangle \,, \\ \chi_{MA} &= |[(\tfrac{1}{2}\tfrac{1}{2})0\tfrac{1}{2}]\tfrac{1}{2}\mu\rangle \,, \end{aligned} \tag{32}$$

where μ is the projection of the total spin of the bag. The isospin wave functions ϕ are defined in an analogous way.

A. Photoproduction of $\Delta(1232)$

Using the Lagrangian of Eqs. (1) and (2) and the wave functions defined in Eqs. (3) and (4) we have evaluated the contributions to the form factors G_{M1} and G_{E2} from the various diagrams in Fig. 1. Specifically, we obtain, for Figs. 1(a), 1(b), 1(c),

$$G^{\Delta N}_{M1}(a) = \frac{8m_N}{\sqrt{3}k_\gamma} f^M_{00}(k_\gamma)\sqrt{Z^N Z^\Delta} \,, \tag{33}$$

$$G^{\Delta N}_{M1}(b) = \frac{2m_N}{27\sqrt{3}\pi^2 k_\gamma}\left[\frac{1}{2f}\right]^2 f^M_{00}(k_\gamma)\int \frac{dk\, k^4 a_{00}{}^2(k)}{\omega}\left[\frac{125}{\omega(\omega+\delta-\omega_\gamma)} + \frac{100}{(\omega+\delta)(\omega+\delta-\omega_\gamma)} + \frac{8}{(\omega+\delta)(\omega-\omega_\gamma)} + \frac{100}{\omega(\omega-\omega_\gamma)}\right] , \tag{34}$$

$$G_{M1}^{\Delta N}(c)=\frac{10m_N}{3\sqrt{3}\pi^3k_\gamma}\left[\frac{1}{2f}\right]^2\int\frac{dk\,dk'}{\omega\omega'}k^3k'^3a_{00}(k)$$
$$\times a_{00}(k')I_{111}(k,k',k_\gamma)\left[\frac{1}{\omega(\omega'-\delta)}+\frac{1}{(\omega+\omega'-\delta)(\omega'-\delta)}+\frac{1}{\omega(\omega+\omega'+\delta)}\right.$$
$$\left.+\frac{5}{\omega'(\omega+\delta)}+\frac{5}{\omega'(\omega+\omega'-\delta)}+\frac{5}{(\omega+\delta)(\omega+\omega'+\delta)}\right]. \quad (35)$$

The photon and pion four-momenta are defined as $(\omega_\gamma,\mathbf{k}_\gamma)$ and $(\omega,\mathbf{k})$, and $\delta=m_\Delta-m_N$ is the nucleon-delta mass difference. The pion decay constant f is related to the strong πN coupling constant by the Goldberger-Treiman relation

$$\frac{1}{2f}=\frac{9}{5}\frac{\omega_0-1}{\omega_0}\frac{f_{\pi NN}}{\mu}, \quad (36)$$

where we have used the bag-model value for g_A. In order to get the proper πN coupling strength we have used $f_{\pi NN}{}^2/4\pi=0.08$. This value gives $f=76$ MeV, which is a factor of 1.2 smaller than the experimental number.

The electromagnetic form factors (γ,BB') and $(\gamma,\pi\pi')$ are defined as

$$f_{ab}^M(k_\gamma)=\tfrac{1}{2}N_aN_b\int_0^R dr\,r^2j_1(k_\gamma r)\left[j_1\left[\frac{\omega_a r}{R}\right]j_0\left[\frac{\omega_b r}{R}\right]+j_0\left[\frac{\omega_a r}{R}\right]j_1\left[\frac{\omega_b r}{R}\right]\right], \quad (37)$$

$$I_{111}(k,k',k_\gamma)=\int_0^\infty dr\,rj_1(kr)j_1(k'r)j_1(k_\gamma r)=\frac{\pi}{32k^2k'^2k_\gamma{}^2}[4k^2k_\gamma^2-(k^2-k'^2+k_\gamma{}^2)^2]\Delta(k,k',k_\gamma), \quad (38)$$

where $\Delta(a,b,c)=1$, whenever a, b, and c obey a triangular relation, and zero otherwise. Since we are considering only nucleons and deltas so far, these equations hold both for pseudoscalar and pseudovector coupling. However, an additional contribution arises from the contact term in PV coupling, Fig. 1(d):

$$G_{M1}^{\Delta N}(d)=-\frac{5m_N}{3\sqrt{3}\pi^2k_\gamma}\left[\frac{1}{2f}\right]^2\int\frac{dk}{\omega}k^3a_{00}(k)C_0(k,k_\gamma)\left[\frac{1}{\omega-\delta}+\frac{6}{\omega}+\frac{5}{\omega+\delta}\right], \quad (39)$$

with the $\gamma\pi B$ vertex form factor

$$C_0(k,k_\gamma)=N_0{}^2\int_0^R dr\,r^2j_1(kr)j_1(k_\gamma r)\left[j_0{}^2\left[\frac{\omega_0 r}{R}\right]-j_1{}^2\left[\frac{\omega_0 r}{R}\right]\right]. \quad (40)$$

The factor $(Z^NZ^\Delta)^{1/2}$ in Eq. (33) describes the bare bag probability in the initial and final states. The renormalization constants are given by[11,23]

$$Z^B=\left[1-\frac{\partial}{\partial E}\Sigma^B(E)\right]^{-1}, \quad (41)$$

where Σ^B is the self-energy of $B=N,\Delta$ due to the interaction with the pion field:

$$\Sigma^N(E)=\frac{1}{12\pi^2}\left[\frac{1}{2f}\right]^2\int dk\frac{k^4}{\omega}a_{00}{}^2(k)\left[\frac{25}{E-m_N-\omega}+\frac{32}{E-m_\Delta-\omega}\right], \quad (42)$$

$$\Sigma^\Delta(E)=\frac{1}{12\pi^2}\left[\frac{1}{2f}\right]^2\int dk\frac{k^4}{\omega}a_{00}{}^2(k)\left[\frac{8}{E-m_N-\omega}+\frac{25}{E-m_\Delta-\omega}\right]. \quad (43)$$

For reasons of consistency we have expanded the renormalization constants $(Z^NZ^\Delta)^{1/2}$ to order $(1/2f)^2$. Consequently only the leading term, Fig. 1(a), will be modified. Such a procedure guarantees current conservation to second order. In previous calculations[11,23] also higher terms [Fig. 1(b), etc.] have been multiplied by the renormalization constants therefore implicitly introducing fourth- and higher-order terms in the coupling constant. Whether or not such a procedure improves the convergence of the perturbation series is not known *a priori;* however, gauge invariance could be violated.

In the case of the quadrupole transitions, $E2$ and $C2$, the contribution of Fig. 1(a), which is dominant in the $M1$ channel, vanishes. Therefore we go beyond the calculations for the $M1$ amplitude by taking into account intermediate states from $0d_{3/2}$ $(\kappa=2)$ excitations. The wave function is given by Eq. (3) and the eigenfrequency ω_2 is determined through the boundary condition $j_2(\omega_2)=-j_1(\omega_2)$, which yields the value $\omega_2=5.1231$ for the lowest mode. We neglect the mass shifts due to the hyperfine interactions between quarks in the presence of a D-state quark, such that

$\Delta \equiv (\omega_2 - \omega_0)/R = 608$ MeV for $R = 1$ fm. For the individual diagrams of Fig. 1 we find the following form factors for the current:

$$G_{E2}^{\Delta N}(a) = 0 \ , \tag{44}$$

$$G_{E2}^{\Delta N}(b) = -\frac{m_N}{90\sqrt{3}\,\pi^2 k_\gamma}\left[\frac{1}{2f}\right]^2 f_{02}^{E}(k_\gamma)\int \frac{dk}{\omega} k^4 a_{00}(k) \times a_{02}^{\dagger}(k)\left[\frac{5}{\omega(\omega+\Delta-\omega_\gamma)} - \frac{2}{(\omega+\delta)(\omega+\Delta-\omega_\gamma)} + \frac{5}{(\omega+\Delta)(\omega+\delta-\omega_\gamma)} - \frac{8}{(\omega+\Delta)(\omega-\omega_\gamma)} + \frac{2}{(\Delta-\omega_\gamma)(\omega-\omega_\gamma)} - \frac{5}{(\Delta-\omega_\gamma)(\omega+\delta-\omega_\gamma)} - \frac{5}{\Delta\omega} + \frac{8}{\Delta(\omega+\delta)}\right] , \tag{45}$$

$$G_{E2}^{\Delta N}(c) = \frac{2m_N}{3\sqrt{3}\pi^3 k_\gamma}\left[\frac{1}{2f}\right]^2 \int \frac{dk\,dk'}{\omega\omega'} k^3 k'^3 a_{00}(k) \times a_{00}(k') I_{102}(k,k',k_\gamma)\left[\frac{1}{\omega(\omega'-\delta)} + \frac{1}{(\omega+\omega'-\delta)(\omega'-\delta)} + \frac{1}{\omega(\omega'+\omega+\delta)} - \frac{1}{\omega'(\omega+\delta)} - \frac{1}{\omega'(\omega+\omega'-\delta)} - \frac{1}{(\omega+\delta)(\omega+\omega'+\delta)}\right] , \tag{46}$$

$$G_{E2}^{\Delta N}(d) = \frac{5m_N}{3\sqrt{3}\pi^2 k_\gamma}\left[\frac{1}{2f}\right]^2 \int \frac{dk}{\omega} k^3 a_{00}(k) C_2(k,k_\gamma)\left[\frac{1}{\omega-\delta} - \frac{2}{\omega} + \frac{1}{\omega+\delta}\right] . \tag{47}$$

The large cancellations among the individual contributions to the electric quadrupole amplitude are obvious from Eqs. (45)–(47). The expressions within the large parentheses vanish exactly in the soft-photon limit ($\omega_\gamma \to 0$) and for a vanishing hyperfine interaction ($\delta \to 0$).

The corresponding charge form factors are

$$G_{C2}^{\Delta N}(a) = 0 \ , \tag{48}$$

$$G_{C2}^{\Delta N}(b) = \xi G_{E2}^{\Delta N}(b; 5, -2, -5, 8, 2, -5, 5, -8) \ , \tag{49}$$

$$G_{C2}^{\Delta N}(c) = \frac{-5\delta m_N}{3\sqrt{3}\pi^3 k_\gamma{}^2}\left[\frac{1}{2f}\right]^2 \int \frac{dk\,dk'}{\omega\omega'} k^3 k'^3 a_{00}(k) a_{00}(k') \times I_{112}(k,k',k_\gamma)\left[\frac{\omega+\omega'}{\omega'(\omega+\delta)} + \frac{\omega'-\omega}{\omega'(\omega+\omega'-\delta)} + \frac{\omega-\omega'}{(\omega+\delta)(\omega+\omega'+\delta)} - \frac{\omega-\omega'}{\omega(\omega+\omega'+\delta)} - \frac{\omega+\omega'}{\omega(\omega'-\delta)} - \frac{\omega'-\omega}{(\omega+\omega'-\delta)(\omega'-\delta)}\right] , \tag{50}$$

$$G_{C2}^{\Delta N}(d) = 0 \ . \tag{51}$$

The terms (b) involving D-state excitations are very similar for $E2$ and $C2$, up to the signs of the individual diagrams as indicated symbolically in Eq. (49) and an overall factor

$$\xi = \frac{5\delta f_{02}^{C}(k_\gamma)}{k_\gamma f_{02}^{E}(k_\gamma)} \ . \tag{52}$$

Because of this change of sign the cancellation observed in the current matrix elements is no longer present for the charge operator. The additional vertex form factors for $BB'\gamma$, $BB'\pi\gamma$, $\pi\pi'\gamma$ are given by

$$f_{02}^{E}(k_\gamma) = N_0 N_2 \int_0^R dr\, r^2 j_1(k_\gamma r)\left[j_1\left[\frac{\omega_0 r}{R}\right] j_2\left[\frac{\omega_2 r}{R}\right] - 5 j_0\left[\frac{\omega_0 r}{R}\right] j_1\left[\frac{\omega_2 r}{R}\right]\right] , \tag{53}$$

$$f_{02}^{C}(k_\gamma) = N_0 N_2 \int_0^R dr\, r^2 j_2(k_\gamma r)\left[j_0\left[\frac{\omega_0 r}{R}\right] j_2\left[\frac{\omega_2 r}{R}\right] - j_1\left[\frac{\omega_0 r}{R}\right] j_1\left[\frac{\omega_2 r}{R}\right]\right] , \tag{54}$$

$$C_2(k,k_\gamma)=N_0^2\int_0^R dr\, r^2 j_1(kr)j_1(k_\gamma r)\left[j_0^2\left[\frac{\omega_0 r}{R}\right]-\frac{1}{5}j_1^2\left[\frac{\omega_0 r}{R}\right]\right], \tag{55}$$

$$\begin{aligned}I_{102}(k,k',k_\gamma)&=\frac{5}{k_\gamma}\int_0^\infty dr\, r[k'j_1(kr)j_0(k'r)-kj_0(kr)j_1(k'r)]j_2(k_\gamma r)\\&=\frac{5\pi}{32k^2k'^2k_\gamma^4}(k^2-k'^2)[4k^2k_\gamma^2-(k^2-k'^2+k_\gamma^2)^2]\Delta(k,k',k_\gamma)\,,\end{aligned} \tag{56}$$

$$I_{112}(k,k',k_\gamma)=\int_0^\infty dr\, r^2 j_1(kr)j_1(k'r)j_2(k_\gamma r)=\frac{3}{k_\gamma}I_{111}+\frac{\pi}{8k^2k'^2k_\gamma}(k_\gamma^2-k'^2-k^2)\Delta(k,k',k_\gamma)\,. \tag{57}$$

B. Photoproduction of R (1440)

For the photoproduction of the Roper we can form two independent isospin amplitudes corresponding to the proton/neutron amplitudes in the usual notation. Since the Roper appears in the final state we have also included it as a possible intermediate state. For the $M1$ form factors we obtain the following contributions from the diagrams of Fig. 1:

$$G_{M1}^{RN,\{^p_n\}}(a)=\frac{4m_N}{k_\gamma}\begin{Bmatrix}1\\-\frac{2}{3}\end{Bmatrix}F_{RN}(k_\gamma)(Z^NZ^R)^{1/2}\,, \tag{58}$$

$$\begin{aligned}G_{M1}^{RN,\{^p_n\}}(b)=\frac{5m_N}{(27\pi)^2k_\gamma}\left[\frac{1}{2f}\right]^2\int\frac{dk}{\omega}k^4\Bigg\{&A_{R0}^-(k)F_{00}(k_\gamma)A_{00}^+(k)\left[\frac{-5}{\omega(\Delta_{RN}-\omega)}\begin{Bmatrix}1\\-4\end{Bmatrix}+\frac{64}{(\Delta_{RN}-\omega)(\Delta_{N\Delta}-\omega)}\begin{Bmatrix}1\\-1\end{Bmatrix}\right.\\&\left.-\frac{64}{\omega(\Delta_{R\Delta}-\omega)}\begin{Bmatrix}1\\-1\end{Bmatrix}-\frac{32}{(\Delta_{R\Delta}-\omega)(\Delta_{N\Delta}-\omega)}\begin{Bmatrix}4\\-1\end{Bmatrix}\right]\\&+A_{R0}^-(k)F_{0R}(k_\gamma)A_{R0}^+(k)\left[\frac{5}{(\Delta_{RN}-\omega)^2}\begin{Bmatrix}1\\-4\end{Bmatrix}+\frac{64}{(\Delta_{R\Delta}-\omega)(\Delta_{NR}-\omega)}\begin{Bmatrix}1\\-1\end{Bmatrix}\right]\\&+A_{RR}^-(k)F_{R0}(k_\gamma)A_{00}^+(k)\left[\frac{5}{\omega^2}\begin{Bmatrix}1\\-4\end{Bmatrix}-\frac{64}{\omega(\Delta_{N\Delta}-\omega)}\begin{Bmatrix}1\\-1\end{Bmatrix}\right]\\&-A_{RR}^-(k)F_{RR}(k_\gamma)A_{RR}^+(k)\frac{5}{\omega(\Delta_{NR}-\omega)}\begin{Bmatrix}1\\-4\end{Bmatrix}\\&+\begin{Bmatrix}27/5\\-18/5\end{Bmatrix}\left[F_{RR}(k_\gamma)A_{R0}^-(k)A_{0N}^+(k)\left[\frac{25}{\Delta_{RN}\omega}+\frac{32}{\Delta_{RN}(\omega+\Delta_{\Delta N})}\right]\right.\\&+F_{RR}(k_\gamma)A_{RR}^-(k)A_{RN}^+(k)\frac{25}{\Delta_{RN}(\omega+\Delta_{RN})}\\&+A_{R0}^-(k)A_{0N}^+(k)F_{NN}(k_\gamma)\left[\frac{25}{\Delta_{RN}(\Delta_{RN}-\omega)}+\frac{32}{\Delta_{RN}(\Delta_{R\Delta}-\omega)}\right]\\&\left.-A_{RR}^-(k)A_{RN}^+(k)F_{NN}(k_\gamma)\frac{25}{\Delta_{RN}\omega}\right]\Bigg\}\,,\end{aligned} \tag{59}$$

$$G_{M1}^{RN,\{{p \atop n}\}}(c)=\frac{-m_N}{72\pi^2}\left[\frac{1}{2f}\right]^2\left\{\begin{matrix}1\\-1\end{matrix}\right\}\int_0^\infty\frac{dk}{\omega}k^4\int_{-1}^{1}dt\frac{1-t^2}{\omega'}$$
$$\times\sum_B d_B\left[\frac{A_{RB}^{+}(k)A_{BN}^{+}(k')+A_{RB}^{+}(k')A_{BN}^{+}(k)}{(\omega+\Delta_{BN})(\omega+\omega'+\Delta_{RN})}+\frac{A_{RB}^{-}(k)A_{BN}^{+}(k')}{(\omega-\Delta_{RB})(\omega'+\Delta_{BN})}+\frac{A_{RB}^{-}(k)A_{BN}^{+}(k')}{(\omega+\Delta_{BN})(\omega'-\Delta_{RB})}\right.$$
$$\left.+\frac{A_{RB}^{-}(k)A_{BN}^{-}(k')}{\omega+\omega'-\Delta_{RN}}\left[\frac{1}{\omega-\Delta_{RB}}+\frac{1}{\omega'-\Delta_{RB}}\right]\right], \tag{60}$$

with

$$k'=|\mathbf{k}-\mathbf{k}_\gamma|=\sqrt{k^2+k_\gamma{}^2-2kk_\gamma t}\ ,\quad \omega'=\sqrt{k'^2+m_\pi{}^2}\ ,\quad \omega=\sqrt{k^2+m_\pi{}^2}\ ,$$
$$\Delta_{B'B}=m_{B'}-m_B\ ,\quad d_B=\begin{cases}\frac{25}{9}, & B=N,R\ ,\\ \frac{8}{9}, & B=\Delta\ .\end{cases} \tag{61}$$

The electromagnetic and strong vertex form factors are given as

$$F_{BB'}(k_\gamma)=\begin{cases}f_{00}, & BB'=NN,N\Delta,\Delta N,\Delta\Delta\ ,\\ \frac{1}{\sqrt{3}}f_{10}, & BB'=NR,RN,\Delta R,R\Delta\ ,\\ \frac{1}{3}f_{11}+\frac{2}{3}f_{00}, & BB'=RR\ ,\end{cases} \tag{62}$$

with f_{ab} defined in Eq. (37),

$$A_{B'B}^{\pm}(k)=\begin{cases}a_{00}^{\pm}(k), & B'B=NN,N\Delta,\Delta N,\Delta\Delta\equiv 00\ ,\\ \frac{1}{\sqrt{3}}a_{10}^{\pm}(k), & B'B=RN,R\Delta\equiv R0\ ,\\ \frac{1}{\sqrt{3}}a_{01}^{\pm}(k), & B'B=NR,\Delta R=0R\ ,\\ \frac{1}{3}a_{11}^{\pm}(k)+\frac{2}{3}a_{00}^{\pm}(k), & B'B=RR\ ,\end{cases} \tag{63}$$

where the index 0 denotes a nucleon or delta, and 1 stands for the Roper resonance. Finally, we obtain additional contributions from the seagull term [Fig. 1(d)] in the case of PV coupling:

$$G_{M1}^{RN,\{{p \atop n}\}}(d)=\frac{2m_N}{3k_\gamma\pi^2}\left\{\begin{matrix}1\\-1\end{matrix}\right\}\left[\frac{1}{2f}\right]^2\int dk\,k^3\sum_B d_B\left[\frac{C_{RB}(k,k_\gamma)A_{BN}^{+}(k)}{\Delta_{NB}-\omega}+\frac{A_{RB}^{-}(k)C_{BN}(k,k_\gamma)}{\Delta_{RB}-\omega}\right], \tag{64}$$

with d_B defined as in Eq. (61) and the vertex form factors

$$c_{ab}=N_aN_b\int_0^R dr\,r^2 j_1(kr)j_1(k_\gamma r)\left[j_0\left[\frac{\omega_a r}{R}\right]j_0\left[\frac{\omega_b r}{R}\right]-j_1\left[\frac{\omega_a r}{R}\right]j_1\left[\frac{\omega_b r}{R}\right]\right], \tag{65}$$

where

$$C_{B'B}(k,k_\gamma)=\begin{cases}c_{00}, & B'B=NN,N\Delta,\Delta N,\Delta\Delta\ ,\\ \frac{1}{\sqrt{3}}c_{10}, & B'B=RN,R\Delta,NR,\Delta R\ ,\\ \frac{1}{3}c_{11}+\frac{2}{3}c_{00}, & B'B=RR\ .\end{cases} \tag{66}$$

IV. RESULTS

In Table I we have listed the results of the individual contributions for photoproduction of the delta resonance. We have varied the bag radius R in the range of 0.6, 0.8, and 1.0 fm. At $R=1$ fm the $M1$ helicity amplitude is dominated by the bare bag contribution. However, for smaller bags the pionic contributions become more important, which is a general feature of bag models. The quadrupole amplitudes $E2$ and $C2$ get no contribution from the bare bag, which is a pure s state, such that the transition is only mediated by the pion cloud. A comparison between pseudoscalar and pseudovector coupling shows that the contribution of terms (a), (b), and (c) remains almost unchanged, because of the equivalence of both models for transitions from ground state to ground state (e.g., πNN, $\pi\Delta N$, $\pi\Delta\Delta$). A difference occurs only for intermediate D states whose contributions are negligible, however. Therefore the PS-PV difference arises almost entirely from the seagull contribution, which is only present in the pseudovector model. Its effect lowers the $M1$ amplitude and almost cancels the $E2$ amplitude. If the seagull term would take the opposite sign, an almost perfect agreement with the experimental $M1$ amplitude would be obtained.[8] However, the same sign as in our calculation was also obtained in a recent investigation[24] evaluating the seagull contributions to magnetic moments and the axial-vector coupling constant. From the seagull charge $(-\gamma_5\gamma_0)$ one does not obtain a contribution for ground-state transitions; therefore, the quadrupole helicity amplitude is almost unchanged in the PV coupling model when calculated via the charge transition $C2$. Obviously, as $C2$ and $E2$ matrix elements are totally different, the electromagnetic current is not conserved. The reason for this is the truncation of the configuration space to nucleons, deltas, and D states in the intermediate state. While this is a good approximation for $C2$, the results for $E2$ are totally spurious and higher contributions, which are hard to calculate, will have a large effect on this amplitude.[15]

A comparison with the experimental data and with the predictions of the constituent-quark model (CQM) shows that the CBM $M1$ amplitude is larger than in the CQM but still smaller than experiment. Our $C2$ calculations are in quite good agreement with the data, both for the CBM and the CQM. In particular, the difference of PS and PV coupling turns out to be very small in this case. Evaluating the matrix element with the $E2$ operator, we observe an almost complete cancellation of pionic and contact currents for PV coupling and a result of the right magnitude but with a wrong sign for PS coupling. In view of the strong cancellation, we were not able to reproduce the result $R_\Delta=-0.92\%$ of Ref. 8 ($R=1$ fm, $E2$ operator, PV coupling), but instead obtained a value smaller by orders of magnitude. Because of the cancellations in the $E2$ matrix elements within the

TABLE I. Helicity amplitudes for $\Delta(1232)$ photoproduction in units of 10^{-3} GeV$^{-1/2}$. Experimental data from Ref. 1 and results of the constituent-quark model (CQM) from Refs. 5, 6, 15, and 29 compared to the predictions of the cloudy bag model for various bag radii R and using pseudoscalar (PS) and pseudovector (PV) coupling, respectively. The specific contributions a–d correspond to the diagrams of Fig. 1. If possible, the ratio R_Δ has been calculated with the results for the $C2$ amplitude (Siegert theorem).

		$A_{3/2}(M1)$		$A_{3/2}(E2)$		$A_{3/2}(C2)$		R_Δ E_{1+}/M_{1+}	
R(fm)	Term	PS	PV	PS	PV	PS	PV	PS	PV
1.0	a	−96	−96						
	b	−52	−52	1.45	0.44	−0.11	0.09		
	c	−46	−46	4.34	4.34	−3.40	−3.40		
	d		26		−4.56				
	Σ	−194	−168	5.79	0.22	−3.52	−3.31	−0.018	−0.020
0.8	a	−60	−60						
	b	−72	−72	1.79	0.39	−0.18	0.07		
	c	−74	−75	3.12	3.12	−5.21	−5.21		
	d		35		−4.81				
	Σ	−206	−171	4.91	−1.30	−5.39	−5.14	−0.026	−0.030
0.6	a	−33	−33						
	b	−102	−102	−1.86	0.27	−0.15	0.06		
	c	−125	−125	1.09	1.09	−7.05	−7.05		
	d		49		−4.43				
	Σ	−260	−211	2.95	−3.06	−7.20	−6.99	−0.028	−0.033
CQM (Ref. 5)		−179							
CQM (Ref. 6)								−0.004	
CQM (Refs. 16 and 29)		−186		−0.14. . . −0.43		−3.3. . . −3.8		−0.02	
Expt (Ref. 1)		−255±10				−3.4±1.2		−0.013±0.005	

highly truncated basis,[16] we believe that this value is completely spurious anyway, and propose to use the predictions obtained via the charge operator. In this case we obtain, for a bag radius $R=1$ fm, a ratio $R_\Delta=-1.6\%$ for pseudoscalar and -1.8% for pseudovector coupling, in good agreement with the experimental data.

In the case of Roper production the contributions of the graphs of Fig. 1 are listed in Table II. Since the Roper is a radial excitation of the nucleon, the bare bag contribution (a) almost vanishes because of orthogonality. Therefore, the $M1$ amplitude for Roper production is much more sensitive to pionic effects and different pion coupling models than in the case of delta production. The dominant contributions arise from the pseudovector seagull term (d) and pionic currents (c). A comparison with the data gives the correct sign in all cases and fair agreement with the PV model at $R\approx 1$ fm. We obtain a qualitative agreement with Ref. 8 for the overall amplitudes, but considerably different results for the individual contributions. All of our calculations give a proton to neutron ratio close to -1, i.e., an almost pure isovector transition, whereas the experimental ratio is more like the ratio of the magnetic moments of protons and neutrons, showing a sizable isoscalar contribu-

TABLE II. Helicity amplitudes for Roper(1440) photoproduction in units of 10^{-3} GeV$^{-1/2}$. For notation see Table I.

		$A^p_{1/2}$		$A^n_{1/2}$		R_R	
R (fm)	Term	PS	PV	PS	PV	PS	PV
1.0	a	-3.9	-3.5	2.6	2.4		
	b	4.6	6.3	0.7	-2.3		
	c	-8.3	0.7	8.3	-0.7		
	d		-38.7		38.7		
	Σ	-7.6	-35.2	11.6	38.0	-0.66	-0.93
0.8	a	-0.6	-0.6	0.4	0.4		
	b	-20.2	-6.7	12.0	2.7		
	c	-35.7	-12.3	35.7	12.3		
	d		-59.6		59.6		
	Σ	-56.5	-79.1	48.2	75.0	-1.17	-1.05
0.6	a	0.4	0.4	-0.3	-0.2		
	b	-69.6	-27.6	35.7	11.8		
	c	-96.5	36.5	96.6	36.5		
	d		-83.6		83.6		
	Σ	-165.6	-146.4	131.9	131.7	-1.26	-1.11
Expt. (Ref. 1)		-69 ± 7		37 ± 19		-1.9 ± 1.1	
CQM (Ref. 5)		-24		16		-1.50	

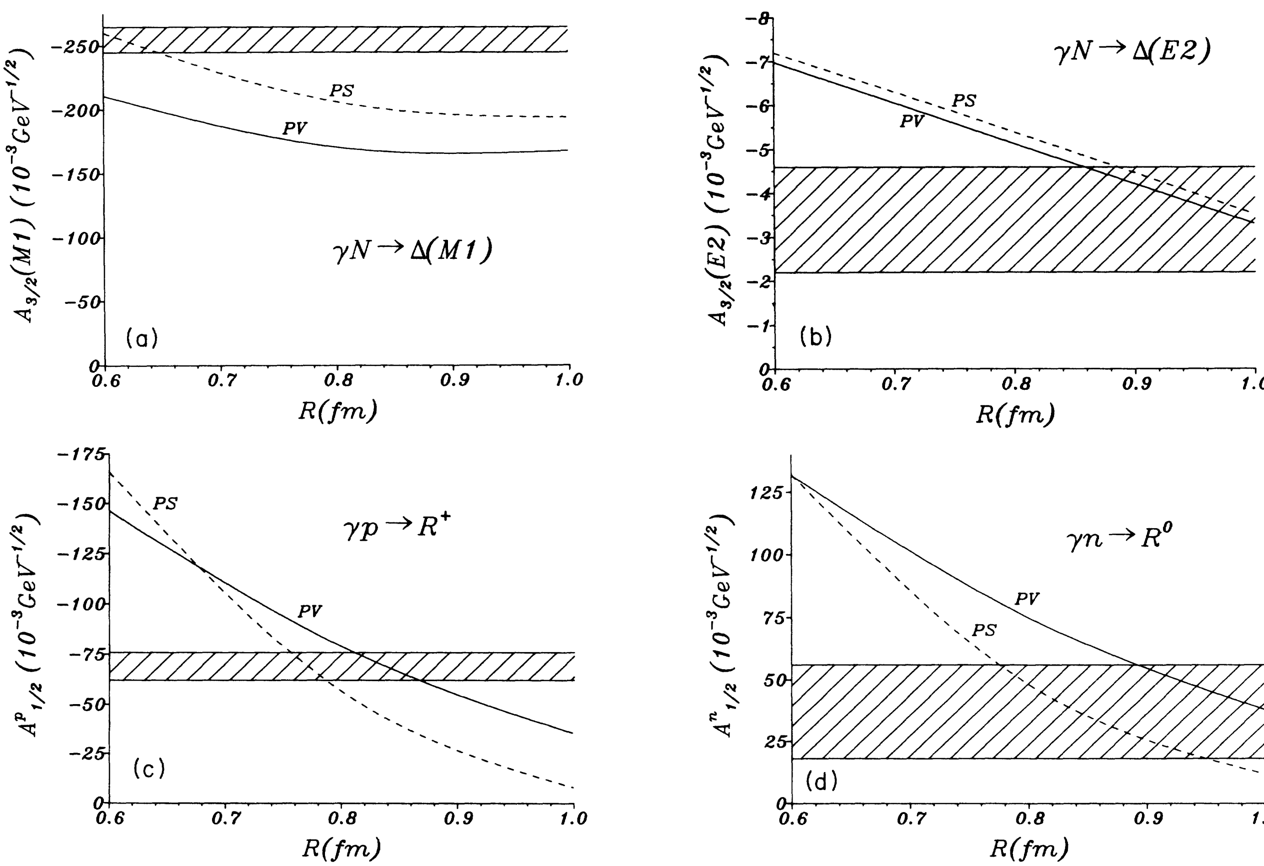

FIG. 2. Helicity amplitudes for photoproduction as functions of the bag radius. The dashed and solid lines are PS and PV calculations, respectively: (a) $A_{3/2}(M1)$ and (b) $A_{3/2}(C2)$ for $\Delta(1232)$ production and (c) $A^p_{1/2}(M1)$ and (d) $A^n_{1/2}(M1)$ for $R(1440)$ production. The numbers are given in units 10^{-3} GeV$^{-1/2}$ and the experimental data from Ref. 1 is shown as error band.

tion. For comparison, the CQM gives a ratio of exactly μ_p/μ_n because of the lack of pionic effects. The absolute magnitudes however are smaller than the experimental value by more than a factor of 2. In view of the large differences between PV and PS couplings we conclude that the truncation of the basis is not well justified in the case of the Roper resonance.

Figure 2 shows the dependence of our results on the bag radius. The comparison with the experimental data indicates a good agreement for a bag radius of $R \approx 0.9$ fm, except for the $M1$ amplitude of the $N\Delta$ transition, which is traditionally underestimated in quark model calculations.

If we had also multiplied the higher-order terms with the wave-function renormalization, see discussion in Sec. III A, the discrepancy between experiment and theory would increase. At $R=1$ fm the predicted amplitude $A_{3/2}(M1)$ would decrease by 9% and 8% for PS and PV coupling, respectively. Although an exact solution of the c.m. problem in the bag model does not exist yet, the prescription of Donoghue and Johnston[25] for the static magnetic moment leads to an increase of 18%. Similar effects could be expected for the electromagnetic transitions to the isobars.

V. SUMMARY AND CONCLUSION

In this contribution we have calculated the photoproduction of delta and Roper resonances within the framework of the cloudy bag model. Although much of our calculation was already performed in Ref. 8, we have repeated this work because of some disagreement concerning both the analytical expressions and the numerical values given in that work. In addition we have compared our results with the original pseudoscalar formulation of the CBM (Ref. 11) and studied in greater detail the small quadrupole transition of the delta. In our calculations we have treated pion loops in first order, taking into account nucleons and deltas in the intermediate states as in standard CBM calculations. Furthermore, for a consistent study of quadrupole excitations, we added intermediate D states in such processes where they can be regarded as admixtures to the initial- and final-state nucleons or deltas. In the case of photoproduction of the Roper we have also allowed this resonance as a possible intermediate state. Higher excitations and also contributions of the Dirac sea have been neglected. Obviously this is also the origin of some problems which remain in our results. We assume that the observed difference between PS and PV models essentially originates from such a truncation of configuration space. In some processes such as s-wave πN scattering[13] or meson-exchange currents[26] and pion photoproduction[14] it can be shown that PV coupling converges faster than PS coupling.[13] However, there seems to be no general rule to determine which one of the two models converges faster to the exact value. In any case, a very useful signature for such convergence can be obtained by comparing both model calculations and, whenever the results agree as for the charge quadrupole excitations of the delta ($C2$), convergence can be assumed. On the other hand, the electric quadrupole ($E2$) has an opposite signature, with values of opposite sign for PS and PV coupling, such that both results should be discarded and replaced by the more stable values obtained for the Coulomb amplitude ($C2$). We conclude that both CQM and CBM models predict $E2/M1$ ratios of the order of $R=-1.5\%$ if the calculation is performed with the charge quadrupole ($C2$) operator, in agreement with the experimental data. In this sense both the hyperfine interaction as residual interaction of gluon exchange and the tensor correlations, due to pion exchange, lead to similar quadrupole deformations and transition moments of the bag.

The predictions for photoexcitation of the Roper resonance[27] are in qualitative agreement with the data for a bag radius $R \approx 0.9$ fm. For smaller radii the contribution of the pion cloud increases rapidly to values considerably higher than the experimental ones. However, the differences between the predictions of the two coupling schemes indicate that the calculations should be repeated in a larger configuration space. Preliminary calculations have shown that these problems are even more pronounced for the Coulomb monopole transition ($C0$) measured by electroproduction of the Roper resonance. It is interesting to note that the analysis of both electroproduction and inclusive electron scattering[28] seems to indicate a strong longitudinal coupling in the region of the Roper resonance. In view of the possible interpretation of this effect as the breathing mode of the nucleon and its relation to the compressibility of the bag, this observation deserves further experimental and theoretical studies.

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